MATHEMATICS PAPER IA

TIME : 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. **10X2 =20**

1. If : R – {0}→ is defined by $f(x) = x^3 - \frac{1}{x^3}$, then show that $f(x) + f\left(\frac{1}{x}\right) = 0$.

2. Find the domain of $f(x) = \frac{\sqrt{2+x} + \sqrt{2-x}}{x}$

3. Find the vector equation of the plane passing through the points (0, 0, 0), (0, 5, 0) and (2, 0, 1).

4. If the vectors $-3\bar{i} + 4\bar{j} + \lambda\bar{k}$ and $\mu\bar{i} + 8\bar{j} + 6\bar{k}$ are collinear vectors then find λ and μ.

5. Find unit vector perpendicular to both $\bar{i} + \bar{j} + \bar{k}$ and $2\bar{i} + \bar{j} + 3\bar{k}$.

6. Find the period of the function $f(x) = 2\sin\frac{\pi x}{4} + 3\cos\frac{\pi x}{3}$

7. prove that $\tan 9° - \tan 27° - \cot 27° + \cot 9° = 4$.

8. Prove that $\frac{\cos hx}{1 - \tan hx} + \frac{\sin hx}{1 - \cot hx} = \sin hx + \cos hx$

9. For any n ×n matrix A, prove that A can be uniquely expressed as a sum of a symmetric and a skew symmetric matrix.

10. If ω is a complex cube root of 1 then show that $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$.

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SECTION B

Short Answer Type Questions
Answer Any Five Of The Following 5 X 4 = 20

11.Let ABCDEF be a regular hexagon with center O. Show that

$\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}=3\overline{AD}=6\overline{AO}$.

12 For any four vectors $\bar{a},\bar{b},\bar{c}$ and $\bar{d}$, $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=[\bar{a}\ \bar{c}\ \bar{d}]\bar{b}-[\bar{b}\ \bar{c}\ \bar{d}]\bar{a}$ and $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=[\bar{a}\ \bar{b}\ \bar{d}]\bar{c}-[\bar{a}\ \bar{b}\ \bar{c}]\bar{d}$.

13. If $A+B+C=\frac{\pi}{2}$ and if none of A, B, C is an odd multiple of π/2, then prove that cot A + cot B + cot C = cotAcotBcotC.

14. Solve the equation $\sqrt{6-\cos x+7\sin^2 x}+\cos x=0$

15. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$ then prove that \` $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$

16.Prove that $\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right)=\frac{abc}{\Delta^3}=\frac{4R}{r^2S^2}$.

17. Show that $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix} = a^3+b^3+c^3-3abc.$

SECTION C

Long Answer Type Questions
Answer Any Five Of The Following 5 X 7 = 35

18. Show that $3.5^{2n+1}+2^{3n+1}+2^{3n+1}$ is divisible by 17

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19. $\bar{a}, \bar{b}, \bar{c}$ are three vectors of equal magnitudes and each of them is inclined at an angle of 60° to the others. If $|\bar{a}+\bar{b}+\bar{c}| = \sqrt{6}$, then find $|\bar{a}|$.

20. If $A+B+C+D=360^0$ then prove that

$$\sin A - \sin B + \sin C - \sin D = -4\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A+D}{2}\right)$$

21 . Solve x + y + z = 9
2x + 5y + 7z = 52
2x + y – z = 0 by Gauss Jordan method

22. Find the complete solution of the system of equations x + y – z = 0
x – 2y + z = 0
3x + 6y – 5z = 0

23. If $f : A \to B$ is a bijection, then $f^{-1} of = I_A, fof^{-1} = I_B$

24. If $(r_2 - r_1)(r_3 - r_1) = 2r_2r_3$. Show that A = 90°.

Solutions

1.If : R – {0}→ is defined by $f(x) = x^3 - \frac{1}{x^3}$, then show that $f(x) + f\left(\frac{1}{x}\right) = 0$.

Sol. Given that $f(x) = x^3 - \frac{1}{x^3}$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$$

$$\therefore f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0$$

2. Find the domain of $f(x) = \frac{\sqrt{2+x} + \sqrt{2-x}}{x}$

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$$\begin{array}{l|l|l} 2+x\ge 0 & 2-x\ge 0 & x\ne 0 \\ \Rightarrow x\ge -2 & \Rightarrow 2\ge x & x\ne 0 \\ & \Rightarrow x\le 2 & \end{array}$$

$\therefore$ Domain of f is $[-2, 2] - \{0\}$

$f(x)=\sqrt{\log_{0.3}(x-x^2)}$

$\log_{0.3}(x-x^2)\ge 0$

$\Rightarrow (x-x^2)\le (0.3)^0$

$\Rightarrow x-x^2\le 1$

$\Rightarrow 0\le x^2-x+1$

$\Rightarrow x^2-x+1\ge 0$

$\Rightarrow x^2-x+1>0,\ \forall x\in R$ $\quad\ldots(1)$

$x-x^2>0$

$\Rightarrow x^2-x<0$

$\Rightarrow x(x-1)<0$

$\Rightarrow 0<x<1$

Since the coefficient of x^2 is +ve

$\therefore\ x \in (0, 1)$ $\quad\ldots(2)$

From (1) and (2)

Domain of f is $R \cap (0, 1) = (0, 1)$

(or) Domain of f is (0, 1)

3.Find the vector equation of the plane passing through the points (0, 0, 0), (0, 5, 0) and (2, 0, 1).

Sol. Let $\bar{a}$ = (0, 0, 0), $\bar{b}$ = (0, 5, 0), $\bar{c}$ =(2, 0, 1)

$\bar{a}=0,\ \bar{b}=5\bar{j},\ \bar{c}=2\bar{i}+\bar{k}$

The vector equation of the plane passing through the points

$\bar{a},\bar{b},\bar{c}$ is $\bar{r}=\bar{a}+s(\bar{b}-\bar{a})+t(\bar{c}-\bar{a}), s,t\in R$

$\bar{r}=s(5\bar{j})+t(2\bar{i}+\bar{k}),\ s,t\in R$

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4. If the vectors $-3\bar{i}+4\bar{j}+\lambda\bar{k}$ and $\mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear vectors then find λ and μ.

Sol. Let $\bar{a}=-3\bar{i}+4\bar{j}+\lambda\bar{k}$, $\bar{b}=\mu\bar{i}+8\bar{j}+6\bar{k}$

From hyp. $\bar{a},\bar{b}$ are collinear then $\bar{a}=t\bar{b}$

$$\Rightarrow -3\bar{i}+4\bar{j}+\lambda\bar{k}=t(\mu\bar{i}+8\bar{j}+6\bar{k})$$

$$-3\bar{i}+4\bar{j}+\lambda\bar{k}=\mu t\bar{i}+8t\bar{j}+6t\bar{k}$$

Comparing i, j, k coefficients on both sides

$$\mu t=-3\Rightarrow \mu=-\frac{3}{t}=-\frac{3}{1/2}=-6\Rightarrow \mu=-6$$

$$8t-4\Rightarrow t=\frac{4}{8}\Rightarrow t=\frac{1}{2}$$

$$6t=\lambda\Rightarrow \lambda=6\cdot\frac{1}{2}\Rightarrow \lambda=3$$

5. Find unit vector perpendicular to both $\bar{i}+\bar{j}+\bar{k}$ and $2\bar{i}+\bar{j}+3\bar{k}$.

Sol. Let $\bar{a}=\bar{i}+\bar{j}+\bar{k}$ and $\bar{b}=2\bar{i}+\bar{j}+3\bar{k}$

$$\bar{a}\times\bar{b}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 1 & 1\\ 2 & 1 & 3\end{vmatrix}$$

$$=\bar{i}(3-1)-\bar{j}(3-2)+\bar{k}(1-2)$$

$$=2\bar{i}-\bar{j}-\bar{k}$$

$$|\bar{a}\times\bar{b}|=\sqrt{6}$$

Unit vector perpendicular to

$$\bar{a}\text{ and }\bar{b}=\pm\frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}=\pm\frac{2\bar{i}-\bar{j}-\bar{k}}{\sqrt{6}}$$

6. Find the period of the function $f\ x\ =2\sin\frac{\pi x}{4}+3\cos\frac{\pi x}{3}$

Period of $\sin\frac{\pi x}{4}$ is $\frac{2\pi}{\pi/4}=8$

Period of $\cos\frac{\pi x}{3}$ is $\frac{2\pi}{\pi/3}=6$

Period of given function is L.C.M of 8, 6

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7. prove that $\tan 9° - \tan 27° - \cot 27° + \cot 9° = 4$.

Sol. Consider,

$$\tan A + \cot A = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{1}{\sin A \cos A}$$

$$= \frac{2}{\sin 2A} = 2\csc 2A$$

$$\tan 81° = \tan(90° - 9°) = \cot 9°$$

$$\tan 63° = \tan(90° - 27°) = \cot 27°$$

$$A = 9° \Rightarrow \tan 9° + \cot 9° = 2\csc 18°$$

$$A = 27° \Rightarrow \tan 27° + \cot 27° = 2\csc 54°$$

$$\text{L.H.S.} = 2(\csc 17° - \csc 54°)$$

$$= 2\left(\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right)$$

$$= 2 \times 4\left(\frac{1}{\sqrt{5}-1} - \frac{1}{\sqrt{5}+1}\right)$$

$$= 8\left(\frac{\sqrt{5}+1-\sqrt{5}+1}{5-1}\right)$$

$$= \frac{8 \times 2}{4} = 4 = \text{R.H.S.}$$

8. Prove that $\frac{\cos hx}{1-\tan hx} + \frac{\sin hx}{1-\cot hx} = \sin hx + \cos hx$

Solution:

$$\frac{\cos hx}{1-\tan hx} + \frac{\sin hx}{1-\cot hx}$$

$$\frac{\cos hx}{1-\frac{\sin hx}{\cos hx}} + \frac{\sin hx}{1-\frac{\cos hx}{\sin hx}} = \frac{\cosh^2 x}{\cos hx - \sin hx} + \frac{\sin h^2 x}{\sin hx - \cos hx}$$

$$= \frac{\cosh^2 x}{\cos hx - \sin hx} - \frac{\sin h^2 x}{\cos hx - \sin hx}$$

$$\frac{\cosh^2 x - \sin h^2 x}{\cos hx - \sin hx} = \frac{\cos hx + \sin x \quad \cos hx - \sin hx}{\cos hx - \sin hx} = \cos hx + \sin hx$$

9. For any n ×n matrix A, prove that A can be uniquely expressed as a sum of a symmetric and a skew symmetric matrix.

Sol. For A square matrix of order n,

A + A′ is symmetric and A – A′ is a skew symmetric matrix and

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

To prove uniqueness, let B be a symmetric matrix and C be a skew-symmetric matrix, such that A = B + C.

Then A′ = (B + C)′ = B′ + C′

= B + (–C) = B – C

and hence $B = \frac{1}{2}(A + A'), C = \frac{1}{2}(A - A')$

10. If ω is a complex cube root of 1 then show that $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$.

Sol. $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 1+\omega+\omega^2 & 1+\omega+\omega^2 & 1+\omega+\omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 0 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} [\because 1+\omega+\omega^2 = 0]$$

$= 0$

11. Let ABCDEF be a regular hexagon with center O. Show that

$$\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{AO}.$$

Sol.

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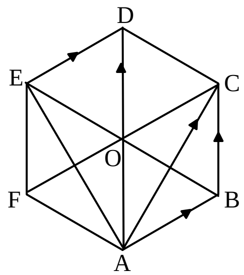

From figure,

$$\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}=$$

$$\overline{AB}+\overline{AE}\ +\overline{AD}+\ \overline{AC}+\overline{AF}$$

$$=\ \overline{AE}+\overline{ED}\ +\overline{AD}+\ \overline{AC}+\overline{CD}$$

$$\because \overline{AB}=\overline{ED}, \overline{AF}=\overline{CD}$$

$$=\overline{AD}+\overline{AD}+\overline{AD}=3\overline{AD}$$

$$=6\overline{AO}(\because \text{O is the center and } \overline{OD}=\overline{AO})$$

12 For any four vectors $\bar{a},\bar{b},\bar{c}$ and $\bar{d}$, $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=[\bar{a}\ \bar{c}\ \bar{d}]\bar{b}-[\bar{b}\ \bar{c}\ \bar{d}]\bar{a}$ and $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=[\bar{a}\ \bar{b}\ \bar{d}]\bar{c}-[\bar{a}\ \bar{b}\ \bar{c}]\bar{d}$.

Sol. Let m = c × d

$$\therefore (\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=(\bar{a}\times\bar{b})\times m$$

$$=(\bar{a}\cdot m)\bar{b}-(\bar{b}\cdot m)\bar{a}$$

$$=(\bar{a}\cdot(\bar{c}\times\bar{d}))\bar{b}-(\bar{b}\cdot(\bar{c}\times\bar{d}))\bar{a}$$

$$=[\bar{a}\ \bar{c}\ \bar{d}]\bar{b}-[\bar{b}\ \bar{c}\ \bar{d}]\bar{a}$$

Again, Let $\bar{a}\times\bar{b}=n$, then

$$(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=n\times(\bar{c}\times\bar{d})$$

$$=(n\cdot\bar{d})\bar{c}-(n\cdot\bar{c})\bar{d}$$

$$=((\bar{a}\times\bar{b})\cdot\bar{d})\bar{c}-((\bar{a}\times\bar{b})\bar{c})\bar{d}$$

$$=[\bar{a}\ \bar{b}\ \bar{d}]\bar{c}-[\bar{a}\ \bar{b}\ \bar{c}]\bar{d}$$

13. If $A+B+C=\frac{\pi}{2}$ and if none of A, B, C is an odd multiple of π/2, then prove that cot A + cot B + cot C = cotAcotBcotC.

Sol. $A+B+C=\frac{\pi}{2}$

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$$A+B=\frac{\pi}{2}-C$$

$$\cot(A+B)=\cot\left(\frac{\pi}{2}-C\right)$$

$$\frac{\cot A\cot B-1}{\cot B+\cot A}=\tan C$$

$$\frac{\cot A\cot B-1}{\cot B+\cot A}=\frac{1}{\cot C}$$

$$\cot C[\cot A\cot B-1]=\cot B+\cot A$$

$$\cot A\cot B\cot C-\cot C\cot A+\cot B$$

$$\cot A\cot B\cot C=\cot A+\cot B+\cot C$$

$$\therefore \cot A+\cot B+\cot C=\cot A\cot B\cot C$$

14. Solve the equation $\sqrt{6-\cos x+7\sin^2 x}+\cos x=0$

Solution:

$$\sqrt{6-\cos x+7\sin^2 x}=-\cos x$$

Squaring on both sides

$$6-\cos x+7\sin^2 x=\cos^2 x$$

$$6-\cos x+7-7\cos^2 x=\cos^2 x$$

$$8\cos^2 x+\cos x-13=0$$

$\cos x=\dfrac{-1\pm\sqrt{1+104}}{16}$ This is not possible

$\because\ \sin x\ \ \cos x\ >1\quad (or)$

$\cos x<-1$ in this $\cos x$

Hence equation has no solution

15. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$ then prove that ` $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$

Solution:

Let $\sin^{-1}x=\alpha$ $\quad\sin^{-1}y=\beta$ $\quad\sin^{-1}z=\gamma$

$\sin\alpha=x$ $\quad\sin\beta=y$ $\quad\sin\gamma=z$

Given $\alpha+\beta+\gamma=\pi\Rightarrow\alpha+\beta=\pi-\gamma$

$$\sin\ \ \alpha+\beta\ \ =\sin\gamma\ and\ \cos\gamma=-\cos\ \ \alpha+\beta$$

$$x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}$$

$$\sin\alpha\sqrt{1-\sin^2\alpha}+\sin\beta\sqrt{1-\sin^2\beta}+\sin\gamma\sqrt{1-\sin^2\gamma}$$

$$\frac{1}{2}\left[\sin^2\alpha+\sin^2\beta+\sin^2\gamma\right]\quad=\quad\frac{1}{2}\left[2\sin\ \alpha+\beta\ \cos\ \alpha-\beta\ +\sin^2\gamma\right]$$

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$$\frac{1}{2}\left[2\sin\gamma\cos\ \alpha-\beta\ +2\sin\gamma\cos\gamma\right]$$

$$\sin\gamma\left[\cos\ \alpha-\beta\ +\cos\ \alpha+\beta\ \right]=2\sin\alpha\sin\beta\sin\gamma=2xyz$$

16. Prove that $\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right)=\frac{abc}{\Delta^3}=\frac{4R}{r^2S^2}$.

Sol. $\frac{1}{r}-\frac{1}{r_1}=\frac{S}{\Delta}-\frac{S-a}{\Delta}=\frac{S-S+a}{\Delta}=\frac{a}{\Delta}$

Similarly we get

$\frac{1}{r}-\frac{1}{r_2}=\frac{b}{\Delta}$ and $\frac{1}{r}-\frac{1}{r_3}=\frac{c}{\Delta}$

$$\text{L.H.S.}=\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right)$$

$$=\frac{a}{\Delta}\frac{b}{\Delta}\frac{c}{\Delta}=\frac{abc}{\Delta^3}$$

$$=\frac{4R\cdot\Delta}{\Delta^3}=\frac{4R}{\Delta^2}=\frac{4R}{(rS)^2}=\text{R.H.S.}$$

17. Show that $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix}$

$= a^3+b^3+c^3-3abc.$

Sol. $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix}$

$R_1 \to R_1+R_3$

$$=\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix}$$

$R_2 \to R_2-R_1$

$$=\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ -c & -a & -b \\ a & b & c \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ -c & -a & -b \\ a & b & c \end{vmatrix}$$

$$= (a+b+c)[(-ac+b^2) -$$
$$(-c^2+ab)+(-bc+a^2)]$$

$$= (a+b+c)(-ac+b^2+c^2-ab-bc+a^2)$$

$$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= a^3+b^3+c^3-3abc$$

18. Show that $3.5^{2n+1}+2^{3n+1}+2^{3n+1}$ is divisible by 17

Sol: Let $S_{(n)} = 3.5^{2n+1}+2^{3n+1}$ be the given statement

$S_{(1)} = 3.5^3+2^4 = 375+16 = 391 = 17\times 23$

This is divisible by 17

Assume S_k is true

$S_k = 3.5^{2k+1}+2^{3k+1}$ is divisible by 17

Let $3.5^{2k+1}+2^{3k+1} = 17m$

$3.5^{2k+1} = 17m-2^{3k+1}$

$S_{k+1} = 3.5^{2k+3}+2^{3k+4}$

$= 3.5^{2k+1}.5^2+2^{3k+1}.2^3$

$= 25\ 17m-2^{3k+1}\ \ +2^{3k+1}.8$

$= 25\times 17m-17(2^{3k+1}) = 17\ \ 25m-2^{3k+1}$ is divisible by 17

Hence S_{k+1} is true

$\therefore S_n$ is true for all $n \in N$

19. $\bar{a},\bar{b},\bar{c}$ are three vectors of equal magnitudes and each of them is inclined at an angle of 60° to the others. If $|\bar{a}+\bar{b}+\bar{c}| = \sqrt{6}$, then find $|\bar{a}|$.

Sol. $|\bar{a}+\bar{b}+\bar{c}| = \sqrt{6}$

$\Rightarrow |\bar{a}+\bar{b}+\bar{c}|^2 = 6$

$\Rightarrow \bar{a}^2+\bar{b}^2+\bar{c}^2+2\bar{a}\bar{b}+2\bar{b}\bar{c}+2\bar{c}\,\bar{a} = 6$

Let $|\bar{a}| = |\bar{b}| = |\bar{c}| = a$

$\Rightarrow a^2+a^2+a^2+2a^2\cos(\bar{a},\bar{b}) + 2a^2\cos(\bar{b},\bar{c})+2a^2\cos(\bar{c},\bar{a}) = 6$

1.
$$\Rightarrow 3a^2 + 2a^2\cos 60° + 2a^2\cos 60° + 2a^2\cos 60° = 6$$
$$\Rightarrow 3a^2 + 6a^2\cos 60° = 6$$
$$\Rightarrow 3a^2 + 6a^2 \times \frac{1}{2} = 6$$
$$\Rightarrow 3a^2 + 3a^2 = 6$$
$$\Rightarrow 6a^2 = 6$$
$$\Rightarrow a^2 = 1 \Rightarrow a = 1 \Rightarrow |\bar{a}| = 1$$

20. If $A+B+C+D = 360^0$ then prove that

$$\sin A - \sin B + \sin C - \sin D = -4\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A+D}{2}\right)$$

Solution:

$$A+B+C+D = 360^0 \quad \Rightarrow \frac{A}{2}+\frac{B}{2}+\frac{C}{2}+\frac{D}{2} = 180^0$$

$$\therefore \frac{A+B}{2} = 180^0 - \left(\frac{C+D}{2}\right)$$

$$\sin A - \sin B + \sin C - \sin D = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) + 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) + 2\cos\left\{180^0 + \overline{\frac{A+B}{2}}\right\}\sin\left(\frac{C-D}{2}\right)$$

$$= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) - 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$2\cos\left(\frac{A+B}{2}\right)\left\{\sin\left(\frac{A-B}{2}\right) - \sin\left(\frac{C-D}{2}\right)\right\}$$

$$2\cos\left(\frac{A+B}{2}\right)\left\{2\cos\left(\frac{A+B+C-D}{4}\right).\sin\left(\frac{A-B-C+D}{4}\right)\right\}$$

$$4\cos\left(\frac{A+B}{2}\right)\cos\left\{\frac{A+C-360^0+A+C}{4}\right\}\sin\left\{\frac{A+D-360^0+A+D}{4}\right\}$$

$$4\cos\left(\frac{A+B}{2}\right)\cos\left\{\frac{A+C}{2} - 90^0\right\}\sin\left\{\frac{A+D}{2} - 90^0\right\}$$

$$4\cos\left(\frac{A+B}{2}\right)\cos\left\{\frac{A+C}{2} - 90^0\right\}\sin\left\{\frac{A+D}{2} - 90^0\right\}$$

$$-4\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A+D}{2}\right)$$

21 . Solve x + y + z = 9
2x + 5y + 7z = 52
2x + y – z = 0 by Gauss Jordan method

Sol.

Augmented matrix $A = \begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{bmatrix}$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_2$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -4 & -8 & -52 \end{bmatrix}$$

$R_1 \to 3R_1 - R_2, R_3 \to 3R_3 + 4R_2$

$$A \sim \begin{bmatrix} 3 & 0 & -2 & -7 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{bmatrix}$$

$R_3 \to R_3\left(-\frac{1}{4}\right)$, we obtain

$$A \sim \begin{bmatrix} 3 & 0 & -2 & -7 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$R_1 \to R_1 + 2R_3, R_2 \to R_2 - 5R_3$, we get

$$A \sim \begin{bmatrix} 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 9 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$R_1 \to R_1\left(\frac{1}{3}\right), R_2 \to R_2\left(\frac{1}{3}\right)$ we have

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

∴ The given equations have a unique solution and solution is x = 1, y = 3, z = 5.

22. Find the complete solution of the system of equations x + y – z = 0
x – 2y + z = 0
3x + 6y – 5z = 0

Sol. Coefficient matrix A = $\begin{bmatrix}1 & 1 & -1\\ 1 & -2 & 1\\ 3 & 6 & -5\end{bmatrix}$

$R_2 \to R_2 - R_1, R_3 \to R_3 - 3R_1$

$A \sim \begin{bmatrix}1 & 1 & -1\\ 0 & -3 & 2\\ 0 & 3 & -2\end{bmatrix}$

⇒det A = 0 as R_2, R_3 are identical.

and rank(A) = 2 as the submatrix $\begin{bmatrix}1 & 1\\ 0 & -3\end{bmatrix}$ is non-singular. Hence the system has non-trivial solution, $\rho(A) < 3$.

The system of equations equivalent to the given system of equations are

$x + y - z = 0$

$3y - 2z = 0$

Let z = k $\Rightarrow y = \frac{2k}{3}, x = \frac{k}{3}$

$\therefore x = \frac{k}{3}, y = \frac{2k}{3}, z = k$ for any real number of k.

23. If $f : A \to B$ is a bijection, then $f^{-1}of = I_A, fof^{-1} = I_B$

Proof: Since $f : A \to B$ is a bijection $f^{-1} : B \to A$ is also a bijection and

$f^{-1}(y) = x \Leftrightarrow f(x) = y \forall y \in B$

$f : A \to B$, $f^{-1} : B \to A \Rightarrow f^{-1}of : A \to A$

Clearly $I_A : A \to A$ such that $I_A(x) = x, \forall x \in A$.

Let $x \in A$

$x \in A, f : A \to B \Rightarrow f(x) \in B$

Let y = f(x)

$y = f(x) \Rightarrow f^{-1}(y) = x$

$(f^{-1}of)(x) = f^{-1}[f(x) = f^{-1}(y) = x = I_A(x)$

$\therefore (f^{-1}of)(x) = I_A(x) \forall x \in A$ $\qquad \therefore f^{1}of = I_A$

$f^{1} : B \to A$, $f : A \to B \to fof^{1} : B \to B$

Clearly $I_B : B \to B$ such that $I_B(y) = y \forall y \in B$

Let $y \in B$

$y \in B, f^{-1} : B \to A = f^{1}(y) \in A$

Let $f^{1}(y)=x$

$f^{1}(y)=x \Rightarrow f(x)=y$

$(fof^{1})(y)=f[f^{1}(y)]=f(x)=y=I_B(y)$

$\therefore (fof^{-1})(y)=I_B(y)\,\forall\, y\in B \qquad \therefore fof^{-1}=I_B$

24. If $(r_2–r_1)(r_3–r_1) = 2r_2r_3$. Show that A = 90°.

Sol. $(r_2 – r_1)(r_3 – r_1) = 2r_2r_3$

$$\Rightarrow \left[\frac{\Delta}{(s-b)}-\frac{\Delta}{(s-a)}\right]\left[\frac{\Delta}{(s-c)}-\frac{\Delta}{(s-a)}\right]$$

$$=2\frac{\Delta}{(s-b)}\frac{\Delta}{(s-c)}$$

$$\Rightarrow \Delta\left[\frac{s-a-s+b}{(s-b)(s-a)}\right]\cdot\Delta\left[\frac{s-a-s+c}{(s-c)(s-a)}\right]$$

$$=\frac{2\Delta^2}{(s-b)(s-c)}$$

$$\Rightarrow (b-a)(c-a)=2(s-a)^2$$

$$\Rightarrow (b-a)(c-a)=2\left(\frac{b+c-a}{2}\right)^2$$

$$\Rightarrow 2(bc-ca-ab+a^2)$$

$$=b^2+c^2+a^2+2bc-2ca-2ab$$

$$\Rightarrow 2a^2=b^2+c^2+a^2$$

$$\Rightarrow b^2+c^2=a^2$$

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