MATHEMATICS PAPER IIB

COORDINATE GEOMETRY AND CALCULUS.

TIME : 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 =20

1) Find the equation of circle with centre (2, 3) and touching the line 3x - 4y + 1= 0.

2. Find 'k' if the following pair of circles are orthogonal.
$x^2+y^2 + 2by-k = 0$, $x^2+y^2+2ax+8=0$

3. A double ordinate of the curve $y^2 = 4ax$ is of length 8a. Prove that the line from the vertex to its ends are at right angles.

4. Find the condition for the line $x\cos\alpha + y\sin\alpha = p$ to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

5. If the angle between asymptotes is 30° then find its eccentricity.

6. Evaluate $\int \sec x\, dx$

7. Evaluate $\int \frac{\sin x}{\sin(a+x)} dx$ on $I \subset R - \{n\pi - a : n \in Z\}$.

8. Evaluate $\int_0^a x(a^2-x^2)^{7/2} dx$

9. Evaluate $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{6n}\right]$

10. Find the general solution of $\sqrt{1-x^2}dy+\sqrt{1-y^2}dx=0$.

SECTION B

SHORT ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING **5 X 4 = 20**

11. Find the equation of tangents the circle $x^2 + y^2 – 10 = 0$ at the points whose abscissa are

12.Find the equation of the circle which passes through the point (0, -3) and intersects the circles given by the equations $x^2 + y^2–6x+3y+5 = 0$, $x^{2+} +y^2– x – 7y = 0$ orthogonally.

13) If the Straight line 2x + 3y = 1 intersects the circle $x^2 + y^2 = 4$ at the points A and B. Find the equation of the circle having AB as diameter.

14. Show that the point of intersection of perpendicular tangents to an ellipse lie on a circle.

15. Find the equation of the hyperbola of given length of transverse axis is 6 whose vertex bisects the distance between the centre and the focus.

16. Evaluate $\int_0^{\pi/4}\log(1+\tan x)dx$

17.find the differential equation corresponding to the family of curves by eliminating arbitrary constants given by the equation $ax^2 + by^2 = 1$; (a, b)

SECTION C

LONG ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING 5 X 7= 35

18. Find the locus of the point whose polars with respect to the circles $x^2 + y^2 - 4x - 4y - 8 = 0$ and $x^2 + y^2 - 2x + 6y - 2 = 0$ are mutually perpendicular.

19. Find the equation of circle passing through the points (3, 4); (3,2); (1,4)

20. Prove that the area of the triangle inscribed in the parabola $y^2 = 4ax$ is $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ sq.units where y_1, y_2, y_3 are the ordinates of its vertices.

21. Evaluate $\int \frac{x^2 + 1}{x^4 + 1} dx$ on R.

22. Evaluate $\int \frac{dx}{1 + \sin x + \cos x}$

23. Show that the area of the region bounded by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$(ellipse) is πab . Also deduce the area of the circle $x^2 + y^2 = a^2$.

24. solve $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$

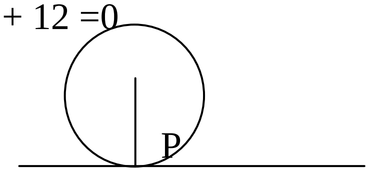
SOLUTIONS

1. Find the equation of circle with centre (2, 3) and touching the line 3x - 4y + 1= 0.
Sol.

Centre C=(2,3).

Radius r =Perpendicular distance from C to 3x-4y +1=0 $= \left|\frac{3(2)-4(3)+1}{\sqrt{3^2+4^2}}\right|$

Equation of circle $(x-h)^2 + (y-k)^2 = r^2$
$(x-2)^2 + (y-3)^2 = 1$
$x^2 + y^2 - 4x - 6y + 12 = 0$



3x – 4y + 1 = 0

2. Find 'k' if the following pair of circles are orthogonal.
$x^2+y^2 + 2by-k = 0$, $x^2+y^2+2ax+8=0$

Sol. Given circles are $x^2+y^2 + 2by-k = 0$, $x^2+y^2+2ax+8=0$

from above equations $g_1 = 0$; $f_1 = b$; $c_1 = -k$

$g_2 = a$; $f_1 = 0$; $c_1 = 8$

since the circles are orthogonal ,

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$2(0)(a) + 2(b)(0) = -k + 8$

$0 = -k + 8$

$K = 8$

3. A double ordinate of the curve $y^2 = 4ax$ is of length 8a. Prove that the line from the vertex to its ends are at right angles.

Sol. Let $P = (at^2, 2at)$ and $P' = (at^2, -2at)$ be the ends of double ordinate PP′. Then

$$8a = PP' = \sqrt{0 + (4at)^2} = 4at \Rightarrow t = 2$$

$\therefore P = (4a, 4a), P' = (4a, -4a)$

Slope of $\overline{AP} \times$ slope of $\overline{AP'}$

$$= \left(\frac{4a}{4a}\right)\left(-\frac{4a}{4a}\right) = -1$$

$\therefore \angle PAP' = \frac{\pi}{2}$

4. Find the condition for the line xcosα + ysinα = p to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Sol. Equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ …(i)

Equation of the line is xcosα + ysinα = p

ysinα = –xcosα + p

$y = -x\frac{\cos\alpha}{\sin\alpha}+\frac{p}{\sin\alpha}$

$\therefore \quad m = -\frac{\cos\alpha}{\sin\alpha}, c = \frac{p}{\sin\alpha}$

Above line is a tangent to the ellipse

$\Rightarrow c^2 = a^2m^2 + b^2$

$\frac{p^2}{\sin^2\alpha} = a^2\frac{\cos^2\alpha}{\sin^2\alpha}+b^2$

or $p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$.

5. If the angle between asymptotes is 30° then find its eccentricity.

Sol: Angle between asymptotes of hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $2\sec^{-1} e$.

$\therefore 2\sec^{-1}e = 30° \Rightarrow \sec^{-1}e = 15°$

$\Rightarrow e = \sec 15° = \frac{1}{\cos 15°} = \frac{1}{\cos(45°-30°)}$

$= \frac{1}{\cos 45°\cos 30° + \sin 45°\sin 30°}$

$= \frac{1}{\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\cdot\frac{1}{2}}$

$= \frac{2\sqrt{2}}{\sqrt{3}+1} = \frac{2\sqrt{2}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$= \frac{2\sqrt{2}(\sqrt{3}-1)}{2}$

$$= \sqrt{2}(\sqrt{3}-1)$$

$$= \sqrt{6}-\sqrt{2}.$$

6. $\int \sec x\, dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x} dx$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx = \log|\sec x + \tan x| + c$$

$$= \log\left|\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right| + c = \log\left|\frac{1+\sin x}{\cos x}\right| + c$$

$$= \log\left|\frac{1-\cos(\pi/2+x)}{\sin(\pi/2+x)}\right| + c$$

$$= \log\left|\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right| + c$$

$$= \log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + c$$

7. $\int \frac{\sin x}{\sin(a+x)} dx$ on $I \subset R - \{n\pi - a : n \in Z\}$.

Sol: $\int \frac{\sin x}{\sin(a+x)} dx = \int \frac{\sin(x+a-a)}{\sin(x+a)} dx$

$$= \int \left[\frac{\sin(x+a)\cos a - \cos(x+a)\sin a}{\sin(x+a)}\right] dx$$

$$= \cos a \int dx - \sin a \int \frac{\cos(x+a)}{\sin(x+a)} dx$$

$$= x\cos a - \sin a - \log|\sin(x+a)| + c.$$

8. $\int_0^a x(a^2-x^2)^{7/2} dx$

Sol. $x = a\sin\theta$ $\qquad a = a\sin\theta$

$dx = a\cos\theta d\theta$ $\theta = \pi/2$

$$= \int_0^{\pi/2} a\sin\theta(a^2 - a^2\sin^2\theta)^{7/2} a\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} a^9\cos^8\theta\sin\theta\, d\theta = a^9\int_0^{\pi/2}\cos^8\theta\sin\theta\cdot d\theta$$

$$= a^9\left(\frac{-\cos^9\theta}{9}\right)_0^{\pi/2} = a^9\left(-0+\frac{1}{9}\right) = \frac{a^9}{9}$$

9. $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{6n}\right]$

Sol: $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{n+5n}\right]$

$$\lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+\ldots+\frac{1}{1+\frac{5n}{n}}\right]$$

$$=\int_0^5 \frac{1}{1+x}dx = \log(1+x)\,_0^5 = \log 6$$

10. Find the general solution of $\sqrt{1-x^2}dy+\sqrt{1-y^2}dx=0$.

Sol. Given d.e. is

$$\sqrt{1-x^2}dy+\sqrt{1-y^2}dx=0$$

$$\sqrt{1-x^2}dy=-\sqrt{1-y^2}dx$$

Integrating both sides

$$\int\frac{dy}{\sqrt{1-y^2}}=-\int\frac{dx}{\sqrt{1-x^2}}$$

$\sin^{-1}y = -\sin^{-1}x + c$

Solution is $\sin^{-1}x + \sin^{-1}y = c$, where c is a constant.

11. Find the equation of tangents the circle $x^2 + y^2 - 10 = 0$ at the points whose abscissa are

Sol. Equation of the circle is $S = x^2 + y^2 = 10$

Let the point be (1, y)

$1 + y^2 = 10 \Rightarrow y^2 = 9$

$Y = \pm 3$.

Co – ordinates of P are (1,3) and (1, -3)

Equation of the tangent at P (1, 3) is $S_1=0$.

$\Rightarrow x. 1 + y. 3 = 10$

$\Rightarrow x + 3y - 10 = 0$

Equation of the tangent of P(1, -3) is $S_2=0$

$\Rightarrow x.1 + y(-3) = 10 \Rightarrow x - 3y - 10 = 0$

12.Find the equation of the circle which passes through the point (0, -3) and intersects the circles given by the equations $x^2 + y^2 – 6x+3y+5 = 0$, $x^{2+} + y^2 – x – 7y = 0$ orthogonally.

Sol. Let circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$-------(1)

(1) is orthogonal to $x^2 + y^2 – 6x+3y+5 = 0$

$\therefore 2g(–3) + 2f\left(\frac{+3}{2}\right) = c + 5$

$–6g + 3f = c + 5$-----------(2)

(1) is orthogonal to $x^2 + y^2 – x – 7y = 0$

$\therefore\ 2g\left(\frac{+1}{2}\right) + 2f\left(\frac{+7}{2}\right) = c$

$–g –7f = c$ ----(3)

Given (1) is passing through (0, –3)

$0 + 9 – 6f + c = 0$

(3) – (2)

$5g – 10f = – 5 \Rightarrow\ g – 2f = – 1$

(iii) + (iv)

$9 – g – 13f = 0 \qquad \Rightarrow g + 13f = 9$

$\underline{g – 2f = –1}$

$15f = 10$

$f = \frac{2}{3} \Rightarrow g = 2.\frac{2}{3} – 1 \Rightarrow g = + \frac{1}{3}$

$\Rightarrow 9 – 6.\frac{2}{3} + c = 0 \Rightarrow\ \ c = – 5$

Therefore , eq of the circles are

$x^2 + y^2 + \frac{4}{3}y + \frac{4}{3}\ x – 5 = 0$

(or) $3x^2 + 3y^2 + 2x + 4y – 15 = 0$

13) If the Straight line $2x + 3y = 1$ intersects the circle $x^2 + y^2 = 4$ at the points A and B. Find the equation of the circle having AB as diameter.

Sol. circle is $S = x^2 + y^2 = 4$

Equation of the line is $L = 2x + 3y = 1$ Equation of circle passing through S=0 and L=0 is $S+\lambda L = 0$

$\Rightarrow (x^2 + y^2 – 4) + \lambda\ (2x + 3y – 1) = 0$

$\Rightarrow x^2 + y^2 + 2\lambda x + 3\lambda y - 4 - \lambda = 0$

$\Rightarrow$ Center $\left(-\lambda, \frac{-3\lambda}{2}\right)$

Centre lies on $2x + 3y - 1 = 0$

$\Rightarrow 2(-\lambda) + 3\frac{-3\lambda}{2} - 1 = 0$

$\Rightarrow \lambda = \frac{-2}{13}$

$\therefore$ Equation of circle be

$13(x^2 + y^2) - 4 \times 13 - 2(2x + 3y - 1) = 0$

$13(x^2 + y^2) - 4x - 6y - 50 = 0.$

14. Show that the point of intersection of perpendicular tangents to an ellipse lie on a circle.

Sol. Equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Let $P(x_1, y_1)$ be the point of intersection of the tangents.

Equation of the tangent is

$y = mx \pm \sqrt{a^2m^2 + b^2}$

This tangent is passing through $P(x_1, y_1)$

$y_1 = mx_1 \pm \sqrt{a^2m^2 + b^2}$

$y_1 - mx_1 = \pm\sqrt{a^2m^2 + b^2}$

$(y_1 - mx_1)^2 = a^2m^2 + b^2$

$m^2x_1^2 + y_1^2 - 2mx_1y_1 - a^2m^2 - b^2x$

$m^2(x_1^2 - a^2) - 2mx_1y_1 + (y_1^2 - b^2) = 0$

This is a quadratic equation in m giving two values for m say m_1 and m_2. These are the slopes of the tangents passing through (x_1, y_1).

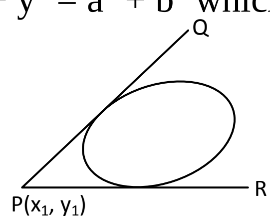
The tangents are perpendicular $\Rightarrow m_1m_2 = -1$

$\frac{y_1^2 - b^2}{x_1^2 - a^2} = -1$

$y_1^2 - b^2 = -x_1^2 + a^2$

$x_1^2 + y_1^2 = a^2 + b^2$

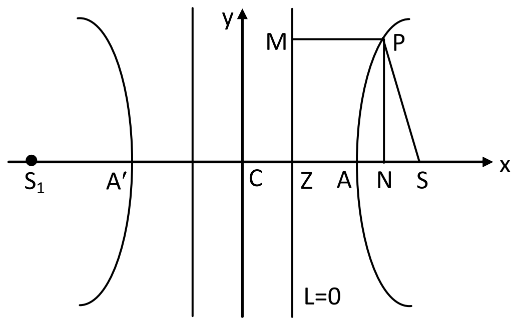
Locus of $P(x_1, y_1)$ is $x^2 + y^2 = a^2 + b^2$ which is a circle.



This circle is called Director circle of the Ellipse.

15. Find the equation of the hyperbola of given length of transverse axis is 6 whose vertex bisects the distance between the centre and the focus.

Sol.



Let the hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Given CA = AS

$a = ae - a \Rightarrow 2a = ae \Rightarrow e = 2$

Length of transverse axis is $2a = 6 \Rightarrow a = 3$

$b^2 = a^2(e^2 - 1) = 9(4 - 1) = 27$

Equation of the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$$\frac{x^2}{9}-\frac{y^2}{27}=1 \Rightarrow 3x^2 - y^2 = 27$$

16. $\int_0^{\pi/4} \log(1+\tan x)dx$

Sol. $I = \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$

$$= \int_0^{\pi/4} \log\left[1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]dx$$

$$= \int_0^{\pi/4} \log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log\left(\frac{1+\tan x+1-\tan x}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log 2-\log(1+\tan x)dx$$

$$= \int_0^{\pi/4} \log 2dx-\int_0^{\pi/4} \log(1+\tan x)dx$$

$$= \log 2(x)_0^{\pi/4}-I$$

$$2I=\frac{\pi}{4}\log 2 \Rightarrow I=\frac{\pi}{8}\log 2$$

17. $ax^2 + by^2 = 1$; (a, b)

Sol.

Given equation is

$ax^2 + by^2 = 1$------(1)

Differentiating w.r.t. x

$\Rightarrow 2ax+2byy_1=0$

$\Rightarrow ax+byy_1=0 ------ 2$

Differentiating w.r.t. x

$\Rightarrow a+b(yy_2+y_1y_1)=0 \Rightarrow a+b(yy_2+y_1^2)=0$

$\Rightarrow ax+bx(yy_2+y_1^2)=0$----(3)

$(3)-(2) \Rightarrow bx(yy_2+y_1^2)-byy_1=0$

$\Rightarrow x(yy_2+y_1^2)-yy_1=0$

18. Find the locus of the point whose polars with respect to the circles $x^2 + y^2 - 4x - 4y - 8 = 0$ and $x^2 + y^2 - 2x + 6y - 2 = 0$ are mutually perpendicular.

Sol. Equation of the circles is

$S= x^2 + y^2 - 4x - 4y - 8 = 0$ - (1)

$S'=x^2 + y^2 - 2x + 6y - 2 = 0$ - (2)

let P (x, y) be any position in the locus.

Equation of the polar of p w.r.to circle (1) is

$xx_1 \; yy_1 - 2(x + x_1) - 2(y + y_1) - 8 = 0$

$x(x_1 -2) + y(y_1 -2) - (2x_1 + 2y_1 + 8) = 0$ (3)

Polar of P w.r. to circle (2) is

$xx_1 + yy_1 - 1(x+ x_1) - 3(y + Y_1) - 2 = 0$

$x_1 + yy_1 - x - x_1 + 3y + 3y_1 - 2 = 0$

$x(x_1 - 1) + y(y_1 + 3) - (x_1 + 3y_1 + 2) = 0$

(3) and (4) are perpendicular

$\Rightarrow a_1 a_2 + b_1 b_2 = 0$

$(x_1 - 2)(x_1 - 1) + (y_1 - 2)(y_1 + 3) = 0$

$\Rightarrow x_1^2 + y_1^2 - 3x_1 + y_1 - 6 = 0$

Locus of $p(x_1, y_1)$ is $x^2 + y^2 - 3x + y - 4 = 0$

19. Find the equation of circle passing through the points (3, 4); (3,2); (1,4)

Let the equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

it is passing through (3, 4); (3,2); (1,4)

$\therefore$ Given points satisfy above equation then

$9 + 16 + 6g + 8f + c = 0$

$25 + 6g + 8f + c = 0$ ______(i)

$9 + 4 + 6g + 4f + c = 0$

$13 + 6g + 4f + c = 0$ ______(ii)

$1 + 16 + 2g + 8f + c = 0$

$17 + 2g + 8f + c = 0$________(iii)

(ii) – (i) we get

$-12 - 4f = 0$ (or) $f = -3$

(ii) – (iii) we get $-4 + 4g - 4f = 0$

$g - f = 1 \Rightarrow g = -2$

Now substituting g, f in equation (i) we get

$25 + 6(-2) + 8(-3) + c = 0$

We get $c = 11$

Required equation of circle be

$X^2 + y^2 - 4x - 6y + 11 = 0$

20. Prove that the area of the triangle inscribed in the parabola $y^2 = 4ax$ is $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ sq.units where y_1, y_2, y_3 are the ordinates of its vertices.

Sol.

Given parabola $y^2 = 4ax$

let $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$, $R(at_3^2, 2at_3)$ be the vertices of ΔPQR.

Area of ΔPQR = $= \frac{1}{2}\begin{vmatrix} at_1^2 - at_2^2 & at_2^2 - at_3^2 \\ 2at_1 - 2at_2 & 2at_2 - 2at_3 \end{vmatrix} = \frac{1}{2}\left|2a^2 (t_1^2 - t_2^2)(t_2 - t_3) - 2a^2 (t_2^2 - t_3^2)(t_1 - t_2)\right|$

$= a^2 \left|(t_1 - t_2)(t_2 - t_3)(t_1 + t_2 - t_2 - t_3)\right|$

$= a^2 |(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|$

$= \frac{a^3}{a} |(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|$

$= \frac{1}{8a} |(2at_1 - 2at_2)(2at_2 - 2at_3)(2at_3 - 2at_1)|$

$= \frac{1}{8a} |(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$

Where $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ are the vertices of ΔPQR.

21. $\int \frac{x^2+1}{x^4+1} dx$ on R.

Sol: $\int \frac{x^2+1}{x^4+1} dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx$

$= \int \frac{\left[1+\frac{1}{x^2}\right]}{\left[x-\frac{1}{x}\right]^2+2} . dx$

$(\because a^2+b^2 = (a+b)^2 - 2ab)$

Take $x - \frac{1}{x} = t$ then $\left(1+\frac{1}{x^2}\right) dx = dt$

$\therefore \int \frac{x^2+1}{x^4+1} dx = \int \frac{dt}{t^2+2} = \int \frac{dt}{t^2+(\sqrt{2})^2}$

$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + c$

$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + c$

$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$.

22. $\int \frac{dx}{1+\sin x+\cos x}$

Sol. $\int \frac{dx}{1+\sin x+\cos x}$

$$= \int \frac{dx}{\left[1 + \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]}$$

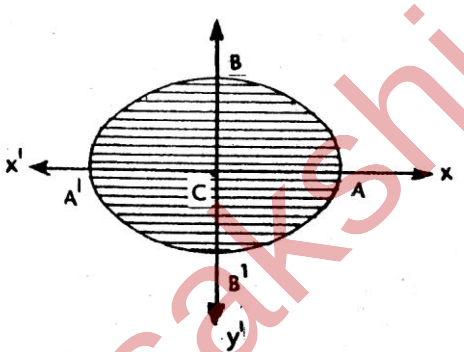
$$= \int \frac{\sec^2\frac{x}{2}dx}{1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$$

$$= \int \frac{\sec^2\frac{x}{2}}{2+2\tan\frac{x}{2}} \text{ put } \tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$= 2\int \frac{dt}{2+2t} = \int \frac{dt}{1+t} \log|1+t| + C$$

$$= \log\left|1+\tan\frac{x}{2}\right| + C$$

23. Show that the area of the region bounded by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$(ellipse) is πab . Also deduce the area of the circle $x^2+y^2=a^2$.



Sol:

The ellipse is symmetrical about X and Y axis Area of the ellipse = 4 Area of CAB= $4.\frac{\pi}{4}ab$

Equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$y = \frac{b}{a}\sqrt{a^2-x^2}$$

$$CAB = \frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\, dn$$

$$= \frac{b}{a}\left(\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)_0$$

$$= \frac{b}{a}\left(0 + \frac{a^2}{2}.\frac{\pi}{2} - ab\right) = \frac{\pi a^2}{4}.\frac{b}{a} = \frac{\pi}{4}ab$$

(from prob. 8 in ex 10(a)) $= \pi ab$

Substituting b = a, we get the circle

$$x^2 + y^2 = a^2$$

Area of the circle = $\pi a \ \ a = \pi a^2$ sq. units.

24. $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$

Sol. $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$

$\frac{dy}{dx} = \frac{x^2y - 2xy^2}{x^3 - 3x^2y}$ which is a homogeneous d.e.

Put y = vx so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{x^3v - 2v^2x^3}{x^3 - 3vx^3}$$

$$= \frac{(v-2v^2)x^3}{(1-3v)x^3} = \frac{v-2v^2}{1-3v}$$

$$x\frac{dv}{dx} = \frac{v-2v^2}{1-3v} - v$$

$$= \frac{v - 2v^2 - v(1-3v)}{1-3v} = \frac{-2v^2 + 3v^2}{1-3v}$$

$$x\frac{dv}{dx} = \frac{v^2}{1-3v} \Rightarrow \frac{1-3v}{v^2}dv = \frac{dx}{x}$$

$$\int\left(\frac{1}{v^2} - \frac{3}{v}\right)dv = \int\frac{dx}{x}$$

$$\frac{-1}{v} - 3\log v = \log x + \log c$$

$$\frac{-x}{y} = 3\log\left(\frac{y}{x}\right) = \log x + \log c$$

$$\frac{-x}{y} - \log\left(\frac{y}{x}\right)^3 = \log xc$$

$$\frac{-x}{y} = \log xc + \log\frac{y^3}{x^3}$$

$$\frac{-x}{y} = \log\left(cx \cdot \frac{y^3}{x^3}\right) = \log\left(\frac{cy^3}{x^2}\right)$$

$$\frac{cy^3}{x^2} = e^{-x/y} \Rightarrow cy^3 = \frac{x^2}{e^{x/y}}$$

$$cy^3 \cdot e^{x/y} = x^2$$