MATHEMATICS PAPER IIA

TIME : 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 =20

1.If $x^2 - 6x + 5 = 0$ and $x^2 - 3ax + 35 = 0$ have a common root, then find a.

2. If α, β, γ are the roots of the equation $x^3 - 3ax + b = 0$, then prove that $\Sigma(\alpha-\beta)(\alpha-\gamma) = 9a$.

3. If the amplitude of $(z - 1)$ is $\pi/2$ then find the locus of z.

4 If $z \neq 0$ find $\operatorname{Arg} z + \operatorname{Arg} \bar{z}$.

5. If $\frac{z_2}{z_1}; z_1 \neq 0$ is an imaginary number then find the value of $\left|\frac{2z_1+z_2}{2z_1-z_2}\right|$.

6.Find the number of positive division of 1080.

7.Find the number of ways in which 4 letters can be put in 4 addressed envelopes so that no letter goes into the envelope meant for it.

8. Show that $C_0 + 2\cdot C_1 + 4\cdot C_2 + 8\cdot C_3 + ... + 2^n \cdot C_n = 3^n$

9. For a binomial distribution with mean 6 and variance 2, find the first two terms of the distribution.

10. Find the mean for the following distribution.

x_i	10	11	12	13
f_i	3	12	18	12

SECTION B

SHORT ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING 5 X 4 = 20

11.If the roots of $ax^2 + bx + c = 0$are real and equal to $\alpha = \frac{-b}{2a}$, $\alpha \neq x \in R$, then *prove that* $ax^2 + bx + c$ and a will have same sign.

12. If $x + iy = \frac{3}{2 + \cos\theta + i\sin\theta}$ then show that $x^2 + y^2 = 4x - 3$.

13. If the letters of the word BRING are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the 59th word.

14. A double decker mini bus has 8 seats in the lower and 10 seats in the upper deck. Find the no. of ways of arranging 18 persons in the bus, if 3 children want to go the upper deck and 4 old people cannot go to the upper deck?

15. resolve $\frac{x^2+1}{(x^2+x+1)^2}$ into partial fractions.

16. A, B, C are 3 newspaper from a city. 20% of the population read A, 16% read B, 14% read C, 8% both B and C, 2% all the three. Find the percentage of the population who read atleast one newspaper.

17. If A, B, C are independent events, show that $A \cup B$ and C are independent events.

SECTION C

LONG ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING 5 X 7 =35

.

18. Solve $18x^3+81x^2+21x+60=0$ given that one root is equal to half the sum of the remaining roots.

19. Show that $\left\{\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right\}^{8/3}=-1$

20. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1 + x)^{th}$ are in A.P. then show that $n^2 - (4r + 1)n + 4r^2 - 2 = 0$.

21. Find the sum of the infinite series $\frac{7}{5}\left(1+\frac{1}{10^2}+\frac{1\cdot3}{1\cdot2}\frac{1}{10^4}+\frac{1\cdot3\cdot5}{1\cdot2\cdot3}\frac{1}{10^6}+...\right)$.

22. The probabilities of three mutually exclusive events are respectively given as $\frac{1+3p}{3},\frac{1-p}{4},\frac{1-2p}{2}$. Prove that $\frac{1}{3}\le p\le\frac{1}{2}$.

23. if A random variable x has the following probability distribution.

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find i) k ii) the mean and iii) p(0< x < 5).

24. The scores of two cricketers A and B in 10 innings are given below. Find who is a better run getter and who is a more consistent player.

Scores of A : x_i	40	25	19	80	38	8	67	121	66	76
Scores of B : y_i	28	70	31	0	14	111	66	31	25	4

SOLUTIONS

1.If $x^2 – 6x + 5 = 0$ and $x^2 – 3ax + 35 = 0$ have a common root, then find a.

Sol: $x^2 – 6x + 5 = 0$

$(x – 5)(x – 1) = 0$

$x = 5, 1$

Now, 5, 1 satisfy

$x^2 – 3ax + 35 = 0$

$25 – 15a + 35 = 0$

$60 = 15 a$

$a = 4$

$1 – 3a + 35 = 0$

$3a = 36$

$a = 12.$

2. If α, β, γ are the roots of the equation $x^3 – 3ax + b = 0$, then prove that $\Sigma(\alpha-\beta)(\alpha-\gamma)=9a$.

Sol: Given α, β, γ are the roots of

$x^3 – 3ax + b = 0$

$\therefore \alpha+\beta+\gamma=0, \alpha\beta+\beta\gamma+\gamma\alpha=-3a, \alpha\beta\gamma=-b$ Now $\Sigma(\alpha-\beta)(\alpha-\gamma)=$

$=\Sigma\left[\alpha^2-\alpha\beta-\alpha\gamma+\beta\gamma\right]$

$=(\alpha^2+\beta^2+\gamma^2)-(\alpha\beta+\beta\gamma+\gamma\alpha)$

$=(\alpha+\beta+\gamma)^2-3(\alpha\beta+\beta\gamma+\gamma\alpha)$ 16. i) If z = x + iy and |z| = 1, then find the locus

$=0-3(-3a)$

$=9a$

$\therefore \Sigma(\alpha-\beta)(\alpha-\gamma)=9a$

of z.

3. If the amplitude of (z – 1) is π/2 then find the locus of z.

Sol: i) z = x + iy

$z-1=(x-1)+iy$

$\text{Tan}^{-1}\frac{y}{x-1}=\frac{\pi}{2}$

$x - 1 = 0$, $y \neq 0$ also $y > 0$.

4 If $z \neq 0$ find $\operatorname{Arg} z + \operatorname{Arg} \bar{z}$.

Sol: $z = x + iy, \bar{z} = x - iy$

$\operatorname{Arg} z = \tan^{-1}\frac{y}{x} \quad \operatorname{Arg}\bar{z} = \tan^{-1}\left(\frac{-y}{x}\right)$

$\operatorname{Arg} z + \operatorname{Arg}\bar{z}$

$\tan^{-1}\frac{y}{x} - \tan^{-1}\left(\frac{-y}{x}\right)$

0 when $\operatorname{Arg} z \neq \pi$

2π when $\operatorname{Arg} z = \pi$

5. If $\frac{z_2}{z_1}; z_1 \neq 0$ is an imaginary number then find the value of $\left|\frac{2z_1 + z_2}{2z_1 - z_2}\right|$.

Sol. $\frac{z_2}{z_1} = ki \left|\frac{2 + \frac{z_2}{z_1}}{2 - \frac{z_2}{z_1}}\right|$

$\left|\frac{2 + ki}{2 - ki}\right| = \frac{\sqrt{4 + k^2}}{\sqrt{4 + k^2}} = 1$

6.Find the number of positive division of 1080.

Sol: $1080 = 2^3 \times 3^3 \times 5^1$

$\therefore$ The number of positive divisions of 1080

$= 3+1 \quad 3+1 \quad 1+1$

$= 4 \times 4 \times 2 = 32$

7.Find the number of ways in which 4 letters can be put in 4 addressed envelopes so that no letter goes into the envelope meant for it.

Sol: Required number of ways is

$4!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 12 - 4 + 1 = 9$.

8. Show that $C_0+2\cdot C_1+4\cdot C_2+8\cdot C_3+...+2^n\cdot C_n=3^n$

Sol. L.H.S.= $C_0+2\cdot C_1+4\cdot C_2+8\cdot C_3+...+2^n\cdot C_n$

$= C_0+C_1(2)+C_2(2^2)+C_3(2^3)+...+C_n(2^n)$

$= (1 + 2)^n = 3^n$

[$(1+x)^n=C_0+C_1\cdot x+C_2x^2+...+C_nx^n$]

9. For a binomial distribution with mean 6 and variance 2, find the first two terms of the distribution.

Sol. Let n, p be the parameters of a binomial distribution

Mean (np) = 6 …(1)

and variance (n pq) = 2 …(2)

then $\frac{npq}{np}=\frac{2}{6}\Rightarrow q=\frac{1}{3}\Rightarrow \because p=1-q=1-\frac{1}{3}=\frac{2}{3}$

From (1) n p = 6

$n\left(\frac{2}{3}\right)=6\Rightarrow n=\frac{18}{2}=9$

First two terms of the distribution are

$p\ \ x=0\ \ ={}^9C_0\left(\frac{1}{3}\right)^9=\frac{1}{3^9}$ and

$p\ \ x=1\ \ ={}^9C_1\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)=\frac{2}{3^7}$

10. Find the mean for the following distribution.

x_i	10	11	12	13
f_i	3	12	18	12

Sol.

x_i	f_i	$f_i\ x_i$	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
10	3	30	1.87	5.61
11	12	132	0.87	10.44
12	18	216	0.13	2.24
13	12	156	1.13	13.56
	N = 45	Σ f_i x_i = 534		$\Sigma f_i\lvert x_i-\bar{x}\rvert$= 31.95

$\therefore$ Mean $(\bar{x}) = \frac{\Sigma f_i x_i}{N} = \frac{534}{45} = 11.87$

11.If the roots of $ax^2 + bx + c = 0$are real and equal to $\alpha = \frac{-b}{2a}$, then $\alpha \neq x \in R,$ $ax^2 + bx + c$ and a will have same sign.

Proof :

The roots of $ax^2 + bx + c = 0$ are real and equal

$\Rightarrow b^2 = 4ac \Rightarrow 4ac - b^2 = 0$

$$\frac{ax^2+bx+c}{a} = x^2 + \frac{b}{a}x + \frac{c}{a}$$

$$= \left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2}$$

$$= \left(x + \frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a^2}$$

$= \left(x + \frac{b}{2a}\right)^2 > 0$ for $x \neq \frac{-b}{2a} = \alpha$

for $\alpha \neq x \in R, ax^2 + bx + c$ and a have the same sign.

12. If $x + iy = \frac{3}{2+\cos\theta + i\sin\theta}$ then show that $x^2 + y^2 = 4x - 3$.

$$x + iy = \frac{3}{2+\cos\theta + i\sin\theta}$$

$$= \frac{3(2+\cos\theta - i\sin\theta)}{(2+\cos\theta)^2 - i^2\sin^2\theta}$$

$$= \frac{3(2+\cos\theta - i\sin\theta)}{4+\cos^2\theta + 4\cos\theta + \sin^2\theta}$$

$$= \frac{6+3\cos\theta - 3i\sin\theta}{5+4\cos\theta}$$

$$= \frac{6+3\cos\theta}{5+4\cos\theta} + \frac{-3i\sin\theta}{5+4\cos\theta}$$

$$x = \frac{6+3\cos\theta}{5+4\cos\theta}, \; y = \frac{-3\sin\theta}{5+4\cos\theta}$$

L.H.S. =

$$x^2 + y^2 = \left(\frac{6+3\cos\theta}{5+4\cos\theta}\right)^2 + \left(\frac{-3\sin\theta}{5+4\cos\theta}\right)^2$$

$$= \frac{36+9\cos^2\theta+36\cos\theta+9\sin^2\theta}{(5+4\cos\theta)^2}$$

$$= \frac{45+36\cos\theta}{(5+4\cos\theta)^2}$$

$$= \frac{9(5+4\cos\theta)}{(5+4\cos\theta)^2}$$

$$x^2 + y^2 = \frac{9}{5+4\cos\theta}$$

R.H.S. =

$$4x-3 = \frac{4\,(6+3\cos\theta)}{5+4\cos\theta} - 3$$

$$= \frac{24+12\cos\theta-15-12\cos\theta}{5+4\cos\theta}$$

$$= \frac{9}{5+4\cos\theta}$$

$\therefore\ x^2 + y^2 = 4x - 3.$

13. If the letters of the word BRING are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the 59th word.

Sol: Given word is BRING.

$\therefore$ The alphabetical order of the letter is :

B, G, I, N, R.

In the dictionary order, first we write all words beginning with B.

Clearly the number of words beginning with B are 4! = 24.

Similarly the number of words begin with G are 4! = 24.

Since the words begin with b and G sum to 48, the 59th word must start with I.

Number of words given by IB = 3! = 6

Hence the 59th word must start with IG.

Number of words begin with IGB = 2! = 2

Number of words begin with IGN = 2! = 2

$\therefore$ Next word is 59th = IGRBN.

14. A double decker mini bus has 8 seats in the lower and 10 seats in the upper deck. Find the no. of ways of arranging 18 persons in the bus, if 3 children want to go the upper deck and 4 old people cannot go to the upper deck?

Sol: Allowing 3 children to the upper deck and 4 old people to the lower deck, we are left with 11 people and 11 seats (7 seats in the upper deck and 4in the lower deck). we can select 7 people in ${}^{11}C_7$ways. The remaining 4 persons go to the lower deck.

Now, we can arrange 10 persons (3 children and 7 others) in the upper deck and 8 persons (4 old people and 4 others) in the lower deck in (10)! and (8)! ways respectively. Hence the required number of arrangements

$$= {}^{11}C_7 \times 10!\ 8!$$

15. resolve $\frac{x^2+1}{(x^2+x+1)^2}$ into partial fractions.

Sol. Let $\frac{x^2+1}{(x^2+x+1)^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2}$

Multiplying with $(x^2 + x + 1)^2$

$x^2 + 1 = (Ax + B)(x^2 + x + 1) + (Cx + D)$

Equating the coefficients of x^3, $A = 0$

Equating the coefficients of x^2,

$A + B = 1 \Rightarrow B = 1$

Equating the coefficients of x,

$A + B + C = 0$

$\Rightarrow 1 + C = 0 \Rightarrow C = -1$

Equating the constant, $B + D = 1$

$\Rightarrow D = 1 - B = 1 - 1 = 0$

$\therefore\ Ax + B = 1, Cx + D = -x$

$$\therefore \frac{x^2+1}{(x^2+x+1)^2} = \frac{1}{x^2+x+1} - \frac{x}{(x^2+x+1)^2}$$

16. A, B, C are 3 newspaper from a city. 20% of the population read A, 16% read B, 14% read C, 8% both B and C, 2% all the three. Find the percentage of the population who read atleast one newspaper.

Sol. Given $p\ A = \frac{20}{100} = 0.2$

$p\ B = \frac{16}{100} = 0.16$

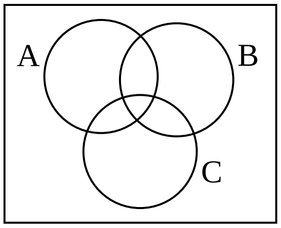
$p\ C = \frac{14}{100} = 0.14$

$$p(A\cap B)=\frac{8}{100}=0.08$$

$$p(B\cap C)=\frac{4}{100}=0.04$$

$$p(A\cap C)=\frac{5}{100}=0.05$$

$$p(A\cap B\cap C)=\frac{2}{100}=0.02$$



$$p(A\cup B\cup C)=p(A)+p(B)+p(C)-p(A\cap B)$$
$$-p(B\cap C)-p(C\cap A)+p(A\cap B\cap C)$$

$$=0.2+0.16+0.14-0.08-0.04-0.05+0.02$$
$$=0.52-0.17=0.35$$

Percentage of population who read atleast one newspaper = 0.35 × 100 = 35%

17. If A, B, C are independent events, show that $A\cup B$ and C are independent events.

Sol. $\because$ A, B, C are independent events.

$\Rightarrow$ A, B ; B, C ; C, A are also independent events.

$$P(A\cap B\cap C)=P(A)P(B)P(C)$$

$$P(A\cap C)=P(A)P(C)$$

$$P(B\cap C)=P(B)P(C)$$

$$P(A\cap B)=P(A)P(B)$$

$$P[(A\cup B)\cap C]=P[(A\cap C)\cup(B\cap C)]$$

$$=P(A\cap C)+P(B\cap C)-P[(A\cap C)\cap(B\cap C)]$$

$$=P(A)P(C)+P(B)P(C)-P(A\cap B\cap C)$$

$$=P(A)P(C)+P(B)P(C)-P(A)P(B)P(C)$$

$$=[P(A)+P(B)-P(A)P(B)]P(C)$$

$$=P(A\cup B)\cdot P(C)$$

$\therefore$ $A\cup B$ and C are independent events.

18. Solve $18x^3 + 81x^2 + 21x + 60 = 0$ given that one root is equal to half the sum of the remaining roots.

Sol: Suppose α, β, γ are the roots of

$18x^3 + 81x^2 + 121x + 60 = 0$

Sum $\alpha + \beta + \gamma = \frac{-81}{18} = \frac{-9}{2}$ ----------------(1)

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{121}{18}$ ----------(2)

$\alpha\beta\gamma = \frac{-60}{18} = \frac{-10}{3}$ -------------(3)

$\because$ One root is equal to half of the sum of the remaining two

Let $\alpha = \frac{1}{2}\ \beta + \gamma$

Substitute in (1)

$\alpha + 2\alpha = -\frac{9}{2} \Rightarrow \alpha = \frac{-3}{2}$

$\because \beta + \gamma = 2\alpha = 2\left(-\frac{3}{2}\right) = -3$

From (3)

$\left(-\frac{3}{2}\right)\ \beta\gamma\ = \frac{-10}{3}$

$\Rightarrow \beta\gamma = \frac{20}{9}$

$\because\ \beta - \gamma\ ^{2} = \ \beta + \gamma\ ^{2} - 4\beta\gamma$

$= \ -3\ ^{2} - 4\left(\frac{20}{9}\right) = \frac{81 - 80}{9} = \frac{1}{9}$

$\therefore \beta - \gamma = \frac{1}{3}$

$\underline{\beta + \gamma = -3}$

Add $2\beta = \frac{1}{3} - 3 = \frac{-8}{3} \Rightarrow \beta = \frac{-4}{3}, \gamma = \frac{-5}{3}$

$\therefore$ The roots of the given equation are

$\frac{-3}{2}, \frac{-4}{3}$ *and* $\frac{-5}{3}$

19. Show that $\left\{\frac{1 + \sin\frac{\pi}{8} + i\cos\frac{\pi}{8}}{1 + \sin\frac{\pi}{8} - i\cos\frac{\pi}{8}}\right\}^{8/3} = -1$

Solution : -

$$\text{LHS} = \left\{\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right\}^{8/3}$$

$$\left\{\frac{1+\cos\left(\frac{\pi}{2}-\pi/8\right)+i\sin\left(\frac{\pi}{2}-\pi/8\right)}{1+\cos\left(\frac{\pi}{2}-\pi/8\right)-1\sin\left(\frac{\pi}{2}-\pi/3\right)}\right\}^{8/3}$$

$$\left\{\frac{1+\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}}{1+\cos\frac{3\pi}{8}-i\sin\frac{3\pi}{8}}\right\}^{8/3} = \left\{\frac{2\cos^2\frac{3\pi}{16}+2i\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}}{2\cos^2\frac{3\pi}{16}-2i\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}}\right\}^{8/3}$$

$$\left[\frac{2\cos\frac{3\pi}{16}\left\{\cos\frac{3\pi}{16}+1\sin\frac{3\pi}{16}\right\}}{2\cos\frac{3\pi}{16}\left(\cos\frac{3\pi}{16}-i\sin\frac{3\pi}{16}\right)}\right]^{\frac{8}{3}}$$

$$\left[\frac{\left(\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)\left(\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)}{\left(\cos\frac{3\pi}{16}-i\sin\frac{3\pi}{16}\right)\left(\cos\left(\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)\right)}\right]^{8/3}$$

$$\left[\frac{\left(\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)^2}{\cos^2\frac{3\pi}{16}+\sin^2\frac{3\pi}{16}}\right]^{8/3}$$

$$\left(\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}\right)^{8/3}$$

$$\cos\pi+i\sin\pi=-1$$

20. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1 + x)^{th}$ are in A.P. then show that $n^2 - (4r + 1)n + 4r^2 - 2 = 0$.

Sol. Coefficient of $T_r = {}^nC_{r-1}$

Coefficient of $T_{r+1} = {}^nC_r$

Coefficient of $T_{r+2} = {}^nC_{r+1}$

Given ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$ are in A.P.

$\Rightarrow 2\,{}^nC_r = {}^nC_{r-1} + {}^nC_{r+1}$

$$\Rightarrow 2\frac{n!}{(n-r)!r!} = \frac{n!}{(n-r+1)!(r-1)!} + \frac{n!}{(n-r-1)!(r+1)!}$$

$$\Rightarrow \frac{2}{(n-r)r} = \frac{1}{(n-r+1)(n-r)} + \frac{1}{(r+1)r}$$

$$\Rightarrow \frac{1}{n-r}\left[\frac{2}{r} - \frac{1}{n-r+1}\right] = \frac{1}{(r+1)r}$$

$$\Rightarrow \frac{1}{n-r}\left[\frac{2n-2r+2-r}{r(n-r+1)}\right] = \frac{1}{r(r+1)}$$

$$\Rightarrow (2n-3r+2)(r+1) = (n-r)(n-r+1)$$

$$\Rightarrow 2nr + 2n - 3r^2 - 3r + 2r + 2 = n^2 - 2nr + r^2 + n - r$$

$$\Rightarrow n^2 - 4nr + 4r^2 - n - 2 = 0$$

$$\therefore n^2 - (4r+1)n + 4r^2 - 2 = 0$$

21. Find the sum of the infinite series $\frac{7}{5}\left(1 + \frac{1}{10^2} + \frac{1\cdot 3}{1\cdot 2}\frac{1}{10^4} + \frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\frac{1}{10^6} + \ldots\right)$.

Sol. $1 + \frac{1}{10^2} + \frac{1\cdot 3}{1\cdot 2}\frac{1}{10^4} + \frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\frac{1}{10^6} + \ldots$

$$= 1 + \frac{1}{1!}\left(\frac{1}{100}\right) + \frac{1\cdot 3}{2!}\left(\frac{1}{100}\right)^2 + \frac{1\cdot 3\cdot 5}{3!}\left(\frac{1}{100}\right)^3 + \ldots$$

Comparing with $(1-x)^{-p/q}$

$$= 1 + \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2$$ p = 1,p+q=3,q= 2

$$\frac{x}{q} = \frac{1}{100} \Rightarrow x = \frac{q}{100} = \frac{2}{100} = 0.02$$

$$\therefore 1 + \frac{1}{10^2} + \frac{1\cdot 3}{1\cdot 2}\cdot\frac{1}{10^4} + \ldots = (1-x)^{-p/q}$$

$$= (1-0.02)^{-1/2} = (0.98)^{-1/2} = \left(\frac{49}{50}\right)^{-1/2} = \left(\frac{50}{49}\right)^{1/2} = \frac{5\sqrt{2}}{7}$$

$$\therefore \frac{7}{5}\left[1 + \frac{1}{10^2} + \frac{1\cdot 3}{1\cdot 2}\frac{1}{10^4} + \frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\frac{1}{10^6} + \ldots\right]$$

$$= \frac{7}{5}\frac{5\sqrt{2}}{7} = \sqrt{2}$$

22. The probabilities of three mutually exclusive events are respectively given as $\frac{1+3p}{3}, \frac{1-p}{4}, \frac{1-2p}{2}$. Prove that $\frac{1}{3} \le p \le \frac{1}{2}$.

Sol. Suppose A, B, C are exclusive events such that

$$P(A) = \frac{1+3p}{3}$$

$$P(B) = \frac{1-p}{4}$$

$$P(C) = \frac{1-2p}{2}$$

We know that

$0 \le P(A) \le 1$

$0 \le \frac{1+3p}{3} \le 1$

$0 \le 1+3p \le 3$

$-1 \le 3p \le 3-1$

$\frac{-1}{3} \le p \le \frac{2}{3}$ ...(1)

$0 \le P(B) \le 1$

$0 \le \frac{1-p}{4} \le 1$

$0 \le 1-p \le 4$

$-1 \le -p \le 4-1$

$1 \ge p \ge -3$

$-3 \le p \le 1$ ...(2)

$0 \le P(C) \le 1$

$0 \le \frac{1-2p}{2} \le 1$

$0 \le 1-2p \le 2$

$-1 \le -2p \le 2-1$

$1 \ge 2p \ge -1$

$\frac{1}{2} \ge p \ge -\frac{1}{2}$

$\frac{-1}{2} \le p \le \frac{1}{2}$ ...(3)

Since A, B, C are exclusive events,

$0 \le P(A \cup B \cup C) \le 1$

$\Rightarrow 0 \le P(A) + P(B) + P(C) \le 1$

$\Rightarrow 0 \le \frac{4+12P+3-3P+6-12P}{12} \le 1$

$\Rightarrow 0 \le \frac{13-3P}{12} \le 1$

$\Rightarrow 0 \le 13 - 3P \le 12$

$\Rightarrow -13 \le -3P \le 12 - 13$

$\Rightarrow 13 \ge 3P \ge 1$

$\Rightarrow \frac{13}{3} \ge P \ge \frac{1}{3}$

$\Rightarrow \frac{1}{3} \le P \le \frac{13}{3}$ ...(4)

Max. of $\left\{\frac{-1}{3}, -3, \frac{-1}{2}, \frac{1}{3}\right\} = \frac{1}{3}$

Min. of $\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\} = \frac{1}{2}$

(1), (2), (3) and (4) holds .

23. if A random variable x has the following probability distribution.

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find i) k ii) the mean and iii) p(0< x < 5).

Sol.

We know that $\sum_{i=1}^{n} p\ x_i\ = 1$

$0 + k + 2k + 2k + 3k + K^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 10k^2 + 9k = 1 \Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow 10k^2 + 10k - k - 1 = 0$

$\Rightarrow 10k\,(k + 1) - 1\,(k + 1) = 0$

$\Rightarrow (10k - 1)\,(k + 1) = 0$

$K = \frac{1}{10}, -1$ Since k > 0 $\quad \therefore$ k = $\frac{1}{10}$

i) k = $\frac{1}{10}$

ii)

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2+k$
$X_i.p(x_i)$	0	k	4k	6k	12k	$5k^2$	$12k^2$	$49k^2+7k$

Mean = $\sum_{i=1}^{n} x_i p\ \ x = x_i$

$= 0 + k + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k$

$= 66k^2 + 30k$

$= 66\left(\frac{1}{100}\right) + 30 \times \left(\frac{1}{10}\right)$

= 0.66 + 3 = 3.66

iii) p(0 < x < 5)

p(0 < x < 5) =

p(x = 1) + p(x = 2) + p(x = 3) + p(x = 4)

= k+ 2k + 2k + 3k = 8k

$= 8\frac{1}{10} = 8\frac{1}{10} = \frac{4}{5}$

24. The scores of two cricketers A and B in 10 innings are given below. Find who is a better run getter and who is a more consistent player.

Scores of A : x_i	40	25	19	80	38	8	67	121	66	76
Scores of B : y_i	28	70	31	0	14	111	66	31	25	4

Sol. For cricketer A : $\bar{x} = \frac{540}{10} = 54$

For cricketer B : $\bar{y} = \frac{380}{10} = 38$

x_i	(x_i- median)	(x_i- median)^2	y_i	(y_i- y median)	(y_i- y median)^2
40	–14	196	28	–10	100
25	29	841	70	32	1024
19	–35	1225	31	–7	49
80	26	676	0	–38	1444
38	–16	256	14	–24	576
8	–46	2116	111	73	5329
67	13	169	66	28	784
121	67	4489	31	–7	49
66	12	144	25	–13	169
76	22	484	4	–34	1156

$\Sigma x_i = 540$		10596	$\Sigma y_i = 380$		10680

Standard deviation of scores of A = $\sigma_x = \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2} = \sqrt{\frac{10596}{10}} = \sqrt{1059.6} = 32.55$

Standard deviation of scores of B = $\sigma_y = \sqrt{\frac{1}{n}\Sigma(y_i - \bar{y})^2} = \sqrt{\frac{10680}{10}} = \sqrt{1068} = 32.68$

C.V. of A = $\frac{\sigma_x}{\bar{x}} \times 100 = \frac{32.55}{54} \times 100 = 60.28$

C.V. of B = $\frac{\sigma_y}{\bar{y}} \times 100 = \frac{32.68}{38} \times 100 = 86$

Since $\bar{x} > \bar{y}$, cricketer A is a better run getter (scorer).

Since C.V. of A < C.V. of B, cricketer A is also a more consistent player.