MATHEMATICS PAPER IIA

TIME : 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 =20

1.Find the changes in the sign of the $x^2 - 5x + 6$ expression and find their extreme values.

$x^2 - 5x + 6$

2.Form the polynomial equation whose roots are the squares of the roots of

$x^3 + 3x^2 - 7x + 6 = 0$

3.Write z = $-7 + i\sqrt{21}$ in the polar form.

4. Find the number of 15^{th} roots of unity, which are also 25^{th} roots of unity.

5.If $1, \omega, \omega^2$ are the cube roots of units then prove that

$x + y + z \quad x + y\omega + z\omega \quad x + y\omega^2 + 2\omega = x^3 + y^3 + z^3 - 3xyz$

6.Find the number of ways of arranging theletters of the word TRIANGLE. So that the relative positions of the vowels and consonants are not distributed

7. Find the number of4 letter words that can be formed using the letters of the word PISTON in which atleast one letter is repeated.

8.Fin the numerically greatest term (s) in the expansion of

$(4 + 3x)^{15}$ when x = $\frac{7}{2}$

9. A poisson variable satisfies p(x = 1) = p(x = 2). Find p(x = 5)

10. Find the mean deviation about the median for the following data.

13, 17, 16, 11, 13, 10, 16, 11, 18, 12, 17

www.sakshieducation.com

SECTION B

SHORT ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING **5 X 4 = 20**

11.Solve the ineqaution $\sqrt{(x-3)(2-x)} < \sqrt{4x^2+12x+11}$.

12. If z = 3 + 5i then show that $z^3 - 10z^2 + 58z - 136 = 0$.

13.Find the number of ways of arranging 8 men and 4 women around a circular table. In how many of them

(i) all the women come together

(ii) no two women come together

14.Find the number of ways of selecting 11 members cricket team from 7 batsman, 6 bowlers and 2 wickets keepers so that team contains 2 wicket keepers and atleast 4 bowlers.

15. Resolve $\frac{x^2-x+1}{(x+1)(x-1)^2}$ into partial fractions

16. Suppose that an unbiased pair of dice is rolled. Let A denote the event that the same number shows on each die. Let B denote the event that the sum is greater than 7. Find (i) $P\left(\frac{A}{B}\right)$, (ii) $P\left(\frac{B}{A}\right)$.

17. State and prove Addition Theorem on Probability.

SECTION C

LONG ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING **5 X 7 =35**

18. Solve the following equations

$$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$$

19. If $\cos\alpha + \cos\square + \cos\square = 0 = \sin\square + \sin\square + \sin\square$ prove that $\cos^2\alpha + \cos^2\square + \cos^2\square = \frac{3}{2} = \sin^2\square + \sin^2\square + \sin^2\square$.

www.sakshieducation.com

20. If the 2nd, 3rd and 4th terms in the expansion of $(a + x)^n$ are respectively 240, 720, 1080, find a, x, n.

21. Prove that

$$({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - ({}^{2n}C_3)^2 + \ldots + ({}^{2n}C_{2n})^2 = (-1)^n\ {}^{2n}C_n$$

22.The range of a random variable x is {0, 1, 2}. Given that $p(x = 0) = 3c^3$, $p(x = 1) = 4c - 10c^2$, $p(x = 2) = 5c - 1$

i) Find the value of c

ii) $p(x < 1)$, $p(1 < x \le 3)$

23. From the prices of shares X and Y given below, for 10 days of trading, find out which share is more stable?

X	35	54	52	53	56	58	52	50	51	49
Y	108	107	105	105	106	107	104	103	104	101

24. Stete and prove Baye's Theorem or Inverse probability Theorem

solutions

1.Find the changes in the sign of the $x^2 - 5x + 6$ expression and find their extreme values.

$x^2 - 5x + 6$

Sol: i) $f(x) = x^2 - 5x + 6$

$f(x) = (x - 3)(x - 2)$

$f(x) < 0$

$2 < x < 3$

and $f(x) > 0$

$(-\infty < x < 2) \cup (3 < x < \infty)$

$$y = \left(x - \frac{5}{2}\right)^2 + \frac{6 - 25}{4}$$

$$y_{min} = -\frac{1}{4}$$

2.Form the polynomial equation whose roots are the squares of the roots of

$x^3 + 3x^2 - 7x + 6 = 0$

Sol: Given equation is

$$f(x) = x^3 + 3x^2 - 7x + 6 = 0$$

Required equations is $f(\sqrt{x}) = 0$

$$\Rightarrow x\sqrt{x} + 3x - 7\sqrt{x} + 6 = 0$$

$$\Rightarrow \sqrt{x}(x-7) = -(3x+6)$$

Squaring on both sides

$$\Rightarrow x(x-7)^2 = (3x+6)^2$$

$$\Rightarrow x(x^2 - 14x + 49) = 9x^2 + 36 + 36x$$

$$\Rightarrow x^3 - 14x^2 + 49x - 9x^2 - 36x - 36 = 0$$

$$i.e., x^3 - 23x^2 + 13x - 36 = 0$$

3.Write z = $-7 + i\sqrt{21}$ in the polar form.

Sol: If z = $-7 + i\sqrt{21}$ = x + iy

then x = $-\sqrt{7}$, y = $\sqrt{21}$, $r = \sqrt{x^2 + y^2}$

$$= \sqrt{7+21} = \sqrt{28} = 2\sqrt{7}$$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{21}}{-\sqrt{7}} = -\sqrt{3}$$

Since the given point lies in the second quadrant, we look for a solution of tan θ = $-\sqrt{3}$ which lies in $\left(\frac{\pi}{2}, \pi\right)$. We find that $\theta = \frac{2\pi}{3}$ is such a solution.

$$\therefore -\sqrt{7} + i\sqrt{21} = 2\sqrt{7}\,\text{cis}\frac{2\pi}{3} \text{ or}$$

$$2\sqrt{7}\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]$$

www.sakshieducation.com

4. Find the number of 15^{th} roots of unity, which are also 25^{th} roots of unity.

Sol: $x = (1)^{1/15}$

$$x = [\cos 2n\pi + i \sin 2n\pi]^{1/15}$$

$$x = \cos\frac{2n\pi}{15} + i\sin\frac{2n\pi}{15}$$

$$n = 0, 1, 2, 3, \ldots 14$$

$$n = 3, m = 5$$

$$x = \cos\frac{2\pi}{25} + i\sin\frac{2\pi}{25}$$

$$n = 9, m = 15$$

$$\cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}$$

$$x = (1)^{1/25}$$

$$x = [\cos 2m\pi + i\sin 2m\pi]^{1/25}$$

$$x = \cos\frac{2m\pi}{25} + i\sin\frac{2m\pi}{25}$$

$$m = 0, 1, 2, 3, \ldots 24$$

$$n = 6, m = 10$$

$$x = \cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}$$

$$n = 12, m = 20$$

$$\cos\frac{8\pi}{5} + i\sin\frac{8\pi}{5}$$

n = 0, m = 0

5 roots common.

5. If $1, \omega, \omega^2$ are the cube roots of units then prove that $x + y + z \quad x + y\omega + z\omega \quad x + y\omega^2 + 2\omega \; = x^3 + y^3 + z^3 - 3xyz$

Solution: -

$$x + y + z \quad x + y\omega + z\omega^2 \quad x + y\omega^2 + z\omega$$

$$x + y + z \quad x^2 + xy\omega^2 + xz\omega + xy\omega + xy\omega + y^2\omega^3 + yz\omega^2 + xz\omega^2 + yz\omega + z^2\omega^3$$

$$x + y + z \quad x^2 + y^2 + z^2 + xy \;\; \omega^2 + \omega \;\; + yz \;\; \omega + \omega^2 \;\; + zx \;\; \omega + \omega^2$$

$$x + y + z \quad x^2 + y^2 + z^2 - xy - yz - zx$$

www.sakshieducation.com

$x^3 + y^3 + z^3 - 3xyz$

6.Find the number of ways of arranging theletters of the word TRIANGLE. So that the relative positions of the vowels and consonants are not distributed

Sol:

In the given, word

Number of vowels is 3

Number of consonants is 5

C C V V C C C V

Since the relative positions of the vowels and consonants are not disturbed.

The 3 vowels can be arranged in their relative positions in

3! Ways and the 5 consonants can be arranged in their

relative positions in 5! Ways.

$\therefore$ The number of required arrangements $= 3!\ \ 5! = 6\ \ 120 = 720$

7. Find the number of4 letter words that can be formed using the letters of the word PISTON in which atleast one letter is repeated.

Sol: The word PISTON has 6 letters

The number of 4 letter words that can be formed using

these 6 letters

(i) When repetition is allowed $= 6^4$

(ii) When repetition is not allowed $= {}^6P_4$

$\therefore$ The number of 4 letter words in which at least one letter repeated is

$= 6^4 - {}^6P_4 = 1296 - 360 = 936$

8.Fin the numerically greatest term (s) in the expansion of $(4 + 3x)^{15}$ when x = $\frac{7}{2}$

www.sakshieducation.com

Sol. Write $(4 + 3x)^{15} = \left[4\left(1+\frac{3}{4}x\right)\right]^{15}$

$$= 4^{15}\left(1+\frac{3}{4}x\right)^{15} \quad \ldots(1)$$

First we find the numerically greatest term in the expansion of $\left(1+\frac{3}{4}x\right)^{15}$

Write X = $\frac{3}{4}x$ and calculate $\frac{(n+1)|x|}{1+|x|}$

Here $|X| = \left(\frac{3}{4}X\right) = \frac{3}{4}\times\frac{7}{2} = \frac{21}{8}$

Now $\frac{(n+1)|x|}{1+|x|} = \frac{15+1}{1+\frac{21}{8}}\cdot\frac{21}{8}$

$$= \frac{16\times 21}{29} = \frac{336}{29} = 11\frac{17}{29}$$

Its integral part $m = \left[11\frac{17}{29}\right] = 11$

T_{m+1} is the numerically greatest term in the expansion $\left(1+\frac{3}{4}x\right)^{15}$ and

$$T_{m+1} = T_{12} = {}^{15}C_{11}\left(\frac{3}{4}x\right)^{4} = {}^{15}C_{11}\left(\frac{3}{4}\cdot\frac{7}{2}\right)^{11}$$

$\therefore$ Numerically greatest term in $(4 + 3x)^{15}$

$$= 4^{15}\left[{}^{15}C_{11}\left(\frac{21}{8}\right)^{11}\right] = {}^{15}C_4\frac{(21)^{11}}{2^3}$$

9. A poisson variable satisfies p(x = 1) = p(x = 2). Find p(x = 5)

Sol. Given p(x = 1) = p(x = 2)

$$p\ \ x = r\ \ = \frac{\lambda^r e^{-\lambda}}{r!}, \lambda > 0$$

$$\frac{\lambda^r e^{-\lambda}}{1!} = \frac{\lambda^2 e^{-\lambda}}{2!}$$

$\lambda = 2$, $(\therefore \lambda > 0)$

$$\therefore p\ \ x = 5\ \ = \frac{2^5 e^{-2}}{5!}$$

$$= \frac{32}{120e^2} = \frac{4}{15e^2}$$

www.sakshieducation.com

10. Find the mean deviation about the median for the following data.

13, 17, 16, 11, 13, 10, 16, 11, 18, 12, 17

Sol. Expressing the given data in the ascending order.

We get 10, 11, 11, 12, 13, 13, 16, 16, 17, 17, 18

Mean (M) of these 11 observations is 13.

The absolute values of deviations are $|x_i - M| = 3, 2, 2, 1, 0, 0, 3, 3, 4, 4, 5$

$\therefore$ Mean deviation about Median = $\dfrac{\sum_{i=1}^{11} |x_i - M|}{n} = \dfrac{3+2+2+1+0+0+3+3+4+4+5}{11}$

$$= \frac{27}{11} = 2.45$$

11.Solve the ineqaution $\sqrt{(x-3)(2-x)} < \sqrt{4x^2 + 12x + 11}$.

Sol. The given inequation is equivalent to the following two inequalities.

$(x - 3)(2 - x) \geq 0$ and

$(x - 3)(2 - x) < 4x^2 + 12x + 11$.

$(x - 3)(2 - x) \geq 0$

$\Leftrightarrow (x-2)(x-3) \leq 0$

$\Leftrightarrow 2 \leq x \leq 3$

$-x^2 + 5x - 6 < 4x^2 + 12x + 11$

$\Leftrightarrow 5x^2 + 7x + 17 = 0$

The discriminant of the quadratic expression $5x^2 + 7x + 17$ is negative.

Hence $5x^2 + 7x + 17 > 0 \ \forall \ x \in \mathbf{R}$.

Hence the solution set of the given inequation is $\{x \in \mathbf{R} : 2 \leq x \leq 3\}$.

12. If z = 3 + 5i then show that $z^3 - 10z^2 + 58z - 136 = 0$.

.

Sol: z = 3 + 5i

$(z - 3)^2 = (5i)^2$

www.sakshieducation.com

$z^2 - 6z + 9 = 25i^2$

$z^2 - 6z + 9 = -25$

$z^2 - 6z + 34 = 0$

$z^3 - 6z^2 + 34z = 0$

$(z^3 - 10z^2 + 58z - 136) + 4z^2 - 24z + 136 = 0$

$(z^3 - 10z^2 + 58z - 136) + 4(z^2 - 6z + 34) = 0$

$\therefore z^3 - 10z^2 + 58z - 136 = 0$

13.Find the number of ways of arranging 8 men and 4 women around a circular table. In how many of them

(i) all the women come together

(ii) no two women come together

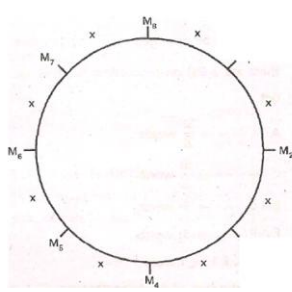
Sol. Total number of persons = 12 (8 men + 4 women)

Therefore, the number of circular permutations is (11) !

(i) Treat the 4 women as one unit. Then we have 8 men + 1 unit of women = 9 entities.

Which can be arranged around a circle in 8! ways. Now, the 4 women among themselves can be arranged in 4! ways. Thus, the number of required arrangements is 8! x 4!.

(ii)First arrange 8 men around a circle in 7! ways. Then there are 8 places in between them as shown in fig by the symbol x (one place in between any two consecutive men). Now, the 4 women can be arranged in these 8 places in ${}^8P_4$ ways.



www.sakshieducation.com

Therefore, the number of circular arrangements in which no two women come together is $7! \times {}^{8}P_{4}$.

14.Find the number of ways of selecting 11 members cricket team from 7 batsman, 6 bowlers and 2 wickets keepers so that team contains 2 wicket keepers and atleast 4 bowlers.

Sol: The required teams can contains the following compositions.

Bowlers	Wicket keepers	Batsmen	Number of ways of selecting team
4	2	5	${}^{6}C_4 \times {}^{2}C_2 \times {}^{7}C_5$ $= 15 \times 1 \times 21 = 315$
5	3	4	${}^{6}C_5 \times {}^{2}C_2 \times {}^{7}C_4$ $= 6 \times 1 \times 35 = 210$
6	2	3	${}^{6}C_6 \times {}^{2}C_2 \times {}^{7}C_3$ $= 1 \times 1 \times 35 = 35$

Therefore, the number of selecting the required cricket team

$= 315+210 + 35 = 560.$

15. $\dfrac{x^2 - x + 1}{(x+1)(x-1)^2}$

Sol. Let $\dfrac{x^2 - x + 1}{(x+1)(x-1)^2} = \dfrac{A}{x+1} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

Multiplying with $(x + 1)(x - 1)^2$

$x^2 - x + 1 =$

$A(x - 1)^2 + B(x + 1)(x - 1) + C(x + 1)$

Put $x = -1$, $1 + 1 + 1 = A(4) \Rightarrow A = \dfrac{3}{4}$

Put $x = 1$, $1 - 1 + 1 = C(2) \Rightarrow C = \dfrac{1}{2}$

Equating the coefficients of x^2

$A + B = 1 \Rightarrow B = 1 - A = 1 - \dfrac{3}{4} = \dfrac{1}{4}$

$$\therefore \frac{x^2 - x + 1}{(x+1)(x-1)^2} = \frac{3}{4(x+1)} + \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$$

16. Suppose that an unbiased pair of dice is rolled. Let A denote the event that the same number shows on each die. Let B denote the event that the sum is greater than 7. Find (i) $P\left(\frac{A}{B}\right)$, (ii) $P\left(\frac{B}{A}\right)$.

Sol. n(S) = 36

Let A be the event of getting the same number on two dice.

n(A) =

{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}=6

$$P(A) = \frac{6}{36}$$

Let B be the event at getting the sum greater than 7.

n(B) = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5),
(6, 4), (5, 6), (6, 5), (6, 6)} = 15

$$P(B) = \frac{15}{36}$$

$$n(A \cap B) = \{(4,4),(5,5),(6,6)\} = 3$$

$$P(A \cap B) = \frac{3}{36}$$

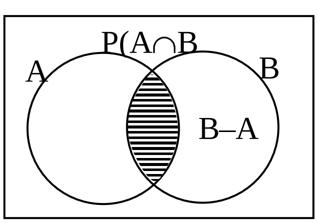
i) $$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{3}{36}}{\frac{15}{36}} = \frac{3}{15} = \frac{1}{5}$$

ii) $$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{3}{36}}{\frac{1}{6}} = \frac{3}{6} = \frac{1}{2}$$

17. State and prove Addition Theorem on Probability.

1. If A, B are two events in a sample space S, then P(A ∪ B) = P(A) + P(B) – P(A ∩ B).

Sol. From the figure (venn diagram) it can be observed that (B – A) ∪ (A ∩ B) = B, (B – A) ∩ (A ∩ B) = ϕ.



$$\therefore \mathrm{P(B)} = \mathrm{P[(B-A)} \cup \mathrm{(A \cap B)]}$$

$$= \mathrm{P(B-A)} + \mathrm{P(A \cap B)}$$

$$\Rightarrow \mathrm{P(B-A)} = \mathrm{P(B)} - \mathrm{P(A \cap B)} \qquad ...(1)$$

Again from the figure, it can be observed that

$$\mathrm{A} \cup \mathrm{(B-A)} = \mathrm{A} \cup \mathrm{B}, \mathrm{A} \cap \mathrm{(B-A)} = \phi$$

$$\therefore \mathrm{P(A \cup B)} = \mathrm{P[A \cup (B-A)]}$$

$$= \mathrm{P(A)} + \mathrm{P(B-A)}$$

$$= \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A \cap B)} \text{ since from (1)}$$

$$\therefore \mathrm{P(A \cup B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A \cap B)}$$

18. Solve the following equations

(i) $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

Sol: This is standard reciprocal equation

Dividing with x^2

$$x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 26 = 0 \text{ .... } 1$$

Put $a = x + \frac{1}{x}$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = a^2 - 2$$

Substituting in (1)

$$a^2 - 2 - 10a + 26 = 0$$

$$\Rightarrow a^2 = 10a + 24 = 0$$

$$\Rightarrow \; a-4 \quad a-6 \; = 0$$

$$a = 4 \; or \; 6$$

Case (i) a = 4

www.sakshieducation.com

$$x+\frac{1}{x}=4$$

$$\Rightarrow x^2+1=4x$$

$$\Rightarrow x^2-4x+1=0$$

$$\Rightarrow x=\frac{4\pm\sqrt{16-4}}{2}=\frac{4\pm 2\sqrt{3}}{2}$$

$$=2\pm\sqrt{3}$$

Case (ii) a = 6

$$x+\frac{1}{x}=6$$

$$x^2+1=6x$$

$$x^2-6x+1=0$$

$$x=\frac{6\pm\sqrt{36-4}}{2}=\frac{6\pm\sqrt{2}}{2}$$

$$x=\frac{2\;\;3\pm 2\sqrt{2}}{2}=3\pm 2\sqrt{2}$$

$\therefore$ The roots are $3\pm 2\sqrt{2}, 2\pm\sqrt{3}$

19. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{2} = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

Sol. $(\cos\alpha+i\sin\alpha)+(\cos\beta+i\sin\beta)+(\cos\gamma+i\sin\gamma)$

$$=(\cos\alpha+\cos\beta+\cos\gamma)+i(\sin\alpha+\sin\beta+\sin\gamma)=0+i0$$

$$(\cos\alpha+i\sin\alpha)+(\cos\beta+i\sin\beta)+(\cos\gamma+i\sin\gamma)=0 \quad ...(1)$$

Let $x=\text{cis}\alpha, y=\text{cis}\beta, z=\text{cis}\gamma$ then

$x + y + z = 0$ by (1), then

$x^2 + y^2 + z^2 = -2(xy + yz + zx)$

www.sakshieducation.com

$$= -2xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

$$= -2xyz[\cos\alpha - i\sin\alpha + \cos\beta - i\sin\beta + \cos\gamma - i\sin\gamma]$$

$$= -2xyz[(\cos\alpha + \cos\beta + \cos\gamma) - i(\sin\alpha + \sin\beta + \sin\gamma)]$$

$$= -2xyz(0 - i0) = 0$$

$$\therefore x^2 + y^2 + z^2 = 0$$

$$\Rightarrow (\cos\alpha + i\sin\alpha)^2 + (\cos\beta + i\sin\beta)^2 + (\cos\gamma + i\sin\gamma)^2 = 0$$

$$\Rightarrow \cos 2\alpha + i\sin 2\alpha + \cos 2\beta + i\sin 2\beta + \cos 2\gamma + i\sin 2\gamma = 0$$

$$\Rightarrow (\cos 2\alpha + \cos 2\beta + \cos 2\gamma) + i(\sin 2\alpha + \sin 2\beta + \sin 2\gamma) = 0$$

$$\therefore \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$$

$$2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1 = 0$$

$$2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 3$$

$$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{2}$$

$$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = \frac{3}{2}$$

$$\therefore \sin^2\alpha + \sin^2\beta + \sin^2\gamma = \frac{3}{2}.$$

20. If the 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(a + x)^n$ are respectively 240, 720, 1080, find a, x, n.

Sol. $T_2 = 240 \Rightarrow {}^nC_1\, a^{n-1}\, x = 240$ …(1)

$T_3 = 720 \Rightarrow {}^nC_2\, a^{n-2}\, x^2 = 720$...(2)

$T_4 = 1080 \Rightarrow {}^nC_3\, a^{n-3}\, x^3 = 1080$ ...(3)

$$\frac{(2)}{(1)} \Rightarrow \frac{{}^nC_2 a^{n-2} x^2}{{}^nC_1 a^{n-1} x} = \frac{720}{240}$$

$$\Rightarrow \frac{n-1}{2}\frac{x}{a} = 3 \Rightarrow (n-1)x = 6a \text{ ...(4)}$$

$$\frac{(3)}{(2)} \Rightarrow \frac{{}^nC_3 a^{n-3} x^3}{{}^nC_2 a^{n-2} x^2} = \frac{1080}{720} \Rightarrow \frac{n-2}{3}\frac{x}{a} = \frac{3}{2} \Rightarrow 2(n-2)x = 9a\text{...(5)}$$

$$\frac{(4)}{(5)} \Rightarrow \frac{(n-1)x}{2(n-2)x} = \frac{6a}{9a} \Rightarrow \frac{n-1}{2n-4} = \frac{2}{3}$$

$$\Rightarrow 3n - 3 = 4n - 8 \Rightarrow n = 5$$

From (4), $(5 - 1)x = 6a \Rightarrow 4x = 6a$

$$\Rightarrow x = \frac{3}{2}a$$

www.sakshieducation.com

Substitute x = $\frac{3}{2}a$, n = 5 in (1)

$$^5C_1 \cdot a^4 \cdot \frac{3}{2}a = 240 \Rightarrow 5 \times \frac{3}{2}a^5 = 240$$

$$a^5 = \frac{480}{15} = 32 = 2^5$$

$$\therefore a = 2, x = \frac{3}{2}a = \frac{3}{2}(2) = 3 \therefore a = 2, x = 3, n = 5$$

21. Prove that

$$({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - ({}^{2n}C_3)^2 + \ldots + ({}^{2n}C_{2n})^2 = (-1)^n \, {}^{2n}C_n$$

Sol. $(x+1)^{2n} = {}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} + {}^{2n}C_2x^{2n-2} + \ldots + {}^{2n}C_{2n}$ ...(1)

$(x-1)^{2n} = {}^{2n}C_0 - {}^{2n}C_1x + {}^{2n}C_2x^2 + \ldots + {}^{2n}C_{2n}x^{2n}$ ...(2)

Multiplying eq. (1) and (2), we get

$$({}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} + {}^{2n}C_2x^{2n-2} + \ldots + {}^{2n}C_{2n})({}^{2n}C_0 - {}^{2n}C_1x + {}^{2n}C_2x^2 + \ldots + {}^{2n}C_{2n}x^{2n}) = (x+1)^{2n}(1-x)^{2n} = [(1+x)(1-x)]^{2n} = (1-x^2)^{2n} = \sum_{r=0}^{2n} {}^{2n}C_r(-x^2)^r$$

Equating the coefficients of x^{2n}

$$({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - ({}^{2n}C_3)^2 + \ldots + ({}^{2n}C_{2n})^2 = (-1)^n \, {}^{2n}C_n$$

22.The range of a random variable x is {0, 1, 2}. Given that $p(x = 0) = 3c^3$, $p(x = 1) = 4c - 10c^2$, $p(x = 2) = 5c - 1$

i) Find the value of c

ii) $p(x < 1)$, $p(1 < x \le 3)$

Sol. $P(x = 0) + p(x = 1) + p(x = 2) = 1$

$3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$3c^3 - 10c^2 + 9c - 2 = 0$

C = 1 satisfy this equation

C = 1 $\Rightarrow$ p(x = 0) = 3 which is not possible dividing with c – 1, we get

$3c^2 - 7c + 2 = 0 \Rightarrow$ $(c - 2)(3c - 1) = 0$

d = 2 or c = 1/3

c = 2 $\Rightarrow$ $p(x = 0) = 3.2^3 = 24$ which is not possible

$\therefore$ c = 1/3

i) p(x < 1) = p(x =0)

$$= 3.c^3 = \left(\frac{1}{3}\right)^3 = 3.\frac{1}{27} = \frac{1}{9}$$

ii) $p(1 < x \le 2) = p(x = 2) = 5c - 1$

$= \frac{5}{3} - 1 = \frac{2}{3}$

iii)

$p(0 < x \le 3) = p(x = 1) + p(x = 2)$

$= 4c - 10c^2 + 5c - 1$

$= 9c - 10c^2 - 1 = 9.\frac{1}{3} - 10.\frac{1}{9} - 1$

$= 3 - \frac{10}{9} - 1 = 2 - \frac{10}{9} = \frac{8}{9}$

23. From the prices of shares X and Y given below, for 10 days of trading, find out which share is more stable?

X	35	54	52	53	56	58	52	50	51	49
Y	108	107	105	105	106	107	104	103	104	101

Sol. Variance is independent of charge of origin.

X	Y	X_i^2	Y_i^2
–15	8	225	64
4	7	16	49
2	5	4	25
3	5	9	25
6	6	36	36
8	7	64	49
2	4	4	16
0	3	0	9
1	4	1	16
–1	1	1	1
$\Sigma X_i = 10$	$\Sigma Y_i = 50$	$\Sigma X_i^2 = 360$	$\Sigma Y_i^2 = 290$

$$V(X) = \frac{\Sigma X_i^2}{n} - (\bar{X})^2 = \frac{360}{10} - \left(\frac{10}{10}\right)^2 = 36 - 1 = 35$$

$$V(Y) = \frac{\Sigma Y_i^2}{n} - (\bar{Y})^2 = \frac{290}{10} - \left(\frac{50}{10}\right)^2 = 29 - 25 = 4$$

Y is stable.

24. Stete and prove Baye's Theorem or Inverse probability Theorem

Statement : If $A_1, A_2, \ldots$ and A_n are *'n'* mutually exclusive and exhaustive events of a random experiment associated with sample space S such that $P(A_i) > 0$ and E is any event which takes place in conjuction with any one of A_i then

$$P(A_k/E) = \frac{P(A_k)P(E/A_k)}{\sum_{i=1}^{n} P(A_i)P(E/A_i)}, \text{ for any } k = 1,2.....n;$$

Proof : Since $A_1, A_2, \ldots, A_n$ are mutually exclusive and exhaustive in sample space S, we have $A_i \cap A_j =$ for i $\neq$ j, $1 \le i, j \le n$ and $A_1 \cup A_2 \cup \ldots \cup A_n = S$.

Since E is any event which takes place in conjuction with any one of A_i, we have

$$E = A_1 \cap E \cup A_2 \cap E \ldots\ldots\ldots \cup A_n \cap E .$$

We know that $A_1, A_2, \ldots\ldots A_n$ are mutually exclusive, their subsets $A_1 \cap E$, $A_2 \cap E$, ... are also mutually exclusive.

Now $P(E) = P(E \cap A_1) + P(E \cap A_2) + \ldots + P(E \cap A_n)$

(by axiom of additivity)

$= P(A_1)\, P(E/A_1) + P(A_2)\, P(E/A_2) + \ldots$

$+ P(A_n)\, P(E/A_n)$

(by multiplication theorem of probability)

$$= \sum_{i=1}^{n} P(A_i)P(E/A_i) \quad \ldots. (1)$$

By definition of conditional probability,

$$P(A_k/E) = \frac{P(A_k \cap E)}{P(E)} \text{ for}$$

$$= \frac{P(A_k)P(E/A_k)}{P(E)}$$

(bymultiplication theorem)

$$= \frac{P(A_k)P(E/A_k)}{\sum_{i=1}^{n} P(A_I)p(E/A_i)} \quad \text{from (1)}$$

Hence the theorem

www.sakshieducation.com