MATHEMATICS PAPER IIA

TIME : 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 =20

1. If $c^2 \neq ab$ and the roots of $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal, then show that $a^3 + b^3 + c^3 = 3abc$ or $a = 0$.

2. If α, β and 1 are the roots of $x^3 - 2x^2 - 5x + 6 = 0$, then find α and β

3. Find the real and imaginary parts of the complex number $\frac{a+ib}{a-ib}$.

4. If $(1-i)(2-i)(3-i)...(1-ni) = x - iy$ then prove that $2 \cdot 5 \cdot 10...(1+n^2) = x^2 + y^2$.

5. Prove that $-\omega$ and $-\omega^2$ are roots of $z^2 - z + 1 = 0$ where ω and ω^2 are the complex cube roots of unity.

6. In how many ways can the letters of the word CHEESE be arranged so that no two E's come together?

7. Find the number of diagonals of a polygon with 12 sides.

8. If $(1+3x-2x^2)^{10} = a_0 + a_1x + a_2x^2 + ... + a_{20}x^{20}$ then prove that
   i) $a_0 + a_1 + a_2 + ... + a_{20} = 2^{10}$
   ii) $a_0 - a_1 + a_2 - a_3 + ... + a_{20} = 4^{10}$

9. If E_1, E_2 are two events with $E_1 \cap E_2 = \phi$, then show that $P(E_1^C \cap E_2^C) = P(E_1^C) - P(E_2)$.

10. Find the variance for the discrete data 6, 7, 10, 12, 13, 4, 8, 12

SECTION B

SHORT ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING 5 X 4 = 20

11. In the expression $\frac{x-p}{x^2-3x+2}$ takes all values of $x \in R$, then find the bounds for p.

12.The points P, Q denote the complex numbers z_1, z_2 in the Argand diagram. O is origin. If $z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$ then show that $\angle POQ = 90°$.

13. A question paper is divided into 3 sections A, B, C containing 3, 4, 5 questions respectively. Find the number of ways of attempting 6 questions choosing atleast one from each section.

14.Find the number of ways in which 5 red balls, 4 black balls of different sizes can be arranged in a row so that (i) no two balls of the same colour come together, (ii) the balls of the same colour come together.

15.The probabilities of three events A, B, C are such that P(A) = 0.3, P(B) = 0.4, P(C) = 0.8, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$, $P(A \cap B \cap C) = 0.09$ and $P(A \cup B \cup C) \geq 0.75$. Show that $P(B \cap C)$ lies in the interval [0.23, 0.48].

16.An urn contains 12 red balls and 12 green balls. Suppose two balls are drawn one after another without replacement. Find the probability that the second ball drawn is green given that the first ball drawn is red.

17.Find the coefficient of x^n in the expansion of $\frac{x-4}{x^2-5x+6}$.

SECTION C

LONG ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING 5 X 7 =35

18.Given that the sum of two roots of $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ is zero, find the roots of the equation

19.Solve $(x - 1)^n = x^n$, n is a positive integer.

20. If R, n are positive integers, n is odd, 0 < F < 1 and if $(5\sqrt{5}+11)^n = R + F$, then prove that

i) R is an even integer and

ii) $(R + F)F = 4^n$.

21. If $x = \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \frac{1\cdot 3\cdot 5\cdot 7}{3\cdot 6\cdot 9\cdot 12} + \ldots$ then prove that $9x^2 + 24x = 11$.

22. In a shooting test the probability of A, B, C hitting the targets are $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$ respectively. It all of them fire at the same target. Find the probability that

i) Only one of them hits the target.

ii) Atleast one of them hits the target.

23. Two dice are rolled at random. Find the probability distribution of the sum of the numbers on them. Find the mean of the random variable.

24. Find the mean deviation about the mean for the following continuous distribution.

Height (in cms)	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	13	26	30	12	10

SOLUTIONS

1. If $c^2 \neq ab$ and the roots of $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal, then show that $a^3 + b^3 + c^3 = 3abc$ or $a = 0$.

Sol: Roots are equal $\Rightarrow$ Discriminant = 0

$$4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

$$a^4 + b^2c^2 - 2a^2bc - b^2c^2 + c^3a + b^3a - a^2bc = 0$$

$$a^4 - 2a^2bc + c^3a + b^3a - a^2bc = 0$$

$$a[a^3 + b^3 + c^3 - 3abc] = 0$$

$$\therefore a^3 + b^3 + c^3 - 3abc = 0 \text{ or } a = 0.$$

2. If α, β and 1 are the roots of $x^3 - 2x^2 - 5x + 6 = 0$, then find α and β

Sol: α, β and 1 are the roots of

$x^3 - 2x^2 - 5x + 6 = 0$

Sum $\alpha + \beta + 1 = 2 \Rightarrow \alpha + \beta = 1$

Product $= \alpha\beta = -6$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= 1 + 24 = 25$

$\alpha - \beta = 5$

$\alpha + \beta = 1$

Adding $2\alpha = 6 \Rightarrow \alpha = 3$

$\alpha + \beta = 1 \Rightarrow \beta = 1 - \alpha = 1 - 3 = -2$

$\therefore \alpha = 3$ and $\beta = -2$

3. Find the real and imaginary parts of the complex number $\frac{a+ib}{a-ib}$.

Sol: $\frac{a+ib}{a-ib} = \frac{(a+ib)(a+ib)}{(a-ib)(a+ib)}$

$= \frac{(a)^2 + (ib)^2 + 2a(ib)}{(a)^2 - (ib)^2}$

$= \frac{a^2 - b^2 + 2iab}{a^2 + b^2}$

$= \frac{a^2 - b^2}{a^2 + b^2} + i\frac{2ab}{a^2 + b^2}$

$\therefore$ Real part = $\frac{a^2 - b^2}{a^2 + b^2}$

Imaginary part = $\frac{2ab}{a^2+b^2}$.

4. If $(1-i)(2-i)(3-i)...(1-ni)=x-iy$ then prove that $2\cdot5\cdot10...(1+n^2)=x^2+y^2$.

Sol: $(1-i)(2-i)(3-i)...(1-ni)=x-iy$

Taking modulus both sides.

$|(1-1)||(2-i)|...|1-ni|=|x-iy|$

$\sqrt{2}\cdot\sqrt{5}......\sqrt{1+n^2}=\sqrt{x^2+y^2}$

$2\cdot5\cdot...\cdot(1+n^2)=x^2+y^2$

5.Prove that $-\omega$ and $-\omega^2$ are roots of $z^2 - z + 1 = 0$ where ω and ω^2 are the complex cube roots of unity.

Sol: $z^2-z+1=0$

$$z=\frac{1\pm\sqrt{1-4}}{2}$$

$$z=\frac{1\pm\sqrt{3}i}{2}$$

$$z=\frac{-[-1\pm\sqrt{3}i]}{2}$$

$$z=-\omega,-\omega^2$$

6. In how many ways can the letters of the word CHEESE be arranged so that no two E's come together?

Sol. The given word contains 6 letters in which one C, one H, 3 E's and one S.

Since no two E's come together, first arrange the remaining 3 letters in 3! ways. Then we can find 4 gaps between them. The 3 E's can be arranged in these 4 gaps in $\frac{{}^4P_3}{3!}$

$\therefore$ The number of required arrangements= 3! X 4= 24

7.Find the number of diagonals of a polygon with 12 sides.

Sol: Number of sides of a polygon = 12

Number of diagonals of a n-sided polygon

$= {}^{n}C_2 - n$

$\therefore$ Number of diagonals of 12 sided polygon

$= {}^{12}C_2 - 12 = 54$.

8.If $(1+3x-2x^2)^{10} = a_0 + a_1x + a_2x^2 + ... + a_{20}x^{20}$ then prove that

i) $a_0 + a_1 + a_2 + ... + a_{20} = 2^{10}$

ii) $a_0 - a_1 + a_2 - a_3 + ... + a_{20} = 4^{10}$

Sol. $(1+3x-2x^2)^{10} = a_0 + a_1x + a_2x^2 + ... + a_{20}x^{20}$

i) Put x = 1

$(1+3-2)^{10} = a_0 + a_1 + a_2 + ... + a_{20}$

$\therefore a_0 + a_1 + a_2 + ... + a_{20} = 2^{10}$

ii) Put x = –1

$(1-3-2)^{10} = a_0 - a_1 + a_2 + ... + a_{20}$

$\therefore a_0 - a_1 + a_2 - a_3 + ... + a_{20} = (-4)^{10} = 4^{10}$

9. If E_1, E_2 are two events with $E_1 \cap E_2 = \phi$, then show that $P(E_1^C \cap E_2^C) = P(E_1^C) - P(E_2)$.

Sol. $P(E_1^C \cap E_2^C) = P\left[(E_1 \cup E_2)^C\right]$

$= 1 - P(E_1 \cup E_2)$

$= 1 - P(E_1) + P(E_2)$

$[\because P(E_1 \cap E_2) = P(\phi) = 0]$

($\because$ from addtion theorem)

$= 1 - P(E_1) - P(E_2)$

$= P(E_1^C) - P(E_2)$

10. Find the variance for the discrete data 6, 7, 10, 12, 13, 4, 8, 12

Sol Mean $\bar{x} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	–3	9
7	–2	4
10	1	1
12	3	9
13	4	16
4	–5	25
8	–1	1
12	3	9
		$\Sigma \lvert x_i - \bar{x} \rvert^2 = 74$

Variance $(\sigma^2) = \frac{\sum_{i=1}^{8}(x_i - \bar{x})^2}{n} = \frac{74}{8} = 9.25$.

11. In the expression $\frac{x-p}{x^2-3x+2}$ takes all values of $x \in R$, then find the bounds for p.

Sol: $y = \frac{x-p}{x^2-3x+2}$

$y(x^2-3x+2) = x-p$

$x^2y + x(-3y-1) + 2y + p = 0$

Discriminant ≥ 0

$(-3y-1)^2 - 4y(2y+p) \geq 0$

$9y^2 + 6y + 1 - 8y^2 - 4p \geq 0$

$y^2 + y(6-4p) + 1 \geq 0$

Discriminant < 0

$(6-4p)^2 - 4 < 0$

$16p^2 - 48p + 36 - 4 < 0$

$16p^2 - 48p + 32 < 0$

$p^2 - 48p + 32 < 0$

$p^2 - 3p + 2 < 0$

$(p-2)(p-1) < 0$

$1 < p < 2$.

12.The points P, Q denote the complex numbers z_1, z_2 in the Argand diagram. O is origin. If $z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$ then show that $\angle POQ = 90°$.

Sol: $z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$

$$\frac{z_1\bar{z}_2 + \bar{z}_1 z_2}{z_2\bar{z}_2} = 0 \Rightarrow \text{Real of } \frac{z_1}{z_2} = 0$$

$$\left(\frac{z_1}{z_2} + \frac{\bar{z}_1}{z_2}\right) = 0$$

Imaginary part of $\left(\frac{z_1}{z_2}\right)$ is k.

$$\left(\frac{z_1}{z_2}\right) + \left(\frac{\bar{z}_1}{z_2}\right) = 0$$

or $\frac{z_1}{z_2}$ is purely imaginary, $\frac{z_1}{z_2} = ki$.

$$\Rightarrow \text{Arg}\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}.$$

13. A question paper is divided into 3 sections A, B, C containing 3, 4, 5 questions respectively. Find the number of ways of attempting 6 questions choosing atleast one from each section.

Sol: A question paper contains 3 sections A, B, C containing 3, 4, 5 questions respectively.

Number of ways of selecting 6 questions out of these 12 questions = ${}^{12}C_6$.

Number of ways of selecting 6 questions from section B and C (i.e. from 9 questions) = ${}^{9}C_6$.

Number of ways of selecting 6 questions from section A and C (i.e. from 8 questions) = ${}^{8}C_6$.

Number of ways of selecting 6 questions from section A and B (i.e. from 7 questions) = ${}^{7}C_6$.

∴ Number of ways of selecting 6 questions choosing atleast one from each section

$$= {}^{12}C_6 - {}^{7}C_6 - {}^{9}C_6 - {}^{8}C_6 = 805.$$

14.Find the number of ways in which 5 red balls, 4 black balls of different sizes can be arranged in a row so that (i) no two balls of the same colour come together, (ii) the balls of the same colour come together.

Sol: Given 5 red balls and 4 black balls are of different sizes.

i) No two balls of the same colour come together :

First arrange 4 black balls in row, which can be done in 4! ways

$\times$ B $\times$B$\times$B$\times$B$\times$

Then we find 5 gaps, to arrange 5 red balls. This arrangement can be done in 5! Ways.

$\therefore$ By principle of counting total number of ways of arranging = 5! × 4!.

ii) The balls of the same colour come together :

Treat all red balls as one unit and all black balls as another unit.

The number of ways of arranging these two units = 2!

The 5 red balls can be arranged in 5! Ways while 4 black balls are arranged in 4! ways.

$\therefore$ By fundamental principal of counting, the required number of ways = 2!×4!×5!.

15.The probabilities of three events A, B, C are such that P(A) = 0.3, P(B) = 0.4, P(C) = 0.8, P(A ∩ B) = 0.08, P(A ∩C) = 0.28, P(A ∩ B ∩ C) = 0.09 and P(A ∪ B ∪ C) ≥ 0.75. Show that P(B ∩ C) lies in the interval [0.23, 0.48].

Sol. $P(A \cup B \cup C) \geq 0.75$

$0.75 \leq P(A \cup B \cup C) \leq 1$

$\Rightarrow 0.75 \leq P(A) + P(B) + P(C) - P(A \cap B) -$
$P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) \leq 1$

$\Rightarrow 0.75 \leq 0.3 + 0.4 + 0.8 - 0.08 - 0.28$
$-P(B \cap C) + 0.09 \leq 1$

$\Rightarrow 0.75 \leq 1.23 - P(B \cap C) \leq 1$

$\Rightarrow -0.75 \geq P(B \cap C) - 1.23 \geq -1$

$\Rightarrow 0.48 \geq P(B \cap C) \geq 0.23$

$\Rightarrow 0.23 \leq P(B \cap C) \leq 0.48$

$\therefore$ P(B ∩ C) lies in the interval [0.23, 0.48].

16.An urn contains 12 red balls and 12 green balls. Suppose two balls are drawn one after another without replacement. Find the probability that the second ball drawn is green given that the first ball drawn is red.

Sol. Total number of balls in an urn n(S) = 24

Let E_1 be the event of drawing a red ball in first draw $P(E_1) = \frac{^{12}C_1}{24} = \frac{1}{2}$

Now the number of balls remaining are 23

Let $\frac{E_2}{E_1}$ be the events of drawing a green ball in the second drawn $P\left(\frac{E_2}{E_1}\right) = \frac{12}{23}$.

$\therefore$ Required probability

$$P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$$

$$= \frac{1}{2} \times \frac{12}{23} = \frac{6}{23}$$

$\therefore$ The probability true the second ball drawn is green given that the first ball drawn is red

$$= \frac{6}{23}$$

17.Find the coefficient of x^n in the expansion of $\frac{x-4}{x^2-5x+6}$.

Sol. Let $\frac{x-4}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}$

Multiplying with $(x-2)(x-3)$

$$x-4 = A(x-3) + B(x-2)$$

$$x = 2 \Rightarrow -2 = A(2-3) = -A \Rightarrow A = 2$$

$$x = 3 \Rightarrow -1 = B(3-2) = B \Rightarrow B = -1$$

$$\frac{x-4}{x^2-5x+6} = \frac{2}{x-2} - \frac{1}{x-3}$$

$$= \frac{2}{-2\left(1-\frac{x}{2}\right)} + \frac{1}{3\left(1-\frac{x}{3}\right)}$$

$$= -\left(1-\frac{x}{2}\right)^{-1} + \frac{1}{3}\left(1-\frac{x}{3}\right)^{-1}$$

$$= -\left(1+\frac{x}{2}+\frac{x^2}{4}+\ldots+\frac{x^n}{2^n}+\ldots\right)$$

$$+\frac{1}{3}\left(1+\frac{x}{3}+\frac{x^2}{9}+\ldots+\frac{x^n}{3^n}+\ldots\right)$$

Coefficient of $x^n = \frac{1}{3^{n+1}} - \frac{1}{2^n}$

18.Given that the sum of two roots of $x^4-2x^3+4x^2+6x-21=0$ is zero, find the roots of the equation

Sol: Let $\alpha, \beta, \gamma, \delta$ are the roots of given equation, since sum of two is zero

$\alpha+\beta=0$

Now $\alpha+\beta+\gamma+\delta=2 \Rightarrow \gamma+\delta=2$

Let $\alpha\beta=p, \gamma\delta=q$

The equation having the roots α, β is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$\therefore x^2+p=0$

The equation having the roots γ, δ is

$x^2-(\gamma+\delta)x+\gamma\delta=0$

$\therefore x^2-2x+q=0$

$\therefore x^4-2x^3+4x^2+6x-21$

$=(x^2+p)(x^2-2x+q)$

$=x^4-2x^3+x^2(p+q)-2px+pq$

Comparing the like terms

$p+q=4, -2p=6$

$-3+q=4 \quad p=-3$

$q=7$

$\therefore x^2-3=0 \Rightarrow x=\pm\sqrt{3}$ *and* $x^2-2x+7=0 \Rightarrow x=\dfrac{2\pm\sqrt{4-28}}{2}$

$=\dfrac{2\pm2\sqrt{6}i}{2}$

$=1\pm\sqrt{6}i$

$\therefore$ Roots are $-\sqrt{3}, \sqrt{3}, 1-i\sqrt{6}$ *and* $1+i\sqrt{6}$

19.Solve $(x-1)^n = x^n$, n is a positive integer.

Sol: $\left(\frac{x-1}{x}\right)^n = 1$

$$\frac{x-1}{x} = 1^{1/n}$$

$$\frac{x-1}{x} = \cos 2m\pi + i\sin 2m\pi^{1/n}$$

$$\frac{x-1}{x} = \cos\frac{2m\pi}{n} + i\sin\frac{2m\pi}{n}$$

$$1 - \frac{1}{x} = e^{i\frac{2m}{n}\pi}$$

$$1 - \cos\frac{2m\pi}{n} - i\sin\frac{2m\pi}{n} = \frac{1}{x}$$

$$2\sin^2\frac{m\pi}{n} - 2i\sin\frac{m\pi}{n}\cos\frac{m\pi}{n} = \frac{1}{x}$$

$$-2\sin^2\frac{m\pi}{n}\left[\cos\frac{m\pi}{n} - i\sin\frac{m\pi}{n}\right] = \frac{1}{x}$$

$$x = \frac{1}{-2i\sin\frac{m\pi}{n}\left[\cos\frac{m\pi}{n} - i\sin\frac{m\pi}{n}\right]}$$

$$x = \frac{\cos\frac{m\pi}{n} + i\sin\frac{m\pi}{n}}{-2i\sin\frac{m\pi}{n}}$$

$\frac{1}{2}\left[1 + i\cot\frac{m\pi}{n}\right]$; m = 1, 2, 3, …(n – 1)

20. If R, n are positive integers, n is odd, 0 < F < 1 and if $(5\sqrt{5}+11)^n = R+F$, then prove that

i) R is an even integer and

ii) $(R + F)F = 4^n$.

Sol. i) Since R, n are positive integers, 0 < F < 1 and $(5\sqrt{5}+11)^n = R+F$

Let $(5\sqrt{5}-11)^n = f$

Now, $11 < 5\sqrt{5} < 12 \Rightarrow 0 < 5\sqrt{5} - 11 < 1$

$\Rightarrow 0 < (5\sqrt{5}-11)^n < 1 \Rightarrow 0 < f < 1 \quad \Rightarrow 0 > -f > -1 \therefore -1 < -f < 0$

R + F – f = $(5\sqrt{5}+11)^n - (5\sqrt{5}-11)^n$

$$= \left[{}^nC_0(5\sqrt{5})^n + {}^nC_1(5\sqrt{5})^{n-1}(11) + {}^nC_2(5\sqrt{5})^{n-2}(11)^2 + \ldots + {}^nC_n(11)^n \right]$$

$$- \left[{}^nC_0(5\sqrt{5})^n - {}^nC_1(5\sqrt{5})^{n-1}(11) + {}^nC_2(5\sqrt{5})^{n-2}(11)^2 + \ldots + {}^nC_n(-11)^n \right]$$

$$= 2\left[{}^nC_1(5\sqrt{5})^{n-1}(11) + {}^nC_3(5\sqrt{5})^{n-3}(11)^2 + \ldots \right]$$

= 2k where k is an integer.

$\therefore$ R + F – f is an even integer.

$\Rightarrow$ F – f is an integer since R is an integer.

But $0 < F < 1$ and $-1 < -f < 0 \Rightarrow -1 < F - f < 1$

$\therefore F - f = 0 \Rightarrow F = f$

$\therefore$ R is an even integer.

ii) (R + F)F = (R + F)f, $\quad \because F = f$

$$= (5\sqrt{5}+11)^n(5\sqrt{5}-11)^n$$

$$= \left[(5\sqrt{5}+11)(5\sqrt{5}-11)\right]^n = (125-121)^n = 4^n$$

$\therefore (R + F)F = 4^n$.

21. If $x = \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \frac{1\cdot 3\cdot 5\cdot 7}{3\cdot 6\cdot 9\cdot 12} + \ldots$ then prove that $9x^2 + 24x = 11$.

Sol. Given $x = \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \frac{1\cdot 3\cdot 5\cdot 7}{3\cdot 6\cdot 9\cdot 12} + \ldots$

$$= \frac{1\cdot 3}{1\cdot 2}\left(\frac{1}{3}\right)^2 + \frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\left(\frac{1}{3}\right)^2 + \ldots$$

$$= 1 + \frac{1}{1}\cdot\frac{1}{3} + \frac{1\cdot 3}{1\cdot 2}\left(\frac{1}{3}\right)^2 + \frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\left(\frac{1}{3}\right)^2 + \ldots - \left[1 + \frac{1}{3}\right]$$

Here p = 1, q = 2, $\frac{x}{q} = \frac{1}{3} \Rightarrow x = \frac{2}{3}$

$$= (1-x)^{-p/q} - \frac{4}{3} = \left(1 - \frac{2}{3}\right)^{-1/2} - \frac{4}{3}$$

$$= \left(\frac{1}{3}\right)^{-1/2} - \frac{4}{3} = \sqrt{3} - \frac{4}{3}$$

$$\Rightarrow 3x + 4 = 3\sqrt{3}$$

Squaring on both sides

$$(3x+4)^2 = (3\sqrt{3})^2 \Rightarrow 9x^2 + 24x + 16 = 27$$

$$\Rightarrow 9x^2 + 24x = 11$$

22. In a shooting test the probability of A, B, C hitting the targets are $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$ respectively. It all of them fire at the same target. Find the probability that

i) Only one of them hits the target.

ii) Atleast one of them hits the target.

Sol. The probabilities that A, B, C hitting the targets are denoted by

$P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$ and $P(C) = \frac{3}{4}$

$\therefore P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(\bar{B}) = 1 - P(B) = 1 - \frac{2}{3} = \frac{1}{3}$

$P(\bar{C}) = 1 - P(C) = 1 - \frac{3}{4} = \frac{1}{4}$

i) Probability that only one of them hits the target

$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A}\bar{B}C)$

$= P(A) \cdot P(\bar{B}) \cdot P(\bar{C}) + P(\bar{A}) \cdot P(B) \cdot P(\bar{C})$

$+ P(\bar{A}) \cdot P(\bar{B}) \cdot P(C)$

($\because$ A, B, C are independent events)

$= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4}$

$= \frac{1+2+3}{24} = \frac{6}{24} = \frac{1}{4}$

ii) Probability that atleast one of them hits the target = $P(A \cup B \cup C)$

= 1 – Probability that none of them hits the target.

$= 1 - P(\bar{A}\ \bar{B}\ \bar{C})$

$= 1 - P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$

$= 1 - \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} = 1 - \frac{1}{24} = \frac{23}{24}$

23. Two dice are rolled at random. Find the probability distribution of the sum of the numbers on them. Find the mean of the random variable.

Sol. When two dice are rolled, the sample space S contains 6 × 6 = 36 sample points.

S = {(1, 1), (1, 2)…(1, 6), (2, 1), (2, 2)…(6,6)}

Let x denote the sum of the numbers on the two dice

Then the range x = {2, 3, 4, …12}

Probability Distribution of x is given by the following table.

$X=x_i$	2	3	4	5	6	7	8	9	10	11	12
$P(X=x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
$X_i.p(x_i)$	2/36	6/36	12/36	20/36	30/36	42/36	40/36	36/36	30/36	22/36	12/36

Mean of x = $\mu = \sum_{1-2}^{12} x_1 p \;\; X = x_1$

$$= 2.\frac{1}{36} + 3.\frac{2}{36} + 4.\frac{3}{36} + 5.\frac{4}{36} + 6.\frac{5}{36} + 7.\frac{6}{36}$$

$$+8.\frac{5}{36} + 9.\frac{4}{36} + 10.\frac{3}{36} + 11.\frac{2}{36} + 12.\frac{1}{36}$$

$$= \frac{1}{36}(2+6+12+20+30+42+40$$

$$+36+30+22+12)$$

$$= \frac{252}{36} = 7$$

24. Find the mean deviation about the mean for the following continuous distribution.

Height (in cms)	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	13	26	30	12	10

Sol.

Height (C.I)	No.of boys (f_i)	Mid point x_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
95-105	9	100	–3	–27	25.3	227.7
105-115	13	110	–2	–26	15.3	198.9
115-125	26	120	–1	–26	5.3	137.8
125-135	30	$130 \rightarrow (A)$	0	0	4.7	141.0
135-145	12	140	1	12	14.7	176.4
145-155	10	150	2	20	24.7	247.0
	N=100			$\Sigma f_i d_i = -47$		1128.8

Mean $(\bar{x}) = A + \frac{\Sigma f_i d_i}{N} \cdot h = 130 + \left(\frac{-47}{100}\right) \cdot 10 = 130 - 4.7 = 125.3$

$\therefore$ Mean Deviation about Mean = $\frac{\sum_{i=1}^{6} f_i |x_i - \bar{x}|}{N} = \frac{1128.8}{100} = 11.29$.