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TS

B

Total No. of Questions : 24
Total No. of Printed Pages : 4

Regd. No.

Part-III

MATHEMATICS, Paper - I (B)

(English version)

Time : 3 Hours] ***[Max. Marks : 75***

Note : *This question paper consists of* ***three*** *sections* ***A, B*** *and* ***C.***

SECTION - A

10×2=20

I . *Very short answer type* questions.

(i) Attempt **all** the questions.

(ii) Each question carries **TWO** marks.

1. Find the equation of the straight line passing through $(-4, 5)$ and cutting off equal and non zero intercepts on the co-ordinate axes.

2. Find the equation of the straight line perpendicular to the line $5x - 3y + 1 = 0$ and passing through the point $(4, -3)$.

3. Find the co-ordinates of the vertex 'C' of ΔABC, if its centroid is the origin and the vertices A, B are $(1, 1, 1)$ and $(-2, 4, 1)$ respectively.

4. Find the angle between the planes $x + 2y + 2z - 5 = 0$ and $3x + 3y + 2z - 8 = 0$.

5. Compute

$$\lim_{x \to a} \frac{\tan(x-a)}{x^2 - a^2} \quad (a \neq 0).$$

6. Compute

$$\lim_{x \to 0} \left(\frac{e^x - 1}{\sqrt{1+x} - 1} \right)$$

7. Find the derivative of $y = \sqrt{2x-3} + \sqrt{7-3x}$.

8. Find the derivative of $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

9. Find Δy and dy for the function $y = x^2 + 3x + 6$ at $x = 10$ and $\Delta x = 0.01$.

10. Verify Rolle's theorem for the function $y = f(x) = x^2 + 4$ in $[-3, 3]$.

SECTION - B

5×4=20

II. *Short answer type questions.*

(i) Attempt any **FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. A(5, 3) and B(3, −2) are two fixed points. Find the equation of the locus of P, so that the area of triangle PAB is 9.

12. When the axes are rotated through an angle 45°, the transformed equation of a curve is $17x^2 - 16xy + 17y^2 = 225$. Find the original equation of the curve.

13. A straight line with slope 1 passes through Q(−3, 5) and meets the straight line $x + y - 6 = 0$ at P. Find the distance PQ.

14. If f, given by $f(x) = \begin{cases} k^2x - k & \text{if } x \geq 1 \\ 2 & \text{if } x < 1, \end{cases}$

is a continuous function on R, then find the values of k.

15. Find the derivative of x^3 from the first principle.

16. A particle is moving along a line according to $S = f(t) = 4t^3 - 3t^2 + 5t - 1$, where S is measured in meters and t is measured in seconds. Find the velocity and acceleration at time t. At what time the acceleration is zero ?

17. Determine the intervals in which $f(x) = \dfrac{2}{(x-1)} + 18x$

$\forall x \in R \setminus \{0\}$ is strictly increasing and decreasing.

SECTION - C 5×7=35

III. *Long answer type* questions.

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **SEVEN** marks.

18. If the equations of the sides of a triangle are $7x + y - 10 = 0$, $x - 2y + 5 = 0$ and $x + y + 2 = 0$, find the orthocentre of the triangle.

19. Show that the lines represented by $(lx + my)^2 - 3(mx - ly)^2 = 0$ and $lx + my + n = 0$ form an equilateral triangle with area $\dfrac{n^2}{\sqrt{3}\left(l^2 + m^2\right)}$.

20. Find the values of 'k', if the lines joining the origin to the points of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the line $x + 2y = k$ are mutually perpendicular.

21. Find the angle between the lines, whose direction cosines are given by the equations $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$.

22. If $y = (\sin x)^{\log x} + x^{\sin x}$, then find $\frac{dy}{dx}$.

23. If the tangent at any point on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ intersects the co-ordinate axes in A and B, then show that the length AB is a constant.

24. A window is in the shape of a rectangle surmounted by a semi-circle. If the perimeter of the window is 20 ft, find the maximum area.
