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TS

B

Total No. of Questions : 24
Total No. of Printed Pages : 4

Regd. No.

Part-III

MATHEMATICS, Paper - I (A)

(English version)

Time : 3 Hours] ***[Max. Marks : 75***

Note : *This question paper consists of* ***three*** *sections* ***A, B*** *and* ***C.***

SECTION - A

10×2=20

I . *Very short answer type* questions.

(i) Answer **all** the questions.

(ii) Each question carries **TWO** marks.

1. $f: R \to R$ defined by $f(x) = \frac{2x+1}{3}$, then this function is injection or not ? Justify.

2. Find the range of the real valued function $f(x) = \sqrt{9-x^2}$.

3. Construct a 3×2 matrix, whose elements are defined by $a_{ij} = \frac{1}{2}|i-3j|$.

4. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$.

5. $a = 2i + 5j + k$ and $b = 4i + mj + nk$ are collinear vectors, then find m and n.

6. OABC is a parallelogram. If OA = a and OC = c, find the vector equation of the side BC.

7. Find the angle between the planes

$r \cdot (2i - j + 2k) = 3$ and $r \cdot (3i + 6j + k) = 4$.

8. Find the period of $\tan(x + 4x + 9x + \ldots\ldots + n^2x)$, where n is any positive integer.

9. If $\sin\alpha = \frac{3}{5}$, where $\frac{\pi}{2} < \alpha < \pi$, evaluate $\cos 3\alpha$.

10. Prove that $(\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$, for any $n \in R$.

SECTION - B

5×4=20

II. *Short answer type* questions.

(i) Answer **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. Examine whether the following system of equations are consistent or inconsistent and if consistent, find the complete solution.

$x + y + z = 1$, $2x + y + z = 2$, $x + 2y + 2z = 1$.

12. a, b, c are non-coplanar vectors. Prove that the following four points are coplanar.

$6a + 2b - c$, $2a - b + 3c$, $-a + 2b - 4c$, $-12a - b - 3c$

13. Find the volume of the tetrahedron,

whose vertices are (1, 2, 1), (3, 2, 5), (2, –1, 0) and (–1, 0, 1).

14. If $0 < A < B < \frac{\pi}{4}$ and $\sin(A+B) = \frac{24}{25}$ and $\cos(A-B) = \frac{4}{5}$, then find the value of $\tan 2A$.

15. If θ_1, θ_2 are solutions of the equation $a\cos 2\theta + b\sin 2\theta = c$, $\tan\theta_1 \neq \tan\theta_2$ and $a + c \neq 0$, then find the values of

(i) $\tan\theta_1 + \tan\theta_2$ *(ii)* $\tan\theta_1 \cdot \tan\theta_2$

16. Prove that $\sin^{-1}\left(\frac{4}{5}\right) + 2\tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{2}$.

17. Prove that $\cot A + \cot B + \cot C = \frac{a^2+b^2+c^2}{4\Delta}$

SECTION - C

5×7=35

III. *Long answer type* questions.

(i) Answer **ANY FIVE** questions.

(ii) Each question carries **SEVEN** marks.

18. Let $f = \{(1, a), (2, c), (4, d), (3, b)\}$ and $g^{-1} = \{(2, a), (4, b), (1, c), (3, d)\}$, then show that $(gof)^{-1} = f^{-1}og^{-1}$.

19. Using Mathematical Induction, prove the statement for all $n \in N$,

$$1\cdot2\cdot3 + 2\cdot3\cdot4 + \ldots\ldots + (\text{upto } n \text{ terms}) = \frac{n(n+1)(n+2)(n+3)}{4}.$$

20. Find the value of x, if

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$$

21. Solve the following system of equations by using Cramer's rule.

$x - y + 3z = 5$, $4x + 2y - z = 0$, $-x + 3y + z = 5$.

22. If $\bar{a} = 2\bar{i} + \bar{j} - 3\bar{k}$, $\bar{b} = \bar{i} - 2\bar{j} + \bar{k}$, $\bar{c} = -\bar{i} + \bar{j} - 4\bar{k}$, and $\bar{d} = \bar{i} + \bar{j} + \bar{k}$, then compute $\left|(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})\right|$.

23. If $A + B + C = \pi$, then prove that

$$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2\left(1 + \sin\frac{A}{2} \cdot \sin\frac{B}{2} \cdot \sin\frac{C}{2}\right).$$

24. If $r_1 = 2$, $r_2 = 3$, $r_3 = 6$ and $r = 1$, prove that $a = 3$, $b = 4$ and $c = 5$.
