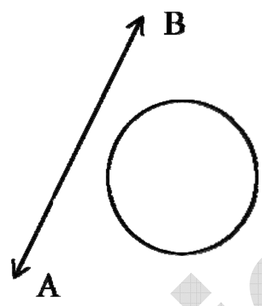
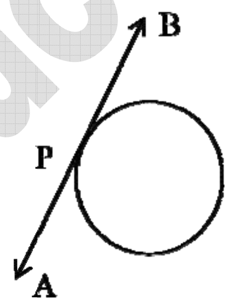
Tangents and Secants to the Circle

A Line and a circle: let us consider a circle and line say AB. There can be three possibilities given as follows.

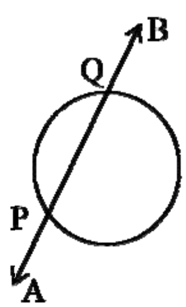
a) Non-intersecting line: The line AB and the circle have no common point. In this case, AB is called a "non-intersecting line" with respect to the circle.



b) **Tangent:** There is only one point P which is common to the line AB and the circle. In this case, AB is called a "tangent" to the circle and the point P is called the point of contact.



c) **Secant:** There are two points P and Q which are common to the line AB and the circle. In this case, AB is said to be a "secant" of the circle.



- The word tangent comes from the latin word “tangere”, which means to touch and was introduced by Danish mathematician “**Thomas finke**” in 1583. The term secant is derived from latin word secare which means to cut.
- **Tangent of a circle:**

i) A tangent to a circle is a line that intersects the circle at only one point.

ii) There is only one tangent at a point of the circle.

iii) We can draw any number of tangents to a circle.

iv) We can draw only two tangents to a circle from a point away from the circle.

v) A tangent is said to touch the circle at the common point.

- **Theorem(1):** The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- **Theorem(2):** If a line in the plane is perpendicular to the radius at its end point on the circle, then the line is tangent to the circle.
- Number of tangents from a circle:
  a) There is no tangent to a circle through a point which lies inside the circle.
  b) There is only one tangent to a circle passing through the point lying on the circle.
  c) There are exactly two tangents to a circle through a point lying outside the circle.
- **Theorem(3):** The lengths of tangents drawn from an external point to a circle are equal.
- **Length of tangent:** let P be an external point of a circle centered at ‘O’ with radius ‘r’ and OP=d units. If the point of contact is A, then length of the tangent PA= $\sqrt{(d^2 - r^2)}$
- **Segment of a circle:**

  In a circle, segment is a region bounded by an arc and a chord.
- Area of segment of a circle= area of the corresponding sector-area of the corresponding triangle.

  $= \frac{x^\circ}{360^\circ} \times \pi r^2$ –area of OAB
- A tangent to a circle intersects it in only one point.
- A line intersecting a circle in two points is called a secant.
- A circle can have two parallel tangents at the most.

- ➔ The common point of a tangent to a circle is called point of contact.
- ➔ We can draw infinite tangents to a given circle.
- ➔ The lengths of tangents drawn from an external point to a circle are equal.
- ➔ In two concentric circles, such that a chord of the bigger circle, that touches the smaller circle is bisected at the point of contact with the smaller circle.
- ➔ IF a circle touches all the four sides of a quadrilateral ABCD at points PQRS, hence AB + CD = BC+DA
- ➔ When the degree measure of the angle at the centre is $x°$, the area of sector is $\frac{x°}{360°} \times \pi r^2$
- ➔ Area of segment of a circle = Area of corresponding sector – area of the corresponding triangles

One Mark Questions

1) **Define tangent of a circle.**

   A line which intersects a circle in only one point is called a tangent to the circle.

2) **How many tangents can be drawn from a point outside a circle.**

   We can draw only two tangents from a point outside a circle.

3) **How can you mark the centre of a circle if the circle is given without centre?**

   We draw any two non parallel chords and again draw the perpendicular bisectors of the chords. The intersecting point of the two perpendicular bisectors is the centre of the circle.

4) **Calculate the length of a tangent from a point 13cm away from the centre of a circle of radius 5cm.**

   Here r = 5cm and d = 13cm

   Length of tangent= $\sqrt{d^2 - r^2}$

   $= \sqrt{13^2 - 5^2}$

   $= \sqrt{144}$

   $= 12$

5) **Write a formula to find area of circle.**

   $A = \pi r^2$

6) **Write the formula to find area of regular hexagon.**

   $A = 6 \times \frac{\sqrt{3}}{4} \times a^2$

7) **Define normal to the circle at a point.**

   The line containing the radius through the point of contact is also called the normal to the circle at the point.

8) **Write a formula to find area of rectangle.**

   A = l × b sq. units

9) **Which is the Longest chord of a circle**

   Diametre

10) **What is meant by secant of a circle?**

If a line touches the circle at two points then it is called secant of the circle

11) What is meant by point of contact?

The tangent where it touches the circle, that point is called point of contact.

12) How many diameters will be there in a circle?

Infinite

13) How many tangents can be drawn through a point inside a circle?

Zero

Short type Questions

1) Two concentric circles having radii 5 cm and 3 cm are drawn. Find the length of the chord of the larger circle which touches the smaller circle.

As shown in the figure

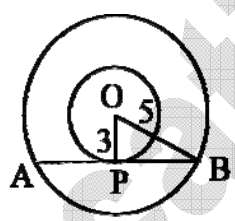
OP = 3 cm,   OB = 5 cm

Since AB touches the smaller circle at P,

We have OP   is perpendicular to AB

$\underline{|OPB} = 90^\circ$

By Pythagoras theorem,



$$OB^2 = OP^2 + PB^2$$

$$PB^2 = OB^2 - OP^2$$

$$= 5^2 - 3^2 = 25 - 9 = 16$$

PB= 4 cm

Since the perpendicular through the center bisects a chord

We have AP=PB = 4cm

AB=AP+PB=4cm+4cm = 8cm

Thus, the length of the required chord = 8cm.

2) A chord of a circle of radius 10cm. subtends a right angle at the centre. Find the area of the   corresponding.

1. Minor segment 2. Major segment

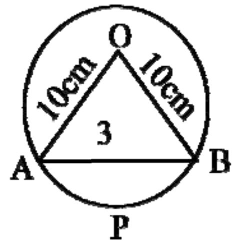
sol: Radius of the circle = 10cm

$\angle AOB = 90°$

Area of the $\Delta OAB = \frac{1}{2} \times OA \times OB$

$=1/2\times10\times10$

$=50cm^2$



Area of sector OAPB = $\frac{x^\circ}{360^\circ}\times\pi r^2$

$= 1/4\times314cm^2$

$= 78.5cm$

i) Area of minor segment=area of the sector OAPB-area of ΔOAB

$= 78.5cm^2-50cm^2$

$= 28.5cm^2$

ii) Area of the circle = πr^2

$= 3.14\times10cm \times 10cm$

$= 314\ cm^2$

Area of the major segment=area of the circle-area of the minor segment

$=314-28.5$

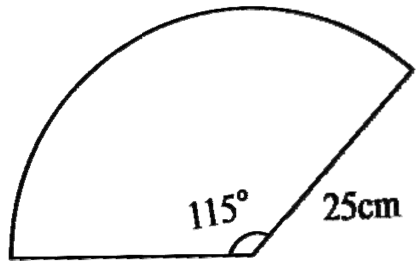
$=285.5cm^2$

3) A car has two wipers which do not overlap .Each wiper has a blade of length 25 cm. Sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Sol: We note that the area cleaned by each sweep of the blade of each wiper is equal.

Area swept by each wiper = Area of the sector.

$= \frac{x^\circ}{360^\circ}\times\pi r^2$



= 115°/ 360°× 22/7 × 25 × 25 = 158125/252 cm^2

Total area cleaned by both wipers = 2× 158125/252 = 1254.96 cm^2

4) Find the area of the shaded region in figure, if ABCD is a square of side 10cm and semi circles are drawn with each side of the square as diameter (use $\pi = 3.14$)

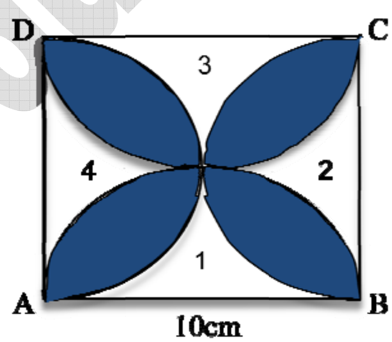
Sol: as in figure, let us name the unshaded regions as 1, 2, 3 and 4 from figure, we observe that area of 1+area of 3

=area of ABCD-areas of two semi circles each of radius 5cm

$=10cm\times10cm-2\times1/2\times\pi r^2$

$=100cm^2-3.14\times5cm\times5cm$

$=100cm^2-78.5cm^2$



$=21.5cm^2$

Similarly, area of 2 + area of 4=2.5cm^2

Area of the shaded part=area of ABCD-area of (1+2+3+4)

$= (100-43)cm^2$

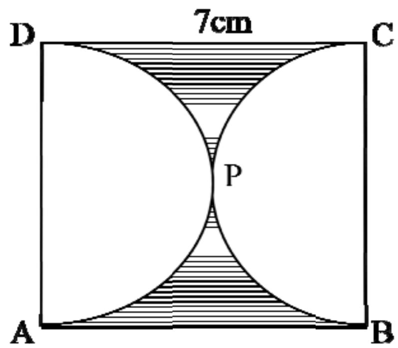
$=57cm^2$

5. Find the area of the shaded region in figure, if ABCD is a square of side 7cm and APD and BPC are semicircles.

Sol: side of the square=7cm

Area of the square=7×7=49cm^2

Radius of each semicircle=7/2cm



Area of APD semicircle=1/2×πr^2

=77/4cm^2

Similarly area of BPC semicircle=77/4cm^2

Sum of areas of two semicircles=38.5cm^2

Area of the shaded part=area of ABCD-sum of areas of semicircles

=49^2-38.5^2

=10.5cm^2

6) Find the area of sector whose radius is 7cm with the given angle:

60°,30°,72°,90°,120°

Sol: If x°=60° then area of sector = $\frac{x°}{360°}\times\pi r^2$

= 77/3cm^2

If x°=30° then area of sector=77/6cm^2

If x°=90° then area of sector=77/2cm^2

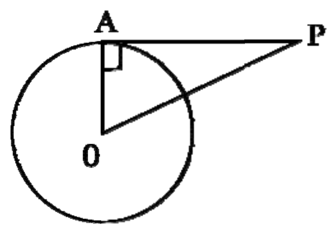
If x°=72° then area of sector=154/5cm^2

7. Find the length of tangent to a circle with centre 'O' and with its radius 6cm from a point P such that OP = 10cm and the point of contact is A.

Sol: Tangent is perpendicular to the radius at a point of contact

$\angle OAP = 90°$

By Pythagoras theorem, $OP^2 = OA^2 + PA^2$



$PA^2 = OP^2 - OA^2$

$= 100-36$

$= 64$

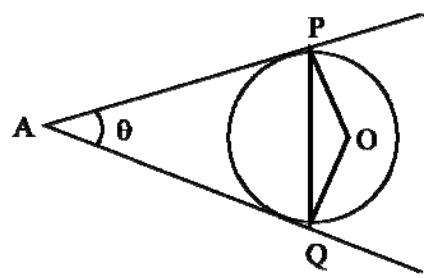
$PA = \sqrt{64}$

$= 8cm$

8. If two tangents AP and AQ are drawn to a circle with centre O from an external point A then prove $\angle PAQ = 2\angle OPQ$.

Sol: $\angle PAQ = \theta$ say

Clearly AP = AQ



$\angle APQ = \angle AQP$

But $\angle PAQ + \angle APQ + \angle AQP = 180°$

$\angle APQ = \angle AQP = 1/2(180° - \theta) = 90° - 1/2\theta$

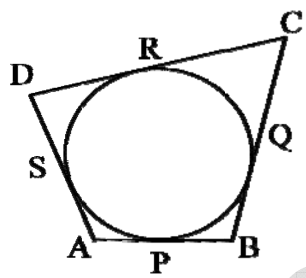
Also $\angle OPQ = \angle OPA - \angle APQ = 1/2\theta = ½\angle PAQ$

$\angle OPQ = 1/2\ \angle PAQ$

$\angle PAQ = 2\ \angle OPQ$

9. If a circle touches all the four sides of a quadrilateral ABCD at points PQRS, then AB + CD = BC + DA.

Sol: the two tangents drawn from a point outside to a circle are equal



AP=AS, BP=BQ, DR=DS, CR=CQ

By adding

AP+BP+DR+CR=AS+BQ+DS+CQ

AB+CD=BC+DA

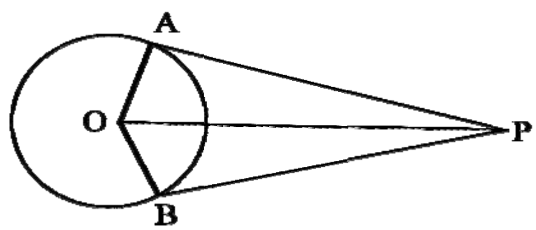
Hence proved.

10. If the radius of a circle is 5cm and the angle inclined between the tangents from a point outside is 60°, then what is the distance between the centre and the point outside.

Sol: From the figure, it is given

OA=5cm, $\angle APB=60°$

Clearly OP is an angular bisector of $\angle P$



$\angle APO=30°$ and $\angle OAP=90°$

In ΔOAP we have

$\sin 30° = OA/OP$

$\frac{1}{2} = 5/OP$

OP=10cm

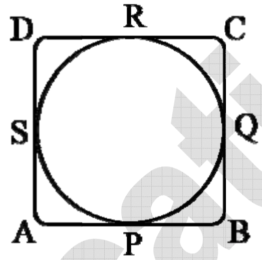
Essay type Questions

1) Prove that the parallelogram circumscribing a circle is a rhombus.

Sol. Given: ABCD is a parallelogram

AB = CD and AD = BC ....(1)

The lengths of tangents to a circle from an external point are equal.



Thus,

AS = AP; BQ = BP; CQ = CR; DS = RD

By adding, we get

AS + BQ + CQ + DS = AP+ BP+ CR + RD

AS + DS + BQ + CQ = AP+ BP+ CR+ RD

AD+ BC = AB + CD

From (1) we have

AD + AD = AB + AB

2A D = 2A B

AD = AB ....(2)

From (1) and (2) we have

AB = BC= CD= AD

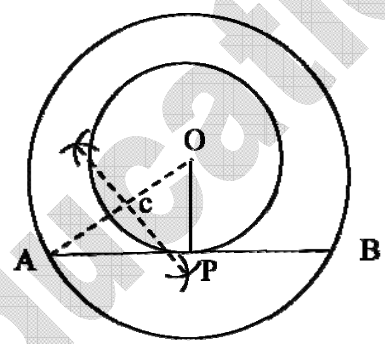
ABCD is a rhombus.

Hence proved.

2) Construct a tangent to a circle of radius 4 cm from a point on the concentric circles of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

Steps of construction;

1) Draw two concentric circles centred at 0 with radii 4 cm and 6 cm
2) Make any point A on the circle with radius 6 cm
3) Join OA and draw a perpendicular bisector to meet OA at C



4) Centred at c with radius AC draw an arc to cut the circle with radius 4 cm at P. AP is a tangent to the circle with radius 4 cm.

We have AB as tangent to the circle with radius 4 cm and P be the point of contact.

Clearly OP is perpendicular to AB; $\angle OPA = 90°$

By Pythagoras theorem

$$OA^2 = OP^2 + AP^2$$

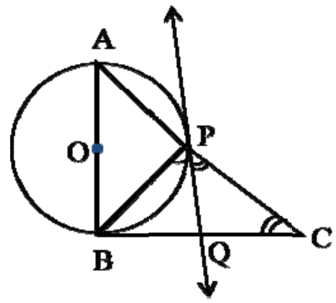
$$AP^2 = OA^2 - OP^2$$

$$= 6^2 - 4^2 = 36-16 = 20$$

$$AP = \sqrt{20} = 2\sqrt{5} = 2 \times 2.236 = 4.472\text{cm (app)}$$

We can verify the measurements are both equal.

3) In a right angle triangle ABC , a circle with a side AB as diameter is drawn to intersect the hypotenuse AC in P. Prove that the tangent to the circle at P bisects the side BC.



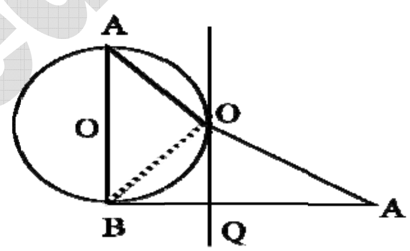
Sol. Given: in

ΔABC, we have ∠B = 90° and AB is the diameter of the circle with centre O. the circle intersect AC in the circle intersect AC in P and the tangent PT at P meets BC in T.

To prove: TB = TC

Construction: joint BP

Proof: ∠APB=90°



Also ∠APB + ∠BPC = 180°

90° + ∠BPC = 180°

∠BPC = 90°

Now ∠ABC = 90°

∠BAC + ∠ACB = 90°

$\angle BPC = \angle BAC + \angle ACB$

$\angle BPC + \angle CPT = \angle BAC + \angle ACB$

But, $\angle BPT = \angle BAC$

$\angle CPT = \angle ACB$

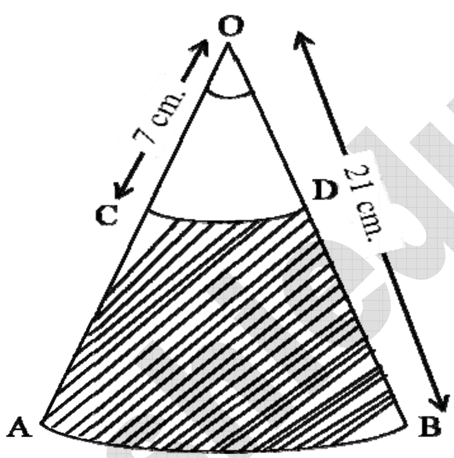
PT = TC

But PT = TB

TB = TC

Hence the theorem

4) AB and CD are respectively arcs of two concentric circles with radii 21cm and 7cm with centre O. If $\angle AOB=30°$, find the area of the shaded region.



Sol: area of the sector OAB = $\frac{x°}{360°} \times \pi r^2$

= 30°/360°×22/7×21×21

= 231/2 cm^2

Area of sector OCD = $\frac{x°}{360°} \times \pi r^2$

=77/6cm^2

Area of shaded portion = area of sector OAB – area of sector OCD

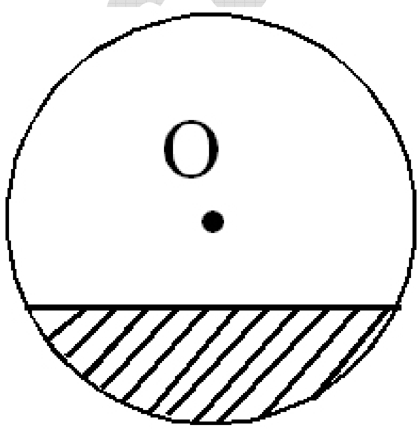
= 231/2cm^2 -77/6cm^2

= 308cm²

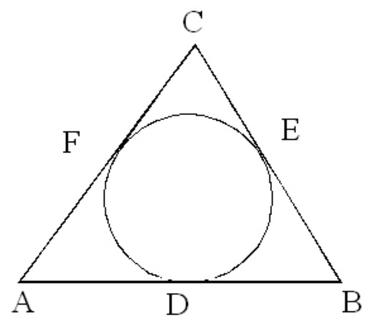
= 102.67cm²

Bit -Bank

1. The length of the tangents from a point A to a circle of radius 3 cm is 4 cm, then the distance between A and the centre of the circle is ____
2. ____ tangents lines can be drawn to a circle from a point outside the circle.
3. Angle between the tangent and radius drawn through the point of contact is ____
4. A circle may have ____parallel tangents.
5. The common point to a tangent and a circle is called ____
6. A line which intersects the given circle at two distinct points is called a ____ line.
7. Sum of the central angles in a circle is___
8. The shaded portion represents ____



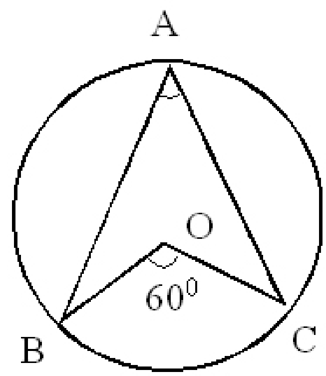
9. If a circle touches all the four sides of an quadrilateral ABCD at points P, Q, R, S then AB + CD = ____
10. If AP and AQ are the two tangents a circle with centre O so that $\angle POQ = 110°$ then $\angle PAQ$ is equal to ____
11. If two concentric circles of radii 5 cm and 3 cm are drawn, then the length of the chord of the larger circle which touches the smaller circle is ____
12. If the semi perimeter of given $\Delta ABC = 28$ cm then AF+BD+CE is ____



13. The area of a square inscribed in a circle of radius 8cm is ____ cm^2.

14. Number of circles passing through 3 collinear points in a plane is ____

15. In the figure ∠BAC ____



16. If the sector of the circle made an at the centre is $x°$ and radius of the circle is r, then the area of sector is ____

17. If the length of the minute hand of a clock is 14cm, then the area swept by the minute hand in 10 minutes ____

18. If the angle between two radii of a circle is 130°, the angle between the tangents at the ends of the radii is ____

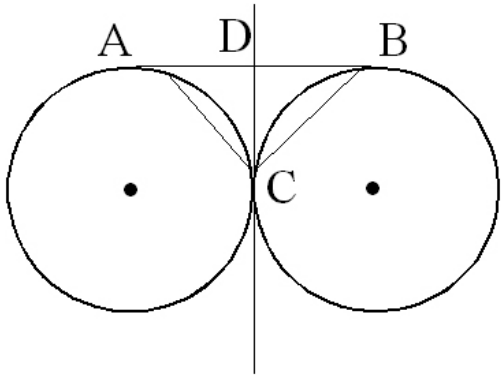
19. If PT is tangent drawn from a point P to a circle touching it at T and O is the centre of the circle, then ∠OPT+∠POT is ____

20. Two parallel lines touch the circle at points A and B. If area of the circle is $25\pi cm^2$, then AB is equal to ____

21. A circle have____ tangents.

22. A quadrilateral PQRS is drawn to circumscribe a circle. If PQ, QR, RS (in cm) are 5, 9, 8 respectively, then PS (in cms) equal to ____

23. From the figure ∠ACB = ____



Answers:

1) 5cm; 2) 2; 3) 90°; 4) 2; 5) Point of contact; 6) Secant line; 7) 360°; 8) Minor segment; 9) BC + AD; 10) 70°; 11) 8 cm; 12) 28cm; 13) 128;14) 1; 15) 30°; 16) $\frac{x^\circ}{360} \times \pi r^2$; 17) $102\frac{2}{3}$; 18) 50°; 19) 90°; 20) 10cm; 21) Infinitely many; 22) 4cm; 23) 90°; 24) 65°; 25) $2\sqrt{a^2 - b^2}$; 26) 6cm; 27) 2cm; 28) equal.