## Applications of Trigonometry

The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.

- The Line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.
- The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level i.e. the case when we raise our head to look at the object.
- The angle of depression of an object viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level i.e. the case when lower our head to look at the object.

- Trigonometry has been used by surveyors for centuries. They use Theodolites to measure angles of elevation or depression in the process of survey.
- When we want to solve the problems of heights and distances, we should consider the following.
i) All the objects such as tower, trees, buildings, ships, mountains etc. Shall be considered as linear for mathematical convenience
ii) The angle of elevation or angle of depression is considered with reference to the horizontal line.
iii) The height of the observer is neglected, if it is not given in the problem.
- The angle of elevation of a tower from a distance ' $d$ ' $m$ from its foot is $\propto^{\circ}$ and hight of the tower is ' $h$ ' $m$ then

$$
\tan \alpha=\frac{h}{d}
$$



- The angle of elevation of the top of a tower as observed from a point on the ground is ' $\propto$ ' and on moving ' $d$ ' meters towards the tower, the angle of elevation is ' $\beta$ ', then the height of the tower $h=\frac{d}{\cot \alpha-\cot \beta}$

- Two men on either side of the tower and in the same straight line with its base notice the angle of elevation of top of the tower to be $\propto$ and $\beta$. If the height of the tower is ' h ' m, then the distance between the two men $d=\frac{h \sin (\alpha+\beta)}{\sin \alpha \cdot \sin \beta}$

- A statue ' $d$ ' $m$ tall stands on the top of a pedestal which is the height of ' $h$ ' $m$. From a point on the ground, the angle of elevation of the top of the statue is $\propto$ and from the same point the angle of elevation of the top of the pedestal is $\beta$, then the height of the statue is $d=\frac{h(\cot \alpha-\cot \beta)}{\cot \beta}$

- Two poles of equal height are standing opposite each other on either side of the road, which is x m wide. From a point between them on the road, the of the poles are $\propto$ and $\beta$ respectively, then the height of the pole $h=\frac{x \tan \alpha \cdot \tan \beta}{\tan \alpha+\tan \beta}$


And the length of $\mathrm{BE}=\frac{x \tan \beta}{\tan \alpha+\tan \beta}$

The length of $\mathrm{DE}=\frac{x \tan \alpha}{\tan \alpha+\tan \beta}$

## Exercise 12.1

1. A tower stands vertically on the ground. From a point which is $\mathbf{1 5}$ meter away from the foot of the tower, the angle of elevation of the top of the tower is $45^{\circ}$. What is the height of the tower?

Sol: $\quad$ Let the light of the tower $=\mathrm{AB}$


Distance between foot of the tower and observation point ' C ' is $\mathrm{BC}=15 \mathrm{mts}$.

Angle of elevation of the top of tower $\angle \mathrm{C}=45^{\circ}$
Form $\triangle \mathrm{ABC}, \tan \mathrm{C}=\frac{A B}{B C}$

$$
\begin{aligned}
& \tan 45^{\circ}=\frac{A B}{B C} \\
& \Rightarrow 1=\frac{A B}{15} \\
& \Rightarrow \mathrm{AB}=15 \mathrm{mts}
\end{aligned}
$$

$\therefore$ Height of the tower $A B=15 \mathrm{~m}$.
2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground by making $30^{\circ}$ angle with the ground. The distance between the foot of the tree and the top of the tree on the ground is $\mathbf{6 m}$. Find the height of the tree before falling down.

Sol: In right triangle ABC ,

(Foot of the tree)
$\operatorname{Cos} 30^{\circ}=\frac{B C}{A C}$

$$
\begin{aligned}
& \Rightarrow \frac{\sqrt{3}}{2}=\frac{6}{A C} \\
& \Rightarrow A C=\frac{12}{\sqrt{3}} \mathrm{~m}
\end{aligned}
$$

$$
\begin{aligned}
& \text { lly } \tan 30^{\circ}=\frac{A B}{B C} \\
& \Rightarrow \frac{1}{\sqrt{3}}=\frac{A B}{6} \Rightarrow A B=\frac{6}{\sqrt{3}} \mathrm{~m} .
\end{aligned}
$$

$\therefore$ Height of the tree $=\mathrm{AB}+\mathrm{AC}$

$$
\begin{aligned}
& =\left(\frac{12}{\sqrt{3}}+\frac{6}{\sqrt{3}}\right) m \\
& =\frac{18}{\sqrt{3}} m=6 \sqrt{3}
\end{aligned}
$$

$\therefore$ The height of the tree before falling down is $=6 \sqrt{ } 3 \mathrm{~m}$.
3.A contractor wants to set up a slide for the children to play in the park, He wants to set it up at the height of 2 m and by making an angle of $30^{\circ}$ with the ground. What should be the length of the slide

Sol: height of the slide $=2 \mathrm{~m}$


Length of the slide $=$ ?

In right triangle $A B C$

$$
\sin 30^{\circ}=\frac{A B}{A C}
$$

$\Rightarrow \frac{1}{2}=\frac{2}{A C}$
$\therefore$ The length of the slide $A C=4 \mathrm{~m}$.
4.Length of the shadow of a 15 meter high pole is $5 \sqrt{ } 3$ meters at 7 ' 0 ' clock in the morning. Then, what is the angle of elevation of the sun rays with the ground at the time?

Sol: Height of the pole $\mathrm{AB}=15 \mathrm{~m}$


Length of the shadow of the pole $B C=5 \sqrt{ } 3 \mathrm{~m}$

Let the angle of elevation of sunrays with ground is $\angle \mathrm{ACB}=\theta$ say.

From right triangle ABC ,

$$
\begin{aligned}
& \tan \theta=\frac{A B}{B C}=\frac{15}{5 \sqrt{3}}=\frac{3}{\sqrt{3}}=\sqrt{3} \\
& \Rightarrow \tan \theta=\sqrt{3} \\
& \Rightarrow \tan \theta=\tan 60^{\circ} \\
& \therefore \theta=60^{\circ} \\
& \therefore \angle A C B=60^{\circ}
\end{aligned}
$$

$\therefore$ The angle of elevation $=60^{\circ}$.
5. You want to erect a pole of height 10 m with the support of three ropes. Each rope has to make an angle $30^{\circ}$ with the pole. What should be the length of the rope?

Sol: Height of the pole $A B=10 \mathrm{~m}$


Let the length of rope to erect the pole $=\mathrm{AC}$

Angle made by the rope with the pole $=30^{\circ}$

From right triangle $\cos \mathrm{A}=\frac{A B}{A C}$

$$
\begin{aligned}
& \Rightarrow \cos 30^{\circ}=\frac{10}{A C} \\
& \Rightarrow \frac{\sqrt{3}}{2}=\frac{10}{A C} \\
& \Rightarrow A C=\frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{20 \sqrt{3}}{3}=\frac{20 \times 1.732}{3} \\
& \Rightarrow A C=11.55 \mathrm{~m}
\end{aligned}
$$

$\therefore$ Length of the rope $=11.55 \mathrm{~m}$.
6. Suppose you are shooting an arrow from the top of a building at a height of $\mathbf{6 m}$ to a target on the ground at an angle of depression of $60^{\circ}$. What is the distance between you and the object.

Sol: Height of a building $\mathrm{AB}=6 \mathrm{~m}$


Angle of depression from top of a building ' B ' to a target ' C ' is $60^{\circ}$
$\angle \mathrm{PBC}=\angle \mathrm{BCA}=60^{\circ} \quad(\because \mathrm{PB} / / \mathrm{AC}$, they are alternate angles $)$
The distance between me and the object $\mathrm{BC}=\mathrm{x}$ say.
From right triangle ABC
$\operatorname{Sin} 60^{\circ}=\frac{A B}{B C}$
$\Rightarrow \frac{\sqrt{3}}{2}=\frac{6}{B C}$
$\Rightarrow B C=\frac{12}{\sqrt{3}}=\frac{12 \sqrt{3}}{\sqrt{3 \cdot \sqrt{3}}}=\frac{12 \sqrt{3}}{3}$
$=4 \sqrt{ } 3 \mathrm{~m}$
$\therefore$ The distance between me and the object is $4 \sqrt{ } 3 \mathrm{~m}$.
7. An electrician wants to repair an electric connection on a pole of height 9 m . He needs to reach 1.8 m below the top of the pole to do repair works. What should be the length of the ladder which he should use, when he climbs it at an angle of $60^{\circ}$ with the ground? What will be the distance between foot of the ladder and foot of the pole?

Sol: Height of electric pole $\mathrm{AB}=9 \mathrm{~m}$.


Length of a ladder $=$ CD say.
Height of electric pole to do repair work $\mathrm{AC}=\mathrm{AB}-\mathrm{BC}$

$$
=9-1.8=7.2
$$

Distance between foot of ladder and the pole $=\mathrm{AD}$

Angle made by ladder with ground at $\mathrm{D}=60^{\circ}$
$\therefore$ from right triangle ACD

$$
\begin{aligned}
& \sin 60^{\circ}=\frac{A C}{C D} \\
& \frac{\sqrt{3}}{2}=\frac{7.2}{C D} \\
& \Rightarrow C D=\frac{7.2 \times 2}{\sqrt{3}}=\frac{14.4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
& =\frac{14.4 \times \sqrt{3}}{3}=4.8 \times 1.732
\end{aligned}
$$

$$
=8.3136 \mathrm{~m}
$$

$$
\begin{aligned}
& \text { lly } \tan 60^{\circ}=\frac{A C}{A D} \\
& \qquad \sqrt{3}=\frac{7.2}{A D} \\
& \Rightarrow A D=\frac{7.2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{7.2 \times 1.732}{3} \\
& =2.4 \times 1.732 \\
& =4.1568 \mathrm{~m}
\end{aligned}
$$

$\therefore$ The distance between foot of the ladder and foot of the pole $=4.1568 \mathrm{~m}$.
8. A boat has to cross a river. It crosses the river by making an angle of $60^{\circ}$ with the bank of the river due to the stream of the river and travels a distance of $\mathbf{6 0 0} \mathbf{m}$ to reach the another side of the river. What is the width of the river?

Sol: Let the width of a river is AB .


Making angle with the bank of river $\angle \mathrm{CAB}=60^{\circ}$

Travel of boat from A to $\mathrm{C}, \mathrm{AC}=600 \mathrm{~m}$.

From right triangle ABC

$$
\cos 60^{\circ}=\frac{A B}{A C}
$$

$\Rightarrow \frac{1}{2}=\frac{A B}{600}$
$\Rightarrow A B=\frac{600}{2}=300 \mathrm{~m}$.
$\therefore$ The width of the river $=300 \mathrm{~m}$.
9. An observer of height 1.8 m is $\mathbf{1 3 . 2} \mathbf{m}$ away from a palm tree. The angle of elevation of the top of the tree from his eyes is $45^{\circ}$. What is the height of the palm tree?

Sol: Height of the observer $\mathrm{AB}=1.8 \mathrm{~m}$.


Height of the palm tree $=C D$ say.
Distance between the palm tree and observer
AC is 13.2 m .
From figure we observed that $\mathrm{AC}=\mathrm{BE}$ and
$\mathrm{AB}=\mathrm{CE}=1.8 \mathrm{~m}$.
$\therefore$ From right triangle $\Delta \mathrm{DBE}$, we get
$\tan 45^{\circ}=\frac{D E}{B E}$
$\Rightarrow 1=\frac{D E}{A C}$

$$
(\because B E=A C=13.2 m)
$$

$\Rightarrow 1=\frac{D E}{13.2}$
$\Rightarrow \mathrm{DE}=13.2 \mathrm{~m}$

Length of the palm tree $C D=C E+E D$
$=1.8+13.2$
$=15 \mathrm{~m}$.
10. in the adjacent figure $A C=6 \mathrm{~cm}, \mathrm{AB}=5 \mathrm{~cm}$ and $\angle \mathrm{BAC}=30^{\circ}$. Find the area of the triangle?


Sol: From the triangle we get $\sin 30^{\circ}=\frac{B D}{A B}=\frac{B D}{5}$

$$
\Rightarrow \frac{1}{2}=\frac{B D}{5} \Rightarrow B D=\frac{5}{2}=2.5 \mathrm{~cm}
$$

$\therefore$ Area of $\triangle \mathrm{ABC}=\frac{1}{2} \times A C \times B D$

$$
=\frac{1}{2} \times 6 \times 2.5=7.5 \mathrm{~cm}^{2}
$$

$\therefore$ Area of $\Delta \mathrm{ABC}=7.5$ sq.cm.
11. A TV tower stands vertically on the side of a road. From a point on the other side directly opposite to the tower, the angle of elevation of the top of tower is $60^{\circ}$. From another point 10 m away from this point, on the tower, the angle of elevation of the top of the tower is $30^{\circ}$. Find the height of the tower and the width of the road?

Sol: Height of the tower is AB say


Width of the road is BD say

Distance between two observation points C and D is $\mathrm{CD}=10 \mathrm{~m}$.

From right triangle $\triangle \mathrm{ABC}$ we get

$$
\tan 60^{\circ}=\frac{A B}{B C} \Rightarrow A B=B C \sqrt{3} \longrightarrow(1)
$$

lly in $\triangle \mathrm{ABD}, \tan 30^{\circ}=\frac{A B}{B D}$

$$
\begin{aligned}
& \frac{1}{\sqrt{3}}=\frac{A B}{B C+C D} \\
& \Rightarrow A B=\frac{B C+C D}{\sqrt{3}} \longrightarrow(2)
\end{aligned}
$$

From (1) \& (2), we get
$\sqrt{3} \cdot B C=\frac{B C+C D}{\sqrt{3}}$

$$
\begin{aligned}
& \Rightarrow 3 \mathrm{BC}=\mathrm{BC}+\mathrm{CD} \\
& \Rightarrow 3 \mathrm{BC}-\mathrm{BC}=\mathrm{CD} \\
& \Rightarrow 2 \mathrm{BC}=\mathrm{CD} \\
& B C=\frac{C D}{2} \\
& B C=\frac{10}{2}=5
\end{aligned}
$$

$$
(\because \text { we know that } C D=10 \mathrm{~m})
$$

$\therefore$ width of the road $\mathrm{BD}=\mathrm{BC}+\mathrm{CD}$

$$
=5+10=15 \mathrm{~m}
$$

Height of the tower $A B=\sqrt{ } 3 . B C$

$$
=\sqrt{3} .5=5 \sqrt{ } 3 \mathrm{~m}
$$

12. A 1.5 m tall boy is looking at the top of a temple which is 30 meter in height from a point at certain distance. The angle of elevation from his eye to the top of the crown of the temple increases from $30^{\circ}$ to $60^{\circ}$ as he walks towards the temple. Find the distance he walked towards the temple.

Sol: height of the temple $A B=30 \mathrm{~m}$.

Height of the Boy PR $=1.5 \mathrm{~m}$.

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The angle of elevation from his eye to the top of the temple is $\angle \mathrm{APC}=30^{\circ}$.

From figure we observed $\mathrm{AC}=\mathrm{AB}-\mathrm{BC}$

$$
\begin{aligned}
& =\mathrm{AB}-\mathrm{PR} \\
& =30-1.5 \\
\mathrm{AC} & =28.5 \mathrm{~m}
\end{aligned}
$$

In right triangle ACQ , we get

$$
\begin{aligned}
& \tan 60^{\circ}=\frac{A C}{Q C}=\frac{28.5}{Q C} \\
& \Rightarrow \sqrt{3}=\frac{28.5}{Q C} \\
& \Rightarrow Q C=\frac{28.5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{28.5 \times \sqrt{3}}{3} \\
& \therefore Q C=9.5 \sqrt{ } 3 .
\end{aligned}
$$

In right triangle APC, we get

$$
\begin{aligned}
& \tan 30^{\circ}=\frac{A C}{P C}=\frac{28.5}{P C} \\
& \Rightarrow \frac{1}{\sqrt{3}}=\frac{28.5}{P C} \\
& \Rightarrow P C=28.5 \sqrt{ } 3 .
\end{aligned}
$$

$\therefore$ The distance walked towards the temple is PQ

$$
\begin{aligned}
& \therefore P Q=P C-Q C \\
& =28.5 \sqrt{ } 3-9.5 \sqrt{ } 3 \\
& =(28.5-9.5) \times \sqrt{ } 3
\end{aligned}
$$

$$
\begin{aligned}
& =19 \times 1.732 \\
& =32.908 \mathrm{~m}
\end{aligned}
$$

13. A statue stands on the top of a 2 m tall pedestal. From a point on the ground, the angle of elevation of the top of the statue is $60^{\circ}$ and from the same point, the angle of elevation of the top of pedestal is $45^{\circ}$. Find the height of the statue.

Sol: Let the height of the statue $A B=h$ say.

(Point on the
ground)

Height of the pedestal $B C=2 \mathrm{~m}$

In right triangle BCP , we get
$\tan 45^{\circ}=\frac{B C}{P C} \quad \quad$ (point on the ground)
$\Rightarrow 1=\frac{B C}{P C} \Rightarrow P C=2 m \longrightarrow(1)$
lly in right triangle ACP, we get

$$
\begin{aligned}
& \tan 60^{\circ}=\frac{A C}{P C} \\
& \Rightarrow \sqrt{3}=\frac{A C}{2} \Rightarrow A C=2 \sqrt{3}
\end{aligned}
$$

$\therefore$ The height of the statue $\mathrm{AB}=\mathrm{AC}-\mathrm{BC}$

$$
\begin{aligned}
& =2 \sqrt{ } 3-2 \\
& =2(\sqrt{ } 3-1) \\
& =2(1.732-1) \\
& =2 \times 0.732 \\
& =1.464 \mathrm{~m}
\end{aligned}
$$

14. From the top of a Building, the angle of elevation of the top of a cell tower is $60^{\circ}$ and the angle of depression to its foot is $45^{\circ}$. If distance of the building from the tower is $7 \mathbf{m}$ then find the height of the tower.

Sol: Height of the building $\mathrm{AB}=\mathrm{h}$ say.


Let $\mathrm{AB}=\mathrm{DE}=\mathrm{h}$
$\mathrm{CE}=\mathrm{x}$ say.

The distance between the tower and building $\mathrm{BD}=7 \mathrm{~m}$.

From the figure $\mathrm{BD}=\mathrm{AE}=7 \mathrm{~m}$.
From right triangle $\mathrm{ACE} \tan 60^{\circ}=\frac{C E}{A E}$

$$
\begin{aligned}
& \sqrt{3}=\frac{C E}{7} \Rightarrow C E=7 \sqrt{3} \\
& x=7 \sqrt{3} \mathrm{~m} .
\end{aligned}
$$

From right triangle ABD , we get

$$
\begin{aligned}
& \tan 45^{\circ}=\frac{A B}{B D} \\
& 1=\frac{A B}{7}=\frac{h}{7}
\end{aligned}
$$

$\therefore \mathrm{h}=7 \mathrm{~m}$. and $\mathrm{AB}=\mathrm{ED}=7 \mathrm{~m}$.
$\therefore$ The height of cell tower $\mathrm{CD}=\mathrm{CE}+\mathrm{ED}$

$$
\begin{aligned}
& =7 \sqrt{ } 3+7 \\
& =7(\sqrt{ } 3+1) \\
& =7(1.732+1) \\
& =7(2.732) \\
& =19.124 \mathrm{~m}
\end{aligned}
$$

15. A wire of length 18 m had been tied with electric pole at an angle of elevation $30^{\circ}$ with the ground. Because it was conversing a long distance, it was cut and tied at an angle of elevation $60^{\circ}$ with ground. How much length of the wire was cut?

Sol: Height of electric pole $=A B=h$ say.

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Length of a wire $=A C=18 \mathrm{~m}$.

From figure

In right triangle ACB , we get
$\sin 30^{\circ}=\frac{A B}{A C}=\frac{h}{18}$
$\Rightarrow \frac{1}{2}=\frac{h}{18} \Rightarrow h=\frac{18}{2}=9 m \longrightarrow(1)$
lly from triangle ADB , we get
$\sin 60^{\circ}=\frac{A B}{A D}=\frac{h}{A D}$

From (1) h = 9m
$\Rightarrow \frac{\sqrt{3}}{2}=\frac{9}{A D}$
$\Rightarrow A D=\frac{18}{\sqrt{3}}=\frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{18 \sqrt{3}}{3}=6 \sqrt{3} \mathrm{~m}$.
$\therefore$ The length of the remaining wire after cutting
$=18-6 \sqrt{ } 3=(18-6 \times 1.732)$
$=18-10.392$
$=7.608 \mathrm{~m}$.
16. The angle of elevation of the top of a building form the foot of the tower is $30^{\circ}$ and the angle of elevation of the top of the tower from the foot of the building is $60^{\circ}$. If the tower is 30 m high, find the height of the building.

Sol: Let the height of the building be $\mathrm{AB}=\mathrm{hm}$

The height of the tower $\mathrm{PQ}=30 \mathrm{~m}$.
From figure in right triangle $\triangle \mathrm{PBQ}$


We get $\tan 60^{\circ}=\frac{P Q}{B Q}$

$$
\begin{aligned}
& \sqrt{3}=\frac{30 m}{B Q} \\
& \Rightarrow B Q=\frac{30}{\sqrt{3}} m \longrightarrow(1)
\end{aligned}
$$

In right triangle $\triangle \mathrm{AQB}$, we get 1

$$
\tan 30=\frac{A B}{B Q}=\frac{h}{B Q}
$$

$$
\Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{B Q}
$$

$\Rightarrow B Q=h \sqrt{3} \longrightarrow(2)$
From (1) \& (2) we get
$h \sqrt{3}=\frac{30}{\sqrt{3}}$
$h=\frac{30}{\sqrt{3} \times \sqrt{3}}=\frac{30}{3}=10 \mathrm{~m}$.
$\therefore$ The height of the Building is 10 m .
17. Two poles of equal heights are standing opposite to each other on either side of the road. Which is $\mathbf{1 2 0}$ feet wide from a point between them on the road, the angles of elevation of the top of the poles are $60^{\circ}$ and $30^{\circ}$ respectively. Find the height of the poles and the distances of the point from the poles.

Sol: The two poles of equal heights are $A B$ and $P Q$ say. Where $A B=P Q=H$ say.


The distance between the two poles AB and PQ is 120 feet.
Take ' D ' is a point between them and let $\mathrm{BD}=\mathrm{hm}$

From figure in right $\triangle \mathrm{ABD}$ we get then $\mathrm{DQ}=(120-\mathrm{h}) \mathrm{m}$

$$
\begin{aligned}
& \tan 60^{\circ}=\frac{A B}{B D} \\
& \sqrt{3}=\frac{A B}{h}
\end{aligned}
$$

$$
\Rightarrow A B=h \sqrt{3} \Rightarrow H=h \sqrt{3} \longrightarrow(1)
$$

lly in right triangle $P Q D$, we get
$\tan 30^{\circ}=\frac{P Q}{D Q}=\frac{H}{120-h}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{H}{120-h}$
$\Rightarrow H=\frac{120-h}{\sqrt{3}} \longrightarrow$
From (1) \& (2) we get
$h \sqrt{3}=\frac{120-h}{\sqrt{3}}$
$\Rightarrow \mathrm{h} \sqrt{3} \times \sqrt{ } 3=120-\mathrm{h}$
$\Rightarrow 3 \mathrm{~h}+\mathrm{h}=120 \Rightarrow 4 \mathrm{~h}=120 \Rightarrow h=\frac{120}{4}=30$
From (1) $\mathrm{H}=30 \sqrt{3} \mathrm{~m}$.
And also $120-\mathrm{h}=120-30=90$.
$\therefore$ The heights of the poles are $30 \sqrt{ } 3$ feet each and the distances of the point form the poles are 30 feet and 90 feet.
18. The angles of elevation of the top of a tower from two points at a distance of $\mathbf{4 m}$ and 9 m . Find the height of the tower from the base of the tower and in the same straight line with it are complementary.

Sol: Height of the tower is AB say


Let $\angle \mathrm{ADB}=\theta$.
Then $\angle \mathrm{ACB}=90-\theta(\because$ given $)$
( $\because \angle \mathrm{ABD}$ and $\angle \mathrm{ACB}$ are
Complementary)
In right triangle ABD
$\tan \theta=\frac{A B}{D B}=\frac{A B}{9} \longrightarrow(1)$
In right triangle $\triangle \mathrm{ABC}$
$\tan \left(90^{\circ}-\theta\right)=\frac{A B}{4} \longrightarrow(2)$
$\cot \theta=\frac{A B}{4}$

Multiplying (1) and (2), we get
$\frac{A B}{9} \times \frac{A B}{4}=\tan \theta \times \cot \theta$
$\frac{A B^{2}}{36}=\tan \theta \times \frac{1}{\tan \theta}=1$
$\Rightarrow \mathrm{AB}^{2}=36$
$\Rightarrow \mathrm{AB}=6 \mathrm{~m}$
$\therefore$ The right of the tower is 6 m .
19. The angle of elevation of a jet plane from a point $A$ on the ground is $60^{\circ}$. After a flight of 15 seconds, the angle of elevation changes to $30^{\circ}$. If the jet plane is flying at a constant height of $1500 \sqrt{ } 3$ meter, find the speed of the jet plane. $(\sqrt{3}=1.732)$

Sol: Let $\mathrm{P}, \mathrm{R}$ be the two positions of the plane and A be the point of observation. It is given that angles of elevation of the plane in A two positions $P$ and $R$ from point A are $60^{\circ}$ and $30^{\circ}$. Respectively

$\Rightarrow \angle \mathrm{PAQ}=60^{\circ}$ and $\angle \mathrm{RAS}=30^{\circ}$.

And also given that plane is flying at a constant height $\mathrm{PQ}=\mathrm{Rs}=1500 \sqrt{ } 3$.

Now, In $\triangle \mathrm{PAQ}$, we get

$$
\tan 60^{\circ}=\frac{P Q}{A Q}=\frac{1500 \sqrt{3}}{A Q}
$$

$$
\begin{aligned}
& \Rightarrow \sqrt{3}=\frac{1500 \sqrt{3}}{A Q} \\
& \Rightarrow A Q=\frac{1500 \sqrt{3}}{\sqrt{3}}=1500 \mathrm{~m} .
\end{aligned}
$$

In $\triangle$ RAS, we get
$\tan 30^{\circ}=\frac{R S}{A S}=\frac{1500 \sqrt{3}}{A S}$
$\Rightarrow \frac{1}{\sqrt{3}}=\frac{1500 \sqrt{3}}{A S}$
$\Rightarrow A S=1500 \sqrt{ } 3 \times \sqrt{ } 3=1500 \times 3=4500$.
$\therefore$ Thus the distance which the plane travels $P R=R S=A S-A Q$

$$
=4500-1500=3000 \mathrm{~m}
$$

$\therefore$ Speed of plane $=\frac{3000}{15}=200 \mathrm{~m} / \mathrm{sec}$.

## Multiple Choice Questions

1. If the angle of elevation of the top of a tower at a distance of 500 m from the foot is $30^{\circ}$. Then the height of the tower is $\qquad$
a) $250 \sqrt{ } 3 \mathrm{~m}$
b) $500 \sqrt{ } 3 \mathrm{~m}$
c) $\frac{500}{\sqrt{3}} m$
d) 250 m
2. A pole 6 m high casts a shadow $2 \sqrt{ } 3 \mathrm{~m}$ long on the ground, then sun's elevation is
$\qquad$
a) $60^{\circ}$
b) $45^{\circ}$
c) $30^{\circ}$
d) $90^{\circ}$
3. The height of the tower is 100 m . When the angle of elevation of sun is $30^{\circ}$, then shadow of the tower is $\qquad$ .
a) $100 \sqrt{ } 3 \mathrm{~m}$
b) 100 m
c) $100(\sqrt{3}-1) \mathrm{m}$
d) $\frac{100}{\sqrt{3}} m$
4. If the height and length of the shadow of a man are the same, then the angle of elevation of the sun is $\qquad$
a) $30^{\circ}$
b) $60^{\circ}$
c) $45^{\circ}$
d) $15^{\circ}$
5. The angle of elevation of the top of a tower, whose height is 100 m , at a point whose distance from the base of the tower is 100 m is $\qquad$ [ ]
a) $30^{\circ}$
b) $60^{\circ}$
c) $45^{\circ}$
d) none of these
6. The angle of elevation of the top of a tree height 2003 m at a point at distance of 200 m from the base of the tree is $\qquad$
a) $30^{\circ}$
b) $60^{\circ}$
c) $45^{\circ}$
d) None of these
7. A lamp post $5 \sqrt{ } 3 \mathrm{~m}$ high casts a shadow 5 m long on the ground. The sun's elevation at this moment is $\qquad$
a) $30^{\circ}$
b) $45^{\circ}$
c) $60^{\circ}$
d) $90^{\circ}$
8. Find the length of shadow of 10 m high tree if the angle of elevation of the sun is $30^{\circ}$
a) 10 m
b) $\frac{10}{\sqrt{3}} m$
c) $10 \sqrt{ } 3 \mathrm{~m}$
d) 20 m
9. If the angle of elevation of a bird sitting on the top of a tree as seen from the point at a distance of 20 m from the base of the tree is $60^{\circ}$. Then the height of the tree is $\qquad$路
a) $20 \sqrt{ } 3 \mathrm{~m}$
b) $10 \sqrt{ } 3 \mathrm{~m}$
c) 20 m
d) 10 m
10. The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of $30^{\circ}$ with horizontal, then the length of the wire is
a) 6 m
b) 8 m
c) 10 m
d) 12 m

Key:

$$
\text { 1) } \mathrm{C} \text {; 2) } \mathrm{A} \text {; 3) } \mathrm{A} \text {; 4) } \mathrm{C} \text {; 5) } \mathrm{C} \text {; 6) } \mathrm{C} \text {; 7) } \mathrm{C} \text {; 8) } \mathrm{C} \text {; 9) } \mathrm{A} \text {; 10) } \mathrm{D} \text {. }
$$

## Fill in the Blanks

1. The ratio of the length of a tree and its shadow is $1: \frac{1}{\sqrt{3}}$. The angle of the sun's elevation is $\qquad$ degrees
2. If two towers of height $h_{1}$ and $h_{2}$ subtend angles of 60 and 30 respectively at the midpoint of the line joining their feet, then $h_{1}: h_{2}$ is $\qquad$
3. The line drawn from the eye of an observer to the object viewed is called
$\qquad$
4. If the angle of elevation of the sun is $30^{\circ}$, then the ratio of the height of a tree with its shadow is $\qquad$
5. From the figure $\theta=$ $\qquad$

6. The angle of elevation of the sun is $45^{\circ}$. Then the length of the shadow of a 12 m high tree is $\qquad$
7. When the object is below the horizontal level, the angle formed by the line of sight with the horizontal is called $\qquad$
8. When the object is above the horizontal level, the angle formed by the line of sight with the horizontal is called $\qquad$
9. The angle of depression of a boat is 60 m high bridge is $60^{\circ}$. Then the horizontal distance of the boat from the bridge is $\qquad$
10. The height or length of an object can be determined with help of $\qquad$
Key
1) $60^{\circ}$;
2) $3: 1$;
3) Line of sight;
4) $1: \sqrt{ } 3$;
5) $60^{\circ}$;
6) 12 m ;
7) angle of depression;
8) angle of elevation;
9) $20 \sqrt{ } 3 \mathrm{~m}$;
10) Trigonometric ratios.
