## Chapter - 6

## Progressions

- Evidence is found that Babylonians some 400 years ago, knew of arithmetic and geometric progressions.
- Among the Indian mathematicians, Aryabhata ( 470 AD ) was the first to give formula for the sum of squares and cubes of natural numbers in his famous work "Aryabhata"
- Indian mathematician Brahmagupta (598 AD), Mahavira (850 AD) and Bhaskara (1114 $-1185 \mathrm{AD})$ also considered the sums of squares and cubes.


## - Arithmetic progression(A.P)

An arithmetic progression (AP) is a list of numbers in which each term is obtained by term adding a fixed number ' $d$ ' to preceding term, except the first term. The fixed number' $d$ ' is called the common difference

Ex: $1,2,7,10,13 \ldots \ldots$ are in AP Here $d=3$

- Let $a_{1}, a_{2}, a_{3}, \ldots . . a_{k}, a_{k+1} \ldots . a_{n} \ldots .$. be an AP.

Let its common difference be $d$, then
$\mathrm{d}=\mathrm{a}_{2}-\mathrm{a}_{1}=\mathrm{a}_{3}-\mathrm{a}_{2}=$ $\qquad$ $=a_{k+1}-a_{k}=$ $\qquad$

- If the first term is ' $a$ ' and the common difference is ' $d$ ' then $a, a+d, a+2 d, a+3 d$, $\ldots .$. is an A.P.
- General term of an A.P.

Let 'a' be the first term and ' $d$ ' be the common difference of an A.P., Then, its nth term or general term is given by $a_{n}=a+(n-1) d$

Ex: The $10^{\text {th }}$ term of the A.P. given by $5,1-3,-7, \ldots \ldots$
is $a_{10}=5+(10-1)(-4)=-31$

- If the number of terms of an A.P. is finite, then it is a finite A.P.

Ex: 13, 11, 9, 7, 5

- If the number of terms of an A.P. is infinite, then it is an infinite A.P.

Ex: $4,7,10,13,16,19, \ldots \ldots$

- Three numbers in AP should be taken as $a-d, a, a+d$.
- Four numbers in AP should be taken as $a-3 d, a-d, a+d, a+3 d$.
- Five numbers in AP should be taken as $a-2 d, a-d, a, a+d, a+2 d$
- Six numbers in AP should be taken as $a-5 d, a-3 d, a-d, a+d, a+3 d, a+5 d$.
- If $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are in AP , then $b=\frac{a+c}{2}$ is called the arithmetic mean if ' a ' and ' c '.
- The sum of the first ' n ' terms of an AP is given by $s_{n}=\frac{n}{2}[2 a+(n-1) d]$.
- If the first and last terms of an AP are 'a' and ' $l$ ', the common difference is not given then $s_{n}=\frac{n}{2}[a+l]$.
- $\mathrm{a}_{\mathrm{n}}=\mathrm{s}_{\mathrm{n}}-\mathrm{s}_{\mathrm{n}-1}$
- The sum of first ' n ' positive integers $s_{n}=\frac{n(n+1)}{2}$.
- Ex: sum of first ' 10 ' positive integers $=\frac{10(10+1)}{2}=55$.


## Geometric progression (G.P.)

A Geometric Progression is a list of numbers in which each term is obtained by multiplying preceding term with a fixed number ' $r$ ' except first term. This fixed number is called common ratio ' $r$ '.

Ex: 3, 9, 27, 81, $\ldots \ldots$ are in G.P.

Here common ratio $r=3$

- A list of numbers $a_{1}, a_{2}, a_{3}, \ldots a_{n} \ldots \ldots$ are in G. P. Then the common ratio $r=\frac{a_{2}}{a_{1}}=\frac{a_{3}}{a_{2}}=\ldots . .=\frac{a_{n}}{a_{n+1}}=\ldots .$.
- The first term of a G.P. by ' $a$ ' and common ratio ' $r$ ' then the G.P is $a$, $a r, ~ a r r^{2}, \ldots .$.
- If the first term and common ratio of a G.P. are $a, r$ respectively then nth term $a_{n}=a r^{n-1}$.


## 1 Mark Questions

1. Do the irrational numbers $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots \ldots .$. form an A.P? If so find common difference?

Sol: Given irrational numbers are $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$.

$$
\begin{aligned}
& \mathrm{d}=\mathrm{a}_{2}-\mathrm{a}_{1} \\
& \Rightarrow \sqrt{8}-\sqrt{2} \\
& \Rightarrow \sqrt{4 \times 2}-\sqrt{2} \\
& \Rightarrow 2 \sqrt{2}-\sqrt{2} \\
& \Rightarrow \sqrt{2} \\
& \sqrt{18}-\sqrt{8}=\sqrt{9 \times 2}-\sqrt{4 \times 2} \\
& \Rightarrow 3 \sqrt{2}-2 \sqrt{2} \\
& \Rightarrow \sqrt{2}
\end{aligned}
$$

Here common difference is same. i.e $\sqrt{2}$
$\therefore$ The numbers are in A.P.
2. Write first four terms of the A.P, when the first term ' $a$ ' and common difference ' $d$ ' are given as follow.
$\mathrm{a}=-1.25, \mathrm{~d}=-0.25$

Sol: $\quad a_{1}=\mathrm{a}=-1.25, \mathrm{~d}=-0.25$
$\mathrm{a}_{2}=\mathrm{a}+\mathrm{d}=-1.25-0.25=-1.50$
$\mathrm{a}_{3}=\mathrm{a}+2 \mathrm{~d}=-1.25+2(-0.25)=-1.75$
$\mathrm{a}_{4}=\mathrm{a}+3 \mathrm{~d}=-1.25+3(-0.25)=-2.00$
$\therefore \mathrm{AP}=-1.25,-1.5,-1.75,-2$.
3. Is the following forms AP? If it, form an AP, find the common difference $d$ and write three more terms.

Sol: $\quad \sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots \ldots$.

Here $\mathrm{a}=\sqrt{ } 2$
$d=a_{2}-a_{1}=\sqrt{8}-\sqrt{2}=\sqrt{2 \times 4}-\sqrt{2}=2 \sqrt{2}-\sqrt{2}=\sqrt{2}$
$d=a_{3}-a_{2}=\sqrt{18}-\sqrt{8}=\sqrt{9 \times 2}-\sqrt{2 \times 4}=3 \sqrt{2}-2 \sqrt{2}=\sqrt{2}$
$d=a_{4}-a_{3}=\sqrt{32}-\sqrt{18}=\sqrt{4 \times 4 \times 2}-\sqrt{9 \times 2}=4 \sqrt{2}-3 \sqrt{2}=\sqrt{2}$
' $d$ ' is equal for all. So it forms an AP

## Next three terms

$a_{5}=a_{4}+d=\sqrt{32}+\sqrt{2}=\sqrt{16 \times 2}+\sqrt{2}=4 \sqrt{2}+\sqrt{2}=5 \sqrt{2}=\sqrt{25 \times 2}=\sqrt{50}$
$a_{6}=a_{5}+d=\sqrt{50}+\sqrt{2}=\sqrt{25 \times 2}+\sqrt{2}=5 \sqrt{2}+\sqrt{2}=6 \sqrt{2}=\sqrt{36 \times 2}=\sqrt{72}$
$a_{7}=a_{6}+d=\sqrt{72}+\sqrt{2}=\sqrt{36 \times 2}+\sqrt{2}=6 \sqrt{2}+\sqrt{2}=7 \sqrt{2}=\sqrt{49 \times 2}=\sqrt{98}$
$\therefore \sqrt{50}, \sqrt{72}, \sqrt{98}$.
4. If an $\mathbf{A P} \mathrm{a}_{\mathrm{n}}=\mathbf{6} \mathbf{+} \mathbf{2}$ find the common difference

Sol: Let $\mathrm{a}_{\mathrm{n}}=6 \mathrm{n}+2$

$$
\begin{aligned}
a_{1}=6 & (1)+2= \\
& =6+2 \\
= & 8 \\
a_{2}=6 & (2)+2=12+2=14 \\
a_{3}=6 & (3)+2=18+2=20 \\
& d=a_{2}-a_{1} \\
= & 14-8 \\
= & 6
\end{aligned}
$$

Common difference $=6$.
5. In G.P. 2, $-6,18,-54 \ldots .$. find $a_{n}$

Sol: $\quad a=2$

$$
r=\frac{a_{2}}{a_{1}}=\frac{-6}{2}=-3
$$

$a_{n}=a . r^{n-1}$
$=2 \cdot(-3)^{\mathrm{n}-1}$
6. The $17^{\text {th }}$ term of an A.P exceeds its $10^{\text {th }}$ term by 7. Find the common difference.

Sol: Given an A.P in which $\mathrm{a}_{17}=\mathrm{a}_{10}+7$

$$
\begin{aligned}
& \Rightarrow \mathrm{a}_{17}-\mathrm{a}_{10}=7 \Rightarrow(\mathrm{a}+16 \mathrm{~d})-(\mathrm{a}+9 \mathrm{~d})=7 \\
& \Rightarrow 7 \mathrm{~d}=7
\end{aligned}
$$

$\Rightarrow d=\frac{7}{7}=1$
7. A man helps three persons. He ask each or them to give their help another three persons. If the chain continued like this way. What are the numbers obtained this series

Sol: First person $=1$
No.of person taken help from $1^{\text {st }}$ person $=3$
No.of person taken help from the persons taken help from first person $=3^{2}=9$
Similarly, no.of persons taken help $27,81,243, \ldots$ progression $1,3,9,27,81,243 \ldots \ldots$ In the above progression $\mathrm{a}_{1}=1, \mathrm{a}_{2}=3, \mathrm{a}_{3}=9$

Common ratio (r) $=\frac{a_{2}}{a_{1}}=\frac{a_{3}}{a_{2}}=\frac{3}{1}=\frac{9}{3}=3$

So, above progression is in G.P.
8. Find the sum of 8 terms of a G.P., whose $n^{\text {th }}$ term is $3^{\text {n }}$.

Sol: In a G.P. $\mathrm{n}^{\text {th }}$ term $\left(\mathrm{a}_{\mathrm{n}}\right)=3^{\text {n }}$
$a_{1}=3^{1}=3$
$\mathrm{a}_{2}=3^{2}=9$
$a_{3}=3^{3}=27 \ldots \ldots$.
$\therefore$ Geometric progression $=3,9,27 \ldots \ldots$

First term $(a)=3$

Common ratio (r) $=\frac{a_{2}}{a_{1}}=\frac{9}{3}=3>1$

No.of terms ( n ) $=8$
Sum of terms $\left(\mathrm{s}_{\mathrm{n}}\right)=\frac{a\left(r^{n}-1\right)}{r-1}$
Sum of 8 terms $\left(s_{8}\right)=\frac{3\left(3^{8}-1\right)}{3-1}$

$$
=\frac{3}{2}\left(3^{8}-1\right)
$$

9. In A.P $\mathbf{n}^{\text {th }}$ tern $\mathbf{a}_{\mathrm{n}}=\mathbf{a}+(\mathbf{n}-1) \mathrm{d}$ explain each term in it.

Sol: $\quad a_{n}=a+(n-1) d$
$\mathrm{a}=$ First term
$\mathrm{n}=$ No.of terms
d = Common difference
$\mathrm{a}_{\mathrm{n}}=\mathrm{n}^{\text {th }}$ term.
10. $6,18,54 \ldots$ is it in G.P. What is the common ratio?

Sol: Given that $6,18,54 \ldots \ldots$.
$r=\frac{a_{2}}{a_{1}}=\frac{18}{6}=3$
$r=\frac{a_{3}}{a_{2}}=\frac{54}{18}=3$
$\frac{a_{2}}{a_{1}}=\frac{a_{3}}{a_{2}}=3$
So, $6,18,54$, i.. is in G.P. Common ratio $=3$.
11. Find sum of series $7,13,19 \ldots .$. upto 35 terms

Sol: Given that 7, 13, 19.....
$a_{1}=7$
$\mathrm{d}=\mathrm{a}_{2}-\mathrm{a},=13-7=6$
No.of terms (n) $=35$

Sum of terms $S_{n}=\frac{n}{2}[2 a+(n-1) d]$

$$
\begin{aligned}
& S_{35}=\frac{35}{2}[2(7)+(35-1) 6] \\
& =\frac{35}{2}[14+34 \times 6] \\
& =\frac{35}{2}[14+204] \\
& =\frac{35}{2} \times 218 \\
& =35 \times 109 \\
& =3815 .
\end{aligned}
$$

12. What is $10^{\text {th }}$ term in the series $\mathbf{3 , 8}, 13 \ldots . . . .$. .

Sol: Given series $3,8,13, \ldots \ldots$ it is in A.P.
First term $(a)=3$
$\mathrm{d}=8-3=5$
$\mathrm{n}=10$
$a_{n}=a+(n-1) d$
$=3+(10-1)(5)$
$=3+45$
$=48$
$\therefore 10^{\text {th }}$ term in given series $\mathrm{a}_{10}=48$.
13. can $x+2, x+4$ and $x+9$ be in A.P. Justify your answer

## Sol: Given terms are:

$$
\begin{aligned}
& x+2, x+4, x+9 \\
& a_{2}-a_{1}=(x+4)-(x+2) \\
& =2 \\
& a_{3}-a_{2}=(x+9)-(x+4) \\
& = \\
& = \\
& =5+9-x-4 \\
& a_{2}-a_{1} \neq a_{3}-a_{2} .
\end{aligned}
$$

$\therefore$ Given terms are not in A.P.
14. In a G.P., first term is $9,7^{\text {th }}$ term is $\frac{1}{81}$ find the common ratio

Sol: G.P. first term $a_{1}=9=3^{2}$
$7^{\text {th }}$ term $a_{7}=\frac{1}{81}=\frac{1}{3^{4}}$

$$
a_{7}=a r^{6}=\frac{1}{81}
$$

$$
\begin{aligned}
& \frac{7^{\text {th }} \text { term }}{1^{\text {st }} \text { term }}=\frac{a_{7}}{a_{1}}=\frac{a r^{6}}{a}=\frac{\frac{1}{81}}{\frac{81}{1}}=\frac{1}{81} \times \frac{1}{9} \\
& \Rightarrow r^{6}=\frac{1}{3^{4}} \times \frac{1}{3^{2}} \\
& r^{6}=\left[\frac{1}{3}\right]^{6}
\end{aligned}
$$

$\therefore$ Common ratio $\mathrm{r}=\frac{1}{3}$.
15. Write the general terms of an AP and GP.

Sol: The general terms of AP are $a, a+d, a+2 d, a+3 d \ldots \ldots$. The general terms of GP are $a, ~ a r, ~ a r^{2}, \operatorname{ar}^{3} \ldots$

## $\underline{2}$ Mark Questions

1. Determine the A.P. whose $3^{\text {rd }}$ term is 5 and the $7^{\text {th }}$ term is 9 .

Sol: we have
$a_{3}=a+(3-1) d=a+2 d=5------$
$a_{7}=a+(7-1) d=a+6 d=9$
solving the pair of linear equations (1) and (2), we get

$$
\begin{gather*}
a+2 d=5------(1) \\
a+6 d=9 \text {------- }(2)  \tag{2}\\
--4 d=-4 \\
d=\frac{-4}{-4} \\
d=1
\end{gather*}
$$

substitute $d=1$ in equ (1)

$$
\begin{aligned}
& a+2 d=5 \Rightarrow a+2(1)=5 \Rightarrow a=5-2=3 \\
& \therefore a=3 \text { and } d=1
\end{aligned}
$$

Hence, the required AP is $3,4,5,6,7, \ldots \ldots$.
2. How many two-digit numbers are divisible by 3?

Sol: The list of two - digit numbers divisible by 3 are $12,15,18 \ldots \ldots 99$. These terms are in A.P.

$$
\left(\because \mathrm{t}_{2}-\mathrm{t}_{1}=\mathrm{t}_{3}-\mathrm{t}_{2}=3\right)
$$

Here, $a=12, d=3, a_{n}=99$

$$
\begin{aligned}
& a_{n}=a+(n-1) d \\
& 99=12+(n-1) 3
\end{aligned}
$$

$$
\begin{aligned}
& 99-12=(n-1) \times 3 \\
& 87=(n-1) 3 \\
& n-1=\frac{87}{3} \\
& n-1=29 \\
& n=29+1=30
\end{aligned}
$$

so, there are 30 two- digit numbers divisible by 3 .
3. Find the respective term of $\mathbf{a}_{1}=5, \mathbf{a}_{4}=9 \frac{1}{2}$ find $\mathbf{a}_{2}, \mathbf{a}_{3}$ in APs

Sol: Given $\mathrm{a}_{1}=\mathrm{a}=5$----- (1)

$$
\begin{equation*}
\mathrm{a}_{4}=\mathrm{a}+3 \mathrm{~d}=9 \frac{1}{2} \tag{2}
\end{equation*}
$$

Solving the equ (1) and equ (2), we get

$$
\begin{aligned}
& \text { equ }(1)-\text { equ }(2) \\
& (\mathrm{a}+3 \mathrm{~d})-\mathrm{a}=9 \frac{1}{2}-5 \\
& \mathrm{a}+3 \mathrm{~d}-\mathrm{a}=4 \frac{1}{2} \\
& 3 \mathrm{~d}=\frac{9}{2} \\
& \mathrm{~d}=\frac{9 \times 1}{2 \times 3} \\
& \therefore \mathrm{~d}=\frac{3}{2} \\
& a_{2}=a+d=5+\frac{3}{2}=\frac{10+3}{2}=\frac{13}{2}
\end{aligned}
$$

$$
\begin{aligned}
& a_{3}=a_{2}+d=\frac{13}{2}+\frac{3}{2}=\frac{16}{2}=8 \\
& \therefore a_{2}=\frac{13}{2}, a_{3}=8 .
\end{aligned}
$$

4. $a_{2}=38 ; a_{6}=-22$ find $a_{1}, a_{3}, a_{4}, a_{5}$

Sol: Given $\mathrm{a}_{2}=\mathrm{a}+\mathrm{d}=38$ -

$$
\begin{equation*}
a_{6}=a+5 d=-22 \tag{1}
\end{equation*}
$$

Equation (2) - equation (1)

$$
\begin{aligned}
& (a+5 d)-(a+d)=-22-38 \\
& a+5 d-a-d=-22-38 \\
& 4 d=-60 \\
& d=\frac{-60}{4} \\
& \therefore d=-15
\end{aligned}
$$

$$
\mathrm{a}_{2}=\mathrm{a}+\mathrm{d}=38
$$

$$
\begin{aligned}
& a+(-15)=38 \Rightarrow a-15=38 \Rightarrow a=38+15 \Rightarrow a=53 \\
& \therefore a_{1}=a=53 \\
& a_{3}=a+2 d=53+2(-15)=53-30=23 \\
& a_{4}=a+3 d \\
& =53+3(-15) \\
& =53-45 \\
& =8 \\
& a_{5}=a+4 d
\end{aligned}
$$

$$
\begin{aligned}
& =53+4(-15) \\
& =53-60 \\
& =-7
\end{aligned}
$$

$\therefore a_{1}=53, a_{3}=23, a_{4}=8, a_{5}=-7$.
5. Which term of the A.P: $3,8,13,18 \ldots \ldots$ is 78 ?

Sol: $\quad a_{n}=78$
$a=3$
$\mathrm{d}=\mathrm{a}_{2}-\mathrm{a}_{1}=8-3=5$
$a_{n}=a+(n-1) d$
$78=3(n-1) 5$
$78=3+5 n-5$
$78=5 n-2$
$5 n=78+2$
$5 n=80$
$n=\frac{80}{5}$
$\mathrm{n}=16$
$\therefore 16^{\text {th }}$ term of the A.P is 78 .
6. Find the $31^{\text {st }}$ term of an AP whose $11^{\text {th }}$ term is 38 and $16^{\text {th }}$ term is 73 .

Sol: Given $\mathrm{a}_{11}=38, \mathrm{a}_{16}=73$ and $\mathrm{a}_{31}=$ ?

$$
\begin{align*}
& \therefore a_{n}=a+(n-1) d \\
&  \tag{1}\\
& \quad a_{11}=a+10 d=38 .
\end{align*}
$$

$$
\begin{equation*}
\mathrm{a}_{16}=\mathrm{a}+15 \mathrm{~d}=73 \tag{2}
\end{equation*}
$$

(2) $-(1) \Rightarrow a+15 d=73$

$$
a+10 d=38
$$



$$
\begin{aligned}
& 5 \mathrm{~d}=35 \\
& \qquad d=\frac{35}{5}=7
\end{aligned}
$$

Substitute d=7 in equ (1)

$$
\begin{aligned}
& a+10 d=38 \\
& a+10(7)=38 \\
& a+70=38 \\
& a=38-70 \\
& a=-32
\end{aligned}
$$

$$
31^{\text {st }} \text { term } \mathrm{a}_{31}=\mathrm{a}+30 \mathrm{~d}
$$

$$
\begin{aligned}
& =-32+30(7) \\
& =-32+210 \\
& =178
\end{aligned}
$$

$\therefore 178$ is the $31^{\text {st }}$ term.
7. Find the sum of $7+10 \frac{1}{2}+\mathbf{1 4}+\cdots--+\mathbf{8 4}$

Sol: Given terms are in A.P.

Here $\mathrm{a}=7, \mathrm{~d}=\mathrm{a}_{2}-\mathrm{a}_{1}=10 \frac{1}{2}-7=3 \frac{1}{2}=\frac{7}{2}, a_{n}=84$

$$
\begin{array}{rl}
\mathrm{a}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}=84 \\
& 7+(\mathrm{n}-1)\left(\frac{7}{2}\right)=84 \\
& (n-1)\left(\frac{7}{2}\right)=84-7 \\
& (n-1)\left(\frac{7}{2}\right)=77 \\
& (n-1)=77 \times \frac{2}{7} \\
\mathrm{n} & \mathrm{n} \\
=1 & 22+1=23 \\
\therefore \mathrm{n}= & 23 \\
s_{n} & =\frac{n}{2}[a+l] \\
s_{23}=\frac{23}{2}[7+84] \\
& =\frac{23}{2}[91] \\
& =\frac{2093}{2}
\end{array}
$$

$$
\therefore S_{23}=1046 \frac{1}{2}
$$

8. In an AP given $a=5, d=3, a_{n}=50$, find $n$ and $s_{n}$

Sol: $\quad a_{n}=a+(n-1) d$

$$
\begin{aligned}
& 50=5+(\mathrm{n}-1) 3 \quad\left(\because \mathrm{a}=5, \mathrm{~d}=3, \mathrm{a}_{\mathrm{n}}=50\right) \\
& 50=5+3 \mathrm{n}-3
\end{aligned}
$$

$$
\begin{aligned}
& 50=3 \mathrm{n}+2 \\
50 & -2=3 \mathrm{n} \\
3 \mathrm{n} & =48 \\
n & =\frac{48}{3} \\
\therefore \mathrm{n}= & 16 \\
& S_{n}=\frac{n}{2}[2 a+(n-1) d] \\
& S_{16}=\frac{16}{2}[2(5)+(16-1)(3)] \\
& =8[10+(15)(3)] \\
& =8[10+45] \\
& =8 \times 55 \\
\therefore \mathrm{~S}_{16}= & 440 .
\end{aligned}
$$

9. In an AP given $a_{3}=15, S_{10}=125$, find $d$ and $a_{10}$

Sol: $\quad \mathrm{a}_{3}=15, \quad \mathrm{~S}_{10}=125$

$$
\begin{equation*}
a_{3}=a+2 d=15 \tag{1}
\end{equation*}
$$

$$
S_{n}=\frac{n}{2}[2 a+(n-1) d]
$$

$$
S_{10}=\frac{10}{2}[2 a+(10-1) d]=125
$$

$$
\begin{align*}
& 5[2 \mathrm{a}+9 \mathrm{~d}]=125 \\
& 2 a+9 d=\frac{125}{5} \\
& 2 \mathrm{a}+9 \mathrm{~d}=25---- \tag{2}
\end{align*}
$$

Equ (1) and equ (2)

$$
\begin{aligned}
& \mathrm{a}+2 \mathrm{~d}=15-----(1) \times 2 \\
& 2 \mathrm{a}+9 \mathrm{~d}=25-----(2) \times 1 \\
& 2 \mathrm{a}+4 \mathrm{~d}=30 \\
& 2 \mathrm{a}+9 \mathrm{~d}=25 \\
& -\quad- \\
& \hline-5 \mathrm{~d}=+5 \\
& d=\frac{+5}{-5}
\end{aligned}
$$

Substitute $d=-1$ in equ (1)

$$
\begin{aligned}
& \mathrm{a}+2 \mathrm{~d}=15 \Rightarrow \mathrm{a}+2(-1)=15 \Rightarrow \mathrm{a}-2=15 \\
& \Rightarrow \mathrm{a}=15+2=17 \\
& a_{10}=a+9 d=17+9(-1) \Rightarrow 17-9=8 \\
& \therefore d=-1 \text { and } a_{10}=8
\end{aligned}
$$

10. The first and the last terms of an $A P$ are 17 and 350 respectively. If the common difference is 9 , how many terms are there and what is their sum?

Sol: Given A.P in which $\mathrm{a}=17$
Last term $=l=350$
Common, difference, $\mathrm{d}=9$
We know that, $\mathrm{a}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d}$

$$
350=17+(n-1)(9)
$$

$$
\begin{aligned}
& 350=17+9 n-9 \\
& 350=9 n+8 \\
& 9 n=350-8 \\
& 9 n=342 \\
& n=\frac{342}{9} \\
& \therefore n=38
\end{aligned}
$$

Now $S_{n}=\frac{n}{2}[a+l]$

$$
\begin{aligned}
& S_{38}=\frac{38}{2}[17+350] \\
& =19 \times 367 \\
& =6973
\end{aligned}
$$

$$
\therefore \mathrm{n}=38 ; \mathrm{S}_{\mathrm{n}}=6973 .
$$

11. Which term of the G.P: $2,2 \sqrt{2}, 4 \ldots \ldots$ is $\mathbf{1 2 8}$ ?

Sol: Here $\mathrm{a}=2, r=\frac{2 \sqrt{2}}{2}=\sqrt{2}$

Let 128 be the $\mathrm{n}^{\text {th }}$ term of the GP
Then $\mathrm{a}_{\mathrm{n}}=\operatorname{ar}^{\mathrm{n}-1}=128$

$$
\begin{aligned}
& 2(\sqrt{ } 2)^{\mathrm{n}-1}=128 \\
& (\sqrt{2})^{n-1}=\frac{128}{2} \\
& (\sqrt{2})^{n-1}=64
\end{aligned}
$$

$$
\begin{aligned}
& 2^{\frac{n-1}{2}}=2^{6} \\
& \frac{n-1}{2}=6 \\
& \mathrm{n}-1=6 \times 2 \\
& \mathrm{n}-1=12 \\
& \mathrm{n}=12+1 \\
& \therefore \mathrm{n}=13
\end{aligned}
$$

Hence 128 is the $13^{\text {th }}$ term of the G.P.
12. Which term of the G.P. is $2,8,32, \ldots .$. is 512 ?

Sol: Given G.P. is $2,8,32, \ldots$. is 512

$$
\begin{aligned}
& \mathrm{a}=2, r=\frac{a_{2}}{a_{1}}=\frac{8}{2}=4 \\
& \mathrm{a}_{\mathrm{n}}=512
\end{aligned}
$$

$$
\mathrm{a}_{\mathrm{n}}=\mathrm{ar}^{\mathrm{n}-1}=512
$$

| 2 | 512 |
| :--- | :--- |
| 2 | 256 |
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 2 |
| 2 | 4 |
| 2 | 2 |
| 1 |  |

$$
\begin{aligned}
& 2(4)^{\mathrm{n}-1}=512 \\
& 2\left(2^{2}\right)^{\mathrm{n}-1}=2^{9} \\
& 2^{2 \mathrm{n}-1}=2^{9} \quad\left(\because a^{m} \times a^{n}=a^{m+n}\right)
\end{aligned}
$$

$$
\begin{aligned}
& 2 \mathrm{n}-1=9 \\
& 2 \mathrm{n}=9+1 \\
& n=\frac{10}{2}=5 \\
& \therefore \mathrm{n}=5
\end{aligned}
$$

512 is the $5^{\text {th }}$ term of the given G.P.
13. $\sqrt{3}, 3,3 \sqrt{3}$. is 729 ?

Sol: Given G.P. is $\sqrt{3}, 3,3 \sqrt{3}$. is 729

$$
\begin{aligned}
& a=\sqrt{ } 3 \\
& r=\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{3 \sqrt{3}}{3}=\sqrt{3} \\
& \begin{array}{l|l|}
3 & 729 \\
3 & 243 \\
3 & 21 \\
3 & 81 \\
3 & 27 \\
3 & 9 \\
\hline & 1
\end{array} \\
& \mathrm{a}_{\mathrm{n}}=729 \\
& \mathrm{a}_{\mathrm{n}}=\mathrm{a} \cdot \mathrm{r}^{\mathrm{n}-1}=729 \\
& \sqrt{3} \cdot(\sqrt{3})^{\mathrm{n}-1}=729 \\
& 3^{\frac{1}{2}} \cdot 3^{\frac{n-1}{2}}=3^{6} \\
& 3^{\frac{1}{2}+\frac{n-1}{2}}=3^{6} \\
& \left(\because a^{m} \times a^{n}=a^{m+n}\right) \\
& 3^{\frac{1+n-1}{2}}=3^{6}
\end{aligned}
$$

$$
\begin{aligned}
& 3^{\frac{n}{2}}=3^{6} \quad\left(\because a^{m}=a^{n} \Rightarrow m=n\right) \\
& \frac{n}{2}=6 \\
& \mathrm{n}=6 \times 2 \\
& \therefore \mathrm{n}=12
\end{aligned}
$$

$\therefore 729$ is the $12^{\text {th }}$ term of the given G.P.
14. In a nursery, there are 17 rose plants in the first row, 14 in the second row, 11 in the third row and so on. If there are 2 rose plants in the last row, find how many rows of rose plants are there in the nursery.

Sol: Number of plants in first row $=17$
Number of plants in second row $=14$
Number of plants in third row $=11$
$\therefore$ The series formed as $17,14,11,8,5,2$; the term are in A.P.
Here $\mathrm{a}=17, \mathrm{~d}=14-17=-3$

$$
\begin{aligned}
& a_{n}=2 \\
& a_{n}=a+(n-1) d=2 \\
& 17+(n-1)(-3)=2 \\
& 17-3 n+3=2 \\
& 20-3 n=2 \\
& 3 n=20-2 \\
& 3 n=18
\end{aligned}
$$

$$
\begin{aligned}
& n=\frac{18}{3} \\
& \therefore \mathrm{n}=6
\end{aligned}
$$

$\therefore$ There are 6 rows in the nursery.
15. Which term of the sequence $-1,3,7,11 \ldots \ldots$ is 95 ?

Sol: Let the A.P. $-1,3,7,11 \ldots . .95$

$$
a=-1 ; d=3-(-1)=3+1=4 ; a_{n}=95
$$

$$
\begin{aligned}
& a+(n-1) d=95 \\
& -1+(n-1)(4)=95 \\
& -1+4 n-4=95 \\
& 4 n-4=95+1 \\
& 4 n-4=96 \\
& 4 n=96+4 \\
& 4 n=100 \\
& n=\frac{100}{4}=25
\end{aligned}
$$

$\therefore 25^{\text {th }}$ term $=95$.
16. A sum of Rs. 280 is to be used to award four prizes. If each prize after the first is Rs. 20 less than its preceding prize. Find the value of each of the prizes.

Sol: The value of prizes form an A.P
$\therefore$ In A.P. $\mathrm{d}=-20$

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{n}}=280 \\
& \mathrm{n}=4
\end{aligned}
$$

$$
\begin{aligned}
& \frac{n}{2}[2 a+(n-1) d]=280 \\
& \frac{4}{2}[2 a+(4-1)(-20)]=280 \\
& 2[2 \mathrm{a}-60]=280 \\
& 2 \mathrm{a}-60=\frac{280}{2} \\
& 2 \mathrm{a}-60=140 \\
& 2 \mathrm{a}=140+60 \\
& 2 \mathrm{a}=200 \\
& a=\frac{200}{2} \\
& \therefore \mathrm{a}=100
\end{aligned}
$$

$\therefore$ The value of each of the prizes $=$ Rs 100 , Rs 80 , Rs 60 , Rs 40.
17. If the $8^{\text {th }}$ term of an A.P. is 31 and the $15^{\text {th }}$ term is $\mathbf{1 6}$ more than the $11^{\text {th }}$ term, find the A.P.

Sol: $\quad$ In an A.P. $a_{8}=31 \Rightarrow a_{8}=a+7 d=31$

$$
\begin{gathered}
a_{15}=16+a_{11} \Rightarrow a+14 d=16+a+10 d \\
14 d-10 d=16+a-a \\
4 d=16 \\
d=\frac{16}{4}=4 \\
a+7 d=31 \text { and } d=4 \\
a+7(4)=31
\end{gathered}
$$

$$
\begin{aligned}
& a+28=31 \\
& a=31-28 \\
& \therefore a=3
\end{aligned}
$$

$\therefore$ A.P. is $3,7,11,15,19$
18. Define Arithmetic progression and Geometric progression.

Sol: Arithmetic Progression: An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number ' d ' to the preceding term, except the first term. The fixed number ' $d$ ' is called the common difference.

Geometric Progression: A geometric progression (G.P) is a list of numbers in which each term is obtained by multiplying preceding term with a fixed number ' $r$ ' except first term. This fixed number is called common ratio (r).

## 4 Mark Questions

1. If the $3^{\text {rd }} \&$ the $9^{\text {th }}$ terms of an A.P. are 4 and -8 respectively. Which term of this AP is zero?

Sol: $\quad a_{3}=4, a_{9}=-8$

$$
\begin{gather*}
a_{3}=a+2 d=4  \tag{1}\\
a_{9}=a+8 d=-8  \tag{2}\\
\begin{array}{c}
(2)-(1) \text { we get } \\
a+8 d=-8 \\
a+2 d=4 \\
-\quad- \\
6 d=-12 \\
d=\frac{-12}{-6} \\
d=-2
\end{array}
\end{gather*}
$$

Substitute $\mathrm{d}=-2$ in the following equations

$$
\begin{aligned}
& a_{4}=a_{3}+d=4+(-2)=4-2=2 \\
& a_{5}=a_{4}+d=2+(-2)=2-2=0
\end{aligned}
$$

$\therefore 5^{\text {th }}$ term of the A.P becomes zero.
2. Find the $20^{\text {th }}$ term from the end of A.P: 3, 8, $13 \ldots \ldots . .253$

Sol: $\quad a=3, d=a_{2}-a_{1}=8-3=5, a_{n}=253$

$$
\begin{aligned}
& a_{n}=a+(n-1) d \\
& 253=3+(n-1)(5) \\
& 253-3=(n-1) 5
\end{aligned}
$$

$$
\begin{aligned}
& \frac{250}{5}=n-1 \\
& \mathrm{n}-1=50 \\
& \mathrm{n}=51
\end{aligned}
$$

$\therefore$ The $20^{\text {th }}$ term from the other end would be $\mathrm{n}-\mathrm{r}+1=51-20+1$

$$
=32
$$

$$
\begin{aligned}
a_{32}= & a+31 d \\
& =3+31(5) \\
& =3+155 \\
& =158
\end{aligned}
$$

The $20^{\text {th }}$ term is 158 .
3. The sum of the $4^{\text {th }}$ and $8^{\text {th }}$ terms of an A.P. is 24 and the sum of the $6^{\text {th }}$ and $\mathbf{1 0}^{\text {th }}$ term is 44. Find the first three terms of the A.P.

Sol: $\quad 4^{\text {th }}+8^{\text {th }}=24$

$$
\begin{aligned}
& \Rightarrow(\mathrm{a}+3 \mathrm{~d})+(\mathrm{a}+7 \mathrm{~d})=24 \\
& \Rightarrow \mathrm{a}+3 \mathrm{~d}+\mathrm{a}+7 \mathrm{~d}=24 \\
& \Rightarrow 2 \mathrm{a}+10 \mathrm{~d}=24 \\
& \Rightarrow 2(\mathrm{a}+5 \mathrm{~d})=24 \\
& \Rightarrow(a+5 d)=\frac{24}{2}
\end{aligned}
$$

$\therefore a+5 d=12$

$$
\begin{equation*}
6^{\mathrm{th}}+10^{\mathrm{th}}=44 \tag{1}
\end{equation*}
$$

$$
\begin{aligned}
& \Rightarrow(\mathrm{a}+5 \mathrm{~d})+(\mathrm{a}+9 \mathrm{~d})=44 \\
& \Rightarrow \mathrm{a}+5 \mathrm{~d}+\mathrm{a}+9 \mathrm{~d}=44 \\
& \Rightarrow 2 \mathrm{a}+14 \mathrm{~d}=44 \\
& \Rightarrow 2(\mathrm{a}+7 \mathrm{~d})=44 \\
& \Rightarrow \mathrm{a}+7 \mathrm{~d}=\frac{44}{2}
\end{aligned}
$$

$\therefore \mathrm{a}+7 \mathrm{~d}=22$
(2) $-(1)=a+7 d=22$ $a+5 d=12$
$\qquad$

$$
2 \mathrm{~d}=10
$$

$$
d=\frac{10}{2}
$$

$$
\therefore \mathrm{d}=5 \text {. }
$$

Substitute d $=5$ in eq (1)
We get, $a+5(5)=12$
$\Rightarrow a+25=12$
$\Rightarrow \mathrm{a}=12-25$
$\Rightarrow \mathrm{a}=-13$
$\therefore$ The first three terms of A.P are

$$
\begin{aligned}
& a_{1}=a=-13 \\
& a_{2}=a+d=-13+5=-8 \\
& a_{3}=a+2 d=-13+2(5)=-13+10=-3 .
\end{aligned}
$$

4. Subba rao started work in $\mathbf{1 9 9 5}$ at an annual salary of Rs $\mathbf{5 0 0 0}$ and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Sol:

| Year | 1995 | 1996 | 1997 | 1998 | 1999 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Subba rao <br> salary | 5000 | 5200 | 5400 | 5600 | 5800 |

$5000,5200,5400,5600,5800 \ldots \ldots \ldots .$. is in A.P.

$$
\begin{aligned}
& \mathrm{a}_{\mathrm{n}}=\mathrm{a}+(\mathrm{n}-1) \mathrm{d} \\
& =5000+(\mathrm{n}-1)(200)=7000 \\
& =5000+200 \mathrm{n}-200=7000 \\
& =200 \mathrm{n}+4800=7000 \\
& =200 \mathrm{n}=7000-4800 \\
& =200 \mathrm{n}=2200 \\
& n=\frac{2200}{200} \\
& \therefore \mathrm{n}=11 . \\
& \therefore \text { The } 11^{\text {th }} \text { is } 7000 .
\end{aligned}
$$

$\therefore$ In the year 2005 his income reaches to Rs 7000 .
5. Given $a=2, d=8, S_{n}=90$. Find $n$ and $a_{n}$.

Sol:

$$
\begin{aligned}
& a_{n}=a+(n-1) d \\
& =2+(n-1) 8
\end{aligned}
$$

$$
\begin{aligned}
& =2+8 n-8 \\
& =8 n-6 \\
& \mathrm{a}=2, \mathrm{~d}=8, \mathrm{~S}_{\mathrm{n}}=90 \\
& S_{n}=\frac{n}{2}[2 a+(n-1) d]=90 \\
& \Rightarrow \frac{n}{2}[2(2)+(n-1) 8]=90 \\
& \Rightarrow \frac{n}{2}[4+8 n-8]=90 \\
& \Rightarrow \mathrm{n}[4+8 \mathrm{n}-8]=90 \times 2 \\
& \Rightarrow 4 \mathrm{n}+8 \mathrm{n}^{2}-8 \mathrm{n}=90 \times 2 \\
& \Rightarrow 4 \mathrm{n}+8 \mathrm{n}^{2}-8 \mathrm{n}=180 \\
& \Rightarrow 8 n^{2}+4 n-8 n=180 \\
& \Rightarrow 8 n^{2}-4 n-180=0 \\
& \Rightarrow 2 \mathrm{n}^{2}-\mathrm{n}-45=0 \\
& \Rightarrow 2 \mathrm{n}^{2}-10 \mathrm{n}+9 \mathrm{n}-45=0 \\
& \Rightarrow 2 \mathrm{n}(\mathrm{n}-5)+9(\mathrm{n}-5)=0 \\
& \Rightarrow(\mathrm{n}-5)(2 \mathrm{n}+9)=0 \\
& \mathrm{n}-5=0 \quad 2 \mathrm{n}+9=0 \\
& \mathrm{n}=5 \quad 2 \mathrm{n}=-9 \\
& n=\frac{-9}{2}
\end{aligned}
$$

But we cannot take negative values so, $n=5$

$$
\begin{aligned}
& \therefore a_{5}=a+4 d=2+4(8) \\
& \quad=2+32=34 . \\
& \therefore n=5 \text { and } a_{5}=34 .
\end{aligned}
$$

6. If the sum of first 7 terms of an A.P is 49 and that of 17 terms is 289 , find the sum of first $\mathbf{n}$ terms.

Sol: The sum of first 7 terms of an A.P $=7$.

$$
\begin{align*}
& S_{n}=\frac{n}{2}[2 a+(n-1) d], \text { where } \mathrm{S}_{\mathrm{n}}=49 \\
& \mathrm{n}=7, \text { then } \Rightarrow 49=\frac{7}{2}[2 a+(7-1) d] \\
& \Rightarrow 49 \times \frac{2}{7}=2 a+6 d \\
& \Rightarrow 14=2 \mathrm{a}+6 \mathrm{~d} \\
& \Rightarrow 14=2(\mathrm{a}+3 \mathrm{~d}) \\
& \quad \Rightarrow a+3 d=\frac{14}{2}=7 \\
& \therefore \mathrm{a}+3 \mathrm{~d}=7--\cdots-(1) \tag{1}
\end{align*}
$$

And the sum of 17 terms is 289 ,

$$
S_{n}=\frac{n}{2}[2 a+(n-1) d], \mathrm{S}_{\mathrm{n}}=289, \mathrm{n}=17
$$

Then $289=\frac{17}{2}[2 a+(17-1) d]$

$$
\begin{aligned}
& \Rightarrow 289 \times \frac{2}{17}=2 a+16 d \\
& \Rightarrow 34=2 \mathrm{a}+16 \mathrm{~d}
\end{aligned}
$$

$$
\Rightarrow 34=2(a+8 d) \Rightarrow a+8 d=\frac{34}{2}=17
$$

$$
\begin{equation*}
\mathrm{a}+8 \mathrm{~d}=17 \tag{2}
\end{equation*}
$$

(1) (2) by solving

$$
\begin{aligned}
& \mathrm{a}+8 \mathrm{~d}=17 \\
& \mathrm{a}+3 \mathrm{~d}=7 \\
& --\quad- \\
& \hline 5 \mathrm{~d}=10 \\
& d=\frac{10}{5} \\
& \therefore \mathrm{~d}=2
\end{aligned}
$$

Substitute d = 2 in eq(1), we get

$$
\begin{aligned}
& a+3 d=7 \Rightarrow a+3 \times 2=7 \Rightarrow a+6=7 \\
& \Rightarrow a=7-6 \\
& \therefore a=1
\end{aligned}
$$

The first ' n ' terms sum $S_{n}=\frac{n}{2}[2 a+(n-1) d]$
$\mathrm{a}=1, \mathrm{~d}=2$, then on substituting, we get

$$
\begin{aligned}
& S_{n}=\frac{n}{2}[2.1+(n-1) 2] \\
& S_{n}=\frac{n}{2}[2+2 n-2] \\
& =\frac{n}{2} \times 2 n=n^{2} \\
& \mathrm{~S}_{\mathrm{n}}=\mathrm{n}^{2}
\end{aligned}
$$

$\therefore$ The sum of first ' $n$ ' terms $\left(\mathrm{S}_{\mathrm{n}}\right)=\mathrm{n}^{2}$.
7. If the sum of the first $\mathbf{n}$ terms of an $A P$ is $\mathbf{4 n}-\mathbf{n}^{\mathbf{2}}$, what is the first term (remember the first term is $S_{1}$ )? What is the sum of first two terms? What is the second term? Similarly, find the $3^{\text {rd }}$, the $10^{\text {th }}$ and the nth terms.

Sol: The sum of the first ' $n$ ' terms of an A.P is $4 n-n^{2}$
First term $\mathrm{a}_{1}=\mathrm{S}_{1}=4 \times 1-1^{2}=4-1=3 \quad(\because \mathrm{n}=1)$
First sum of the two terms $=4 \times 2-2^{2}=8-4=4$

$$
S_{3}=4 \times 3-3^{2}=12-9=3 \quad a_{2}=S_{2}-S_{1}=4-3=1
$$

$\therefore$ Third term $\left(\mathrm{a}_{3}\right)=\mathrm{S}_{3}-\mathrm{S}_{2}=3-4=-1$

$$
\begin{aligned}
& S_{10}=4 \times 10-10^{2}=40-100=-60 \\
& S_{9}=4 \times 9-9^{2}=36-31=-45
\end{aligned}
$$

Tenth term $\left(\mathrm{a}_{10}\right)=\mathrm{S}_{10}-\mathrm{S}_{9}=-60-(-45)=-60+45=15$

$$
\mathrm{S}_{\mathrm{n}}=4 \mathrm{n}-\mathrm{n}^{2}
$$

$$
S_{n-1}=4(n-1)-(n-1)^{2}
$$

$$
=4 n-4-\left(n^{2}-2 n+1\right)
$$

$$
=-n^{2}+6 n-5
$$

The nth term $\mathrm{a}_{\mathrm{n}}=\mathrm{S}_{\mathrm{n}}-\mathrm{S}_{\mathrm{n}-1}$

$$
\begin{aligned}
& \Rightarrow a_{n}=4 n-n^{2}-\left(-n^{2}+6 n-5\right) \\
& =4 n-n^{2}+n^{2}-6 n+5 \\
& =5-2 n
\end{aligned}
$$

$\therefore \mathrm{S}_{1}=3, \mathrm{~S}_{2}=4, \mathrm{a}_{2}=1, \mathrm{a}_{3}=-1, \mathrm{a}_{10}=-15, \mathrm{a}_{\mathrm{n}}=5-2 \mathrm{n}$.
8. A sum of Rs $\mathbf{7 0 0}$ is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than it's preceding prize, find the value of each of the prizes.

Sol: First term $=$ Rs a
Each price is Rs 20 less than it's preceding prize, then the remaining prize of gift (a-20), (a-40) ----------- (a - 120), then
a, (a-20), (a-40) --------- (a-120) forms an A.P
so, $S_{n}=\frac{n}{2}\left[a+a_{n}\right]$. Here $\mathrm{S}_{\mathrm{n}}=700, \mathrm{n}=7, \mathrm{a}=\mathrm{a}, \mathrm{a}_{\mathrm{n}}=\mathrm{a}-120$, on substituting these values we get

$$
\begin{aligned}
& 700=\frac{7}{2}[a+a-120] \\
& 700 \times \frac{2}{7}=2 a-120 \\
& 200=2 \mathrm{a}-120 \\
& 320=2 \mathrm{a} \\
& a=\frac{320}{2} \\
& \therefore \mathrm{a}=160
\end{aligned}
$$

$\therefore$ Each value of the prize Rs 160 , Rs 140 , Rs 120 , Rs 100 , Rs 80 , Rs 60 , Rs 40 .
9. The number of bacteria in a certain culture triples every hour if there were 50 bacteria present in the culture originally. Then, what would be number of bacteria in fifth, tenth hour.

Sol: The no of bacteria in a culture triples every hour.
$\therefore$ No of bacteria in first hour $=50$
No of bacteria in second hour $=3 \times 50=150$
No of bacteria in third hour $=3 \times 150=450$
$\therefore 50,150,450 \ldots$... would forms an G.P.
First term (a) $=50$
Common ratio $(\mathrm{r})=\frac{t_{2}}{t_{1}}=\frac{150}{50}=3$
nth term $a_{n}=a^{n-1}$
No of bacteria in $5^{\text {th }}$ hour $=50 \times 3^{5-1}=50 \times 81=4050$
No of bacteria in $10^{\text {th }}$ hour $=50 \times 3^{10-1}=50 \times 19683$

$$
=984150
$$

$\therefore 3^{\text {rd }}, 5^{\text {th }}, 10^{\text {th }}$ hours of bacteria number $=450,4050,984150$.
10. The $4^{\text {th }}$ term of a G.P is $\frac{2}{3}$ the seventh term is $\frac{16}{81}$. Find the Geometric series.

Sol: The $4^{\text {th }}$ term of G.P $=\frac{2}{3}$, and the seventh term is $\frac{16}{81}$.

$$
\begin{aligned}
\text { i.e. } a r^{3} & =\frac{2}{3} \longrightarrow(1) \\
a r^{6} & =\frac{16}{81} \longrightarrow(2)
\end{aligned}
$$

$\frac{(2)}{(1)}$,then we get

$$
\begin{aligned}
& \frac{a r^{6}}{a r^{3}}=\frac{\frac{16}{81}}{\frac{2}{3}} \\
& \Rightarrow r^{3}=\frac{8}{27} \\
& \Rightarrow r^{3}=\left(\frac{2}{3}\right)^{3} \\
& \therefore r=\frac{2}{3}
\end{aligned}
$$

Now substitute $r=\frac{2}{3}$ in eq(1), we get

$$
\begin{aligned}
& a .\left(\frac{2}{3}\right)^{3}=\frac{2}{3} \Rightarrow a \times \frac{8}{27}=\frac{2}{3} \\
& \therefore a=\frac{9}{4}, r=\frac{2}{3}
\end{aligned}
$$

Then A.P. $\mathrm{a}, \mathrm{ar}, \mathrm{ar}^{2}, \mathrm{ar}^{3}, \ldots \ldots$.

$$
\begin{aligned}
& \frac{9}{4}, \frac{9}{4} \times \frac{2}{3}, \frac{9}{4} \times \frac{2^{2}}{3^{2}}, \frac{9}{4} \times \frac{2^{3}}{3^{3}} \\
& \Rightarrow \frac{9}{4}, \frac{3}{2}, 1, \frac{2}{3}, \ldots \ldots \ldots .
\end{aligned}
$$

11. If the geometric progressions $162,54,18 \ldots \ldots \ldots$ and $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}, \ldots \ldots \ldots$. . have their $n$ n term equal. Find its value of $\mathbf{n}$ ?

Sol: $162,54,18 \ldots \ldots$.
Here $\mathrm{a}=162, r=\frac{a_{2}}{a_{1}}=\frac{54}{162}=\frac{1}{3}$

The $\mathrm{n}^{\text {th }}$ term $=\operatorname{ar}^{\mathrm{n}-1}=162\left(\frac{1}{3}\right)^{n-1}$.

$$
\frac{2}{81}, \frac{2}{27}, \frac{2}{9}, .
$$

Here $a=\frac{2}{81}, r=\frac{a_{2}}{a_{1}}=\frac{\frac{2}{27}}{\frac{2}{81}}=\frac{2}{27} \times \frac{81}{2}=3$

$$
\begin{equation*}
n^{\text {th }} \text { term }=a \cdot r^{n-1}=\frac{2}{81} \cdot(3)^{n-1} \tag{2}
\end{equation*}
$$

Given that $\mathrm{n}^{\text {th }}$ terms are equal

$$
\begin{array}{ll}
\Rightarrow 162 \times\left(\frac{1}{3}\right)^{n-1}=\frac{2}{81} \times(3)^{n-1} & (\because \text { From }(1) \&(2)) \\
\Rightarrow 3^{n-1} \times 3^{n-1}=162 \times \frac{81}{2} & \\
\Rightarrow 3^{n-1+n-1}=81 \times 81 \\
\Rightarrow 3^{2 n-2}=3^{4} \times 3^{4} & {\left[a^{m} \cdot a^{n}=a^{m+n}\right]} \\
\Rightarrow 3^{2 n-2}=3^{8} & \\
\Rightarrow 2 n-2=8
\end{array}
$$

[if the bases are equal, exponents are also equal]

$$
\begin{aligned}
& 2 n=8+2 \\
& n=\frac{10}{2} \\
& \therefore n=5
\end{aligned}
$$

$\therefore$ The $5^{\text {th }}$ terms of the two G.P. s are equal.
12. Find the $12^{\text {th }}$ term of a G.P. whose $8^{\text {th }}$ term is 192 and the common ratio is 2 .

Sol: Given G.P $\mathrm{a}_{8}=192$ \& $\mathrm{r}=2$

$$
\begin{aligned}
& \quad \mathrm{a}_{\mathrm{n}}=\mathrm{a} \cdot \mathrm{r}^{\mathrm{n}-1} \\
& \mathrm{a}_{8}=\mathrm{a}(2)^{8-1}=192 \\
& \quad a \cdot 2^{7}=192 \Rightarrow a=\frac{192}{2^{7}}=\frac{192}{128}=\frac{12}{8}=\frac{3}{2} \\
& \therefore a_{12}=a . r^{11}=\frac{3}{2} \times(2)^{11} \\
& =3 \times 2^{10}=3 \times 1024 \\
& =3072
\end{aligned}
$$

13. In an A.P $2^{\text {nd }}, 3^{\text {rd }}$ terms are $14 \& 18$ and find sum of first 51 terms?

Sol: $2^{\text {nd }}$ term:
$a+d=14$
$a+2 d=18$

$$
\begin{array}{r}
a+d=14  \tag{2}\\
a+2 d=18 \\
-\quad- \\
-d=-4 \\
d=4
\end{array}
$$

Substitute d=4 in eq (1)

$$
\begin{aligned}
& \mathrm{a}+4=14 \\
& \mathrm{a}=14-4 \\
& \mathrm{a}=10 \\
& S_{n}=\frac{n}{2}[2 a+(n-1) d] \\
& S_{n}=\frac{51}{2}[2(10)+(51-1)(4)]
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{51}{2}[20+(50)(4)] \\
& =\frac{51}{2}[20+200] \\
& =\frac{51}{2}[220] \\
& =51 \times 110 \\
& =5610
\end{aligned}
$$

$\therefore$ The sum of first 51 term $=5610$.
14. In an A.p, the sum of the ratio of the $m$ and $n$ terms in $m^{2}: n^{2}$, then show that $m^{\text {th }}$ term and $n^{\text {th }}$ terms ratio is $(2 m-1):(2 n-1)$.
Sol: AP, first term $=\mathrm{a}$
Common difference $=\mathrm{d}$

$$
\begin{aligned}
& S_{m}=\frac{m}{2}[2 a+(m-1) d] \\
& S_{n}=\frac{n}{2}[2 a+(n-1) d]
\end{aligned}
$$

Given $\frac{S_{m}}{S_{n}}=\frac{m^{2}}{n^{2}}$

$$
\begin{aligned}
& \frac{\frac{m}{2}[2 a+(m-1) d]}{\frac{n}{2}[2 a+(n-1) d]}=\frac{m^{2}}{n^{2}} \\
& {[2 \mathrm{a}+(\mathrm{m}-1) \mathrm{d}] \mathrm{n}=[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}] \mathrm{m}} \\
& 2 \mathrm{a}(\mathrm{n}-\mathrm{m})=\mathrm{d}[(\mathrm{n}-1) \mathrm{m}-(\mathrm{m}-1) \mathrm{n}] \\
& 2 \mathrm{a}(\mathrm{n}-\mathrm{m})=\mathrm{d}(\mathrm{n}-\mathrm{m}) \\
& \mathrm{d}=2 \mathrm{a} \\
& \frac{T_{m}}{T_{n}}=\frac{a+(m-1) 2 a}{a+(n-1) 2 a}=\frac{a+2 a m-2 a}{a+2 a n-2 a} \\
& =\frac{2 a m-a}{2 a n-a}=\frac{a(2 m-1)}{a(2 n-1)} \\
& \frac{T_{m}}{T_{n}}=\frac{2 m-1}{2 n-1}
\end{aligned}
$$

15. The sum of $n, 2 n, 3 n$ terms of an A.P are $S_{1}, S_{2}, S_{3}$ respectively prove $\mathrm{S}_{3}=\mathbf{3}\left(\mathrm{S}_{2}-\mathrm{S}_{1}\right)$
Sol: In an A.P. first term is a and the common difference is d .

$$
\begin{aligned}
& S_{1}=\frac{n}{2}[2 a+(n-1) d] \longrightarrow(1) \\
& S_{2}=\frac{2 n}{2}[2 a+(2 n-1) d] \longrightarrow(2) \\
& S_{3}=\frac{3 n}{2}[2 a+(3 n-1) d] \longrightarrow(3) \\
& S_{2}-S_{1}=\frac{2 n}{2}[2 a+(2 n-1) d]-\frac{n}{2}[2 a+(n-1) d] \\
& S_{2}-S_{1}=\frac{n}{2}[2 a+(3 n-1) d] \\
& 3\left(S_{2}-S_{1}\right)=\frac{3 n}{2}[2 a+(3 n-1) d]=S_{3} \\
& \mathrm{~S}_{3}=3\left(\mathrm{~S}_{2}-\mathrm{S}_{1}\right)
\end{aligned}
$$

## Multiple Choice Questions

1. The $n^{\text {th }}$ term of G.P is $\mathbf{a}_{\mathrm{n}}=\mathbf{a r}{ }^{\mathrm{n}-1}$ where ' $r$ ' represents
a) First term
b) Common difference
c) Common ratio
d) Radius
2. The $n^{\text {th }}$ term of a G.P is $2(0.5)^{n-1}$ then $r=$
a) 5
b) $\frac{1}{7}$
c) $\frac{1}{3}$
d) 0.5
3. In the A.P $10,7,4 \ldots-62$, then $11^{\text {th }}$ term from the last is $\qquad$
a) -40
b) -23
c) -32
d) 10
4. Which term of the G.P $\frac{1}{3}, \frac{1}{9}, \frac{1}{27} \ldots \ldots$ is $\frac{1}{2187}$ ?
a) 12
b) 8
c) 7
d) None
5. $n-1, n-2, n-3, \ldots \ldots a_{n}=\ldots$.
a) $n$
b) 0
c) -1
d) $n^{2}$
6. In an A.P $\mathrm{a}=-7, \mathrm{~d}=5$ then $\mathrm{a}_{18}=\ldots$
a) 71
b) 78
c) 87
d) 12
7. $2+3+4+\ldots .+100=\ldots$.
a) 5050
b) 5049
c) 5115
d) 1155
8. $-1, \frac{1}{4}, \frac{3}{2}, \ldots . . s_{81}=\ldots$.
a) 3418
b) 8912
c) 3963
d) 3969
9. In G.P, $1^{\text {st }}$ term is 2 , $Q$ common ratio is -3 then $7^{\text {th }}$ term is
a) 1458
b) -1458
c) 729
d) -729
10. $1,-2,4,-8, \ldots$ is a .... Progression
a) A.P
b) G.P
c) Both
d) None of these
11. Common difference in $\frac{1}{2}, 1, \frac{3}{2} \ldots \ldots$
a) $\frac{-1}{2}$
b) $\frac{1}{2}$
c) 2
d) -2
12. $\sqrt{3}, 3,3 \sqrt{3} \ldots \ldots$ is a
a) A.P
b) G.P
c) Harmonic progression
d) Infinite progression
13. $a=\frac{1}{3}, d=\frac{4}{3}$, the $\mathbf{8}^{\text {th }}$ term of an A.P is $\qquad$ [ ]
a) $\frac{7}{3}$
b) $\frac{29}{3}$
c) $\frac{29}{9}$
d) $\frac{29}{24}$
14. Arithmetic progression in which the common difference is 3 . If $\mathbf{2}$ is added to every term of the progression, then the common difference of new A.P. [ ]
a) 5
b) 6
c) 3
d) 2
15. In an A.P. first term is $\mathbf{8}$ common difference is 2 , then which term becomes zero
a) $6^{\text {th }}$ term
b) $7^{\text {th }}$ term
c) $4^{\text {th }}$ term
d) $5^{\text {th }}$ term
16. $4,8,12,16$, $\qquad$
$\qquad$ series
a) Arithmetic
b) Geometric
c) Middle
d) Harmonic
17. Next 3 terms in series 3, 1, -1, -3. $\qquad$
a) $-5,-7,-9$
b) $5,7,9$
c) $4,5,6$
d) $-9,-11,-13$
18. If $x, x+2 \& x+6$ are the terms of G.P. then $x$ $\qquad$ [ ]
a) 2
b) -4
c) 3
d) 7
19. In G.P. $a_{p+q}=m, a_{p-q}=n$. Then $a_{p}=$
a) $m^{2} n$
b) $\frac{m}{n}$
c) $\sqrt{m n}$
d) $m \sqrt{n}$
20. $3+6+12+24 \ldots \ldots$. Progression, the $n^{\text {th }}$ term is $\qquad$
a) $3.2^{\mathrm{n}-1}$
b) $-3.2^{\mathrm{n}-1}$
c) $2^{n+1}$
d) $2.3^{\mathrm{n}-1}$
21. $\mathbf{a}_{12}=37, d=3$, then $S_{12}=$ $\qquad$
a) 264
b) 246
c) 4
d) 260
22. In the garden, there are 23 roses in the first row, in the $2^{\text {nd }}$ row there are 19. At the last row there are 7 trees, how many rows of rose trees are there?
a) 10
b) 9
c) 11
d) 7
23. From 10 to 250 , how many multiples of $\mathbf{4}$ are $\qquad$
a) 40
b) 60
c) 45
d) 65
24. The taxi takes Rs. 30 for $\mathbf{1}$ hour. After for each hour Rs. 10, for how much money can be paid \& how it forms progression
a) Geometric progressions
b) Harmonic progression
c) Series Progressions
d) Arithmetic progression

Key

1) C
2) $D$
3) C
4) C
5) B
6) B
7) $B$
8) $D$
9) A
10) $B$
11) $B$
12) $B$
13) B
14) C
15) D
16) A
17) A
18) A
19) C
20) A 21) B
21) $B$
22) $B$
23) D

## Bit Blanks

1. The sum of first 20 odd numbers $\qquad$
2. $10,7,4, \ldots . . a_{30}=$ $\qquad$
3. $1+2+3+4+\ldots .+100=$ $\qquad$
4. In the G.P $25,-5,1,-\frac{1}{5} \ldots \mathrm{r}=$ $\qquad$
5. The reciprocals of terms of G.P will form $\qquad$
6. If $-\frac{2}{7}, \mathrm{x},-\frac{7}{2}$ are in G.P. Then $\mathrm{x}=$ $\qquad$
7. $1+2+3+\ldots .+10=$ $\qquad$
8. If $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are in G.P, then $\frac{b}{a}=$ $\qquad$
9. $x, \frac{4 x}{3}, \frac{5 x}{3}, \ldots . . a_{6}=$
10. In a G.P $\mathrm{a}_{4}=$ $\qquad$
11. $\frac{1}{1000}, \frac{1}{100}, \frac{1}{10}, 1$ are in $\qquad$
12. The $10^{\text {th }}$ term from the end of the A.P; $4,9,14 \ldots \ldots 254$ is $\qquad$
13. In a G.P $\mathrm{a}_{\mathrm{n}-1}=$ $\qquad$
14. In an A.P $\mathrm{s}_{\mathrm{n}}-\mathrm{s}_{\mathrm{n}-1}=$ $\qquad$
15. $1.2+2.3+3.4+\ldots .5$ terms $=$ $\qquad$
16. In a series $a_{n}=\frac{n(n+3)}{n+2}, a_{17}=-\cdots-$
17. $-3,-\frac{1}{2}, 2, \ldots$ A.P, the $\mathrm{n}^{\text {th }}$ term $\qquad$
18. $a_{3}=5 \& a_{7}=9$, then find the A.P $\qquad$
19. The $\mathrm{n}^{\text {th }}$ term of the G.P $2(0.5)^{\mathrm{n}-1}$, then the common ratio $\qquad$
20. $4,-8,16,-32$ then find the common ratio is $\qquad$
21. The nth term $t_{n}=\frac{n}{n+1}$ then $t_{4}=$ $\qquad$
22. In an A.P $1=28, s_{n}=144 \&$ total terms are 9 , then the first term is
23. In an A.P $11^{\text {th }}$ term is 38 and $16^{\text {th }}$ term is 73 , then common difference of A.P is
24. In a garden there are 32 rose flowers in first row and 29 flowers in $2^{\text {nd }}$ row, and 26 flowers in $3^{\text {rd }}$ row, then how many rose trees are there in the $6^{\text {th }}$ row is $\qquad$ 25. $-5,-1,3,7 \ldots$ Progression, then $6^{\text {th }}$ term is
25. In Arithmetic progression, the sum of $n^{\text {th }}$ terms is $4 n-n^{2}$, then first term is $\qquad$

## Key

1) 400 ;
2) -77 ;
3) 5050 ;
4) $\left(-\frac{1}{5}\right)$;
5) Geometric Progression;
6) $\pm 1$;
7) 55 ;
8) $\frac{c}{b}$;
9) $\frac{8 x}{3}$;
10) $\mathrm{ar}^{3}$;
11) G.P.;
12) 209 ;
13) $a r^{n-2}$;
14) $a_{n}$;
15) 70 ;
16) $\frac{340}{19}$;
17) $\frac{1}{2}(5 n-11)$;
18) $3,4,5,6,7$;
19) 0.5 ;
20) -2 ;
21) $\frac{4}{5}$;
22) 4 ;
23) 7 ;
24) 17 ;
25) 15 ; 26) 3 .
