Chapter –10

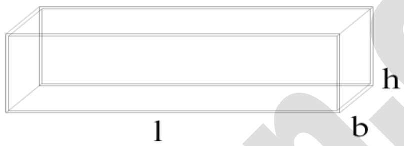
Mensuration

Cuboid:

l: length, b:breadth ,h:height

Lateral Surface Area(LSA) or Curved Surface Area (CSA)

=2h (l+b)

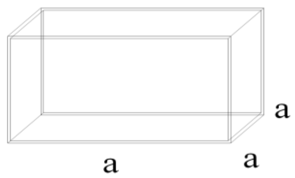


Total Surface Area (TSA) =2(lb+bh+hl)

Volume=lbh

Cube:

a: Side of the cube

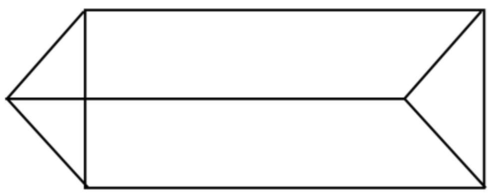


Lateral surface Area(LSA) =$4a^2$

Total surface Area(TSA)= **$6a^2$**

Volume= **a^3**

Right Prism:

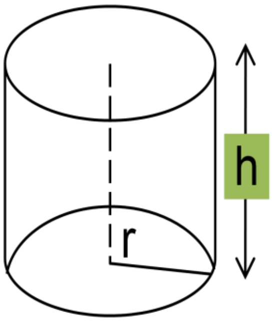


LSA= perimeter of base $\times$ height

TSA= LSA+2(area of the end surface)

Volume= Area of base $\times$ height

Regular Circular Cylinder



r: radius of the base

h: height

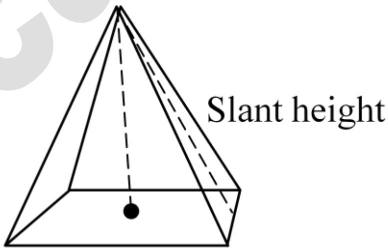
LSR = 2πrh

TSA = **2**πr(h+r)

Volume= $\pi r^2 h$

Right Pyramid

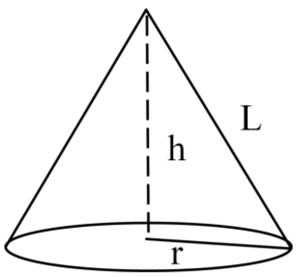
LSA=$\frac{1}{2}$(Perimeter of base) $\times$ slant height



TSA= LSA+ area of the end surface

Volume = $\frac{1}{3}$Area of base $\times$ height.

Right circular cone:



r : radius of the base, h:height ; l: slant height

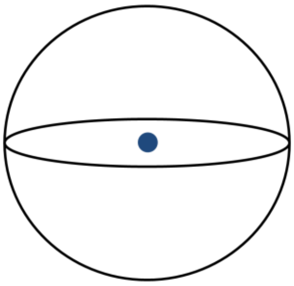
LSA = πrl

TSA = $\pi r(l + r)$

Volume $\frac{1}{3}\pi r^2 h$

Slant height $l = \sqrt{h^2 + r^2}$

Sphere:



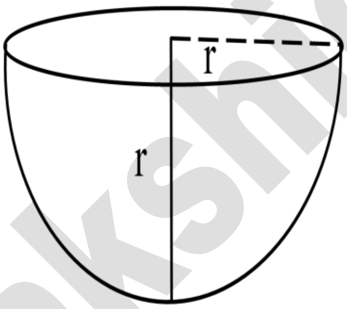
r : radius

LSA = $4\pi r^2$

TSA = $4\pi r^2$

Volume $\frac{4}{3}\pi r^3$

Hemisphere:



r : radius

LSA = $2\pi r^2$

TSA = $3\pi r^2$

Volume = $\frac{2}{3}\pi r^3$

- If A sphere, a cylinder and a cone are of the same radius and same height them the ratio of their curved surface areas are 4 : 4 : √5
- If A cylinder and cone have bases of equal radii and are of equal heights, then their volumes are in the ratio of 3 : 1
- If A sphere is inscribed in a cylinder then the surface of the sphere equal to the curved surface of the cylinder

1 Mark Problems

1. Write the formula to find the volume of cylinder.

A. $\pi r^2 h$

Where r : radius of the base

h : height.

2. Find the ratio between lateral surface area and total surface area of cube?

A. lateral surface area of cube = $4a^2$

Total surface area of cube = $6a^2$

$4a^2 : 6a^2$

$4 : 6$

$2 : 3$

3. The diagonal of a cube is 6√3 cm then find its lateral surface area?

A. The diagonal of a cube is = $\sqrt{3}a$

$6\sqrt{3} = \sqrt{3}a$

$6 = a$

Lateral surface area of cube = $4a^2$

$= 4 \times 6^2$

$= 4 \times 6 \times 6$

$= 4 \times 36$

= 144 sq.cm.

4. What is the largest chord of the circle?

A. The 'Diameter' is the largest chord of the circle.

5. Find the volume of a sphere of radius 2.1cm.

A. Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1$$

$= 38.808\ cm^3$.

6. Find the total surface area of a hemisphere of diameter 7cm.

A. $r = \frac{7}{2} cm$

T.S.A $= 3\pi r^2$

$$= 3 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{231}{2}$$

$= 115.5\ cm^2$.

7. What is the lateral surface area of cube.

A. $4a^2$, where a: side of the cube.

8. Find the total surface area of regular circular cylinder.

A. $2\pi r(r + h)$

Where, r : radius of the base

h : height.

9. Find the circumference of a circle of radius 8.4 cm

A. r = 8.4 cm

Circumference, $c = 2\pi r$

$$= 2 \times \frac{22}{7} \times 8.4cm$$

= 52.8 cm.

10. In a cuboid l = 5cm, b = 3cm, h = 2cm. Find its volume?

A. volume v = lbh

$= 5 \times 3 \times 2$

$= 15 \times 2$

$= 30\ cm^3$.

11. Find the surface area of sphere of radius 2.1cm.

A. Radius of sphere (r) = 2.1cm

Surface area of sphere = $4\pi r^2$

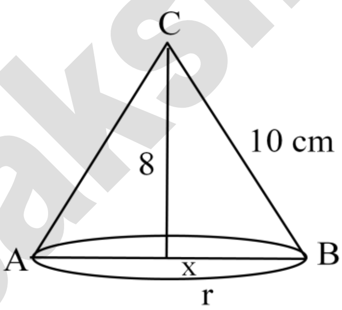
$$= 4 \times \frac{22}{7} \times (2.1)^2$$

$$= 4 \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10}$$

$$= \frac{1386}{25}$$

$= 55.44 cm^2$.

12. In the figure find r.



A. $10^2 = 8^2 + r^2$

$100 = 64 + r^2$

$r^2 = 100 - 64$

$r^2 = 36$

$r = \sqrt{36}$

$r = 6$

$\therefore\ r = 6cm.$

13. In a hemisphere r = 8cm, find CSA.

A. r = 8cm

CSA = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 8 \times 8\ cm^2$$

$$= \frac{2816}{7} cm^2$$

14. The area of the base of a cylinder is 616 sq.cm. Then find its radius

A. The area of the base of a cylinder = 616

We know that area base cylinder = $\pi\ r^2$.

$\therefore\ \pi\ r^2 = 616$

$$\frac{22}{7} \times r^2 = 616$$

$$r^2 = 616 \times \frac{7}{22}$$

$r^2 = 196$

$r = \sqrt{196}$

r = 14 cm.

15. Find T.S.A of a solid hemisphere whose radius is 7cm.

A. Total surface area of hemisphere = $3\pi r^2$

$$= 3 \times \frac{22}{7} \times 7 \times 7$$

$= 21 \times 22$

$= 462$ sqcm.

16. The diagonal of square is $7\sqrt{2}$cm. Then find its area.

A. The diagonal of a square = $\sqrt{2}.a$, a is the side of a square

$\sqrt{2}.a = 7\sqrt{2}$

$$a = \frac{7\sqrt{2}}{\sqrt{2}}$$

A = 7

Area of square = a × a

$= 7 \times 7$

$= 49\ cm^2$.

17.If the ratio of radii of two spheres is 2 : 3. Then find the ratio of their surface areas.

A. Lateral surface area of sphere = $4\pi r^2$

Ratio of radii of two spheres = 2: 3 ⇒ 2x : 3x

$4\ \pi\ (2x)^2 : 4\pi\ (3x)^2$

$4\pi\ 4x^2 : 4\pi\ 9x^2$

4 : 9

The ratio of surface areas are 4 : 9.

18. Find the surface area of hemispherical bowl whose radius is 21 cm.

A. surface area of Hemisphere = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 21 \times 21$$

$= 44 \times 63$

$= 2772\ cm^2$.

19. Find the T.S.A. of cube whose edge is 1cm.

A. T.S.A of cube = $6a^2 = 6(1)^2$

$= 6\ cm^3$.

2 Marks Problems

1.The radius of a conical tent is 7 meters and its height is 10 meters. Calculate the length of canvas used in making the tent if width of canvas is 2m.

A. If the the radius of conical tent is given (r) = 7 metes

Height (h) = 10m.

$\therefore$ So, the slant height of the cone $l^2 = r^2 + h^2 \Rightarrow l =$

$$= \sqrt{r^2 + h^2}$$

$$= \sqrt{49 + 100}$$

$$= \sqrt{149} = 12.2m.$$

Now, surface area of the tent = πrl

$$= \frac{22}{7} \times 7 \times 12.2m^2$$

$$= 268.4 \text{ m}^2.$$

Area of canvas used = 268.4m^2

It is given the width of the canvas = 2m.

Length of canvas used $= \frac{Area}{width} = \frac{268.4}{2} = 134.2cm.$

2.An oil drum is in the shape of a cylinder having the following dimensions: diameter is 2m and height is 7 meters. The painter charges Rs. 3 per m^2 to paint the drum. Find the total charges to be paid to the printer for 10 drums?

A. It is given that diameter of the (oil drum) cylinder = 2 m

Radius of cylinder $= \frac{d}{2} = \frac{2}{2} = 1m.$

Total surface area of a cylindrical drum = $2 \times \pi r (r + h)$

$$= 2 \times \frac{22}{7} \times 1(1+7)$$

$$= 2 \times \frac{22}{7} \times 8$$

$$\frac{352}{7} m^2$$

$$= 50.\ 28m^2$$

So, the total surface area of a drum = $50.28m^2$

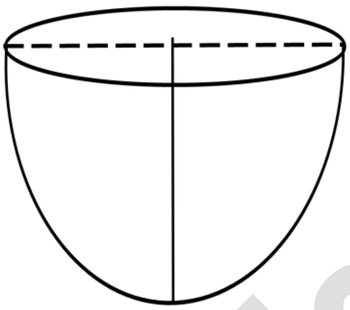
Painting charge per $1m^2$ = Rs. 3

Cost of painting of 10 drums = $50.28 \times 3 \times 10$

= Rs. 1508.40.

3.A company wanted to manufacture 1000 hemispherical basins from a thin steel sheet. If the radius of hemispherical basin is 21cm, find the required area of steel sheet to manufacture the above hemispherical basins?

A. Radius of the hemispherical basin (r) = 21cm



Surface area of a hemispherical basin = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 21 \times 21$$

$$= 2772\ cm^2$$

So, surface area of a hemispherical basin = $2772\ cm^2$

Hence, the steel sheet required for one basin = $2772\ cm^2$

Total area of steel sheet required for 1000

basins = 2772×1000

$= 2772000\ cm^2$

$= 277.2m^2$.

4. Find the volume and surface area of a sphere of radius 2.1cm.

A. Radius of sphere (r) = 2.1 cm

Surface area of sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (2.1)^2$$

$$= 4 \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10}$$

$$= \frac{1386}{25}$$

$= 55.44 cm^2$.

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1$$

$= 38.808 cm^3$.

5. Find the volume and the total surface area of a hemisphere of radius 3.5cm.

A. Radius of sphere (r) is 3.5 cm $= \frac{7}{2} cm$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{539}{6}$$

$= 89.83\ cm^3$

Total surface area = $3\pi r^2$

$$= 3 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

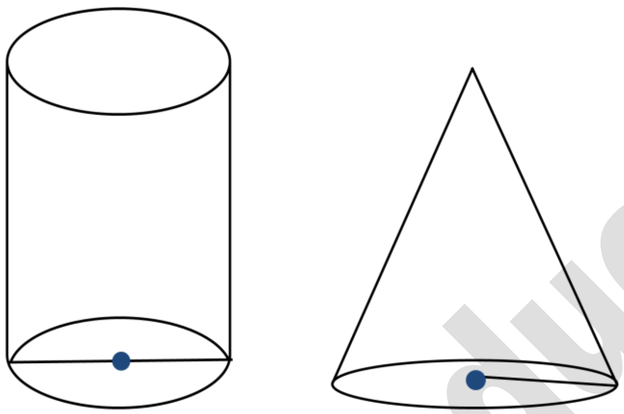
$$= \frac{231}{2}$$

$= 115.5cm^2$.

6.The lateral surface area of a cylinder is equal to the curved surface area of a cone. If the radius be the same, find the ratio of the height of the cylinder and slant height of the cone.

A. Given, L.S.A of cylinder = C.S.A of the cone.

The dimensions are:



Cylinder	**cone**
Radius = r	Radius = r
Height = h	slant height = l
L.S.A = $2\pi rh$	C.S.A = πrl

If radius is same,

$$2\pi rh = \pi rl$$

$$\Rightarrow \frac{h}{l} = \frac{\pi r}{2\pi r} = \frac{1}{2}$$

$\Rightarrow$ h: l = 1: 2.

$\therefore$ The ratio of height of cylinder and height of cone is 1: 2.

7.A cylinder and cone have bases of equal radii and are of equal height. Show that their volumes are in the ratio of 3: 1.

A. **Given dimensions are:**

Cone	**cylinder**
Radius = r	Radius = r
Height = h	Height = h
Volume (v) $=\frac{1}{3}\pi r^2 h$	Volume (v) = πr^2h

Ratio of volumes $=\pi r^2 h:\frac{1}{3}\pi r^2 h$

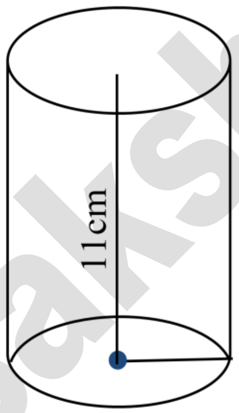
$=1:\frac{1}{3}$

= 3: 1

Hence, their volumes are in the ratio = 3 : 1.

8.A solid iron rod has a cylindrical shape, its height is 11cm. and base diameter is 7cm. Then find the total volume of 50 rods.

A.



Diameter of the cylinder (d) = 7cm

Radius of the base (r) $=\frac{7}{2}=3.5cm$

Height of the cylinder (h) = 11cm

Volume of the cylinder v = $\pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 11$$

= 423.5 cm^3

$\therefore$ Total volume of 50 rods

= 50 $\times$ 423.5 cm3

= 21175 cm^3.

9. The curved surface area of a cone is 4070cm^2 and its diameter is 70cm. What is slant height?

A. The curved surface area of a cone = 4070 cm^2

Its diameter (d) = 70 cm

Radius (r) $= \frac{d}{2} = \frac{70}{2}$

= 35cm

Now πrl = 4070 cm^2

$$= \frac{22}{7} \times 35 \times l = 4070 cm^2$$

$$l = 4070 \times \frac{7}{22} \times \frac{1}{35}$$

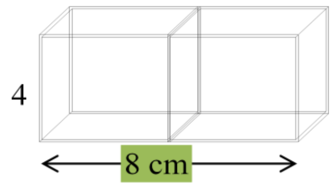
= 37cm

$\therefore$ The slant height (l) = 37cm.

10.Two cubes each of volume $64cm^3$ are joined end to end together. Find the surface area of the resulting cuboid.

A. volume of cube (v) = $64cm^3$

$S^3 = 64cm^3 = (4cm)^3$



$\Rightarrow S = 4cm$

Length of the cuboid = 4cm + 4cm

= 8cm

Surface area of the cuboid

$= 2(lb + lh + bh)$

$= 2\,[8 \times 4 + 8 \times 4 + 4 \times 4]\ cm^2$

$= 2\,[32 + 32 + 16]\ cm^2$

$= 160\ cm^2$.

12.A hemisphere is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the length of the cube. Determine the surface area of the remaining solid.

A. Let the length of the edge of the cube = a units

T.S.A of the given solid d = 5 × Area of each surface + Area of hemisphere

Square surface:

Side = a units

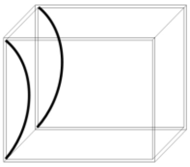
Area = a^2 sq units

Hemisphere:

Diameter = a units

Radius= $\frac{a}{2}$

C.S.A = $2\pi r^2$



$= 2\pi \frac{a^2}{4}$

$= \frac{\pi a^2}{2} sq.units$

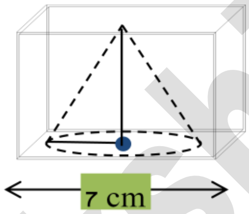
Total surface area $= 5a^2 + \frac{\pi a^2}{2}$

$= a^2 \left[5 + \frac{\pi}{2}\right] sq.units.$

13.Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 7cm.

A. Radius of the cone with the largest volume that can

be cut from a cube of edge 7cm $= \frac{7}{2} cm$



Height of the cone = edge of the cube = 7cm

$\therefore$ Volume of the cone $v = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7$

= $89.83 cm^3$

14.A cone of height 24cm and radius of base 6cm is made up of modeling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

A. volume of cone $=\frac{1}{3}\times\pi\times6\times6\times24\,cm^3$

If r is the radius of the sphere, then its volume is $\frac{4}{3}\pi r^3$

Since the volume of clay is in the form of the same, we have

$$\frac{4}{3}\pi r^3=\frac{1}{3}\pi\times6\times6\times24$$

$$r^3=\frac{6\times6\times24}{4}$$

$$r^3 = 6 \times 6 \times 6$$

$$r^3 = 6^3$$

r = 6 cm.

$\therefore$ The radius of the sphere is 6cm.

15.A hemispherical bowl of internal radius 15cm. contains a liquid. The liquid is to be filled into cylindrical bottles of diameter 5cm and height 6cm. How many bottles are necessary to empty the bowl?

A. Volume of hemisphere= $\frac{2}{3}\pi r^3$

Internal radius of hemisphere (r) = 15cm

$\therefore$Volume liquid contained in hemisphere bowl= $\frac{2}{3}\times\pi\times(15)^3\,cm^3$

$$= 2250\pi\ cm^3$$

This liquid is to be filled in cylinder bottles and the height of each bottle (h) = 6cm

Radius of cylindrical bottle (R) $=\frac{5}{2}cm$

$\therefore$Volume of 1 cylindrical bottle = πR^2h

$$= \pi \times \left(\frac{5}{2}\right)^2 \times 6$$

$$= \pi \times \frac{25}{4} \times 6cm^3$$

$$= \frac{75}{2}\pi cm^3$$

Number of cylindrical bottles required $= \dfrac{volume\ of\ hemispherical\ bowl}{volume\ of\ 1\ cylindrical\ bottle}$

$$= \frac{2250\pi}{\frac{75}{2}\pi}$$

$$= \frac{2\times 2250}{75}$$

= 60.

16.A metallic sphere of radius 4.2cm is melted and recast into the shape of a cylinder of radius 6cm. Find the height of the cylinder.

A. Let the height, of the cylinder = h cm

Volume of cylinder = volume of sphere

$$= \pi \times 6^2 \times h = \frac{4}{3} \times \pi (4.2)^3 \Rightarrow h = \frac{4}{3} \times \frac{\pi \times 4.2 \times 4.2 \times 4.2}{\pi \times 6 \times 6}$$

h = 4 ×0.7 × 0.7 × 1.4

h = 2.744cm.

17.Find the area of required cloth to cover the heap of grain in conical shape whose diameter is 8cm and slant height of 3m.

A. diameter (d) = 8m

Radius (r) $= \dfrac{d}{2} = \dfrac{8}{2} = 4$

Slant height (l) = 3

Surface area of cone = πrl

$$= \frac{22}{7} \times 4 \times 3$$

$$= \frac{22 \times 12}{7} = \frac{264}{7}$$

$$= 37.71$$

Area of the cloth to cover the heap of the grain = 37.71

18. If the total surface area of a cube is 600 sq.cm. Then find its diagonal.

A. Total surface area = $6l^2$

$$6l^2 = 600$$

$$l^2 = \frac{600}{6}$$

$$l = \sqrt{100}$$

$$l = 10$$

Diagonal of cube = $\sqrt{3}l$

$$= \sqrt{3} \times 10$$

Diagonal = $10\sqrt{3}$ cm

19. The diagonal of a cube is $6\sqrt{3}$ cm. Then find its volume.

A. Diagonal of a cube = $\sqrt{3}l$

$$\sqrt{3}l = 6\sqrt{3}$$

$$l = \frac{6\sqrt{3}}{\sqrt{3}} = 6$$

Volume of cube = l^3

$$= 6 \times 6 \times 6$$

Volume = 216 cm^3.

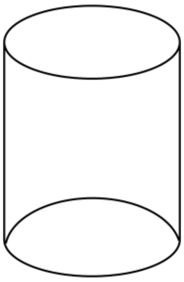
Extra Problems

1.A sphere, a cylinder and a cone are of the same radius and same height. Find the ratio of their curved surface areas?

A. Let r be the common radius of a sphere, a cone and a cylinder.

Height of sphere= its diameter = 2r

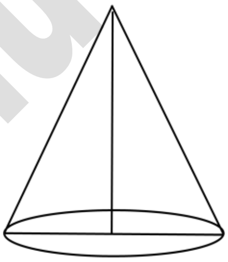
Then, the height of the cone = height of cylinder = height of sphere = 2r



Let l be the slant height of cone $=\sqrt{r^2+h^2}$

$$=\sqrt{r^2+(2r)^2}$$

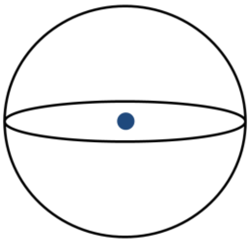
$$=\sqrt{5}r$$



$\because S_1$= curved surface area of sphere = $4\pi r^2$

S_2 = curved surface area of cylinder = $2\pi rh = 2\pi r \times 2r$

$$= 4\pi r^2$$



S_3= curved surface area of cone = πrl

$$= \pi r \times \sqrt{5}r$$

$= \sqrt{5}\ \pi r^2$

Ratio of curved surface area as

$\therefore S_1 : S_2 : S_3 = 4\pi r^2 : 4\ \pi r^2 : \sqrt{5}\pi r^2$

$= 4 : 4 : \sqrt{5}$.

2. A right circular cylinder has base radius 14cm and height 21cm. Find

(i) Area of base or area of each end

(ii) Curved surface area

(iii) Total surface area

(iv) Volume of the right circular cylinder

A. Radius of the cylinder (r) = 14cm

Height of the cylinder (h) = 21cm

(i) Area of base (area of each end) $\pi r^2 = \frac{22}{7} \times (14)^2$

$$= \frac{22}{7} \times 196$$

$$= 616\, cm^2$$

(ii) curved surface area = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 21$$

$= 1848\ cm^2$.

(iii) Total surface area = 2 × area of the base + curved surface area

$= 2 \times 616 + 1848$

$= 3080\ cm^2$

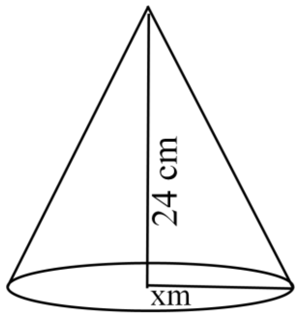
(iv) volume of cylinder = $\pi r^2 h$ = area of the base × height

$= 616 \times 21$

$= 12936\ cm^3$.

3.A joker's cap is in the form of right circular cone whose base radius is 7cm and height is 24cm. Find the area of the sheet required to make 10 such caps.

A. Radius of the base (r) = 7cm



Height of the cone (h) = 24cm

$\therefore$ Slant height (l) $= \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 24^2}$

$= \sqrt{49 + 576}$

$= \sqrt{625}$

= 25cm

Thus, lateral surface area of the joker cap = πrl

$= \frac{22}{7} \times 7 \times 25$

= 550 cm^2

$\therefore$ Total area of the sheet required to make10 such caps

= 10 $\times$ 550 cm^2

= 5500cm^2.

4.A heap of rice is in the form of a cone of diameter 12 m and height 8m. Find its volume? How much canvas cloth is required to cover the heap?

A. Diameter of the conic heap of rice = 12m

$\therefore$ Its radius (r) $= \frac{12}{2}m = 6m$

Its volume (v) $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 6^2 \times 8$

$= 301.44m^3$

$\therefore$ Volume $= 301.44m^3$.

The lateral height (l) = $\sqrt{r^2 + h^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10m$

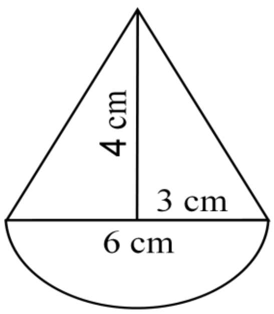
Required canvas cloth to cover the heap = curved surface area of heap

$= \pi rl = 3.14 \times 6 \times 10\ m^2$

$= 188.4m^2$

5.A toy is in the form of a cone mounted on a hemisphere. The diameter of the base and the height of the cone are 6cm and 4cm respectively. Determine the surface area of the toy.

A. Diameter of the cone (d) = 6cm



Radius of the cone (r) $= \frac{d}{2} = \frac{6}{2} = 3cm$

Height of the cone (h) = 4cm

Slant height (l) = $\sqrt{r^2 + h^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

= 5cm.

$\therefore$ Surface area of cone = πrl

$= 3.14 \times 3 \times 5$

$= 47.1 \text{ cm}^2$

Surface area of hemisphere = $2\pi r^2$

$= 2 \times 3.14 \times 3 \times 3$

$= 56.52 \text{ cm}^2$

Thus, total surface area of the toy

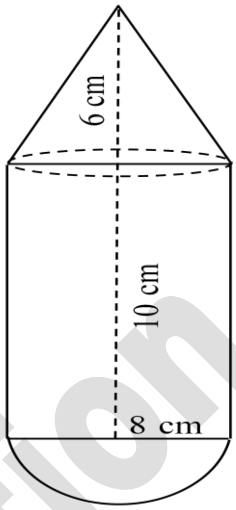
= SA of cone + SA of hemisphere

$= 47.1 \text{ cm}^2 + 56.52 \text{ cm}^2$

$= 103.62\text{cm}^2$.

6.A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. The radius of the common base is 8cm and the heights of the cylindrical and conical portions are 10cm and 6cm respectively. Find the total surface area of the solid.

A. Radius of the hemisphere (r) = 8cm



Surface area of hemisphere = $2\pi r^2$

$= 2 \times 3.14 \times 8 \times 8$

$= 401.92 \text{ cm}^2$

Height of the cylinder (h) = 10cm

Surface area of cylinder = $2\pi rh$

$= 2 \times 3.14 \times 8 \times 10$

$= 502.4\text{cm}^2$

Height of the cone (h) = 6cm.

Slant height (l) = $\sqrt{r^2 + h^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

= 10 cm

Surface area of the cone = πrl

$= 3.14 \times 8 \times 10$

$= 251.2cm^2$

$\therefore$ Total surface area of the solid

= SA of hemisphere + SA of cylinder + SA of cone

= 401.92 + 502.4 + 251.2

$= 1155.52cm^2$

(If we take $\pi = \frac{22}{7}$ we get $1156.58cm^2$)

7.A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the capsule is 14mm and width is 5mm. Find its surface area.

A. length of the cylinder = AB

= 14mm – 2 × 2.5mm

= 14mm – 5mm = 9mm

Curved surface area of cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.5 \times 9$$

$= 141.43 \text{ mm}^2$.

Curved surface area of hemisphere = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 2.5 \times 2.5$$

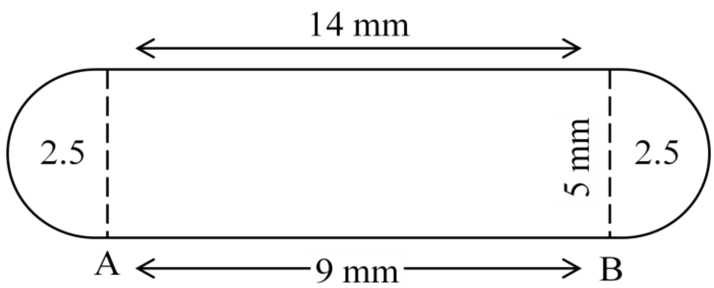
$= 39.29 \text{ mm}^2$.

$\therefore$ Total surface area of the capsule

= CSA of cylinder + 2 × CSA of hemisphere

$= 141.43 \text{ mm}^2 + 2 \times 39.29 \text{ mm}^2$

$= 220.01 \text{ mm}^2$.



8.A storage tank consists of a circular cylinder with a hemisphere struck on either end. If the external diameter of the cylinder be 1.4m and its length be 8m. Find the cost of painting it on the outside at rate of Rs. 20 per m^2.

A. The external diameter of the cylinder = 1.4m

$\therefore$ Its radius (r) $= \frac{1.4}{2} m = 0.7m$

Its length or height (h) = 8m

Curved surface area of each hemisphere = $2\pi r^2$

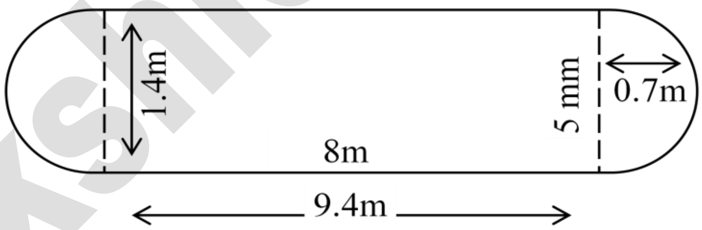
$= 2 \times 3.14 \times 0.7 \times 0.7$

$= 3.08m^2$

Curved surface area of cylinder = $2\pi rh$

$= 2 \times 3.14 \times 0.7 \times 8$

$= 35.17m^2$.



$\therefore$ Total surface area of the storage tank

$= 35.17m^2 + 2 \times 3.08m^2$

$= 35.17m^2 + 6.16m^2$

$= 41.33m^2$

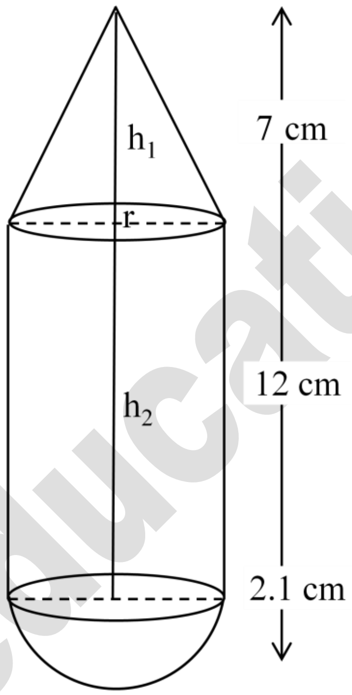
The cost of painting it on the outside at rate of Rs 20 per 1m^2

= Rs 20 × 41.33

= Rs. 826.60.

9.A solid toy is in the form of a right circular cylinder with hemispherical shape at one end and a cone at the other end. Their common diameter is 4.2 cm and the height of the cylindrical and conical portions are 12cm and 7cm respectively. Find the volume of the solid toy.

A.



Let height of the conical portion h_1= 7cm

The height of cylindrical portion h_2= 12cm

radius (r) $= \frac{4.2}{2} = 2.1 = \frac{21}{10} cm$

volume of the solid toy

= volume of the cone + volume of the cylinder + volume of the hemisphere.

$$= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2 + \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left[\frac{1}{3}h_1 + h_2 + \frac{2}{3}r\right]$$

$$= \frac{22}{7} \times \left[\frac{21}{10}\right]^2 \times \left[\frac{1}{3} \times 7 + 12 + \frac{2}{3} \times \frac{21}{10}\right]$$

$$= \frac{22}{7} \times \frac{441}{100} \times \left[\frac{7}{3} + \frac{12}{1} + \frac{7}{5}\right]$$

$$= \frac{22}{7} \times \frac{441}{100} \times \left[\frac{35 + 180 + 21}{15}\right]$$

$$= \frac{22}{7} \times \frac{441}{100} \times \frac{236}{15}$$

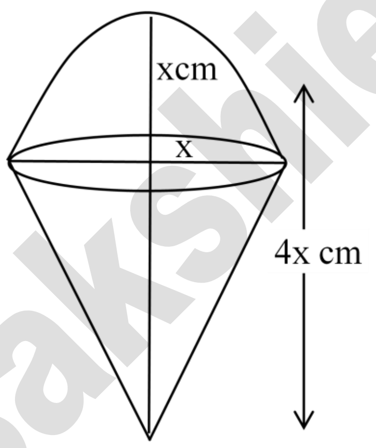
$$= \frac{27258}{125}$$

$= 218.064 cm^3$.

10.A cylindrical container is filled with ice-cream whose diameter is 12cm and height is 15cm. The whole ice-cream is distributed to 10 children in equal cones having hemispherical tops. If the height of the conical portion is twice the diameter of its base find the diameter of the ice- cream cone.

A. Let the radius of the base of conical ice cream = x cm.

$\therefore$ Diameter = 2x cm



Then, the height of the conical ice-cream

= 2(diameter) = 2(2x) = 4xcm

Volume of ice – cream cone

= volume of conical portion + volume of hemispherical portion

$$=\frac{1}{3}\pi r^2 h+\frac{2}{3}\pi r^3$$

$$=\frac{1}{3}\pi r^2 (4x)+\frac{2}{3}\pi x^3$$

$$=\frac{4\pi x^3+2\pi x^3}{3}=\frac{6\pi x^3}{3}$$

$= 2x^3 cm^3$

Diameter of cylindrical container = 12cm

Its height (h) = 15 cm

$\therefore$ Volume of cylindrical container = $\pi r^2 h$

$= \pi(6)^2(15)$

$= 540\pi\ cm^3$.

Number of children to whom ice-creams is given = 10

$$\frac{volume\ of\ cylindrical\ container}{volume\ of\ one\ ice-cream\ cone}=10$$

$$\Rightarrow \frac{540\pi}{2\pi x^3}=10$$

$$\Rightarrow 2\pi x^3 \times 10 = 540\pi$$

$$\Rightarrow x^3=\frac{540}{2\times 10}=27$$

$\Rightarrow x^3 = 27$

$\Rightarrow x^3 = 3^3$

$\therefore$ x = 3

$\therefore$ Diameter of ice-cream cone 2x = 2(3) = 6cm.

11.An iron pillar consists of a cylindrical portion of 2.8cm height and 20cm in diameter and a cone of 42cm height surmounting it. Find the weight of the pillar if $1cm^3$ of iron weighs 7.5g.

A. Height of the cylinder portion = 2.8m

= 280 cm

Diameter of the cylinder = 20cm

Radius of the cylinder $= \frac{20}{2} cm = 10 cm$

Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 10 \times 10 \times 280 cm^3$$

= 88000 cm^3

Height of the cone (h) = 42cm

Radius of the cone (r) = 10cm

Volume of the cone (v) $= \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 42 cm^3$$

= 4400 cm^3

Volume of the pillar

= $88000cm^3$ + $4400cm^3$

= 92400 cm^3

Weight of $1cm^3$ of iron = 7.5g

Weight of the pillar = 7.5 × 92400g

= 693000g

= 693 kg.

12.A pen stand is made of wood in the shape of cuboid with three conical depressions to hold the pens. The dimensions of the coboid are 15cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5cm and depth is 1.4cm. Find the volume of wood in the entire stand.

A. It is given that the dimensions of the wooden cuboid pen stand are

l=15cm, b = 10cm, h= 3.5 cm

volume of the cuboid (v_1) = lbh

$= 15 \times 10 \times 3.5\ cm^3 = 525\ cm^3$

Radius of each conical depression (r) = 0.5cm

Depth of each conical depression (h) = 1.4cm

Volume of each conical depression (v_2) $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4 cm^3$$

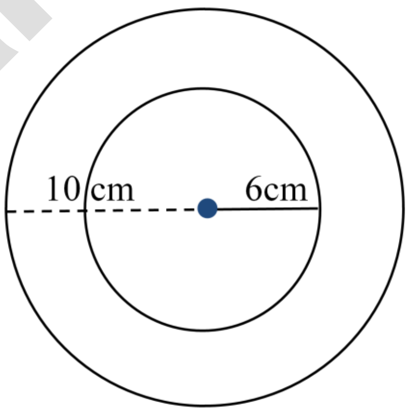
$= 0.367cm^3$

Volume of three conical depressions = 0.367 × 3 = 1.101

Volume of wood in entire stand = 525 cm^3-1.101

$= 523.899 = 523.9cm^3$

13.The diameter of the internal and external surfaces of a hollow hemispherical shell are 6cm and 10cm. Respectively it is melted and recast into a solid cylinder of diameter 14cm. Find the height of the cylinder.



A. Radius of hollow hemispherical shell $=\frac{10}{2}=5cm=R$

Internal radius of hollow hemispherical shell $=\frac{6}{2}=3cm=r$

Volume of hollow hemispherical shell

= External volume – Internal volume

$$=\frac{2}{3}\pi R^3-\frac{2}{3}\pi r^3$$

$$=\frac{2}{3}\pi\left(R^3-r^3\right)$$

$$=\frac{2}{3}\pi\left(5^3-3^3\right)$$

$$=\frac{2}{3}\pi(125-27)$$

$$=\frac{2}{3}\pi\times 98cm^3$$

$$=\frac{196\pi}{3}cm^3\longrightarrow(1)$$

Since, this hollow hemispherical shell is melted and recast into a solid cylinder. So their volumes must be equal

Diameter of cylinder = 14cm

So, radius of cylinder = 7cm

Let the height of cylinder = h

$\therefore$ volume of cylinder = $\pi r^2 h$

$$= \pi \times 7 \times 7 \times h \text{ cm}^3$$

$$= 49\, \pi h \text{ cm}^3 \rightarrow (2)$$

According to given condition

Volume of hollow hemispherical shell = volume of solid cylinder

$$\frac{196}{3}\pi=49\pi h \qquad \text{[from equation (1) and (2)]}$$

$$h = \frac{196}{3 \times 49}$$

$$h = \frac{4}{3} cm$$

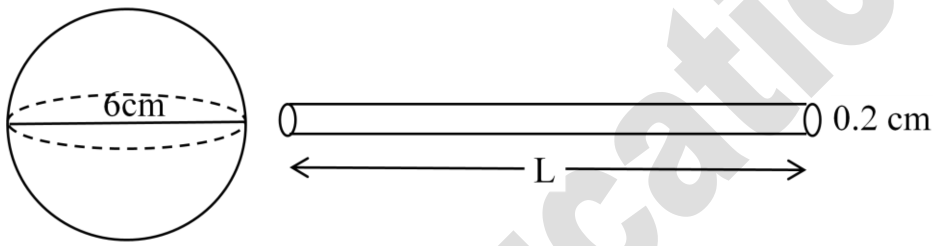
Hence, height of the cylinder = 1.33cm.

14.The diameter of a metallic sphere is 6cm. It is melted and drawn into a wire having diameter of the cross section as 0.2cm. Find the length of the wire?

A. We have diameter of metallic sphere = 6cm

$\therefore$ Radius of metallic sphere = 3cm.

Also we have,



Diameter of cross-section of cylindrical wire = 0.2cm

Radius of cross- section of cylindrical wire = 0.10m

Let the length of wire be 'l'cm.

Since the metallic sphere is covered into a cylindrical shaped wire of length h cm.

$\therefore$ volume of the metal used in wire = volume of sphere

$$\pi r^2 h = \frac{4}{3} \pi r^3$$

$$\pi \times (0.1)^2 \times h = \frac{4}{3} \times \pi \times 3^3$$

$$\pi \times \left(\frac{1}{10}\right)^2 \times h = \frac{4}{3} \times \pi \times 27$$

$$\pi \times \frac{1}{100} \times h = \frac{4}{3} \times \pi \times 27$$

$$h = \frac{36\pi \times 100}{\pi}$$

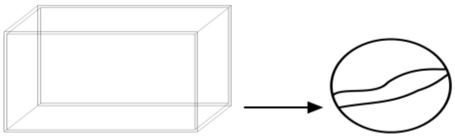
h = 3600cm

h = 36m.

Therefore the length of wire is 36m.

15.How many spherical balls can be made out of a solid cube of lead whose edge measures 44cm and each ball being 4cm in diameters?

A. Side of lead cube = 44cm



Radius of spherical ball $= \frac{4}{2} cm = 2cm$

Now, volume of spherical ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 2^3 cm^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 8 cm^3$$

Volume of x spherical ball $= \frac{4}{3} \times \frac{22}{7} \times 8 \times x\, cm^3$

Its is clear that volume of x spherical balls = volume of lead cube

$$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times 8 \times x = (44)^3$$

$$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times 8 \times x = 44 \times 44 \times 44$$

$$x = \frac{44 \times 44 \times 44 \times 3 \times 7}{4 \times 22 \times 8}$$

x = 11× 11× 3 × 7

$x = 121 \times 21$

$x = 2541$

Hence, total number of spherical balls = 2541.

16.A women self help group (DWACRA) is supplied a rectangular solid (cuboid shape) of wax with diameters 66cm, 42cm, 21cm, to prepare cylindrical candles each 4.2cm in diameters and 2.8cm of height. Find the number of candles.

A. volume of wax in rectangular filed = lbh

$= (66 \times 42 \times 21)\ cm^3$

Radius of cylindrical bottle $= \frac{4.2}{2} cm = 2.1 cm.$

Height of the cylindrical candle = 2.8cm.

Volume of candle = $\pi r^2 h$

$$= \frac{22}{7} \times (2.1)^2 \times 2.8$$

Volume of x cylindrical wax candles $= \frac{22}{7} \times 2.1 \times 2.1 \times 2.8 \times x$

$\therefore$ Volume of x cylindrical candles = volume of wax in rectangular shape

$$\because \frac{22}{7} \times 2.1 \times 2.1 \times 2.8 \times x = 66 \times 42 \times 21$$

$$x = \frac{66 \times 42 \times 21 \times 7}{22 \times 2.1 \times 2.1 \times 2.8}$$

$x = 1500$

Hence, the number of cylindrical wax candles is 1500

17.Metallic spheres of radius 6cm, 8cm and 10cm, respectively are melted to form a single solid sphere. Find the radius of resulting sphere.

Sol: Radius of first sphere = (r_1) = 6cm.

Radius of second sphere = (r_2) = 8cm.

Radius of third sphere = (r_3) = 10cm

Let the radius of resulting sphere = r cm

Sum of volumes of 3 spheres = volumes of resulting sphere.

$$\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi\left(r_1^3 + r_2^3 + r_3^3\right) = \frac{4}{3}\pi r^3$$

$$r_1^3 + r_2^3 + r_3^3 = r^3$$

$$6^3 + 8^3 + 10^3 = r^3$$

$$216 + 512 + 1000 = r^3$$

$$r^3 = 1728$$

$$r^3 = 12^3$$

$$\therefore r = 12\text{cm}.$$

18.A well of diameter 14cm is dug 15m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 7cm to form an embankment. Find the height of the embankment.

Sol: Well is in the shape of cylinder.

Depth of well (b_1) = 15m/s

Diameter of well (d_1) = 14 mm

Radius $(r_1) = \frac{d_1}{2} = \frac{14}{2} = 7\,meters$

Width of circular ring (w) = 7 meters

Radius of outer circle $(r_2) = r_1 + w$ = 7 +7 = 14 m

Radius of inner circle (r_1) = 7 mts

Radius of the earth taken from well = volume of circular ring

$$\Rightarrow \pi \times r_1^2 \times h_1 = \pi\left(r_2^2 - r_1^2\right)\times h_2$$

$$\Rightarrow \frac{22}{7}\times 7^2 \times 15 = \frac{22}{7}\left(14^2 - 7^2\right)\times h_2$$

$$\Rightarrow \frac{22}{7}\times 49 \times 15 = \frac{22}{7}(196-49)\times h_2$$

$$\Rightarrow 49\times 15 = 147\times h_2$$

$$\Rightarrow h_2 = \frac{49\times 15}{147}$$

$\Rightarrow$ h_2= 5 meters.

19.How many silver coins 1.75 cm in diameter and thickness 2mm, need to be melted to form a cuboid of dimensions 5.5cm × 10 cm × 3.5cm?

Sol: Let the number of silver coins needed to melt = n

Then total volume of n coins = volume of the cuboid.

$n\times \pi r^2 h = lbh$ [$\because$ The shape of the coin is a cylinder and $v = \pi r^2 h$]

$$n\times\frac{22}{7}\times\left[\frac{1.75}{2}\right]^2\times\frac{2}{10} = 5.5\times 10\times 3.5 \qquad \left[\because 2mm = \frac{2}{10}cm, r = \frac{d}{2}\right]$$

$$n = 55\times 3.5\times\frac{7\times 2\times 2\times 10}{22\times 1.75\times 1.75\times 2}$$

$$= \frac{55\times 35\times 7\times 4}{22\times 2\times 1.75\times 1.75}$$

$$= \frac{5\times 35\times 7}{1.75\times 1.75}$$

$$= \frac{175\times 7}{1.75\times 1.75} = \frac{100\times 1}{0.25} = 400$$

$\therefore$ 400 silver coins coins are needed.

20.A solid metallic sphere of diameter 28cm is melted and recast into a number of smaller cones, each of diameters $4\frac{2}{3}$ cm and height 3cm. Find the number of cones so formed.

Sol: let the no.of small cones = n

Then, total volume of n cones = volume of sphere

Diameter = 28cm

Cones:

Radius, r = $\frac{diameter}{2}$

$$=\frac{4\frac{2}{3}}{2}=\frac{\frac{14}{3}}{2}=\frac{7}{3}cm$$

Height, h = 3cm

Volume, $v=\frac{1}{3}\pi r^2 h$

$$=\frac{1}{3}\times\frac{22}{7}\times\frac{7}{3}\times\frac{7}{3}\times 3$$

$$=\frac{22\times 7}{3\times 3}=\frac{154}{9}cm^3$$

Total volume of n-cones $=n.\frac{154}{9}cm^3$

Sphere:

Radius, $r=\frac{d}{2}=\frac{28}{2}=14cm$

Volume, $v=\frac{4}{3}\pi r^3$

$$=\frac{4}{3}\times\frac{22}{7}\times 14\times 14\times 14$$

$$= \frac{88 \times 28 \times 14}{3} cm^3$$

$$\therefore n.\frac{154}{9} = \frac{88 \times 28 \times 14}{3}$$

$$n = \frac{88 \times 28 \times 14}{3} \times \frac{9}{154} = 672$$

$\therefore$ No.of cones formed d = 672.

Multiple Choices

1. Area of circle with d as diameter is _____ sq.units **[ ]**

A) $\frac{\pi d^2}{4}$ B) πr^2 C) $\frac{\pi d^3}{2}$ D) None

2. Number of diameters of a circle is _________ **[ ]**

A) 2 B) 3 C) 4 D) infinite

3. The ratio between the volume of a cone and a cylinder is **[ ]**

A) 1 :2 B) 2 : 1 C) 1 : 3 D) None

4. Heap of stones is example of ________ **[ ]**

A) circle B) cone C) triangle D) curue

5. Volume of a cylinder = 88cm^3, r = 2cm then h = _____ cm **[ ]**

A) 8.5 B) 7 C) 6.4 D) None

6. Area of Ring = ______ **[ ]**

A) $R^2 - r^2$ B) $\pi(R^2 - r^2)$ C) $\pi(R^2 + r^2)$ D) None

7. Book is an example of __________ **[ ]**

A) cube B) cuboid C) Cone D) cylinder

8. The edge of a pencil gives an idea about **[ ]**

A) curue B) secant C) cone D) cylinder

9. In a cylinder d = 40cm, h = 56cm then CSA= ____ cm^2 **[ ]**

A) 7040 B) 70.40 C) 704 D) None

10. If each side of a cube is doubled then its volume becomes _______ times **[ ]**

A) 6 B) 2 C) 8 D) 6.0

11. r = 2.1cm then volume of the sphere is _______cm^3 **[ ]**

A) 19.45 B) 55.44 C) 38.88 D) None

12. **The volume of right circular cone with radius 6cm and height 7cm is ______ cm^3** [ ]

A) 164 B) 264 C) 816 D) None

13. **Laddu is in ________ shape** [ ]

A) circular B) spherical C) conical D) None

14. **In a cylinder r = 1 cm, h = 7cm, then TSA = ______ cm^2** [ ]

A) 53.18 B) 51.09 C) 99.28 D) 50.28

15. **The base of a cylinder is ____________** [ ]

A) triangle B) pentagon C) circle D) None

16. **In a cylinder r = 10cm, h = 280cm then volume = ______ cm^3** [ ]

A) 88000 B) 8800 C) 880 D) None

17. **Volume of cube is 1728 cm then its edge is ______ cm** [ ]

A) 21 B) 18 C) 12 D) 16

18. **If d is the diameter of a sphere then its volume is _______ cubic units** [ ]

A) $\frac{1}{6}\pi d^3$ B) $\frac{1}{12}\pi d^3$ C) $\frac{1}{9}\pi r^3$ D) All

19. **Volume of cylinder is ____________** [ ]

A) $\pi r^2 h$ B) $\pi^2 r h^2$ C) $2\pi r h$ D) $2\pi r (r + h)$

20. **Circumference of semi circle is ______ units** [ ]

A) $\frac{22}{7}r$ B) $2\pi r$ C) πr2 D) $\frac{\pi r^2}{2}$

Key:

1. A;	2. D;	3.C;	4.B;	5. B;	6. B;	7. B;
8. C;	9. A;	10. C;	11. C;	12. B;	13. B;	14. D;
15. C;	16. A;	17. C;	18. A;	19. A;	20. A.	

Bit Blanks

1. The area of the base of a cylinder is 616 sq.units then its radius is _______
2. Volume of hemisphere is ________
3. T.S.A of a cube is $216cm^2$ then volume is _____ cm^3
4. In a square the diagonal is _____ times of its side.
5. Volume of sphere with radius r units is _______ cubic units.
6. In the cone l^2 = ________
7. Number of radii of a circle is _____
8. Number of a edges of a cuboid is _________
9. Diagonal of a cuboid is _________
10. In a hemisphere r = 3.5cm Then L.S.A = _____ cm^2
11. L.S.A of cone is __________
12. Rocket is a combination of _________ and ___________
13. Volume of cone is __________ (or) ___________
14. The surface area of sphere of radius 2.1 cm is ______ cm^2
15. In a cone r = 7cm, h = 21cm Then l = _______cm
16. The base area of a cylinder is 200 cm^2 and its height is 4cm then its volume is _______ cm^3
17. The diagonal of a square is 7√2cm. Then its area is _______ cm^2
18. The ratio of volume of a cone and cylinder of equal diameter and height is _____
19. In a cylinder r = 1.75cm, h = 10cm, then CSA = ________cm^2
20. T.S.A of cylinder is _________ sq.units.

Key:

1) 14cm; 2) $\frac{2}{3}\pi r^3$; 3) 216; 4) $\sqrt{2}$; 5) $\frac{4}{3}\pi r^3$;

6) $r^2 + h^2$; 7) infinite; 8) 12; 9) $\sqrt{l^2+b^2+h^2}$;

10) 77; 11) πrl; 12) cone, cylinder;

13) $\frac{1}{3}\times \textit{volume of cylinder}, \frac{1}{3}\times\pi r^2 h$; 14) 55.44; 15) $\sqrt{490}$;

16) 800; 17) 49; 18) 1 : 3; 19) 110; 20) $2\pi r(h + r)$.