Chapter –8

Similar Triangles

Key Concepts:

1.A polygon in which all sides and angles are equal is called a regular polygon.

2. **Properties of similar Triangles:**

a) Corresponding sides are in the same ratio

b) Corresponding angles are equal

3.All regular polygons having the same number of sides are always similar

4.All squares and equilateral triangles are similar.

5.All congruent figures are similar but all similar figures need not be congruent.

6.**Thales Theorem (Basic proportionality Theorem):** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

7.If a line divides two sides of a triangle in the same ratio. Then the line is parallel to the third side.

8.**AAA criterion for similarity:** In two triangles, if the angles are equal, then the sides opposite to the equal angles are in the same ratio (or proportional) and hence the two triangles are similar.

9.**SSC criterion for similarity:** if in two triangles the sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the triangles are similar.

10.**SAS criterion for similarity:** if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

11.If the areas of two similar triangles are equal, then they are congruent.

12.**Pythagoras theorem (Baudhayan Theorem):** In a right angle triangle, the square of hypotenuse is equal to the sum of the squares of the other two sides.

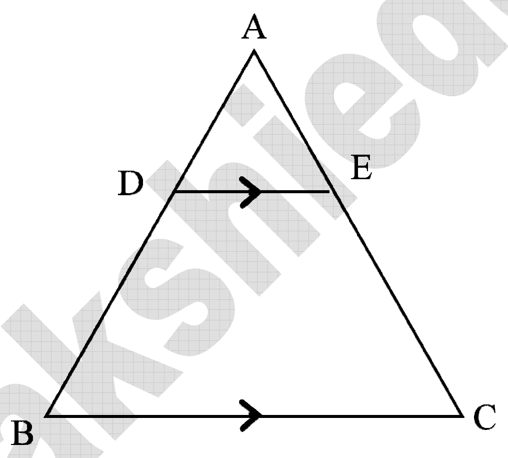
Short Questions

1. In ΔABC, DE//BC and $\frac{AD}{DB}=\frac{3}{5}$, AC = 5.6. Find AE.

Sol: In ΔABC, DE//BC

$$\Rightarrow \frac{AD}{DB}=\frac{AE}{EC} \text{ (By Thales Theorem)}$$

$$\frac{AD}{DB}=\frac{3}{5}(Given), so \frac{AE}{EC}=\frac{3}{5}$$



Given AC = 5.6; AE : EC = 3:5

$$\frac{AE}{AC-AE}=\frac{3}{5}$$

$$\frac{AE}{5.6-AE}=\frac{3}{5}$$

5AE = 3 (5.6 - AE) (cross multiplication)

8AE = 16.8

$\Rightarrow AE = \frac{16.8}{8} = 2.1cm$

2.In a trapezium ABCD, AB//DC. E and F are points on non – parallel sides AD and BC respectively such that EF//AB show that $\frac{AE}{ED} = \frac{BF}{FC}$.

A. Let us join AC to intersect EF at G.

AB//DC and EF//AB (Given)

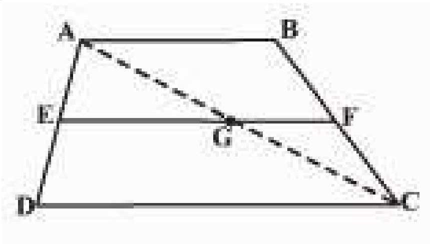
⇒EF//DC (Lines parallel to the same line are parallel to each other)

In ΔABC, EG//DC

So, $\frac{AE}{ED} = \frac{AG}{GC}$ (By Thales Theorem) $\rightarrow$ (1)

Similarly In ΔCAB GF//AB

$\frac{CG}{GA} = \frac{CF}{FB}$ (By Thales Theorem)



$\frac{AG}{GC} = \frac{BF}{FC} \longrightarrow (2)$

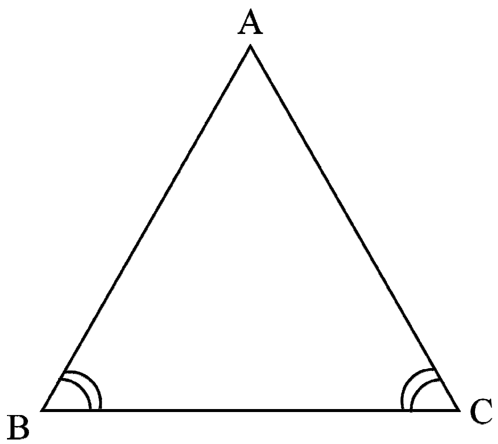
From (1) and (2)

$\frac{AE}{ED} = \frac{BF}{FC}$

3.Prove that in two triangles, if the angles are equal, then the sides opposite to the equal angles are in the same ratio (or proportional) and hence the two triangles are similar.

Sol: **Given:** In triangles ABC and DEF

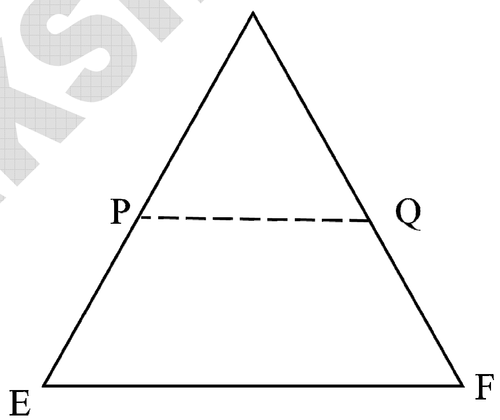
$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$



RTP: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Construction: locate points P and Q on DE and DF respectively such that AB = DP and AC = DQ. Join PQ.

Proof: $\Delta ABC \cong \Delta DPQ$



$\angle B = \angle P = \angle E$ and PQ//EF

$$\frac{DP}{PE}=\frac{DQ}{QF}$$

$$i.e\frac{AB}{DE}=\frac{AC}{DF}$$

Similarly $\frac{AB}{DE}=\frac{BC}{EF}$ and so $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$

Hence proved

4. Prove that if the areas of two similar triangles are equal then they are congruent.

Sol: $\Delta ABC \sim \Delta PQR$

$$So\frac{ar(\Delta ABC)}{ar(\Delta PQR)}=\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{AC}{PR}\right)^2$$

But $\frac{ar(\Delta ABC)}{ar(\Delta PQR)}=1$ (areas are equal)

$$\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{AC}{PR}\right)^2=1$$

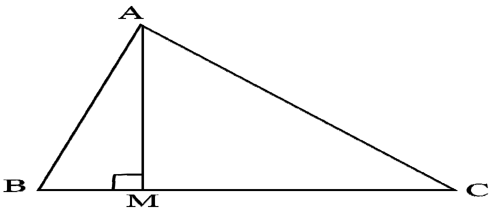
So $AB^2 = PQ^2$; $BC^2 = QR^2$; $AC^2 = PR^2$

From which we get AB= PQ, BC = QR, AC = PR

$\therefore \Delta ABC \cong \Delta PQR$ (by SSS congruency)

5.In a right angle triangle the square of hypotenuse is equal to the sum of the squares of the other two sides (Pythagoras theorem, (BAUDHAYAN THEOREM)).

Sol: **Given:** ΔABC is a right angle triangle



RTP: $Ac^2 = AB^2 + BC^2$

Construction: Draw $BD \perp AC$

Proof: $\Delta ADB \sim \Delta ABC$

$\frac{AD}{AB} = \frac{AB}{AC}$ (sides are proportional)

$AD.AC = AB^2 \rightarrow (1)$

Also $\Delta BDC \sim \Delta ABC$

$\Rightarrow \frac{CD}{BC} = \frac{BC}{AC}$

$CD.AC = BC^2 \rightarrow (2)$

(1) + (2)

$AD.AC + CD.AC = AB^2 + BC^2$

$AC (AD + CD) = AB^2 + BC^2$

$AC.AC = AB^2 + BC^2$

$AC^2 = AB^2 + BC^2$

6.The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Sol: **Given:** $\Delta ABC \sim \Delta PQR$

RPT: $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{RP}\right)^2$

Construction: Draw $AM \perp BC$ and $PN \perp QR$

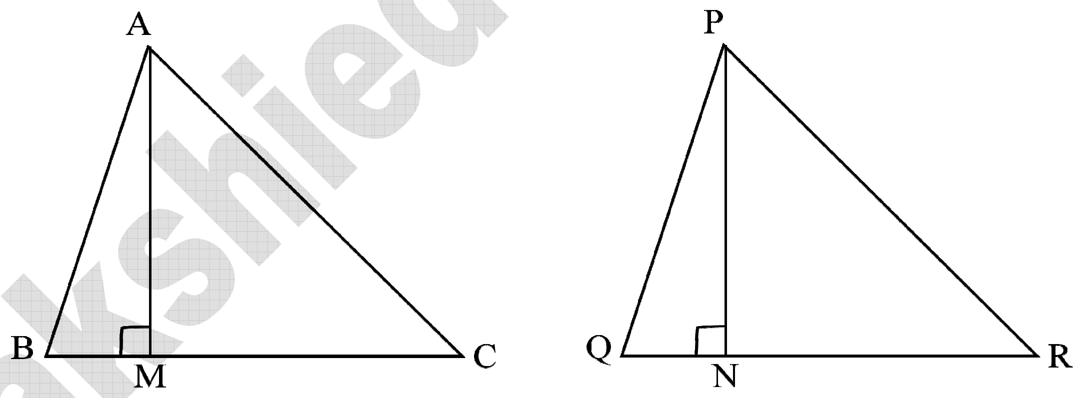
Proof: $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2}\times BC\times AM}{\frac{1}{2}\times QR\times PN} = \frac{BC\times AM}{QR\times PN} \longrightarrow (1)$

In ΔABM & ΔPQN

$\angle B = \angle Q$ $(\because \Delta ABC \sim \Delta PQR)$

$\angle M = \angle N = 90°$

$\therefore \Delta ABM \sim \Delta PQN$ (by AA similarity)



$\frac{AM}{PN} = \frac{AB}{PQ} \longrightarrow (2)$

also ΔABC & ΔPQR (Given)

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \longrightarrow (3)$

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ} [now (1), (2) \& (3)]$$

$$= \left(\frac{AB}{PQ}\right)^2$$

Now by using (3), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$ Hence proved

7.Prove that the sum of the squares of the sides of a Rhombus is equal to the sum & squares of its diagonals.

Sol: in rhombus ABCD

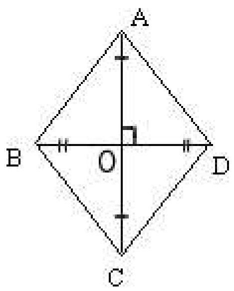
AB = BC = CD = DA and

Diagonals of rhombus perpendicularly bisect each other at 'o'

So, $OA = OC \Rightarrow OA = \frac{A}{2}$

$$OB = OD \Rightarrow OD = \frac{BD}{2}$$

In ΔAOD, ∠AOD = 90°



$AD^2 = OA^2 + OD^2$ (Pythagoras Theorem)

$$= \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$= \frac{AC^2}{4} + \frac{BD^2}{4}$$

$$AD^2 = \frac{AC^2 + BD^2}{4}$$

$4AD^2 = AC^2 + BD^2$

$AD^2 + AD^2 + AD^2 + AD^2 = AC^2 + BD^2$

But AB = BC = CD = AD (Given)

$\therefore AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

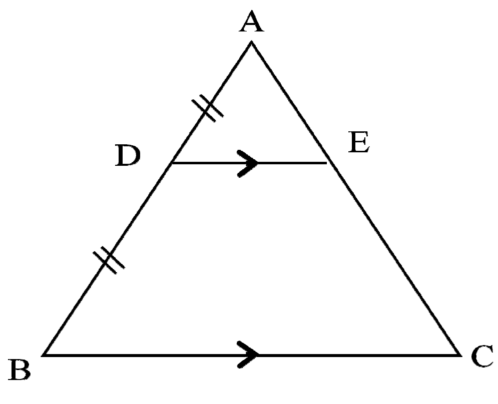
8.Prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side (using basic proportionality theorem)

Sol: **Given:** In ΔABC, D is the mid-point of AB and DE//BC

To prove: AE = CE

Proof: by Thales theorem

$$\frac{AD}{DB} = \frac{AE}{EC} \longrightarrow (1)$$



But D is the mid – point of AB

$\Rightarrow AD = DB$

$$\frac{AD}{DB}=1$$

From (1) we get

$$\frac{AE}{EC}=1$$

AE = CE

$\therefore$ AC is bisected by the parallel line

9.Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Sol: **Given:** $\Delta ABC \sim \Delta DEF$ and AM and DN are their corresponding medians.

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)}=\frac{AM^2}{DN^2}$

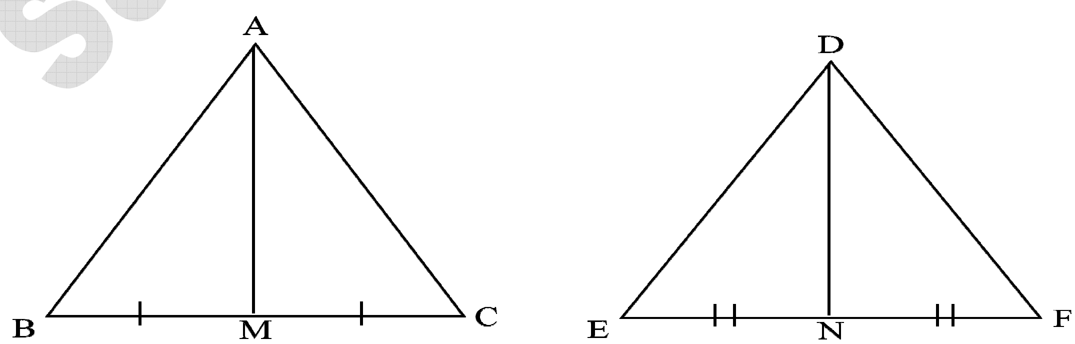
Proof: It is given that $\Delta ABC \sim \Delta DEF$

By the theorem an areas of similarity triangles

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)}=\frac{AB^2}{DE^2}$$

$$\frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{AC^2}{DE^2}\longrightarrow(1)$$

Also $\frac{AB}{DE}=\frac{BC}{EF}=\frac{2BM}{2EN}=\frac{BM}{EN}$



$$\Rightarrow \frac{AB}{DE} = \frac{BM}{EN}$$

Clearly ∠ABM = ∠DEN

SAS similarity criterion,

ΔABC ~ ΔDEF

$$\frac{AB}{DE} = \frac{AM}{DN} \longrightarrow (2)$$

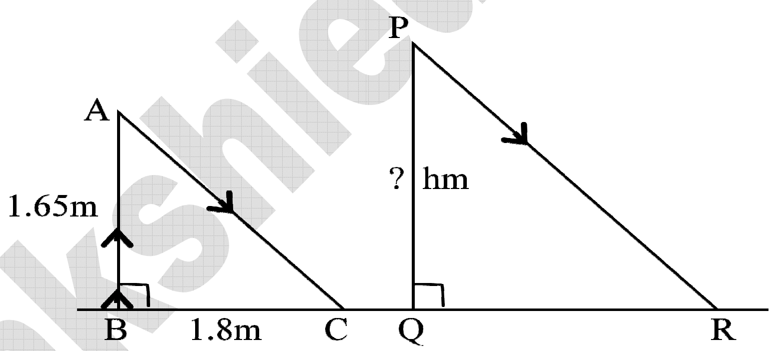
From (1) and (2) we get

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AM^2}{DN^2}$$

Hence proved

10.A person 1.65m tall casts 1.8m shadow. At the same instance, a lamp-posts casts a shadow of 5.4m. Find the height of the lamppost?

Sol: In ΔABC and ΔPQR



∠B = ∠Q = 90°

∠C = ∠R AC//PR, (all sun's rays are parallel at any instance)

ΔABC ~ ΔPQR (by AA similarly)

$$\frac{AB}{PQ} = \frac{BC}{QR}$$ (corresponding parts of similar triangles)

$$\frac{1.65}{PQ}=\frac{1.8}{5.4}$$

$$PQ=\frac{1.65\times54}{1.8}=4.95m$$

Height of the lamp post = 4.95m.

11. The perimeters of two similar triangles are 30cm and 20cm respectively. If one side of the first triangle is 12cm, determine the corresponding side of the second triangle

Sol: Let the corresponding side of the second triangle be x m

We know that,

The ratio of perimeters of similar triangles = ratio of corresponding sides

$$\Rightarrow\frac{30}{20}=\frac{12}{x}\Rightarrow x=8cm$$

$\therefore$ Corresponding side of the second triangle = 8cm

12. ΔABC ~ ΔDEF and their areas are respectively 64cm^2 and 121cm^2 If EF =15.4 cm, Then Find BC.

Sol: $$\frac{ar(\Delta ABC)}{ar(\Delta DEF)}=\left(\frac{BC}{EF}\right)^2$$

$$\frac{64}{121}=\left(\frac{BC}{15.4}\right)^2$$

$$\frac{8}{11}=\frac{BC}{15.4}\Rightarrow BC=\frac{15.4\times8}{11}=11.2cm$$

13.ΔABC ~ ΔDEF, BC = 3cm, EF = 4cm and area of ΔABC = 54cm Determine the area of ΔDEF.

Sol: ΔABC ~ ΔDEF BC = 3cm, EF = 4cm

Area of ΔABC = $54cm^2$

By the theorem on areas of similar triangles,

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\frac{54cm^2}{ar(\Delta DEF)} = \frac{9cm^2}{16cm^2}$$

∴ Area of ΔDEF = 96 cm^2

14.The areas of two similar triangles are $81cm^2$ and 49 cm^2 respectively. If the altitude of the bigger triangle is 4.5cm. Find the corresponding altitude of the similar triangle.

Sol: We know that the ratio of areas of two similar triangles is equal to square of the ratio of their corresponding altitudes

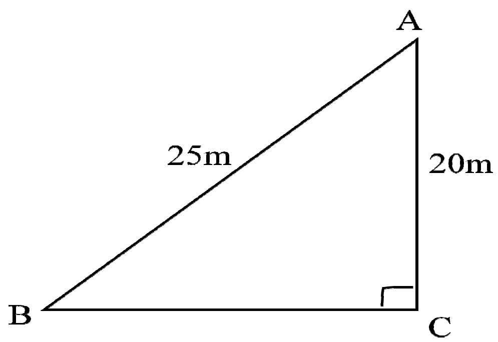
$$\Rightarrow \frac{area\ of\ bigger\ triangle}{area\ of\ smaller\ triangle} = \left(\frac{altitude\ of\ bigger\ triangle}{altitude\ of\ smaller\ triangle}\right)^2$$

$$\Rightarrow \frac{81}{49} = \left(\frac{4.5}{x}\right)^2 \Rightarrow x = 3.5cm$$

Corresponding altitude of the smaller triangle = 3.5cm.

15.A ladder 25m long reaches a window of building 20m above the ground. Determine the distance of the foot of the ladder from the building.

Sol: **In** ΔABC, $\angle C = 90°$



$AB^2 = AC^2 + BC^2$ (By Pythagoras Theorem)

$25^2 = 20^2 + BC^2$

$BC^2 = 625 - 400 = 225m$

$BC = \sqrt{225} = 15m$

$\therefore$ The distance of the foot of the ladder from the building is 15m.

16.The hypotenuse of a right triangle is 6m more than twice of the shortest side if the third side is 2m less than the hypotenuse. Find the sides of the triangle.

Sol: Let the shortest side be x m

Then hypotenuse = (2x + 6) m, third side = (2x + 4)m

By Pythagoras Theorem we have

$(2x + 6)^2 = x^2 + (2x + 4)^2$

$4x^2 + 36 + 24x = x^2 + 4x^2 + 16 + 16x$

$x^2 - 8x - 20 = 0$

$(x - 10)(x + 2) \Rightarrow x = +10, x = -2$

But x can't be negative as side of a triangle

x = 10m

Hence the sides of the triangle are 10m, 26m, 24m.

17. Prove that three times the square of any side of an equilateral triangle is equal to four times the square of altitude

Sol: In ΔABC, AB = BC = CA, AD ⊥ BC

To prove: $3AB^2 = 4AD^2$

Proof: we have AB = AC (Given)

AD = AD (common side)

∠ADB = ∠ADC (Given)

ΔADB ≅ ΔADC (RHS congruently property)

$$\Rightarrow BD = CD = \frac{1}{2}BC = \frac{1}{2}AB$$

Δ ADB is right triangle

By Baudhayana Theorem

$AB^2 = AD^2 + BD^2$

$$= AD^2 + \left(\frac{1}{2}AB\right)^2 = AD^2 + \frac{1}{4}AB^2$$

$$AD^2 = AB^2 - \frac{1}{4}AB^2$$

$$AD^2 = \frac{1}{4}AB^2$$

$3\ AB^2 = 4AD^2$ Hence proved

Essay Type Questions

1.Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points then the other two sides are divided in the same Ratio (proportionality theorem / Thales Theorem).

Sol: **Given:** In ΔABC, DE//BC which intersects sides AB and AC at D and E respectively

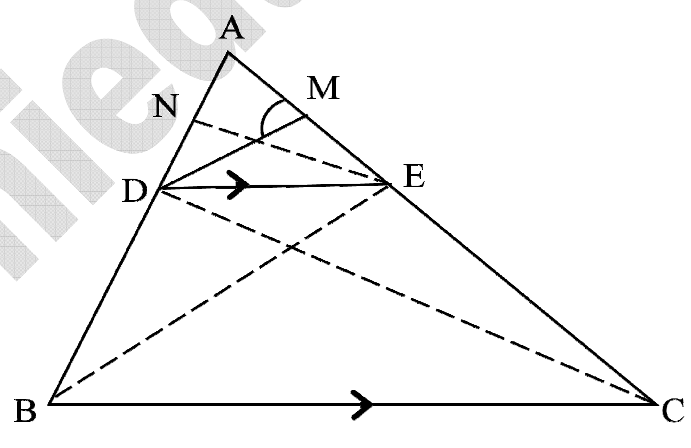
RTP: $\frac{AD}{DB}=\frac{AE}{EC}$

Construction: Join B, E and C, D and then draw

DM ⊥ AC and EN ⊥ AB

Proof: Area of ΔADE $=\frac{1}{2}\times AD\times EN$

Area of ΔBDE $=\frac{1}{2}\times BD\times EN$



SO $\frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta BDE}=\frac{\frac{1}{2}\times AD\times EN}{\frac{1}{2}\times BD\times EN}=\frac{AD}{BD}\longrightarrow(1)$

Again Area of ΔADE $=\frac{1}{2}\times AE\times DM$

Area of ΔCDE $= \frac{1}{2} \times EC \times DM$

$$\frac{ar(\Delta ABE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{CE} \longrightarrow (2)$$

Observe that ΔBDE and ΔCDE are on the same base DE and between same parallels BC and DE

So ar (ΔBDE) = ar (ΔCDE) → (3)

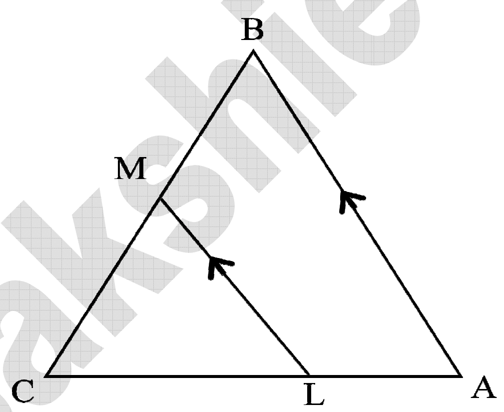
From (1) (2) & (3) we have

$$\frac{AD}{DB} = \frac{AE}{CE}$$

Hence proved

2.In the given figure LM//AB AL = x – 3, AC = 2x, BM = x – 2 and BC = 2x + 3 find the value of x.

Sol: in ΔABC, LM//AB



$\Rightarrow \frac{AL}{LC} = \frac{BM}{MC}$ (By Thales Theorem)

$$\frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)}$$

$$\frac{x-3}{(x+3)}=\frac{x-2}{x+5}$$ (cross multiplication)

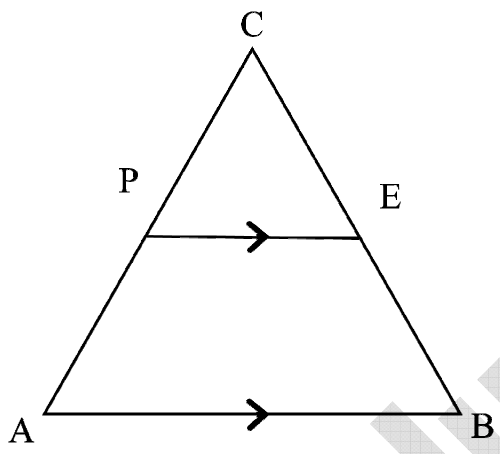
(x - 3) (x + 5) = (x - 2) (x + 3)

$x^2 + 2x – 15 = x^2 + x – 6$

$2x – x = -6 + 15 \Rightarrow x = 9.$

3. What values of x will make DE//AB in the given figure.

Sol: In ΔABC, DE//AB



$$\frac{CD}{AD}=\frac{CE}{CB}$$

$$\frac{x+3}{8x+9}=\frac{x}{3x+4}$$

(x + 3) (3x + 4) = x (8x + 9)

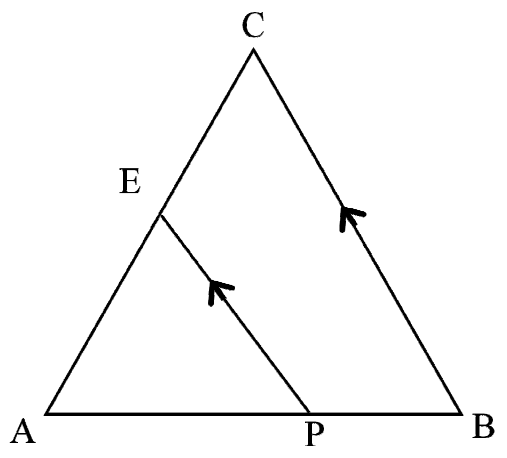
$3x^2 + 13x + 12 = 8x^2 + 9x$

$5x^2 – 4x – 12 = 0$

$(x – 2) (5x + 6) = 0 \Rightarrow x = 2;\ x = \frac{-6}{5}$

4.In ΔABC, DE//BC, AD = x, DB = x - 2, AE = x + 2 and EC = x – 1 Find value of x.

Sol: In ΔABC, DE//BC



$$\frac{AD}{PB} = \frac{AE}{AC} \text{(by thales theorem)}$$

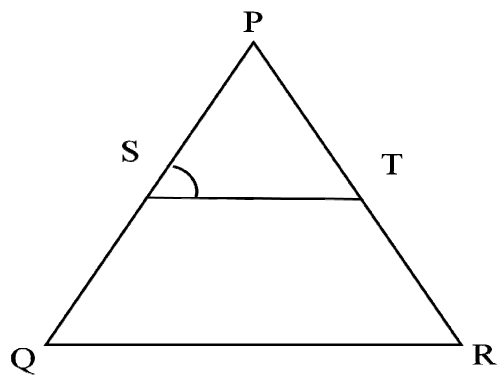
$$\frac{x}{x-2} = \frac{x+2}{x-1}$$

x(x - 1) = (x + 2) (x - 2)

$x^2 - x = x^2 - 4 \Rightarrow x = 4$.

5.In ΔPQR, ST is a line such that $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ prove that ΔPQR is isosceles triangle.

Sol: In ΔPQR, ST is a line such that $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ



By the converse theorem of Thales,

ST//QR

$\Rightarrow \angle PST = \angle PQR$ (corresponding angles) $\rightarrow$ (1)

$\angle PQR = \angle PRQ$ (Given) $\rightarrow$ (2)

From (1) and (2) we get

$\angle PST = \angle PRQ$

$\Rightarrow$ PQ = QR (sides opposite to the equal angles)

$\therefore$ Δ PQR is an isosceles triangle

6.Prove that a line drawn through the mid-point of one side of a Triangle parallel to another side bisects the third side (using basic proportionality theorem)

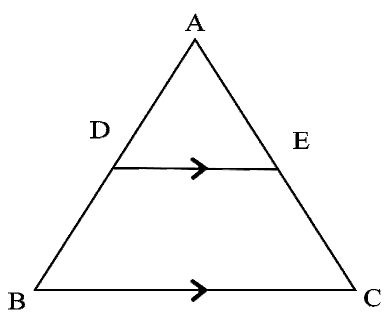
Sol: In ΔABC, D is the mid-point of AB and DE//BC

To prove: AE = EC

$\frac{AD}{DB} = \frac{AE}{EC}$ (By Thales theorem) $\rightarrow$ (1)

But D is the mid-point of AB

$\Rightarrow$ AD = DB



$\frac{AD}{DB} = 1$

$\frac{AE}{EC} = 1$ from (1)

$\Rightarrow$ AE = EC

AC is bisected by the parallel line.

One Mark Questions

1. Define regular polygon?

A. A polygon in which all sides and angles are equal is called a regular polygon.

2. Write the properties of similar triangles?

A. Corresponding sides are in the same ratio corresponding angles are equal

3. Which figures are called similar figures?

A. The geometrical figures which have same shape but not same size.

4. Which figures are called congruent figures?

A. The geometrical figures which have same size and same shape.

5. When do you say that two triangles are similar?

A. Two triangles are said to be similar if their

i) Corresponding angles are equal

ii) Corresponding sides are in the same ratio.

6. Sides of two similar triangles are in the ratio 2:3. Find the ratio of the areas of the triangle

A. 4 : 9

7. Write the basic proportionality theorem.

A. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

8. Write the converse of basic proportionality theorem.

A. If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side

9. Write AAA axiom.

A. In two triangles, if the angles are equal, then the sides opposite to the equal angles are in the same ratio (or proportion) and hence the two triangles are similar.

10. Write SSS criterion

A. If in two triangles the sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the triangles are similar.

11. Write SAS criterion.

A. If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, then the two triangles are similar.

12. State Pythagoras theorem

A. In a right triangle, the square of hypotenuse is equal to the sum of the squares of the other two sides.

13. Which is the longest side in a right triangle?

A. Hypotenuse

14. If the side of an equilateral triangle is 'a' then find height?

A. $\frac{\sqrt{3}}{2}$ a units

Fill in the Blanks

1. In ΔABC if $\angle D = 90°$ and $BD \perp AC$ then BD^2= ________
2. All squares and equilateral triangles are _______
3. Example of similar figures is _________________
4. Example of non similar figures is _______________
5. If a line divides two sides of a triangle in the same ratio. Then the line is parallel to the ____________
6. In ΔABC, $BC^2 + AB^2 = AC^2$ Then ________ is a right angle
7. If D is the midpoint of BC in ΔABC then $AB^2 + AC^2$ =____________
8. In ΔABC, D and E are mid points of AB and AC then DE : BC is __________.
9. The diagonal of a square is ____ times to its side
10. If ABC ~ PQR Than AB : AC ________
11. The ratio of corresponding sides of two similar triangles is 3 : 4 Then the ratio of their areas is _____________
12. Basic proportionality theorem is also known as ________ theorem
13. Area of an equilateral triangle is_____________
14. _______________ is the longest side of right angled triangle.

Key

1) AD.DC; 2) Similar; 3) Bangles if different sizes, any two squares;

4) Any two walls, square and rhombus; 5) Third side; 6) $\angle B$;

7) $2AD^2 + 2BD^2$; 8) 1 :2; 9) $\sqrt{2}$; 10) PQ : PR; 11) 9 : 16;

12) Thales; 13) $\frac{\sqrt{3}}{4}a^2$; 14) Hypotenuse.