Chapter -6

Refraction of Light at Curved Surfaces

SYNOPSIS

A lens is formed when a transparent material is bounded by two surfaces of which one or both surfaces are spherical. Lenses can be of various types. Biconvex lens, Biconcave lens, Plano convex lens, Plano concaves lens and concave – convex lens.

In lenses, light undergoes the phenomena of refraction. If u,v and f are the object distance, Image distance and focal length then the lens formula is $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

If R_1, R_2 are the radius of curvatures of the lens, 'n' is the absolute refractive index then

$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$ known as the Lens-Maker's formula.

In concave lens, only virtual image is formed. In convex lens both virtual and real images are formed. The position of image changed in convex lens by changing the position of object.

2 Mark Questions

1. A man wants to get a picture of a zebra. He photographed a white donkey after fitting a glass, with black stripes, on to the lens of his camera. What photo will he get? Explain? [AS1]

A. 1. He will get a photograph which consists of Black and White strips.

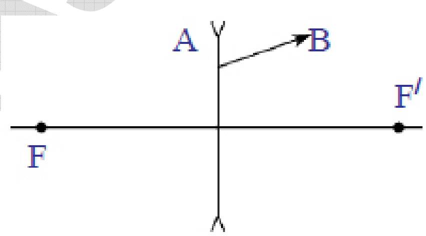
2. Black strips opera with more intensity and white strips appear with less intensity because the reflected rays from white donkey entered into camera through the lens having black stripes, these black stripes don't allow the rays inside. So photograph of white donkey will obtained with reduced brightness.

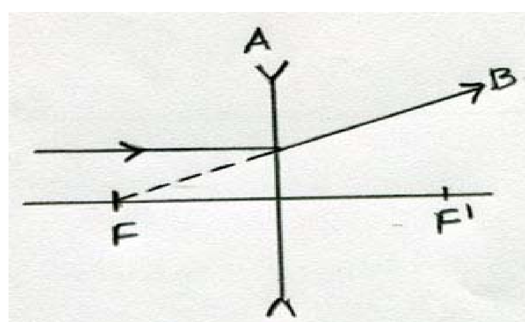
2. A convex lens is made up of three different materials as shown in figure. How many of images does it form? [AS2]

A. The lens has been made up of three different materials. These three different materials will have three different refractive indices. Hence three different images will be formed.

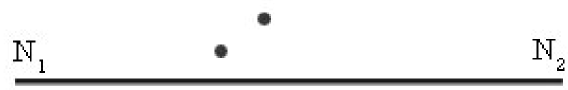
3. Figure shows ray AB that has passed through a divergent lens. Construct the path of the ray up to the lens if the position of focal is known?

A. A light passing parallel to principal axis, after refraction appears to be diverging from the focus of lens.





4. Figure shows a point light source and its image produced by a lens with an optical axis N_1N_2. Find the position of the lens and its focal using a ray diagram?



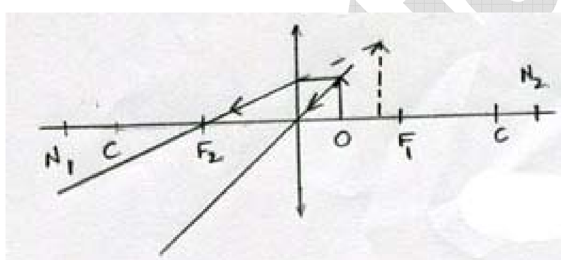
A. 1. The object is in between focus and optic centre

2. Lens used us is convex lens

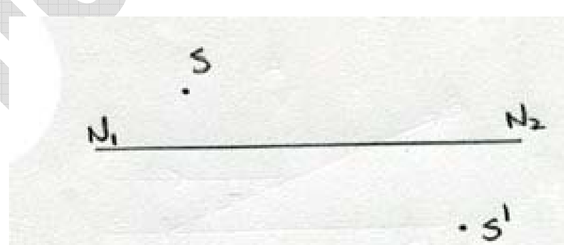
3. Image is virtual, Erect, Magnified.

4. Object is at 'O'; image is at I, ll_1 is the lens.

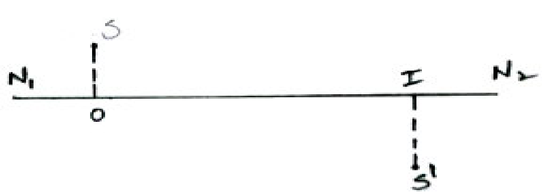
5. F is the focal point



5. Find the focus by drawing a ray diagram using the position of source 'S' and image 'S^1' given in figure?

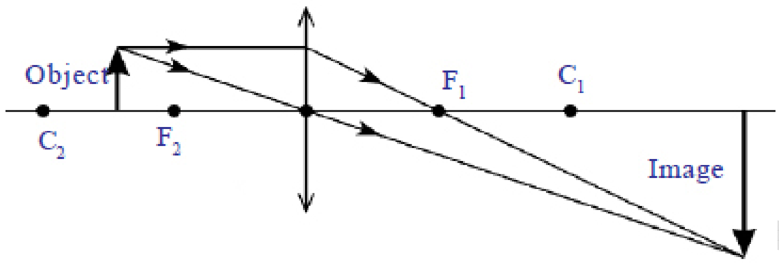


A. 1. Here the image should be inverted, and magnified because 'SO' < 'S^1I'.



2. The object is placed between curvature (C) and focal point F. The image formed beyond centre of curvature (C_2).

3. Image produced is Inverted, Real, Magnified.



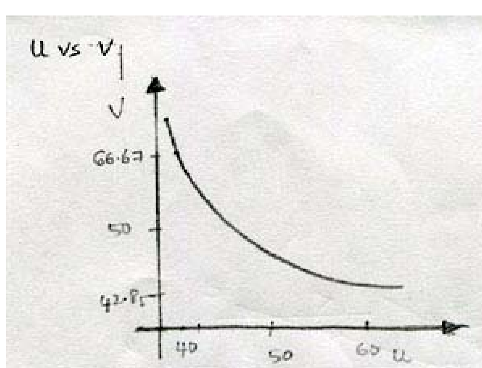
6. Use the data obtained by activity 2 in table 1 of this lesson and draw the graphs of u vs. v and $\frac{1}{u} vs \frac{1}{v}$

A. Graph of u – v using data obtained by activity 2 take focal length of lens be f(say) = 25 cm.

$$\frac{1}{f} = \frac{1}{u} - \frac{1}{v}$$

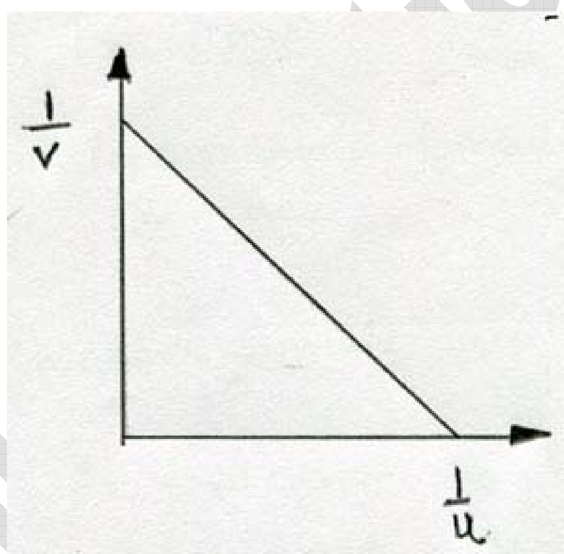
Object distance (u)	Image distance (v)	Focal length (f)
60cm	42.85cm	25cm
50cm	50cm	25cm
40cm	66.67cm	25cm

Graph: u vs. v



Graph $\frac{1}{u} vs \frac{1}{v}$

S.No.	u(cm)	V(cm)	$\frac{1}{u}(cm^{-1})$	$\frac{1}{v}(cm^{-1})$
1	60	42.85	$\frac{1}{60} = 0.01667$	$\frac{1}{42.85} = 0.0233$
2	50	50	$\frac{1}{50} = 0.02$	$\frac{1}{50} = 0.02$
3	40	66.67	$\frac{1}{40} = 0.025$	$\frac{1}{66.67} = 0.015$



1 Mark Questions

1. Write the lens makers formula and explain the terms in it? [AS1]

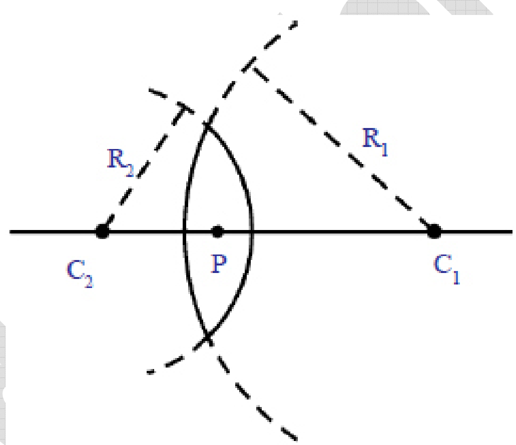
A. Lens maker's formula is $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

f = focal length of lens

n = refractive index of lens

R_1 = Radius of curvature of first surface on lens

R_2 = Radius of curvature of second surface on lens.



2. Assertion [A]: A person standing on the land appears taller than actual height to a fish inside a pond.

Reason [R]: Light bends away from the normal as it enters air from water.

Which of the following are correct? Explain?

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true and R is not correct explanation of A.

c) A is the true but R is false

d) Both A and R are false

e) A is false but R is true.

A. Option (c) is correct.

Assertion is correct because the person in air the light ray (from fish) travels from denser to rarer medium. So, the person appears taller than his actual height.

Reason is wrong because as the light ray travels from air to water light ray bends towards the normal.

3. Can a virtual image be photographed by camera? (AS2)

A. Yes. Virtual image can be photographed by camera.

4. Suppose you are inside the water in a swimming pool near an edge. A friend is standing on an edge. Do you find your friend taller or shorter than his usual height? Why? [AS7]

A. My friend appears to be taller than usual height because the light travelling from denser to rarer medium the rays bends away from the normal. So it is actually Virtual image of my friend appears to be larger and taller than actual height due to refraction.

4 Mark Questions

1. Two convergent lenses are to be placed in the path of parallel rays so that the rays remain parallel after passing through both lenses. How should the lenses be arranged? Explain with neat diagram. [AS1]

A. 1. Let the lenses are l_1 and l_2

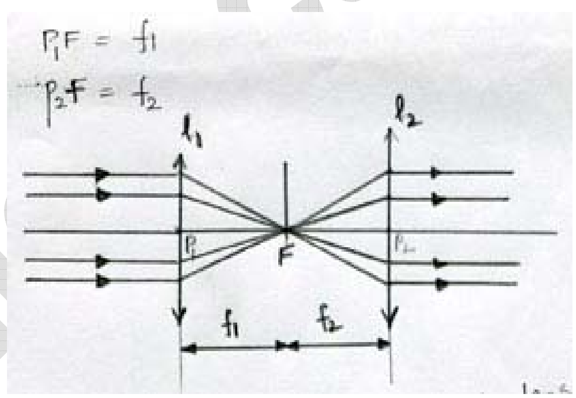
2. P_1, P_2 are the optic centers of lenses l_1 and l_2

3. Assume focal lengths of two lenses are to be f_1 and f_2

4. F is the focus of both lenses

$P_1F = f_1$

$P_2F = f_2$



From above figure, two convergent lenses are to be placed in a path of parallel rays, so that the rays remain parallel after passing through both lenses by separating the lenses at a distance of $f_1 + f_2$ units apart.

2. How do you verity experimentally that the focal length of a convex lens is increased when it is kept in water? [AS1]

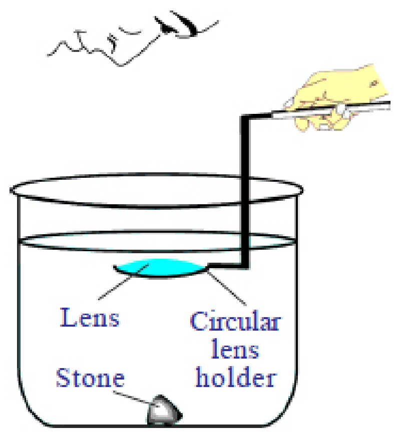
A. 1.Take a convex lens whose focal lengths is known

2.Take a glass tumbler, whose height is known and nearly four times of focal length of lens

3.Keep a black stone inside the vessel

4.Now pour water into the vessel up to a height such that the height of water level from the top of surface of stone is greater than focal length of lens.

5.Now dip the lens horizontally using a circular lens holder.



6.Set the distance between stone and lens that is equal to or less than focal length of lens.

7.Now look at the stone through lens.

8.We can see the image of the stone if the distance between lens and stone is less than the focal length of lens.

9.Now increase the distance between lens and stone until you can't see the image of stone.

10.We have dipped the lens to a certain height which is greater than the focal length of lens in air but we can see the image. This shows that focal length of lens has increased in water.

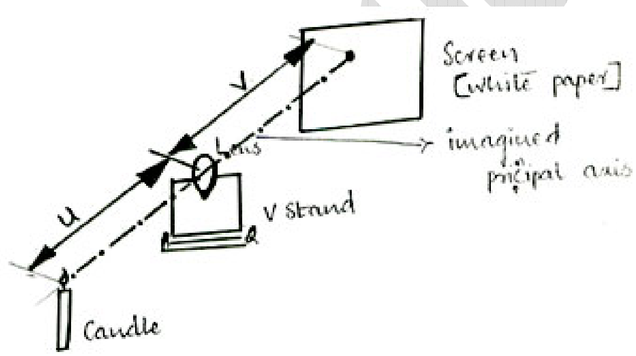
3. How do you find the focal length of a lens experimentally? [AS1]

A. 1.Take a 'v' stand and place it on a long table at the middle

2.Place a convex lens on 'v' stand. Imagine the principal axis of lens.

3.Light a candle and ask your friend to take the candle far away from the lens along the principal axis.

4.Adjust a screen (a sheet of white paper placed perpendicular to axis) which is on other side of lens until you get an image on it.



5.Measure the distance of image from the v stand of lens (image distance 'v') and also measure the distance between candle and stand of lens (object distance 'u'). Record the values in table.

S.No	Object distance(u)	Image distance (v)	Focal length 'f'

6.Now place the candle at a distance of 60cm from the lens, try to get an image of candle flame on other side on screen. Adjust the screen till you get a clear image.

7.Now measure ‘V’ (image distance) and object distance ‘U’ and values are tabulated.

8.Repeat the experiment for various object distances like 50cm, 40cm, 30cm etc. Measure the distances of images and tabulate the values.

9.Using formula $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$, find f in all cases we will observe the value ‘f’is equal in all cases.

10.The value ‘f’ is the focal length of lens.

4. Harsha tells Siddhu that the double convex lens behaves like a convergent lens. But Siddhu knows that Harsha’s assertion is wrong and corrected Harsha by asking some questions. What are the questions asked by Siddhu? [AS2]

A. 1.If the refractive index of medium is greater than the refractive index of lens, then double convex lens behaves unlike to convergent lens.

2.What happens to the rays when the object is kept in between the pole and focal point.

3.How do the air bubbles in water behave?

5. You have a lens. Suggest an experiment to find out the focal length of lens? [AS3]

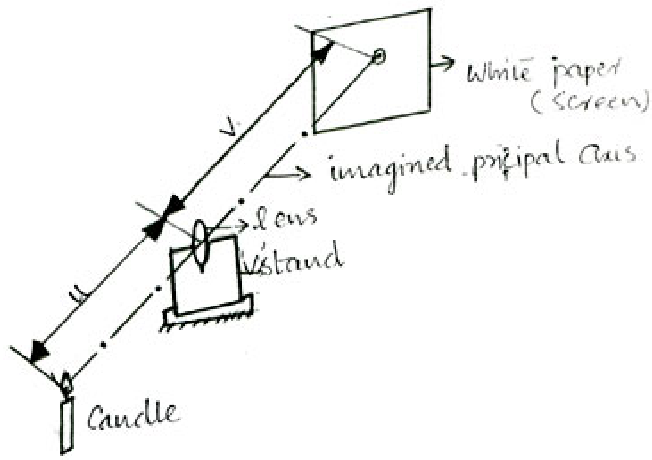
A. 1.Take a v stand and place it on a long table at the middle.

2. Place a convex lens on v stand Imagine the principal axis of lens.

3. Light a candle and ask your friend to take the candle far away from the lens along the principal axis.

4. Adjust a screen (a sheet of paper perpendicular to axis) which is on other side of lens until you get an image on it.

5. Measure the distance of image of V-stand of lens (image distance v) and object distance [u], tabulate the values.



6.Now place the candle at a distance of 60cm from lens, try to get an image of candle flame on the other side on screen. Adjust the screen till you get a clear image.

7.Measure the image distance 'v' and object distance 'u' and record the values in table.

S.No	Object distance (u)	Image distance (v)	Focal length(f)

8.Repeat the experiment for various object distances like 50cm, 40cm, 30cm etc. Measure the image distances in all class and tabulate them.

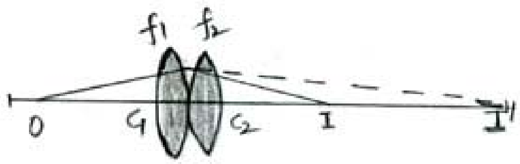
9.Using the formula $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$. Find f in all cases. The value of 'f' is the focal lengths of lens.

6. Let us assume a system that consists of two lenses with focal length f_1 and f_2 respectively. How do you find the focal length of system respectively, when-

i.Two lenses are touching each other.

ii.They are separated by distance 'd' common principal axis [AS3]

A. i.When two lenses are in contact with each other.



1.Focal lengths of two lenses l_1, l_2 is f_1, f_2 (say) respectively.

2.Lenses are made of same material with refractive index 'n'

3.$u = OC_1$

$v_1 = I_1C_1$

(I_1 = image of object 'o' produced by lens of focal length f_1)

+ $v_2 = -I_1C_1$ ($\because$ the image of object 'o' acts as a object to second lens of focal length f_2)

$\approx -I_2C_1$

$V_2 = C_2I$ (I is the image formed by I^1)

$$\therefore \frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{f_1}; \frac{1}{u_2} + \frac{1}{v_2} = \frac{1}{f_2}$$

$$\frac{1}{OC_1} + \frac{1}{I_1C_1} = \frac{1}{f_1} \text{ ——(1); } \frac{1}{-C_1I_1} + \frac{1}{IC_1} = \frac{1}{f_2} \text{ —(2)}$$

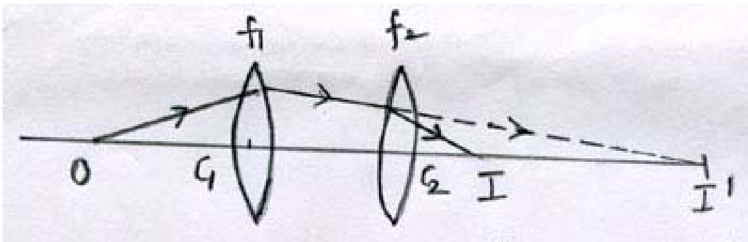
Adding (1) and (2) we get

$$\frac{1}{OC_1} + \frac{1}{C_1I} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

Effective focal length $F=\frac{f_1 f_2}{f_1+f_2}$

ii. Combined focal length of two thin lenses separated by a distance d



Let the lens be l_1, l_2 and focal lengths of lens are f_1 and f_2 respectively

Let the object be o

I^1 be the image formed by object o by lens l_1

I be the image formed by image I^1 by lens l_2

So object distance for first lens $(l_1) = u_1 = OC_1$

Image distance for lens $l_1 = v_1 = I^1 C_1$

Object distance for second lens $= I^1 C_2 = u_2$

Images distance for second lens $= I C_2 = v_2$

$I_1 C_1 = I_1 C_2 + d$

So $\frac{1}{f_1}=\frac{1}{u}+\frac{1}{v_1}$

$$\frac{1}{f_1}=\frac{1}{OC_1}+\frac{1}{I_1C_1}\longrightarrow(1)$$

Similarly

$$\frac{1}{f_2}=\frac{1}{u_2}+\frac{1}{v_2}$$

$$=\frac{1}{-I_1C_2}+\frac{1}{IC_2}$$

$$= \frac{1}{-(I_1C_1 - d)} + \frac{1}{IC_2} \longrightarrow (2)$$

And $\frac{1}{OC_1} + \frac{1}{IC_2} = \frac{1}{F}$ ____(3)

Solving (1) and (2) and substituting (3) we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F} + \frac{d}{f_1 f_2}$$

$$\frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{F}$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

7. Collect the information about the lenses available in a optical shop. Find out how the focal length of lens may be determined by given 'power' in lens. [AS4]

A. Lenses available in an optical shop are-

1) Plano convex lens
2) Double convex lens
3) Plano concave lens
4) Double concave lens
5) Cylindrical lens
6) Achromatic lens
7) Aspheric lens
8) IR lens
9) UV lens
10) Atoric lens

To know focal length:

If given power is 'p', then focal length

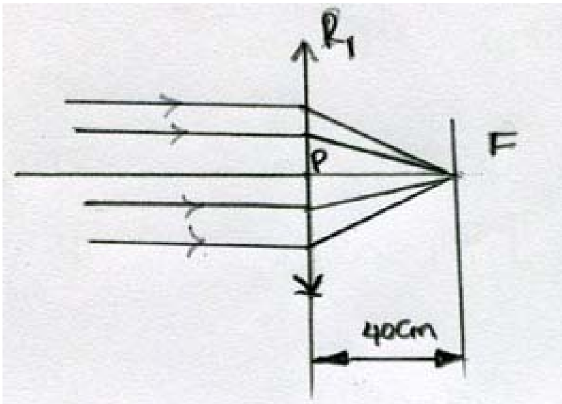
$$f = \frac{100}{p} cm$$

'p' is positive, lens in Biconvex

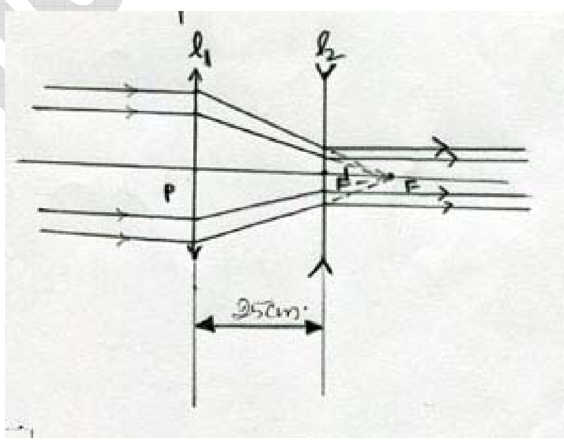
'p' is negative, lens is Biconcave

8. A parallel beam of rays is incident on a convergent lens with a focal length of 40 cm. where should a divergent lens with a focal length of 15cm be placed for the beam of rays to remain parallel after passing through two lenses? Draw a ray diagram? (AS5)

A. A parallel beam of rays when incident on a convergent lens, after refraction they meet at focus of lens.



Divergent lens of focal length 15cm should be placed after convergent lens at a distance of (40-15) cm from convergent lens. So the rays remain parallel.



(Or)

Let focal length of convergent lens = f_1 = 40cm

Focal length of divergent lens = f_2 = –15 cm

Let the 'd' is the distance between two lens

Then $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

$$\frac{1}{\infty} = \frac{1}{40} + \frac{1}{-25} - \frac{d}{40 \times (-15)} \; (for\ emergent\ parallel\ beam\ F = \infty)$$

$$0 = \frac{1}{40} - \frac{1}{25} + \frac{d}{40(15)}$$

$$\frac{1}{15} = \frac{1}{40} + \frac{d}{40(15)}$$

$$d = 40(15)\left[\frac{1}{15} - \frac{1}{40}\right]$$

$$d = \frac{40 - 15}{40 \times 15} \; (40 \times 15)$$

d = 25cm.

Distance between two lens = 25cm

9. Draw ray diagrams for the following positions and explain the nature and position of image. [AS6]

i) Object placed at c_2

ii) Object placed between F_2 and centre p.

A. i. object is placed at c_2

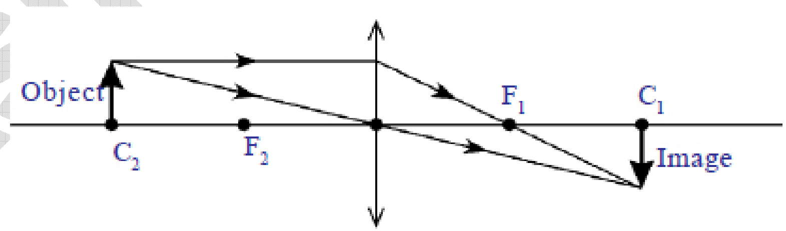


Image:

i. Real , inverted same size of object

ii. Formed at c_1.

Object placed between $F_2$and centre p:

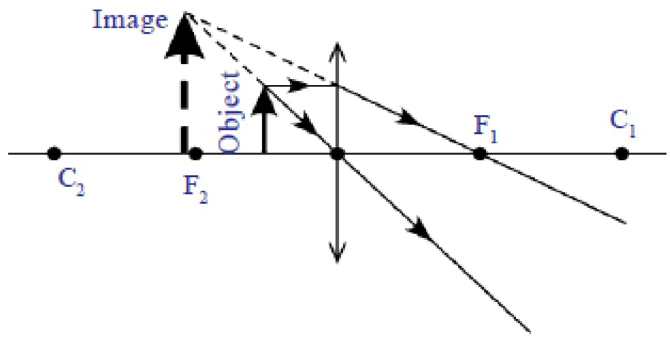


Image:

i) Virtual, magnified, erect image

ii) Image formed between C_2 and F_2

10. How do you appreciate the coincidence of the experimental facts with the results obtained by a ray diagram in terms of behavior of images formed by lenses? [AS7]

A. 1. The behavior of images formed by lenses i.e., image in real/virtual; inverted/erect; magnified/diminished when the object is kept at different places from lens.

2. The ray diagrams are based on the fact that the light travels in a straight line and takes the shortest way for its travel [Fermat's principal]

Problem

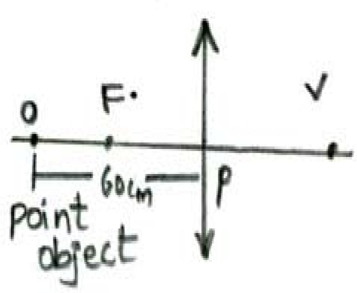
1. **The focal length of a converging lens is 20cm. An object is 60cm from the lens. Where the image will be formed and what kind of image it is? [AS1]**

A. Given

Focal length of converging lens (f) = 20 cm

Object distance, u = 60cm

Image distance, v = v (say)



Lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

u = –60 w.r.t sign convention

$$\frac{1}{v} - \frac{1}{-60} = \frac{1}{20}$$

$$\frac{1}{v} + \frac{1}{60} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{60}$$

$$\frac{1}{v} = \frac{1}{30}$$

v = 30 cm

The image is Real, Diminished and Inverted.

2. A double convex lens has two surfaces of equal radii 'R', and refractive index n = 1.5. Find the focal length 'f'? (AS1)

Sol: Radii of curvature of both surfaces are equal-

$R_1 = R_2 = R$

Refractive index of material n = 1.5

Focal length of lens, f =?

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

For double convex lens w.r.t. sign convention

R_1 is +ve; R_2 is – ve

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{-R_2}\right)$$

$$= (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$= (n-1)\left[\frac{1}{R} + \frac{1}{R}\right] \qquad [\because R_1 = R_2 = R]$$

$$= (n-1)\left(\frac{2}{R}\right) \qquad [\because Given\, n = 1.5]$$

$$= (1.5-1)\left(\frac{2}{R}\right)$$

$$= \frac{(0.5)(2)}{R}$$

$$\frac{1}{f} = \frac{1}{R}$$

f = R.

Focal length of double convex lens is R.

3. Find the refractive index of the glass which has symmetrically convergent lens if its focal length is equal to radius of curvature of its surface? [AS7]

A. Given

Lens is symmetrical converging lens-

$R_1 = R_2 = R$

Focal length of lens, f = R

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

According sign convention-

$R_1 = R$ and $R_2 = -R_2$

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{-(R_2)}\right]$$

$$= (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$= (n-1)\left[\frac{1}{R} + \frac{1}{R}\right] \qquad [\because R_1 = R_2 = R]$$

$$= (n-1)\left(\frac{2}{R}\right)$$

$$\frac{1}{f} = \frac{(n-1)2}{R}$$

$$f = \frac{R}{2(n-1)}$$

$$\cancel{R} = \frac{\cancel{R}}{2(n-1)}$$

$2(n-1) = 1$

$$(n-1) = \frac{1}{2}$$

$$n = 1 + \frac{1}{2}$$

n = 1.5

Refractive index of glass n = 1.5

4. Find the radii of curvature of convexo concave. Convergent lens made of glass with refractive index 1.5 having focal length of 24cm. One of the radii of curvature is double the other. [AS7]

Sol: Refractive index of material = n = 1.5

Given

"Convexo –concave convergent lens"

Focal length of f=24cm

Given one of the radius of curvature is double to other so

$R_2 = 2R_1$

According to sign convention R_1 is positive

R_2 is positive

Using lens maker's formula

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= (n-1)\left(\frac{1}{R_1} - \frac{1}{2R_1}\right)$$

$$= (n-1)\left[\frac{1}{2R_1}\right]$$

$$\frac{1}{24} = (1.5-1)\left[\frac{1}{2}.R_1\right]$$

$$\frac{1}{24} = (0.5)(0.5)\left[\frac{1}{R_1}\right]$$

R_1= 24 (0.5) (0.5)

= 6

R_1= 6cm

Given R_2= $2R_1$

= 2 × 6

= 12cm

Ans: R_1= 6cm

R_2= 12cm

5. When the distance between two point sources of light is 24cm, where should a convergent lens with focal length of f=9cm be placed between them to obtain the images of both sources at same point? [AS7]

Sol. Distance between the two point sources d = 24 cm

Focal length of lens f = 9 cm

Lens is separated from first object at a distance = x (say)

From first point source

$$\frac{1}{f}=\frac{1}{u_1}+\frac{1}{v_1}$$

$$\frac{1}{9}=\frac{1}{x}+\frac{1}{v}\longrightarrow(i)$$

Similarly from second point source $\frac{1}{f}=\frac{1}{u_2}+\frac{1}{v_2}$

Here v_2 is negative w.r.t sign conversion

$$\frac{1}{9}=\frac{1}{24-x}-\frac{1}{v}\longrightarrow(ii)$$

$[\because u_1+u_2=d=24]$

$[x + u_2 = 24]$

$[u_2 = 24 - x]$

(i) + (ii)

$$\Rightarrow\frac{1}{9}+\frac{1}{9}=\frac{1}{x}+\frac{1}{v}+\frac{1}{24-x}-\frac{1}{v}$$

$$\frac{2}{9}=\frac{1}{x}+\frac{1}{24-x}$$

$$\frac{2}{9}=\frac{24-x+x}{(24-x)x}$$

$$\frac{2}{9}=\frac{24}{(24-x)x}$$

$24x - x^2 = 12 \times 9$

$x^2 - 24x = -108$

$x^2 - 24x + 108 = 0$

$x^2 - 18x - 6x + 108 = 0$

$(x - 6)(x - 18) = 0$

$x = 6, 18$

Lens should be placed at 6cm or 18 cm.

Fill in the Blanks

1. The rays from the distant object, falling on the convex lens pass through ________.
2. The ray passing through the _____ of the lens is deviated.
3. Lens formula is given by _______.
4. The focal length of the Plano convex lens is 2R where R is the radius of curvature of the surface. Then the refractive index of the material of the lens is ________.
5. The lens which can form real and virtual images is _______.
6. If the focal length is positive then the lens is ______.
7. If the focal length is negative then the lens is ______.
8. _______ image cannot be seen by eyes.
9. _______ image captured on screen.
10. _______ lens is called converging lens.

Key: 1. Focal point, 2. Pole, 3. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$,

4. 3/2, 5. Convex 6. Convex,

7. Concave 8. Virtual 9. Real

10.convex.

Objective type Questions

1. **Which of the following material cannot be used to make a lens? [ ]**

A) Water B) Glass C) Plastic D) Clay

2. **Which of the following is true? [ ]**

A) The distance of virtual image is always greater than the object distance for convex lens

B) The distance of virtual image is not greater than the object distance for convex lens

C) Convex lens always forms a virtual image

D) Convex lens always forms a real image

3. **Focal length of the Plano convex lens is ______ when radius of curvature R and n is refractive index of lens. [ ]**

A) f = R B) $f = \frac{R}{2}$ C) $f = \frac{R}{n-1}$ D) $f = \frac{n-1}{R}$

4. **The value of focal length of lens is equal to the value of image distance when the rays are- [ ]**

A) Passing through the optic centre

B) Parallel to principal axis

C) Passing through focus

D) In all cases

5. **Which of the following is the Lens-Maker's formula? [ ]**

A) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ B) $\frac{1}{f} = (n+1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

C) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ D) $\frac{1}{f} = (n+1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

Key:

1. D, 2. B, 3. C, 4.D, 5.C

Match the Following

I. Group-I — **Object position for convex lens** | **Group-II** — **image position**

1. Beyond centre of curvature [ ] a) Real, inverted, magnified
2. At center of curvature [ ] b) Real, inverted, same size
3. Between centre of curvature and focus [ ] c) Real, inverted, diminished
4. Between focus/ optic centre [ ] d) Converged to focus
5. Parallel beam [ ] e) Virtual, erect, magnified

Key:

1. c, 2. b, 3. a, 4. e, 5. d

II. Group-I | **Group-II**

1. Lens-Maker's Formula [ ] a) $n = \frac{\sin i}{\sin r}$
2. Lens Formula [ ] b) $F = \frac{f_1 f_2}{f_1 + f_2}$
3. Optical power [ ] c) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

4. Snell's Law [ ] d) $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

5. When two lens are touching

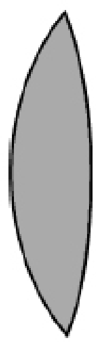
each other then focal length [ ] e) $p=\frac{1}{f(m)}$

Key:

1. c, 2. d, 3. e, 4. a, 5. b

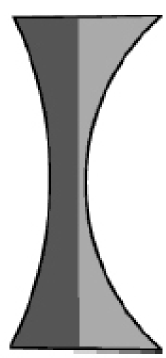
Important Images

1. Biconvex lens



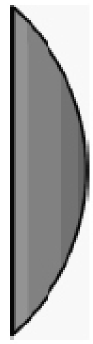
Biconvex

2. Biconcave lens



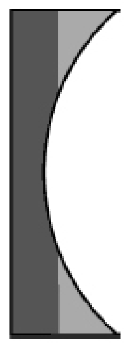
Biconcave

3. Plano convex



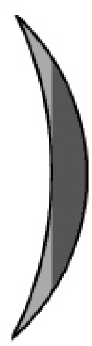
Plano-convex

4. Plano concave



Plano-concave

5. Concavo - convex



Concavo-convex