TWO PORT NETWORKS

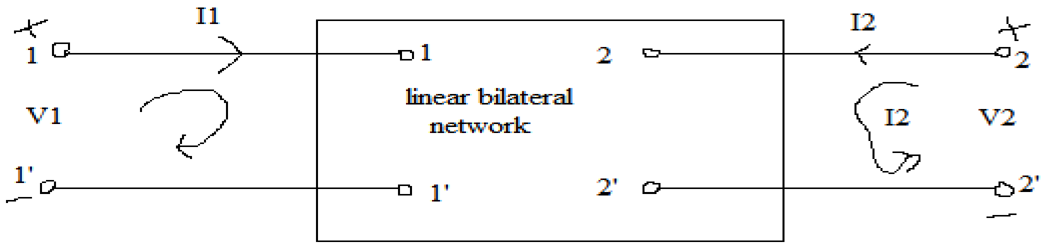
Introduction:

A port is normally referred to a pair of terminals of a network through which we can have access to network either for a source for measuring an output. We have already seen the methods of calculating current in any part of the network. Frequently the problem is more restricted in nature and may be that of calculating the response at a terminal pair designated as output terminals, when the excitation is applied at another terminal pair designated as input terminals. It is the problem of the external behavior of network. The network having only two pairs of terminals such as input and output terminals through which it is accessible, and also these are called two port networks. We will study the relation between the input and output voltages and currents and define different sets of two port parameters.

If we relate the voltage of one port to the current of the same port, we get driving point (input or output) immittance. On the other hand, if we relate the voltage of one port to the current at another port, we get transfer immittance. Immittance is a general term used to represent either the impedance or the admittance of a network. We have discussed the driving point and transfer immittance of one port network. For one port network we have only driving point impedance / admittance and transfer immittances. A general network with two pairs of terminals is a very important building block in control systems, transmission systems, and communication systems.

General Two Port Networks:

We will consider a general two port network composed of linear, bilateral elements and no independent sources. Dependent sources are permitted. It is represented as a block box accessible terminal pairs as shown in fig.



The terminal pair $(1 - 1^1)$ represent port1 and is called input port or sending end and the terminal pair $(2 - 2^1)$ represent port 2 and is called output port or receiving end. The voltage and current at port 1 are V1, I1 and at port 2 are V2, I2. The polarities of V1 and V2 and the directions of I1 and I2 are customarily selected as shown in fig. out of the four variables V1, I1, V2 and I2 only two are independent. The other two are expressed in terms of the independent variables in terms of network parameters. This can be done in number of ways.

S.NO	NAME of PARAMETERS	EXPRESSED (dependent)	INTERMS of (independent)	EQUATIONS
1	Open circuit Impendence parameters	V1 , V2	I1, I2	$\begin{matrix} V1 \\ V2 \end{matrix} = \begin{matrix} Z11 & Z12 \\ Z21 & Z22 \end{matrix} \begin{matrix} I1 \\ I2 \end{matrix}$
2	Short circuit Admittance parameters	I1, I2	V1, V2	$\begin{matrix} I1 \\ I2 \end{matrix} = \begin{matrix} Y11 & Y12 \\ Y21 & Y22 \end{matrix} \begin{matrix} V1 \\ V2 \end{matrix}$
3	Transmission parameters (ABCD)	V1, I1	V2, I2	
4	Hybrid parameters (h-parameters)	V1, I2	I1, V2	$\begin{matrix} V1 \\ I2 \end{matrix} = \begin{matrix} h11 & h12 \\ h21 & h22 \end{matrix} \begin{matrix} I1 \\ V2 \end{matrix}$

Short circuit Admittance Parameter:

Consider the general two port network and assume that the network is made up of n loops including the two external loops. If I1, I2, - - - - -, In represent the loop currents, the network equations in loop method of analysis can be written as ZI = V i.e.,

$$\begin{matrix} Z11 & Z12 & Z1n \\ Z21 & Z22 & Z2n \\ Zn1 & Zn2 & Znn \end{matrix} \begin{matrix} V1 \\ I1 \end{matrix} = \begin{matrix} A & B \\ C & D \end{matrix} \begin{matrix} V2 \\ -I2 \end{matrix} \quad \begin{matrix} I1 \\ I2 \\ In \end{matrix} = \begin{matrix} V1 \\ V2 \\ Vn \end{matrix}$$ ----------- (1)

By Cramer's rule we get,

I1 = V1 (A11 / Dz) + V2 (A21 / Dz) + - - - - -

I2 = V1 (A12 / Dz) + V2 (A22 / Dz) + - - - - - -

Where Dz is the determinant of the loop impedance matrix [Z] and Aij is the cofactor

Aij = $(-1)^{(i+j)} * |\, Z\, | with\ ith\ row\ and\ jth\ column$

Network is a passive network with no independent sources, so source voltages i.e.,

V3 = V4 = V5 = - - - - - = Vn = 0.

Heance ,

I1 = V1 (A11 / Dz) + V2 (A21 / Dz)

I2 = V1 (A12 / Dz) + V2 (A22 / Dz) ------------------ (2)

Since the dimensions of (Aij / Dz) is an admittance, we can write equations (2) as

$I_1 = Y_{11}\ V_1 + Y_{12}\ V_2$

$I_2 = Y_{21}\ V_1 + Y_{22}\ V_2$ ---------------- (3)

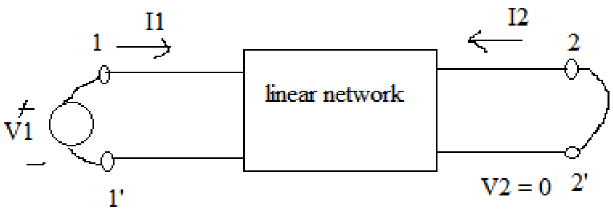
Where $Y_{11} = A_{11} / D_Z$ and $Y_{12} = A_{12} / D_Z$

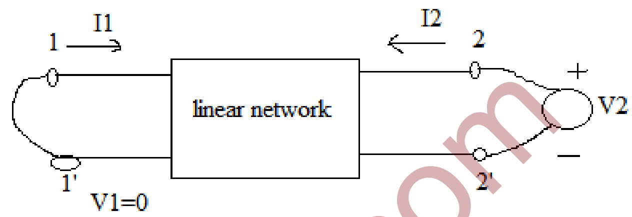
$Y_{21} = A_{21} / D_Z$ and $Y_{22} = A_{22} / D_Z$

These parameters are called as Admittance (Y) parameters. These can be determined by equating V_1 & V_2 equal to zero i.e. by short circuiting the ports (1) & (2). Since each parameter is admittance and is obtained by short circuiting one of the ports, these parameters are known as short circuit admittance parameters.

The short circuit admittance parameters are obtained by short circuiting one of the ports and are defined as fallows.

If port (2) is short circuited as in fig i.e. $V_2 = 0$, then





From equation (3) we have,

$Y_{11} = I_1 / V_1 \mid V_2 = 0$ -- short circuit driving point admittance at port (1)

$Y_{21} = I_2 / V_1 \mid V_2 = 0$ -- short circuit transfer admittance between port (1) & (2) - - - (4)

If port (1) is short circuited as in fig i.e. $V_1 = 0$, then

$Y_{12} = I_1 / V_2 \mid V_1 = 0$ -- short circuit transfer admittance between port (2) & (1)

$Y_{22} = I_2 / V_2 \mid V_1 = 0$ -- short circuit driving point admittance at port (2) - - - (5)

For bilateral networks $Y_{12} = Y_{21}$

Hence the two port network can be described in terms of short circuit parameters as from equation

$$\begin{matrix} I_1 \\ I_2 \end{matrix} = \begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix} \quad \begin{matrix} V_1 \\ V_2 \end{matrix} \text{ - - - - - (6)}$$

Open Circuit Impedance Parameters:

For the general two port network, consider the nodal equations with n nodes as

$$\begin{matrix} Y_{11} & Y_{12} & Y_{1n} \\ Y_{21} & Y_{22} & Y_{2n} \\ Y_{n1} & Y_{n2} & Y_{nn} \end{matrix} \quad \begin{matrix} V_1 \\ V_2 \\ V_n \end{matrix} \quad = \quad \begin{matrix} I_1 \\ I_2 \\ I_n \end{matrix} \text{ -------- (i)}$$

Since there are no sources inside the network except the two current sources I_1 and I_2 at node (1) and node (2), the remaining current sources I_3, ----- , I_n are all set to zero.

By Cramer's rule solving,

$$V_1 = (A_{11}/ D_Y) I_1 + (A_{21}/ D_Y) I_2$$

$$V_2 = (A_{12}/ D_Y) I_1 + (A_{22}/ D_Y) I_2 \text{ ------- (ii)}$$

WhereD_Y, is the determinant of nodal admittance matrix [Y] and A_{ij} is the cofactor of D_Y with ith row and jth column removed from D_Y.

Since the terms (A_{ij}/ D_Y) has the dimension of impedence

From equation (ii) as fallows,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \text{ ------- (iii)}$$

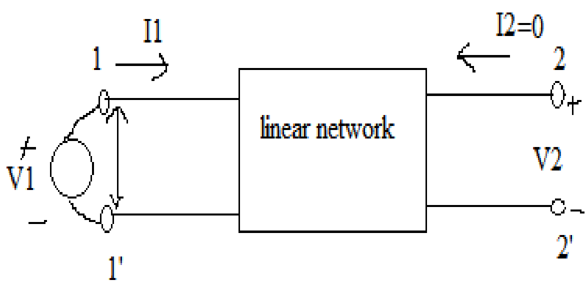
Where $Z_{11} = (A_{11}/ D_Y)$

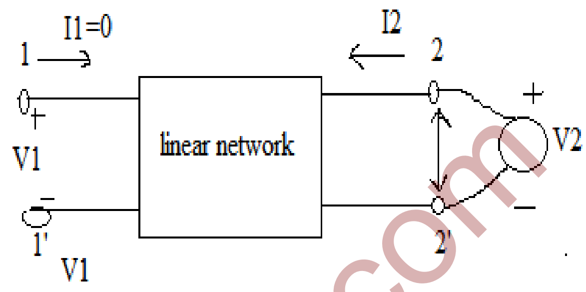
$Z_{12} = (A_{21}/ D_Y)$

$Z_{21} = (A_{12}/ D_Y)$ and

$Z_{22} = (A_{22}/ D_Y)$

These parameters are called impedance parameters. They can be obtained by equating I_1 and I_2 to zero i.e., by open circuiting ports (1) & (2) as shown in fig.





If port (2) is open circuited that is $I_2 = 0$ then

$Z_{11} = V_1 \,/\, I_1 \mid I_2 = 0$ driving point impedance at port (1)

$Z_{21} = V_2 \,/\, I_1 \mid I_2 = 0$ Transfer impedence between ports (2) & (1) - - - (iv)

If port (1) is open circuited that is $I_1 = 0$ then

$Z_{12} = V_1 \,/\, I_2 \mid I_1 = 0$ Transfer impedence between ports (1) & (2)

$Z_{22} = V_2 \,/\, I_2 \mid I_2 = 0$ driving point impedence at port (2) - - - - - (v)

For bilateral networks, $Z_{12} = Z_{21}$

Hence the two port network can be described in terms of open circuit impedance parameters as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{- - - - - (vi)}$$

Relationship between Y and Z Parameters:

We can possible to express the relationship between Y and Z parameters and also vice versa.

From equation (vi), I_1, I_2 expressed as,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{matrix} I_1 \\ I_2 \end{matrix} = \begin{matrix} \frac{Z_{22}}{D_Z} & \frac{-Z_{21}}{D_Z} \\ -Z_{\frac{12}{D_Z}} & Z_{\frac{11}{D_Z}} \end{matrix} \begin{matrix} V_1 \\ V_2 \end{matrix}$$

From that we can show that,

$Z = Y^{-1}$

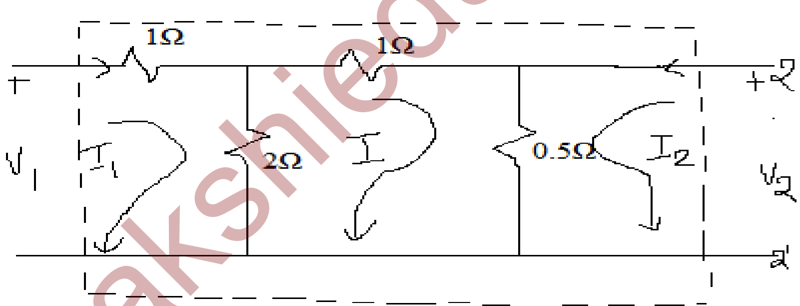
$Z_{11} = Y_{22}/ D_Y$

Z_{12} =- Y_{21}/ D_Y

Z_{21} =- Y_{12}/ D_Y and

Z_{22} =- Y_{11}/ D_Y

Example 1: For the two port network determine Z and Y parameters.



From the fig, the loop equations are

1st loop,

I_1 + 2 (I_1 - I) = V_1

3 I_1 -2I = V_1 ---- (1)

2nd loop,

2 (I-I_1) + 1 * I + (I + I_2) (0.5) = 0

-2 I_1 + 3.5I + 0.5 I_2 = 0 ----(2)

3rd loop,

0.5 I + 0.5 I_2 = V_2 ---- (3)

From equation (2), 3.5 I = 2 I_1 - 0.5 I_2

I = (4/7) I_1 - (1/7) I_2 ---- (4)

Substitute (4) in (1) & (3)

3 I_1 - 2 [(4/7) I_1 - (1/7) I_2] = V_1

(13/7) I_1 + (2/7) I_2 = V_1 -----(5)

[(4/7) I_1 - (1/7) I_2 + I_2] 0.5 = V_2

(2/7) I_1 + (3/7) I_2 = V_2 ------ (6)

From equations (5) and (6) write the matrix form,

$$\begin{matrix} V1 \\ V2 \end{matrix} = \begin{matrix} \frac{13}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} \end{matrix} \begin{matrix} I1 \\ I2 \end{matrix}$$

Therefore from that, Z = $\begin{matrix} Z11 & Z12 \\ Z21 & Z22 \end{matrix} = \begin{matrix} \frac{13}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} \end{matrix}$

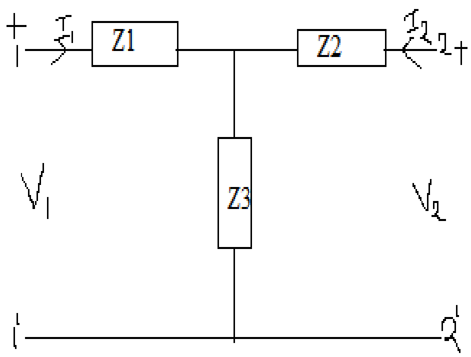
Y parameters = Y = Z^{-1} = $\begin{matrix} \frac{13}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} \end{matrix}^{-1}$

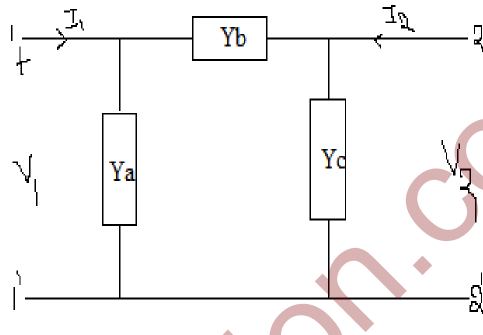
Solve the above and we get, Y = (7/5) $\begin{matrix} \frac{3}{7} & \frac{-2}{7} \\ \frac{-2}{7} & \frac{13}{7} \end{matrix} = \begin{matrix} \frac{3}{5} & \frac{-2}{5} \\ \frac{-2}{5} & \frac{13}{5} \end{matrix}$

Y = $\begin{matrix} Y11 & Y12 \\ Y21 & Y22 \end{matrix} = \begin{matrix} \frac{3}{5} & \frac{-2}{5} \\ \frac{-2}{5} & \frac{13}{5} \end{matrix}$

Example 2: For the given π-network (delta connected network) determine the equivalent T- network (star connected network) using two port equations.

T – Network and π-network show in below fig.





The open circuit parameters are determined as given below

T-network:

Port 2 is open circuited i.e. $I_2 = 0$

$$V_1 = I_1 (Z_1 + Z_3)$$

$$V_2 = I_1 Z_3$$

$$Z_{11} = V_1/I_1 \mid I_2 = 0 = Z_1 + Z_3$$

$$Z_{21} = V_2/I_1 \mid I_2 = 0 = Z_3$$

Port 1 is open circuited i.e. $I_1 = 0$

$$V_2 = I_2 (Z_2 + Z_3)$$

$$V_1 = I_2 Z_3$$

$$Z_{22} = V_2/I_2 \mid I_1 = 0 = Z_2 + Z_3$$

$$Z_{21} = V_1/I_2 \mid I_2 = 0 = Z_3$$

Therefore Z- parameters of T-network, $\begin{matrix} Z11 & Z12 \\ Z21 & Z22 \end{matrix} = \begin{matrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{matrix}$

π-network:

The short circuit parameters are determined as fallows

Port 2 is short circuited i.e. $V_2 = 0$

$$I_1 = V_1 (Y_a + Y_b)$$

$$I_2 = - V_1 Y_b$$

$$Y_{11} = I_1 / V_1 \mid V_2 = 0 = Y_a + Y_b$$

$$Y_{21} = I_2 / V_1 \mid V_2 = 0 = - Y_b$$

Port 1 is short circuited i.e. $V_1 = 0$

$$I_2 = V_2 (Y_a + Y_c)$$

$$I_1 = - V_2 Y_b$$

$$Y_{22} = I_2 / V_2 \mid V_1 = 0 = Y_c + Y_b$$

$$Y_{12} = I_1 / V_2 \mid V_1 = 0 = - Y_b$$

The Y parameters of a π-network = $\begin{matrix} Y11 & Y12 \\ Y21 & Y22 \end{matrix} = \begin{matrix} Y_a + Y_b & - Y_b \\ - Y_b & Y_c + Y_b \end{matrix}$

In order the two networks are equivalent to Z parameters both networks must be equal. $\begin{matrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{matrix} = \begin{matrix} Y_a + Y_b & - Y_b \\ - Y_b & Y_c + Y_b \end{matrix}^{-1}$

$$= (1/Y_a Y_b + Y_c Y_b + Y_a Y_c) \begin{matrix} Y_a + Y_b & - Y_b \\ - Y_b & Y_c + Y_b \end{matrix}$$

$$Z_3 = (Y_b / Y_a Y_b + Y_c Y_b + Y_a Y_c) = [(1/Z_b) / (1/Z_a Z_b) + (1/Z_c Z_b) + (1/Z_a Z_c)]$$

$$= Z_a Z_c / (Z_a + Z_b + Z_c)$$

$$Z_1 + Z_3 = Y_c + Y_b / \sum Y_a Y_b$$

$$Z_1 = Y_c / \sum Y_a Y_b = Z_a Z_b / (Z_a + Z_b + Z_c)$$

$Z_2 + Z_3 = Y_a + Y_b / \sum Y_a Y_b$

$Z_2 = Y_a / \sum Y_a Y_b = Z_c Z_b / (Z_a + Z_b + Z_c)$

This gives delta to star conversion, or π to T conversion. The star to delta conversion can also be obtained in a similar way. Express $Z_a, Z_b \& Z_c$ interms of $Z_1, Z_2 \& Z_3$

$$\begin{matrix} Y_a + Y_b & -Y_b \\ -Y_b & Y_c + Y_b \end{matrix} = \begin{matrix} Z_1 + Z_3 & Z_2 \\ Z_2 & Z_2 + Z_3 \end{matrix}^{-1}$$

$$= (1/Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1) \begin{matrix} Z_1 + Z_3 & -Z_2 \\ -Z_2 & Z_2 + Z_3 \end{matrix}$$

$Y_b = Z_3 / \sum Z_2 Z_1$

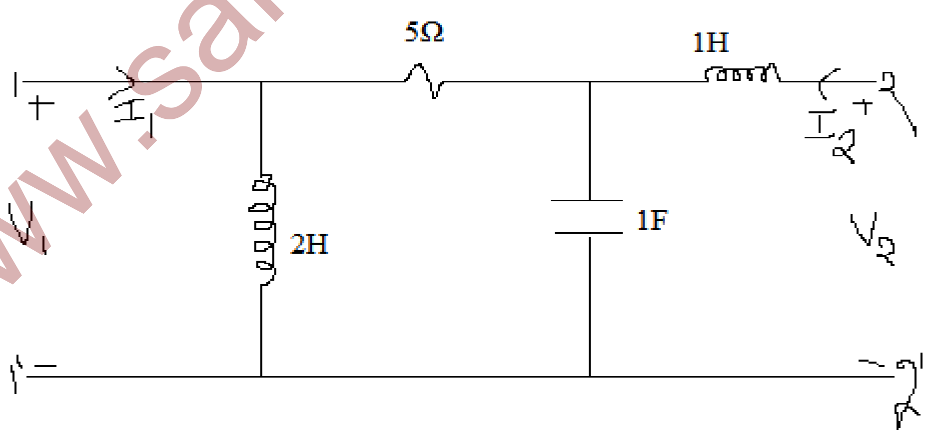
Therefore $Z_b = \sum Z_2 Z_1 / Z_3 = Z_1 + Z_2 + (Z_2 Z_1 / Z_3)$

$Y_a + Y_b = Z_2 + Z_3 / \sum Z_2 Z_1$

Similarly, $Z_a = Z_1 + Z_3 + (Z_3 Z_1 / Z_2)$

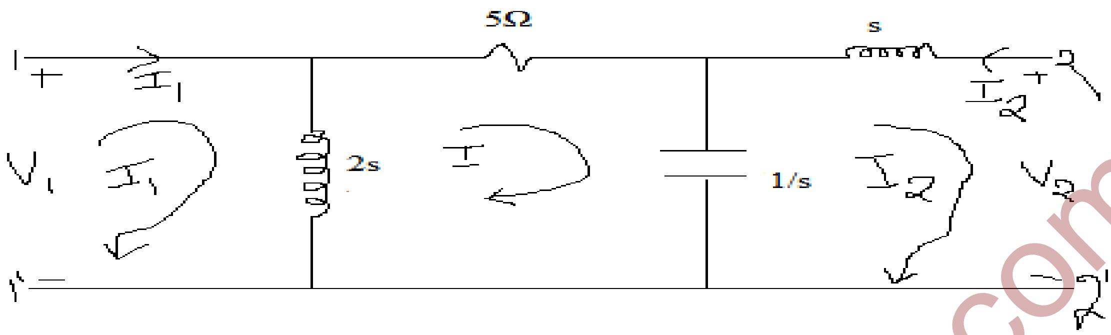
$Z_c = Z_3 + Z_2 + (Z_2 Z_3 / Z_1)$

Example 3: For the fallowing two port network, determine the impedance parameters



The impedance parameters, $\begin{matrix} V1 \\ V2 \end{matrix} = \begin{matrix} Z11 & Z12 \\ Z21 & Z22 \end{matrix} \begin{matrix} I1 \\ I2 \end{matrix}$

The loop equations are,



$(I1$- I) 2s = $V1$ ---------- (1)

(I -I_1) 2s + 5I + (I +I_2) = V_2----- (2)

s I_2 + (1/s) (I +I_2) = V_2----- (3)

From equation (2)

(5 + 2s + 1/s) I = 2s I_1 - 1/s I_2

(5 s+ $2s^2$ + 1) I = $2s^2$ I_1 - I_2

I = $\frac{2s^2}{2s^2+5s+1}I_1 - \frac{1}{2s^2+5s+1}I_2$ ------ (4)

Substituting for I in equations (1) & (3) we get

2s I_1 – 2s [$\frac{2s^2}{2s^2+5s+1}I_1$ - $\frac{1}{2s^2+5s+1}I_2$] = V_1

I_1 [2s -$\frac{2s^2}{2s^2+5s+1}$] + I_2 [$\frac{2s}{2s^2+5s+1}$] = V_1

Solve the above and we get the equation,

$\frac{2s\,(5s+1)}{2s^2+5s+1}I_1 + \frac{2s}{2s^2+5s+1}I_2 = V_1$ ------ (5)

Therefore $Z11 = \frac{2s\,(5s+1)}{2s^2+5s+1}$ $Z12 = \frac{2s}{2s^2+5s+1}$

From substitute I in equation (3)

$\frac{1}{s}$ I + (s +$\frac{1}{s}$) $I_2 = V_2$

$\frac{1}{s}$ [$\frac{2s^2}{2s^2+5s+1}$ I_1 - $\frac{1}{2s^2+5s+1}$ I_2] + $\frac{s^2+1}{s}$ $I_2 = V_2$

$\frac{2s}{2s^2+5s+1}$ I_1 + $\frac{s^2+1}{s}$ - $\frac{1}{s(2s^2+5s+1)}$ $I_2 = V_2$

$\frac{2s}{2s^2+5s+1}$ I_1 + $\frac{2s^3+5s^2+3s+1}{2s^2+5s+1}$ $I_2 = V_2$

$Z21 = \frac{2s}{2s^2+5s+1}$