# Transmission (ABCD) and HYBRID PARAMETERS

Transmission or ABCD Parameters:

The transmission parameters relate the voltages and current at port1 sending end to voltage and current port2 receiving end. These are commonly employed in determining the sending end quantities in terms of receiving end quantities of a transmission line in power systems. In power system network, the power is received at the receiving end i.e. port2 and the current direction for I_2 at port2 will be flowing away from the port (which is opposite to the direction of I_2 conventionally assumed). Hence current at port2 is taken as $-I_2$ the equations are
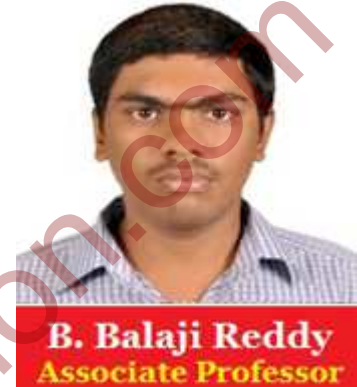
$$V_1 = A\,V_2 - B\,I_2$$

$$I_1 = C\,V_2 - D\,I_2 \text{ ------- (1)}$$

This can be represented in matrix form as shown in below,

$$\begin{matrix} V1 \\ I1 \end{matrix} = \begin{matrix} A & B \\ C & D \end{matrix} \begin{matrix} V2 \\ -I2 \end{matrix}$$

These parameters can be determined by conducting open circuit and short circuit tests at port2

When port2 is on open circuit i.e. $I_2 = 0$.

From equation (1) we get,

$$V_1 = A\,V_2$$

Or $A = \frac{V_1}{V_2} \mid I_2 = 0$

$$I_1 = C\,V_2$$

Or $C = \frac{I1}{V_2} \mid I_2 = 0$ -------------- (2)

From the above equations it is observed that A is dimensionless quantity and C is an admittance and is expressed as in Mhos.

When port2 is short circuited i.e., $V_2 = 0$.

From equation (1) we get,

$$V_1 = -B \, I_2$$

$$\text{Or } B = \frac{V_1}{-I_2} \mid V_2 = 0$$

$$I_1 = -D \, I_2$$

$$\text{Or } D = \frac{I1}{-I_2} \mid V_2 = 0 \text{ -------------- (3)}$$

From the above equation it is observed that B is an impedance and is expressed in Ohms and D is dimensionless quantity.

Relation between ABCD and Z Parameters:

We can also represent Z parameters in the form of ABCD parameters.

We know that Z parameters,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \text{ -------- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \text{ --------- (2)}$$

From equation (2), $I_1 = \frac{V_2 - Z_{22} I_2}{Z_{21}}$ ---------- (3)

Substituting (3) in (1) we get,

$$V_1 = Z_{11}[\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2] + Z_{12} I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 - [\frac{Z_{22} Z_{11}}{Z_{21}} - Z_{12}] I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 - \frac{D_Z}{Z_{21}} I_2 \text{ -------------- (4)}$$

$$\text{Where } D_Z = \frac{Z_{22} Z_{11} - Z_{12}Z_{21}}{Z_{21}}$$

Comparing (3) & (4) we get the equations,

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{22} \, Z_{11} - Z_{12} Z_{21}}{Z_{21}}$$

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}} \quad \text{--------------- (5)}$$

The reverse transformation can be written as,

$$Z_{11} = \frac{A}{C}$$

$$Z_{21} = Z_{12} = \frac{1}{C}$$

$$Z_{22} = \frac{D}{C} \quad \text{-------------- (6)}$$

Relation between ABCD and Y Parameters:

We can also represent Y parameters in the form of ABCD parameters.

We know that Y parameters,

$$I_1 = Y_{11} \, V_1 + Y_{12} \, V_2 \text{ --------- (1)}$$

$$I_2 = Y_{21} \, V_1 + Y_{22} \, V_2 \text{ ---------- (2)}$$

From equation (2), $Y_{21} \, V_1 = I_2 - Y_{22} \, V_2$

$$\text{Therefore } V_1 = \frac{-Y_{22}}{Y_{21}} \, V_2 + \frac{1}{Y_{21}} I_2 \text{ --------- (3)}$$

Substitute equation (3) in (1) we get

$$I_1 = Y_{11}[\frac{-Y_{22}}{Y_{21}} \, V_2 + \frac{1}{Y_{21}} I_2] + Y_{12} \, V_2$$

$$= Y_{11} \frac{-Y_{22}}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2 + Y_{12} \, V_2$$

$$= [\ Y_{12} - \frac{Y_{11}Y_{22}}{Y_{21}} \ ] \ V_2 + \frac{Y_{11}}{Y_{21}} \ I_2$$

Therefore, $I_1 = - [\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{21}} \ ] \ V_2 + \frac{Y_{11}}{Y_{21}} \ I_2$ ---------- (4)

Comparing (4) with (1) ,

$A = -\frac{Y_{22}}{Y_{21}}$

$B = -\frac{1}{Y_{21}}$

$C = - [\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{21}}]$

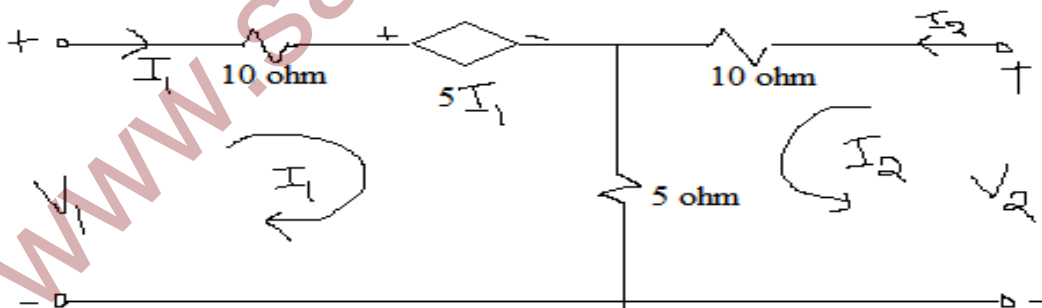$D = -\frac{Y_{11}}{Y_{21}}$ ------------- (5)

The reverse transformation is,

$Y_{11} = \frac{D}{B}$

$Y_{12} = Y_{21} = \frac{1}{B}$

$Y_{22} = \frac{A}{B}$ --------- (6)

Example 1: Determine the two port parameters of given network

Writing the loop equations of the network

Loop1, $10 \ I_1 + 5I_1 + 5 \ (I_1 + I_2 \ ) = \ V_1$

$20 I_1 + 5 I_2 = V_1$ ----- (1)

Loop2, $10 I_2 + 5 (I_1 + I_2) = V_2$

$5 I_1 + 15 I_2 = V_2$ ------- (2)

$Z_{11} = 20$

$Z_{21} = Z_{12} = 5$

$Z_{22} = 15$

Y- parameters, $Y = Z^{-1} = \begin{matrix} 20 & 5 \\ 5 & 15 \end{matrix}^{-1} = \begin{matrix} 3/55 & -1/55 \\ -1/55 & 4/55 \end{matrix}$

ABCD Parameters:

From equation (2), $5 I_1 = V_2 - 15 I_2$

$$I_1 = \frac{1}{5} V_2 - 3 I_2$$

Therefore C = 0.2 mho

D = 3

Substitute I_1 in equation (1) we get, $20 [\frac{1}{5} V_2 - 3 I_2] + 5 I_2 = V_1$

$$V_1 = 4 V_2 - 55 I_2$$

Therefore A = 4

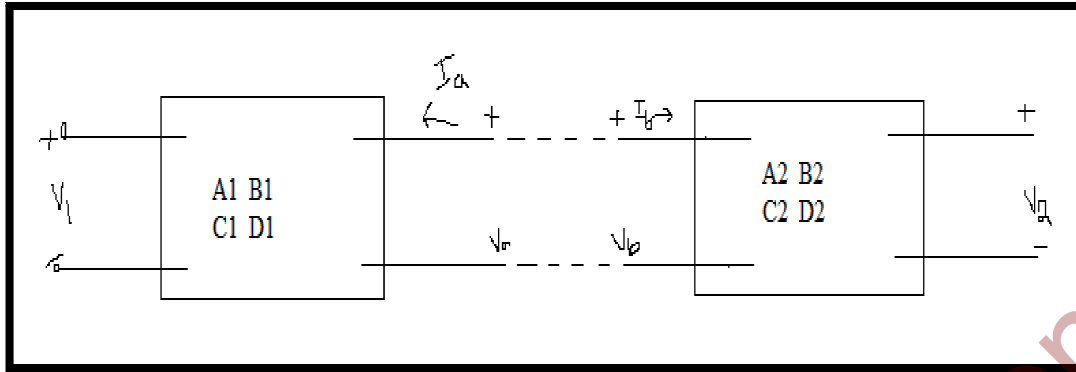B = 55Ω

For a linear bilateral network AD- BC = 1 ➔ 12 – 11 = 1

Interconnection of Two Port Networks:

Consider two transmission line networks with transmission parameters A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 are connected in cascade as shown in fig,

From the above networks,

$$= \qquad \text{--------- (1)}$$

$$= \qquad \text{------------ (2)}$$

In the above two networks are connected in cascade, the total A, B, C, D parameters are calculated.

For cascade connection, =

And =

Substituting (2) in (1) we get,

$$= \qquad \text{------- (3)}$$

The A, B, C, D parameters of a cascade network is,

$$\text{------- (4)}$$

This can be extended to any number of networks connected in cascade.

HYBRID PARAMETERS:

The hybrid parameters are well suited for use with transistor circuits because they can be conventionally measured by experiment. The hybrid parameters are defined by writing the equations for , interms of , .

$$V_1 = h_{11} \ I_1 + h_{12} \ V_2$$

$$I_2 = h_{21} \ I_1 + h_{22} \ V_2$$

When port2 is short circuit i.e., $V_2 = 0$,

$h_{11} = \frac{V_1}{I_1} \mid V_2 = 0$ = short circuit driving point or input impedance in Ω.

$h_{21} = \frac{I_2}{I_1} \mid V_2 = 0$ = short circuit forward current gain.

When port1 is open circuit i.e. $I_1 = 0$

$h_{12} = \frac{V_1}{V_2} \mid I_1 = 0$ = open circuit reverse voltage gain.

$h_{22} = \frac{I_2}{V_2} \mid I_1 = 0$ = open circuit admittance in mhos.

   Since the parameters are impedence, short circuit forward current gains, voltage gain and admittance they are called Hybrid parameters. They also relate the voltage at port1, current at port2 with current at port1 and voltage at port2.

When they are referred to transistors $h_{11}, h_{12}, h_{21}, h_{22}$ becomes h_i, h_r, h_f and h_o respectively.

Where the subscript denote the i- input, r- reverse, f- forward, o- output.

RELATION BETWEEN Z Parameters and HYBRID Parameters:

$$V_1 = h_{11} \ I_1 + h_{12} \ V_2 \ \text{------ (1)}$$

$$I_2 = h_{21} \ I_1 + h_{22} \ V_2 \ \text{--------- (2)}$$

From Z Parameters,

$$V_1 = Z_{11} \ I_1 + Z_{12} \ I_2 \ \text{-------- (3)}$$

$$V_2 = Z_{21} \ I_1 + Z_{22} \ I_2 \ \text{--------- (4)}$$

From equation (4) we get,

$$I_2 = \frac{-Z_{21}}{Z_{22}} \ I_1 + \frac{1}{Z_{22}} V_2 \ \text{----- (5)}$$

Comparing equations (2) & (5),

$$h_{21} = \frac{-Z_{21}}{Z_{22}}$$

$$h_{22} = \frac{1}{Z_{22}}$$

Substituting I_2 in (3) we get,

$$V_1 = Z_{11} I_1 + Z_{12} \left[\frac{-Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2\right]$$

$$V_1 = \left[Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}}\right] I_1 + \frac{Z_{12}}{Z_{22}} V_2 \ \ ----- (6)$$

Comparing (1) & (6) we get,

$$h_{11} = \frac{Z_{11}Z_{22} - Z_{12} Z_{21}}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

RELATION BETWEEN Y Parameters and HYBRID Parameters:

From the Y parameters,

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \ \ -------- (7)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \ \ --------- (8)$$

From equation (7) we get,

$$Y_{11} V_1 = I_1 - Y_{12} V_2$$

$$V_1 = \frac{1}{Y_{11}} I_1 + \frac{Y_{12}}{Y_{11}} V_2 \ \ ------ (9)$$

Comparing equation (1) & (9),

$$h_{11} = \frac{1}{Y_{11}}$$

$$h_{12} = -\frac{Y_{12}}{Y_{11}}$$

Substitute V_1 in (8) we get,

$$I_2 = Y_{21} \left[\frac{1}{Y_{11}} \ I_1 + \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22} \ V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \left[Y_{22} - \frac{Y_{21}Y_{12}}{Y_{11}}\right] V_2 \ \text{---------- (10)}$$

Comparing (2) & (10)

$$h_{21} = \frac{Y_{21}}{Y_{11}}$$

$$h_{22} = \frac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{11}}$$

Example 1: A two port network has tha fallowing parameters

$Z_{22} = 40 \ \Omega$, $Z_{11} = 30 \ \Omega$ and $Y_{12} = 0.05 \Omega$

Find out the ABCD Parameters.

Given data,

$$Z_{22} = \frac{D}{C} = 40$$

$$Z_{11} = \frac{A}{C} = 30$$

$$Y_{22} = \frac{A}{B} = 0.05$$

$$Z_{22} - \frac{1}{Y_{22}} = \frac{D}{C} - \frac{B}{A} = \frac{AD - BC}{AC} = 40 - \frac{1}{0.05} = 20 = \frac{1}{AC}$$

$$\frac{Z_{11}}{Z_{22} - \frac{1}{Y_{22}}} = \frac{1}{AC} * AC = A^2$$

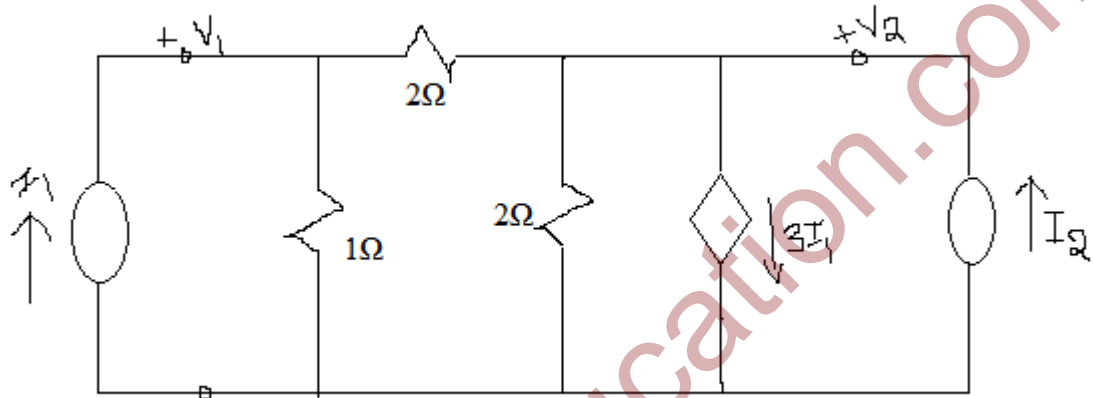$$A^2 = \sqrt{\frac{Z_{11}}{Z_{22} - \frac{1}{Y_{22}}}} = \sqrt{\frac{30}{20}}$$

A = 1.225

C = $\frac{1.225}{30}$ = 0.0408 mhos

B = $\frac{A}{0.05}$ = 1.225 * 20 = 24.5 Ω

D = 40 * C = 40 * 0.0408 = 1.632

Example 2: Find the Hybrid parameters of the network shown in fig.

Writing the equations of the network,

At node1, $\frac{V_1}{1} + \frac{V_1 - V_2}{2} = I_1$

At node2, $\frac{V_2 - V_1}{2} + \frac{V_2}{2} + 3I_1 = I_2$

$$4 V_1 - \frac{V_2}{2} = I_2$$

The hybrid parameters are ,

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

When port2 is short circuit i.e., $V_2 = 0$,

$h_{11} = \frac{V_1}{I_1} \mid V_2 = 0 = \frac{2}{3}$ ohm
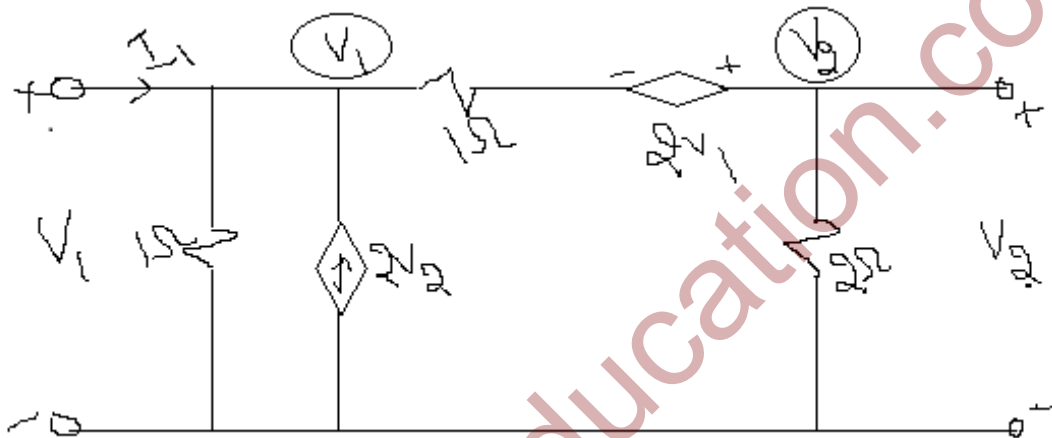
$h_{21} = \frac{I_2}{I_1} \mid V_2 = 0 = \frac{8}{3}$

When port1 is open circuit i.e. $I_1 = 0$

$h_{12} = \frac{V_1}{V_2} | I_1 = 0 = \frac{1}{3}$

$h_{22} = \frac{I_2}{V_2} | I_1 = 0 = \frac{5}{6}$ mho

Example 2: Determine 'h' parameters for the network shown in figure below.

$$V_1 = h_{11} \ I_1 + h_{12} \ V_2$$

$$I_2 = h_{21} \ I_1 + h_{22} \ V_2$$

The nodal equations at nodes 1 and 2.

At node1,

$V_1 - I_1 - 2 V_2 + V_1 - (V_2 - 2V_1) = 0$

$V_1 - 2V_2 + 3V_1 - V_2 = I_1$

$4V_1 - 3 V_2 = I_1$ ------------- (1)

At node2,

$\frac{V_2}{2} - \frac{V_2 - 2 V_1 - V_1}{1} - I_2 = 0$

$-3V_1 + 3 \ \frac{V_2}{2} = I_2$ ----------- (2)

h_{11} $= \frac{V_1}{I_1} \mid V_2 = 0 = \frac{1}{4}\Omega.$

h_{21} $= \frac{I_2}{I_1} \mid V_2 = 0 = \frac{-3}{4}$

h_{12} $= \frac{V_1}{V_2} \mid I_1 = 0 = \frac{3}{4}$ mho

h_{22} $= \frac{I_2}{V_2} \mid I_1 = 0 = \frac{-3}{4}$