3φ Unbalanced Systems

Unbalanced Systems:

A system is said to be balanced system if the impedances or phase angle or frequencies of three phases is same otherwise it is called as unbalanced system.

There are two types of unbalanced systems. Those are

1. Three phase four wire system (star connection with neutral)
2. Three phase three wire system(Delta or Star connection with Neutral)

1. Three Phase Four Wire System:

The three phase three wire unbalanced system can be solved by any one of the following methods.

i) Star to Delta conversion method
ii) Loop or Mesh analysis method.
iii) Milliman's Method

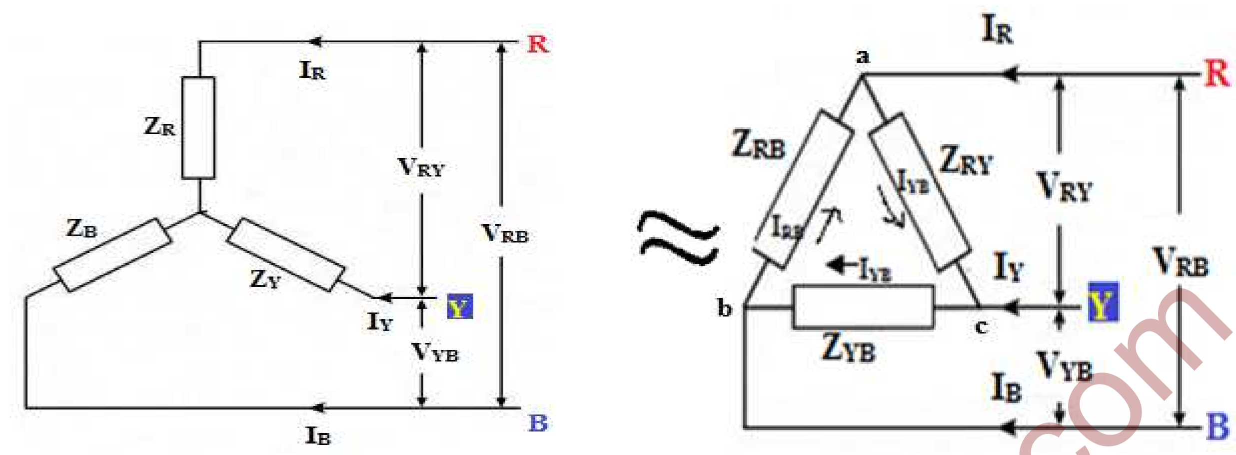
STAR TO DELTA CONNECTION:

Star to Delta conversion method is used to solve 3Φ, 3 wire unbalanced system. Let us consider the 3 Φ star connection without neutral as shown below. Let the phase sequence be R, Y & B.

Let Z_R, Z_Y, Z_B are the impedances of R, Y, B phases. I_R, I_Y, I_B are currents through R, Y, B phases respectively.

V_{RN}, V_{BN}, V_{YN} be the phase voltages (V_{PH}) and V_{RY}, V_{YB}, V_{BR} be the line voltages (V_L).

$$V_{RN} \neq V_{BN} \neq V_{YN} \neq V_{PH}$$



Z_{RY} , Z_{RB} and Z_{YB} are the branch impedances and are determined as

$Z_{RY} = Z_R + Z_Y + (Z_R\ Z_Y)/\ Z_B$

$Z_{RB} = Z_R + Z_B + (Z_R\ Z_B)/\ Z_Y$

$Z_{YB} = Z_Y + Z_B + (Z_Y\ Z_B)/\ Z_R$

Brach Currents:

$I_{RY,}$ $I_{YB,}$ I_{BR} are the Brach currents and are determined as $I_{RY} = V_{RY}\angle 0° \ /\ Z_{RY}$

$I_{YB} = V_{YB}\angle -120° \ /\ Z_{RY}$

$I_{RB} = V_{RB}\angle -240° \ /\ Z_{RY}$

Line currents:

I_R, I_Y, I_B are the line currents and are determined as

At point 'a' $I_{RB} + I_R = I_{RY}$

$I_R = I_{RY} - I_{RB}$

At point 'b' $I_{YB} + I_Y = I_{RB}$

$I_Y = I_{RB} - I_{YB}$

At point 'c' $I_{RY} + I_Y = I_{YB}$

$I_Y = I_{YB} - I_{RY}$

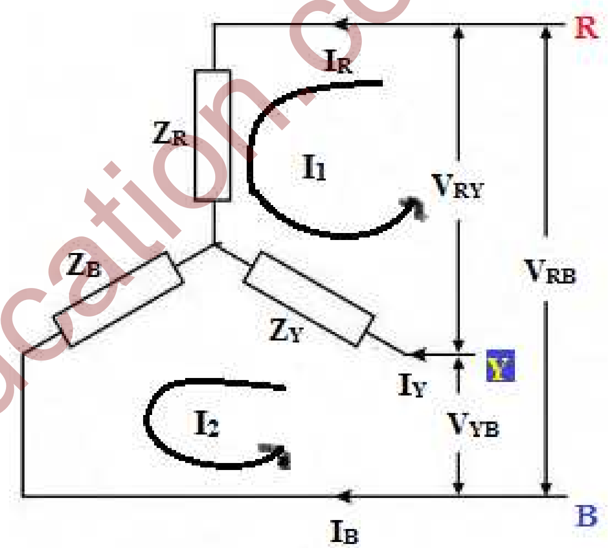
The voltage across Z_R is $V_{ZR} = I_R Z_R$

Voltage across Z_Y is $V_{ZY} = I_Y Z_Y$

Voltage across Z_B is $V_{ZB} = I_B Z_B$

LOOP OR MESH ANALYSIS:

The loop or mesh analysis method is used to solve the 3Φ, star without neutral system. Let us consider a star without neutral as shown below. Let the phase sequence as RYB



Let Z_R, Z_Y, Z_B are the impedances of R, Y, B phases.

Voltage across R & Y is $V_{RY}\angle 0°$

Voltage across Y & B is $V_{YB}\angle -120$

Voltage across R& B is $V_{RB}\angle -240°$

I_R, I_Y, I_B are the line or phase currents

Applying KVL to loop 1

$$V_{RY}\angle 0 = I_1 Z_R + (I_1 - I_2) Z_Y$$

$$V_{RY}\angle 0 = I_1 Z_R + I_1 Z_Y - I_2 Z_Y$$

$$V_{RY}\angle 0 = I_1 (Z_R + Z_Y) - I_2 Z_Y$$

Find I_1 from above equation

$$I_1 = \frac{V_{RY}\angle 0 + Z_Y I_2}{Z_R + Z_Y}$$

Applying KVL to loop 2

$$V_{YB}\angle{-120} = (I_2 - I_1)\, Z_Y + Z_B\, I_2$$

$$V_{YB}\angle{-120} = -\, I_1\, Z_Y + (Z_B + Z_Y)\, I_2$$

Now by substituting I1 in above equation we get I_2

From circuit branch currents are $I_R = I_1$

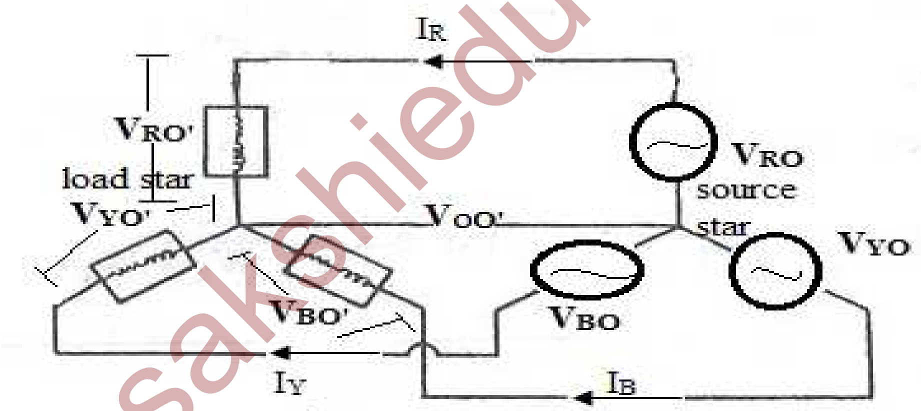
$$I_Y = I_1 - I_2$$

$$I_B = -I_2$$

Milliman’s Theorem:

Consider a 3Φ star without neutral is excited by star connected supply as shown in fig.

Let Z_R, Z_Y, Z_B are the impedances of R, Y, B phases. I_R, I_Y, I_B are the currents of R, Y, B phases.



According to milliman’s theorem,

The voltage at load star point o’ w.r.t source point o is $V_{OO'}$

$$V_{OO'} = \frac{\dfrac{V_{RO}\angle 0}{Z_R} + \dfrac{V_{YO}\angle{-120}}{Z_Y} + \dfrac{V_{BO}\angle{-240}}{Z_B}}{\dfrac{1}{Z_R} + \dfrac{1}{Z_Y} + \dfrac{1}{Z_B}}$$

Voltage across Z_R of load is $V_{RO'} = V_{OO'} – V_{RO}$

Voltage across Z_Y of load is $V_{YO'} = V_{OO'} – V_{YO}$

Voltage across Z_B of load is $V_{BO'} = V_{OO'} – V_{BO}$

Branch currents are $I_R = V_{RO'} / Z_R = (V_{OO'} – V_{RO}) / Z_R$

$I_Y = V_{YO'} / Z_Y = (V_{OO'} – V_{YO}) / Z_Y$

$I_B = V_{BO'} / Z_B = (V_{OO'} – V_{BO}) / Z_B$

Measurement of power in 3 –Φ system (Balanced or unbalanced system):

The power in 3-Φ system can be measured by using following methods

1. Three wattmeter method
2. Two wattmeter method
3. Single wattmeter method

THREE WATTMETER METHOD:

In this method, three wattcmeters are connected in each of three phases of load whether star or delta connected. The current coil of each wattmeter carries the current of one coil only and pressure coil measure the phase voltage of the phase as shown below in fig

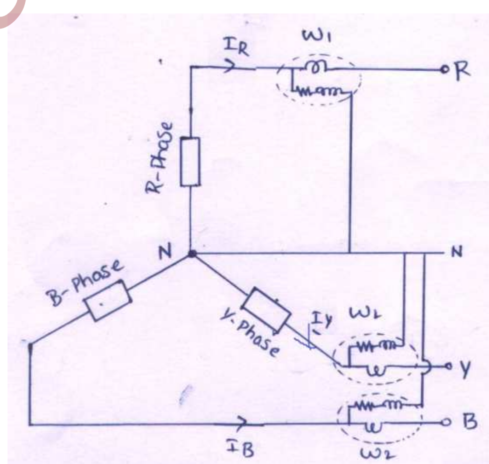


Fig Three wattmeter method – Star

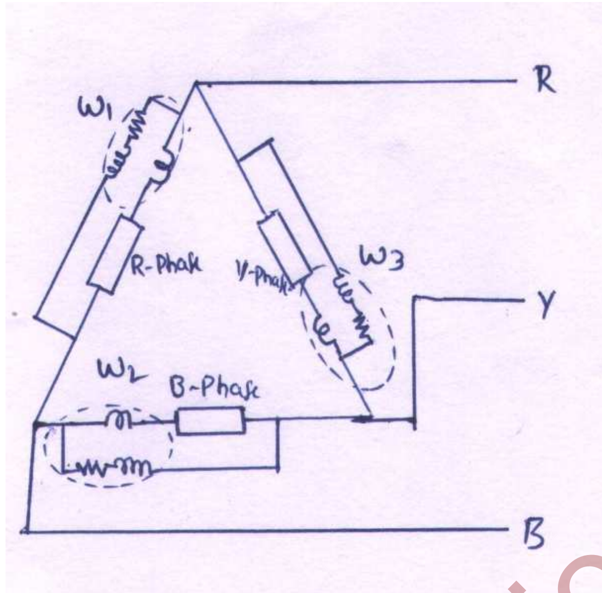
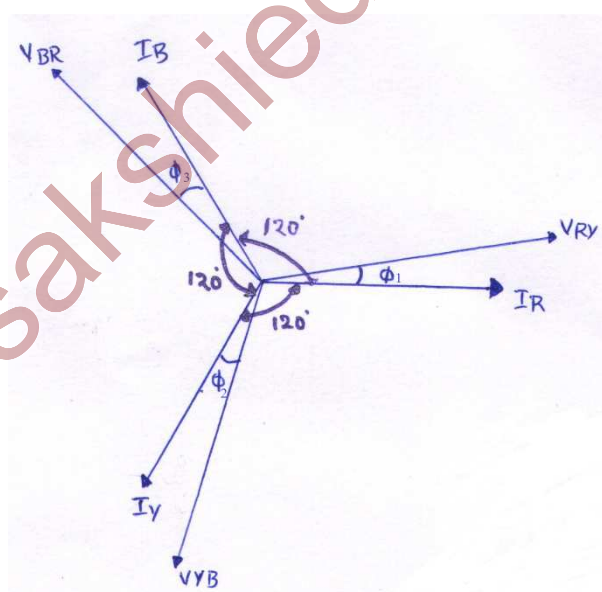


Fig Three wattmeter method- Delta

The total power in load is given by algebraic sum of the readings .Let W_1,W_2, W_3 are the readings of wattcmeters then the total power supplied to 3 –Φ load is P= $W_1+W_2+W_3$.



Phasor diagram

The Three wattmeter method is suitable for measurement of 3-Φ unbalanced power . Let us consider I_R, I_Y, I_B are the currents of R, Y, B phases respectively which are nothing but phase and line currents. From circuit $V_{RN,}$ $V_{BN,}$ V_{YN} be the phase voltages (V_{PH}) and $V_{RY,}$ $V_{YB,}$ V_{BR} be the line voltages (V_L) .

Current through wattmeter 1 is I_R and voltage across pressure coil of wattmeter 1 is V_{RN} now reading in wattmeter 1 is

$$W_1 = V_{RN} I_R \cos\Phi_1$$

$$W_1 = V_{PH} I_{PH} \cos\Phi_1$$

Current through wattmeter 2 is I_Y and voltage across pressure coil of wattmeter 1 is V_{YN} now reading in wattmeter 2 is

$$W_2 = V_{YN} I_y \cos\Phi_2$$

$$W_2 = V_{PH} I_{PH} \cos\Phi_2$$

Current through wattmeter 3 is I_B and voltage across pressure coil of wattmeter 3 is V_{BN} now reading in wattmeter 3 is

$$W_3 = V_{YN} I_y \cos\Phi_3$$

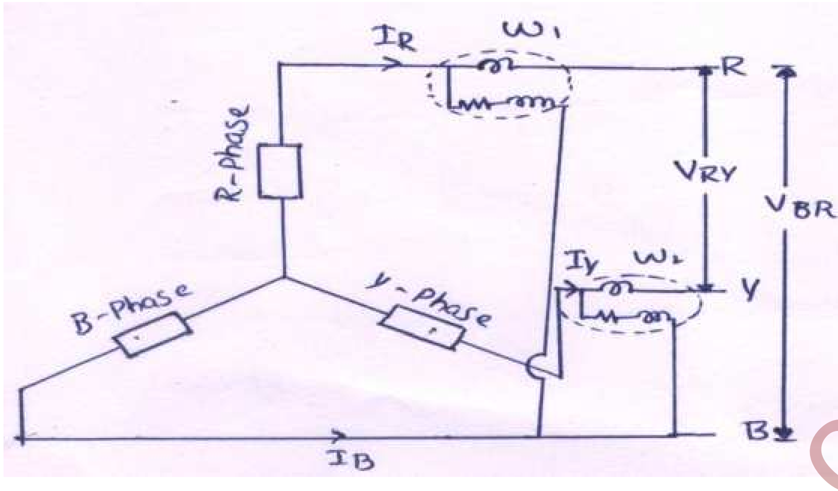
$$W_3 = V_{PH} I_{PH} \cos\Phi_3$$

Total power measured by three wattcmeters is $P = W_1 + W_2 + W_3$

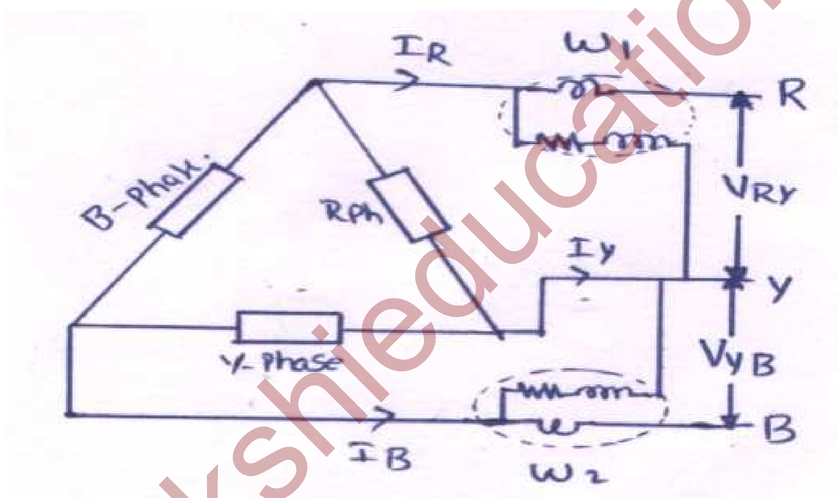
$$P = V_{PH} I_{PH} \cos\Phi_1 + V_{PH} I_{PH} \cos\Phi_2 + V_{PH} I_{PH} \cos\Phi_3$$

Two Wattmeter Method:

The two wattmeter method is suitable for both balanced and unbalanced load. In this method, the current coils of two wattcmeters are inserted in any two Phases and pressure coils of each joined to third phase.

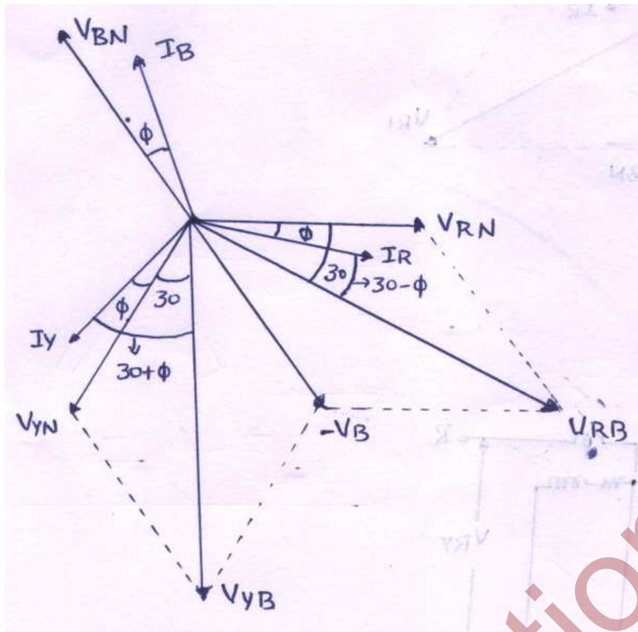


Two wattmeter method- Star



Two wattmeter method- Delta

The total power absorbed by the 3Φ balanced load is the sum of powers obtained by wattcmeters W_1 and W_2. When load is assumed as inductive load, the vector diagram for such a balanced star connected load is shown below



Vector Diagram

Let V_{RN}, V_{BN}, V_{YN} are the phase voltages and I_R, I_Y, I_B are currents (phase or line) . Since load is inductive, the current lags their respective phase voltages by phase angle (Φ).

Let the current through wattmeter $w_1 = I_R$

Potential difference across pressure coil of wattmeter $w_1 = V_{RB} = V_{RN} - V_{BN}$

From vector diagram phase angle between V_{RB} and I_R is 30-Φ.

∴ Reading of wattmeter $W_1 = V_{RB} I_R \cos(30-\Phi)$

$$= V_L I_L \cos(30-\Phi) \quad (1)$$

Similarly current through wattmeter $w_2 = I_Y$

Potential difference across pressure coil of wattmeter 2 $W_2 = V_{YB}$

$$= V_Y - V_B$$

The phase difference / angle between V_{YB} and I_Y is 30+Φ

∴ Reading of wattmeter $W_2 = V_{YB} I_Y \cos(30+\Phi)$

$$= V_L I_L \cos(30+\Phi) \quad (2)$$

Total power (P) = w_1+w_2

$= V_L I_L \cos(30-\Phi) + V_L I_L \cos(30+\Phi)$

$P = \sqrt{3}\, V_L I_L \cos\Phi$ watts. ——————— (3)

Hence the sum of two wattcmeters gives the total power absorbed by the 3Φ load.

Similarly to find Power factor

$w_1 - w_2 = V_L I_L \cos(30-\Phi) + V_L I_L \cos(30+\Phi)$

$w_1 - w_2 = V_L I_L \sin\Phi$ ——————— (4)

Dividing equation (3) by (4)

$$\frac{\sqrt{3}(w1-w2)}{w1+w2} = \frac{\sqrt{3} VL\ IL\ \sin\Phi}{\sqrt{3}\ VL\ IL\ \cos\Phi}$$

$$\frac{\sqrt{3}\,(w1-w2)}{w1+w2} = \mathrm{Tan}\Phi$$

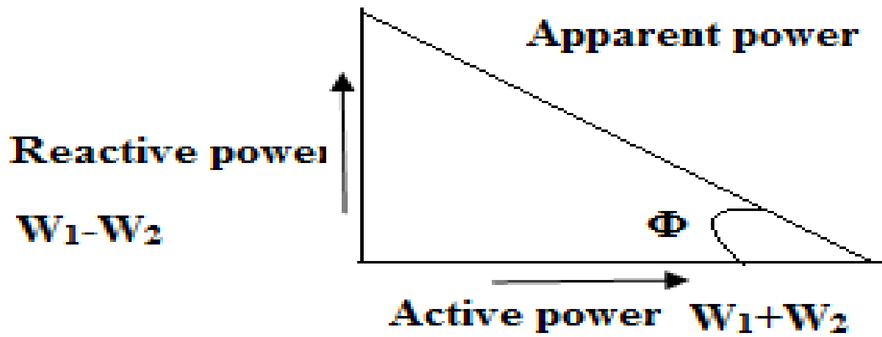
∴Phase angle $\Phi = \pm\left(\tan^{-1}\frac{\sqrt{3}(w1-w2)}{w1+w2}\right)$ + for lagging or inductive loads

- for leading or capacitive loads

Power factor is nothing but COS Φ = $\mathrm{COS}\pm\left(\tan^{-1}\frac{\sqrt{3}(w1-w2)}{w1+w2}\right)$

Reactive Power Measurement with Two Wattmeter Method:-

We know that $\frac{\sqrt{3}\,(w1-w2)}{w1+w2} = \mathrm{Tan}\Phi$



Power triangle

In balanced condition, from above relations and power triangle the reactive power is given by √3 times the difference of readings of wattcmeters used.

$$\text{Reactive power} = \sqrt{3}\,(W_1 - W_2)\ \text{var}$$

We know the value of (W_1-W_2) from eqn (4)

⇨ Reactive power = $\sqrt{3}\,(V_L\, I_L \sin\Phi)$ var

Variations in wattmeter readings in 2 wattmeter method due to power factor:-

We know that, for balanced inductive load

Reading of wattmeter 1 is $W_1 = V_L\, I_L \cos(30-\Phi)$

Reading of wattmeter 2 is $W_2 = V_L\, I_L \cos(30+\Phi)$

From above equation, it is clear that readings of wattcmeters not only depend on load but also depends on its phase angle i.e.

i. When $\Phi = 0°$ i.e. power factor $=\cos\Phi$ = unity (resistive load)

Then $W_1 = W_2 = \text{Cos } 30°$

The readings of both wattcmeters are same.

ii. When $\Phi = 60°$ i.e power factor $=\cos\Phi = 0.5$ lag

Then $W_1 = V_L I_L \cos(30°-60°) = V_L I_L \cos 30°$

$W_2 = V_L I_L \cos(30°+60°) = 0$

Hence wattmeters 1 only read power

iii. When $90 > \Phi > 60$ i.e $0.5 > \cos \Phi > 0$

When phase angle is 60 to 90, the wattmeter W_1 readings are positive but readings of wattmeter W_2 are reversed. For getting the total power , the readings of W_2 is to be subtracted from that of W_1 .

iv. When $\Phi = 90°$ i.e power factor = 0

Then $W_1 = V_L I_L \cos(30°-90°) = V_L I_L \cos 60°$

$W_2 = V_L I_L \cos(30°+90°) = -V_L I_L \sin 30°$

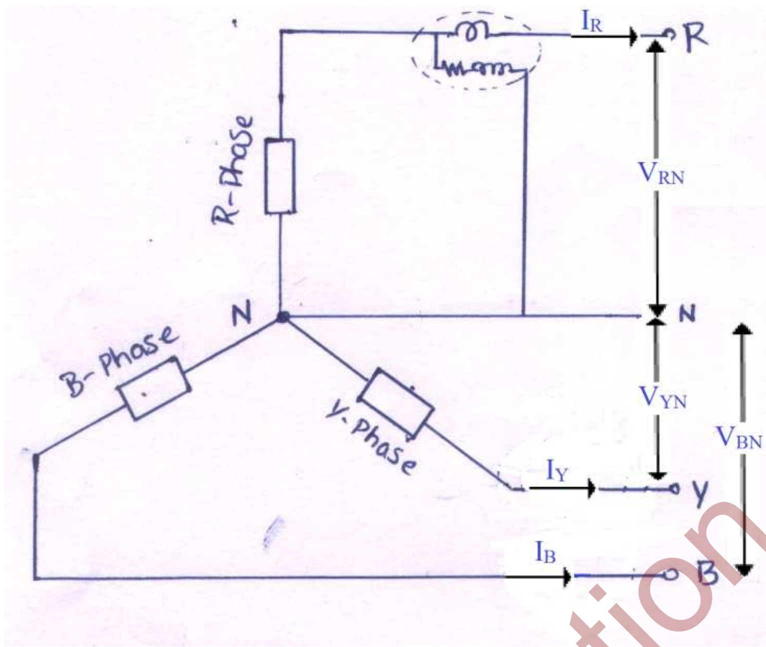
These two readings are equal in magnitude but opposite in sign

$\therefore$ Total power = $W_1 + W_2 = 0$

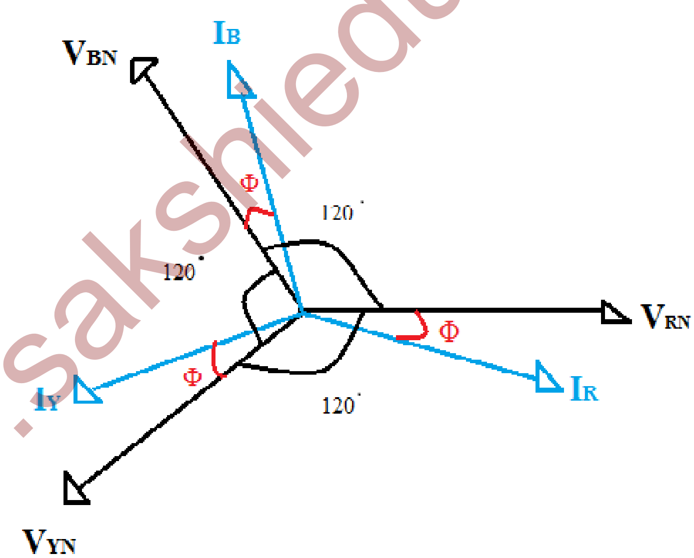
Single Wattmeter Method:

The single wattmeter method is used to measure the power of 3-Φ *balanced system*. Let Z_R, Z_Y, Z_B are the impedances of R, Y, B phases. I_R, I_Y, I_B are currents through R, Y, B phases respectively.

V_{RN}, V_{BN}, V_{YN} be the phase voltages (V_{PH}) and V_{RY}, V_{YB}, V_{BR} be the line voltages (V_L).



Single wattmeter method



Vector diagram

From above diagram, the current through wattmeter is I_R, voltage across pressure coil is V_{RN}. Now wattmeter reading is

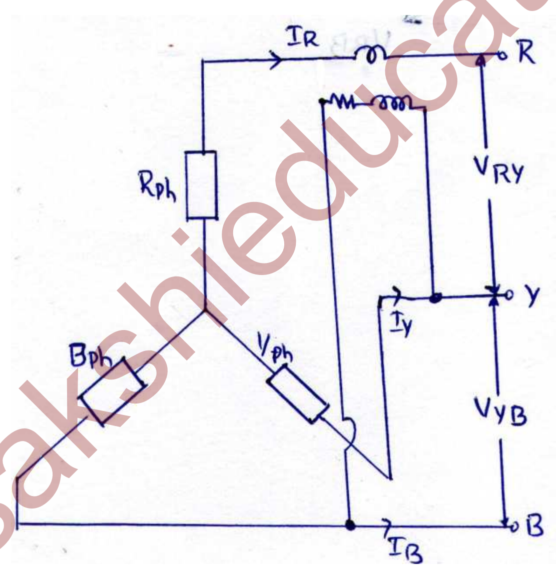
$$W = V_{RN} I_R \cos\Phi = V_{PH} I_{PH} \cos \Phi$$

$$\text{Total power} = 3^* V_{PH} I_{PH} \cos\Phi$$

$$= \sqrt{3} V_L I_L \cos \Phi \qquad \because V_L=\sqrt{3}V_{PH} \text{ \& } I_L=I_{PH}$$

Measurement of Reactive Power in Single Wattmeter Method:

The reactive power of 3Φ circuit can be measured by using compensated wattmeter. The circuit diagram of 3Φ star connection with compensated wattmeter is shown below.

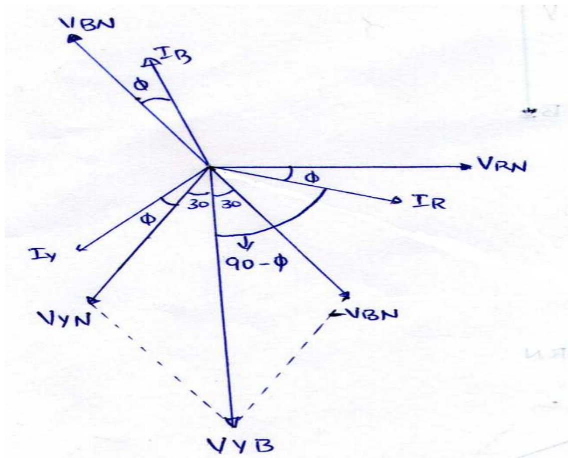


Let Z_R, Z_Y, Z_B are the impedances of R, Y, B phases. I_R, I_Y, I_B are currents through R, Y, B phases respectively.

V_{RN}, V_{BN}, V_{YN} be the phase voltages (V_{PH}) and V_{RY}, V_{YB}, V_{BR} be the line voltages (V_L) .

$$V_{RN}=V_{BN}=V_{YN}=V_{PH}$$

$$V_{RY}=V_{YB}=V_{BR}=V_{LINE}$$



Vector diagram

Current through the current coil of wattmeter is I_R

Voltage across pressure coil of wattmeter $= V_{YB} = V_Y - V_B$

$$\text{Wattmeter reading} = \sqrt{3}\, V_{PH}\, I_{PH} \sin \Phi$$

$$= \sqrt{3}\,(\sqrt{3}\, V_{PH}\, I_{PH} \sin\Phi\,)$$

$$Q = 3\, V_{PH}\, I_{PH} \sin\Phi$$