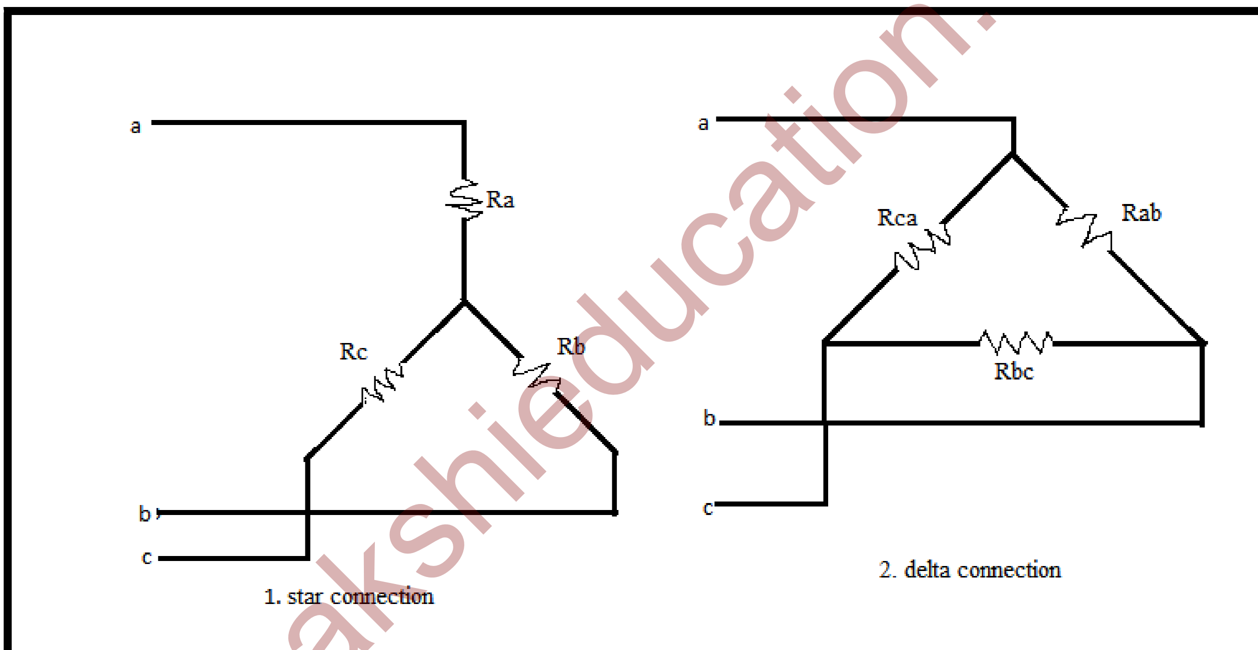
STAR – DELTA TRANSFORMATION

If there are three resistances are connected to a common point in the form as shown in fig (1). They are said to be star connected and if they are connected as shown in fig (2) they are said to be delta connected.

- In order to reduce the networks, it may be necessary to replace a star connected set of resistances by an equivalent delta connected set of resistances vice versa.



- The star delta transformation technique is useful in solving complex networks. Basically, any three circuit elements, i.e. Resistive, Inductive or capacitive, may be connected in two different ways. One way of connecting these elements is called the star connection, or the Y connection. The other way of connecting these elements is called delta connection or Δ connection.

The equivalence between the above two networks is obtained by equating the effective resistance between the corresponding terminals for the two networks.

Equating the resistances between corresponding pairs of terminals, Between a & b

$$Ra + Rb = Rab\,(Rbc + Rca) / (Rab + Rbc + Rca) \quad \text{------------ (1)}$$

Between b & c

$$Rb + Rc = Rbc\,(Rca + Rab) / (Rab + Rbc + Rca) \quad \text{------------ (2)}$$

Between c & a

$$Rc + Ra = Rca\,(Rab + Rbc) / (Rab + Rbc + Rca) \quad \text{------------ (3)}$$

Subtracting (2) from (1) we get

$$Ra - Rc = Rca\,(Rab - Rbc) / (Rab + Rbc + Rca) \quad \text{------------ (4)}$$

Adding (3) and (4) we get

$$Ra = Rca\,.Rab / (Rab + Rbc + Rca) \quad \text{------------ (5)}$$

Similarly,

$$Rb = Rbc\,.Rab / (Rab + Rbc + Rca) \quad \text{------------ (6)}$$

$$Rc = Rca\,.Rbc / (Rab + Rbc + Rca) \quad \text{------------ (7)}$$

Equations (5), (6) & (7) to transform delta – star i.e. we can obtain an equivalent star connected resistances for the given delta connected resistances.

From the above equations, we get

$$RaRb + RbRc + RcRa = Rab\,.Rbc\,.Rca / (Rab + Rbc + Rca) \quad \text{------------ (8)}$$

Dividing (8) by Ra i.e. equation (5)

$$RaRb + RbRc + RcRa = Rbc\,.Ra$$

Cancel Ra on both sides & we get

$$Rb + Rc + (Rb.Rc/Ra) = Rbc \quad \text{------------- (9)}$$

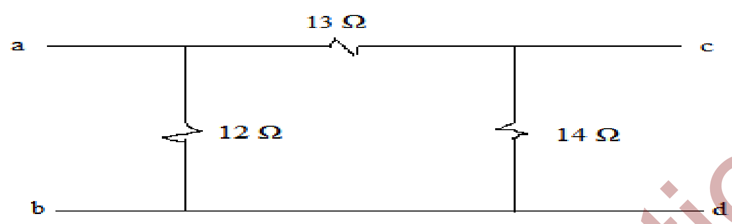
Simillarly dividing equation (8) by equation Rb & Rc, we got

$$Ra + Rb + (Ra.Rb/Rc) = Rab \quad \text{-------------} \ (10)$$

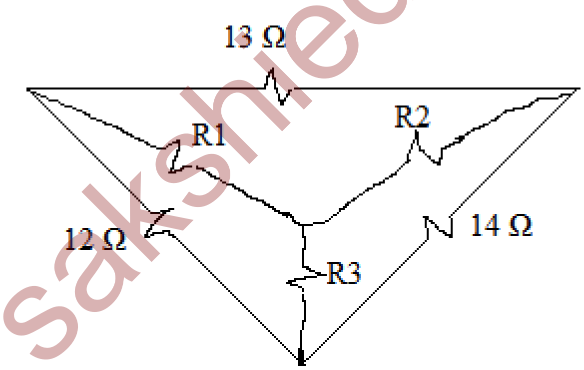
$$Rc + Ra + (Ra.Rc/Rb) = Rca \quad \text{-------------} \ (11)$$

From the above equations (9),(10) ,(11) we can replace a star connected resistances by an equivalent delta connected resistances.

Example 1: Obtain the star connected equivalent for the delta connected circuit



The above circuit is in delta connected and can be replaced as star as shown in fig

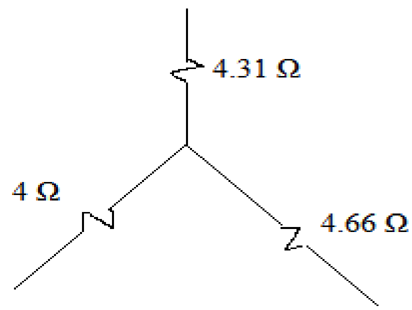


Performing Δ - Y transformation, we obtain

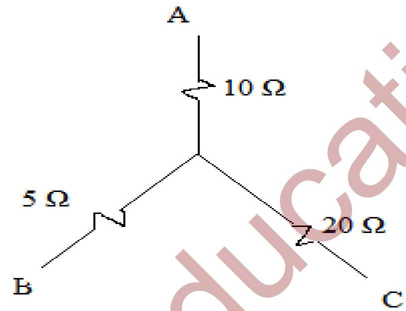
$$R1 = (13 * 12) / (14 + 13 + 12) = 4\Omega$$

$$R2 = (13 * 14) / (14 + 13 + 12) = 4.66\Omega$$

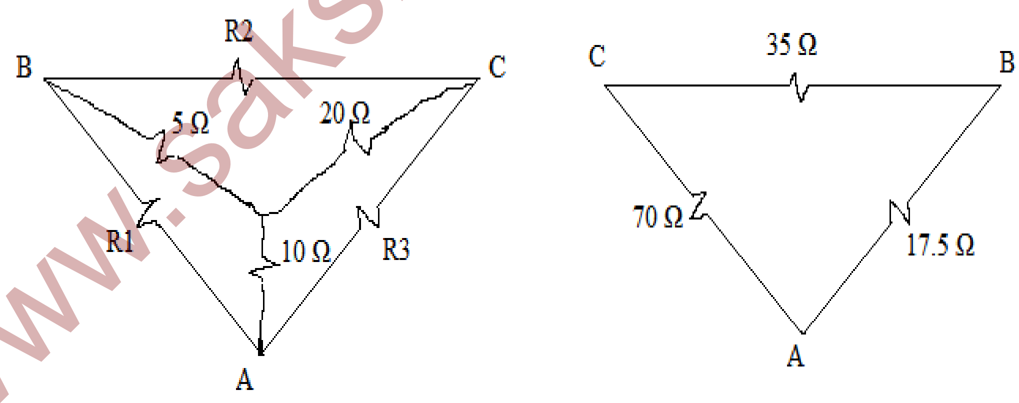
$$R3 = (14 * 12) / (14 + 13 + 12) = 4.31\Omega$$



Example 2: Obtain the delta connected equivalent for the star connected circuit.



The above circuit can be replaced by delta connected circuit as shown in fig



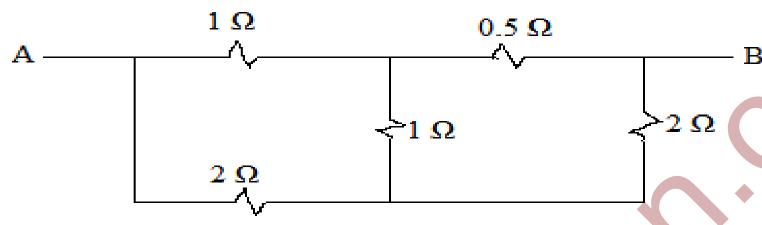
R1 = (20 * 10 + 20 * 5 + 10 * 5) / 20 = 17.5Ω

R1 = (20 * 10 + 20 * 5 + 10 * 5) / 10 = 35Ω

R1 = (20 * 10 + 20 * 5 + 10 * 5) / 5 = 70Ω

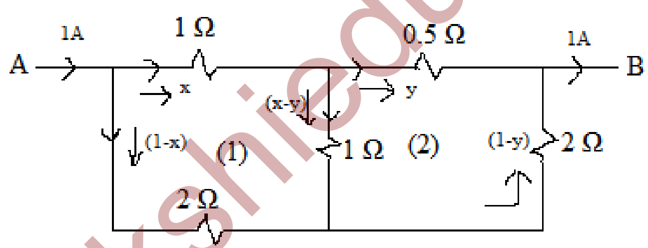
Example 3: Determine the equivalent resistances between A, B in the circuit as shown in fig

- Without using the Y - Δ transformation
- Using Y - Δ transformation



Without using the Y - Δ transformation:

Let the input current be 1A,



Applying KVL to the two loops, we get

Loop (1): $1x + 1\,(x\text{-}y) - 2\,(1\text{-}x) = 0$

$$4x - y - 2 = 0 \quad \text{---------------} \; (1)$$

Loop (2): $0.5y - 2(1\text{-}y) - 1(x\text{-}y) = 0$

$$-x + 3.5y - 2 = 0$$

$$x - 3.5y + 2 = 0 \; \text{----------------} \; (2)$$

on solving (1) & (2) by Cramer's rule, we get

$$\begin{bmatrix} 4 & -1 \\ 1 & -3.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 4 & -1 \\ 1 & -3.5 \end{vmatrix} = 4\,(-3.5) - (-1)\,(1) = -13$$

$$\Delta 1 = \begin{vmatrix} 2 & -1 \\ -2 & -3.5 \end{vmatrix} = 2\,(-3.5) - (-2)\,(-1) = -9$$

$$\Delta 2 = \begin{vmatrix} 4 & 2 \\ 1 & -2 \end{vmatrix} = (-2)\,(4) - (1)\,(2) = -10$$

$$x = \Delta 1 / \Delta = (-9) / (-13) = 0.6923A$$

$$y = \Delta 2 / \Delta = (-10) / (-13) = 0.7692A$$

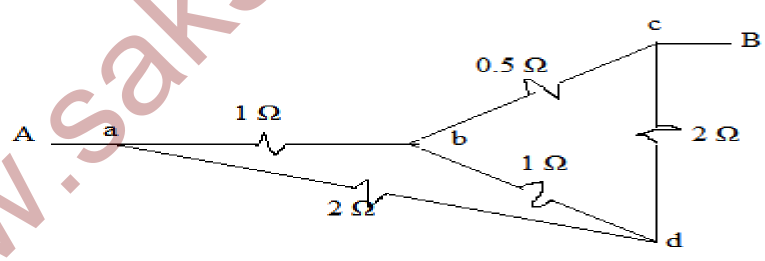
Potential drop from A to B = 1x + 0.5y = 1.0769V

Current, I = 1A and V = 1.0769V

Req = V / I = 1.0769V

Using Y - Δ transformation:

The circuit can be modified as,



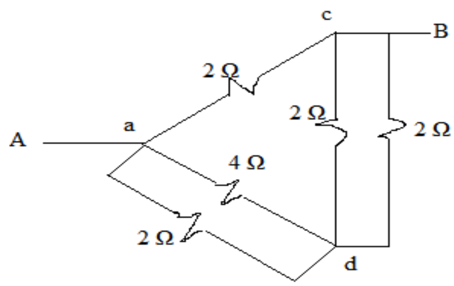
Replacing star acd to equivalent delta,

Rab * Rbc + Rbc * Rbd + Rbd * Rab = 1 (0.5) + 0.5 (1) + 1(1) = 2Ω

Rcd = (Rab * Rbc + Rbc * Rbd + Rbd * Rab) / Rab = 2 / 1 = 2Ω

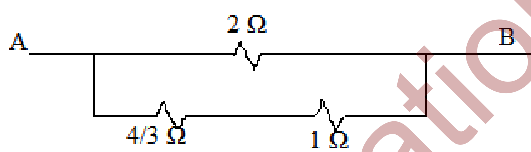
Rad = (Rab * Rbc + Rbc * Rbd + Rbd * Rab) / Rbc = 2 / 0.5 = 4Ω

Rab = (Rab * Rbc + Rbc * Rbd + Rbd * Rab) / Rbd = 2 / 1 = 2Ω



2Ω | | 2 Ω are in parallel, equivalent resistance is, (2 * 2) / (2 + 2) = 1Ω

4Ω | | 2 Ω are in parallel, equivalent resistance is, (4 * 2) / (4 + 2) = 4/3Ω

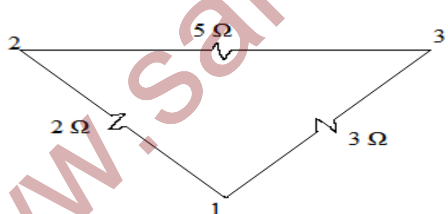


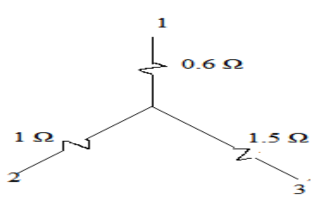
4/3Ω and 1Ω are in series, (4/3) + 1 = 7/3Ω

Equivalent resistance, Rab = (2 * (7/3)) / (2 + (7/3)) = 1.0769Ω

Example 4: Find the voltage to be applied across AB in order to drive a current of 5A into the circuit Y-Δ transformation.

First convert 123 delta to star,



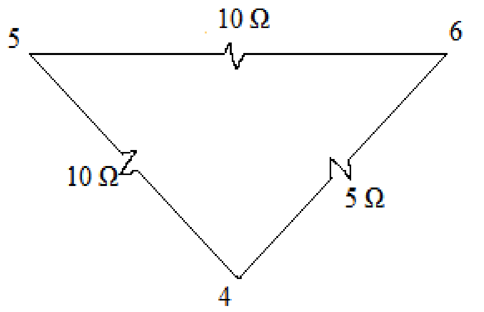


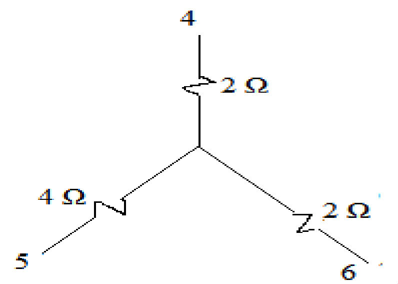
Ra1 = 2 * 3 / (2 + 5 + 3) = 0.6Ω

Ra2 = 2 * 5 / (2 + 5 + 3) = 1Ω

Ra3 = 5 * 3 / (2 + 5 + 3) = 1.5Ω

Similarly convert 456 delta to star,



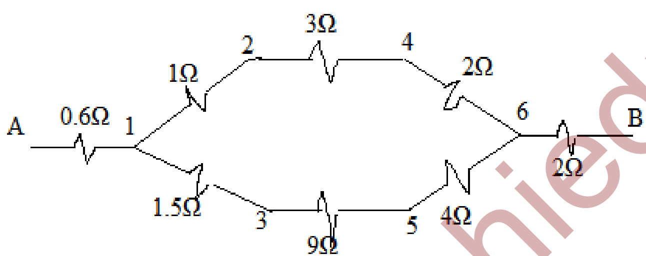


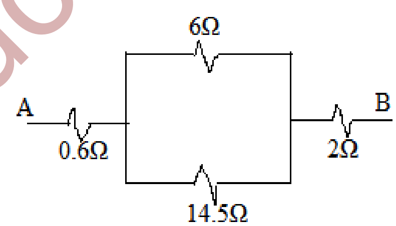
Ra4 = 10 * 5 / (10 + 10 + 5) = 2Ω

Ra2 = 10 * 10 / (10 + 10 + 5) = 4Ω

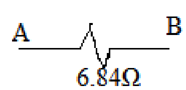
Ra3 = 5 * 10 / (10 + 10 + 5) = 2Ω

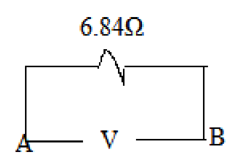
Now replace the delta part of the circuit with star circuit as shown below





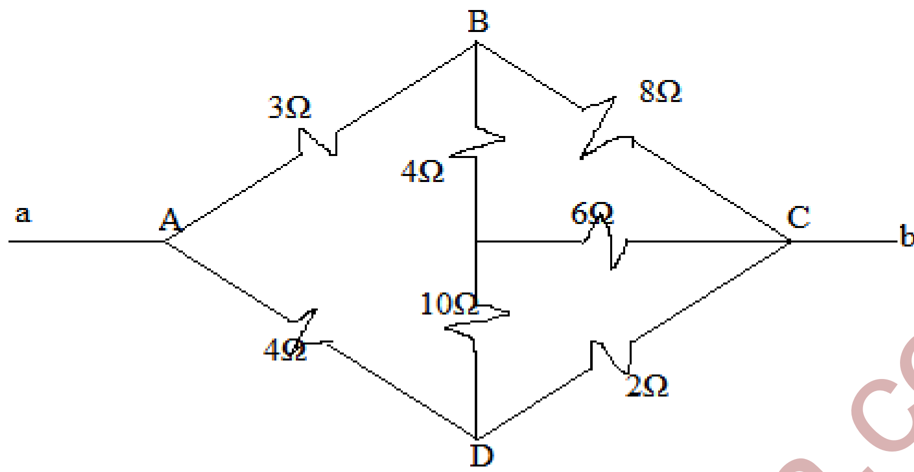
Now, 6 * 14.5 / (6 + 14.5) = 4.24Ω



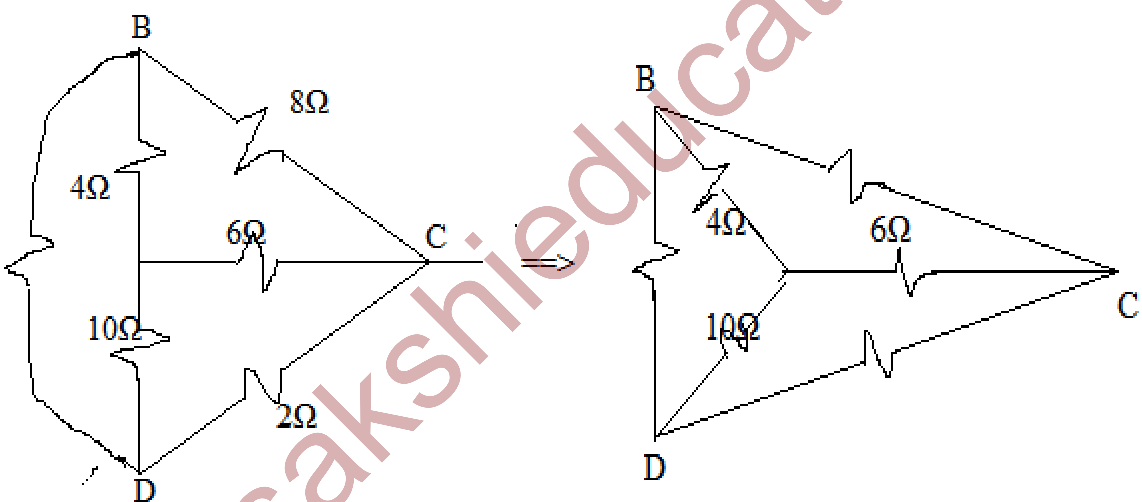


V = IR = 5 (6.84) = 34.2V

Example: Determine the resistance between ab in the circuit shown in fig.



First convert equivalent star to delta



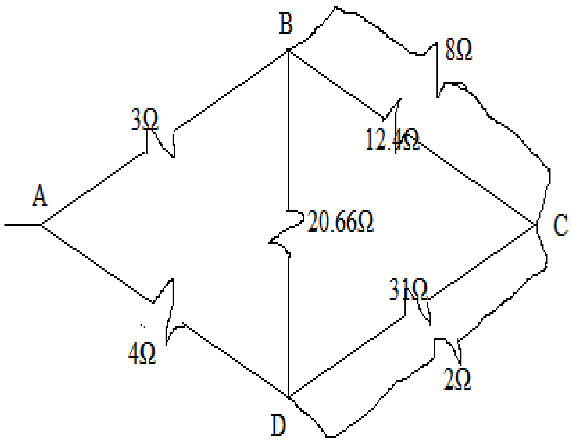
4 * 6 + 6 * 10 + 10 * 4 = 24 + 60 + 40 = 124 Ω

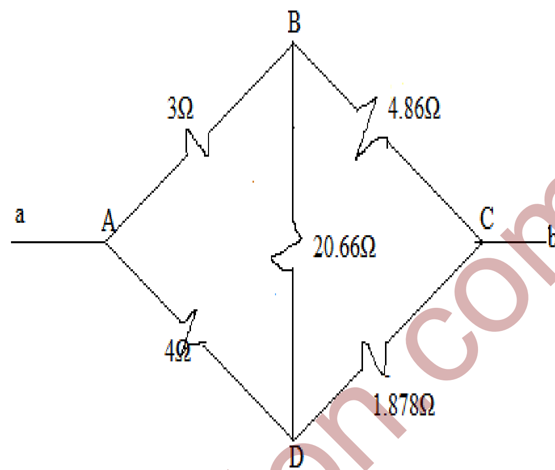
R_{BD} = 124 / 6 = 20.66 Ω

R_{BC} = 124 / 10 = 12.4 Ω

R_{CD} = 124 / 4 = 31 Ω

The circuit can be redrawn as

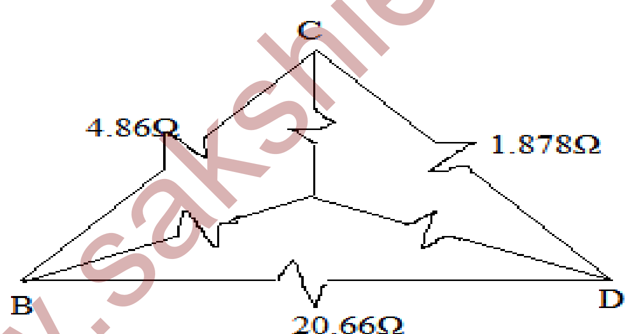




$$\frac{12.4*8}{12.4+8} = \frac{99.2}{20.4} = 4.86\Omega$$

$$\frac{31*2}{31+2} = 1.878\ \Omega$$

Now transform delta to star



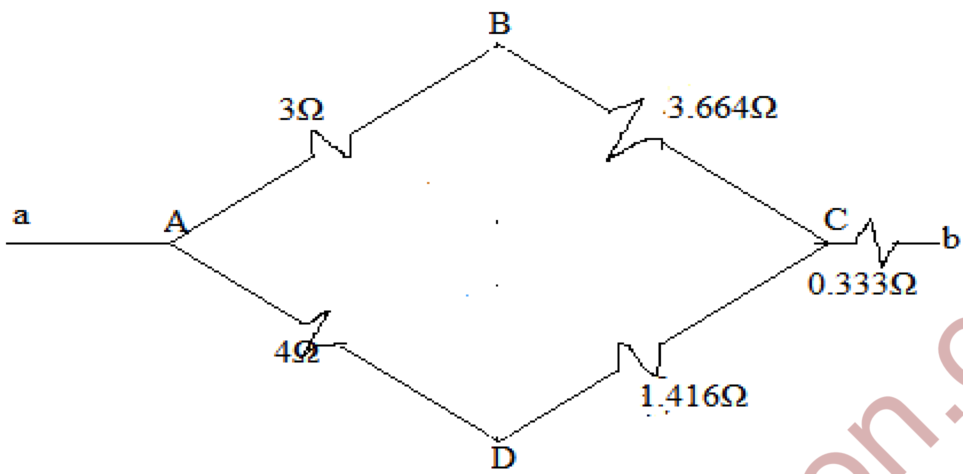
$$R_{BC} + R_{CD} + R_{DB} = 4.86 + 1.878 + 20.66 = 27.398\ \Omega$$

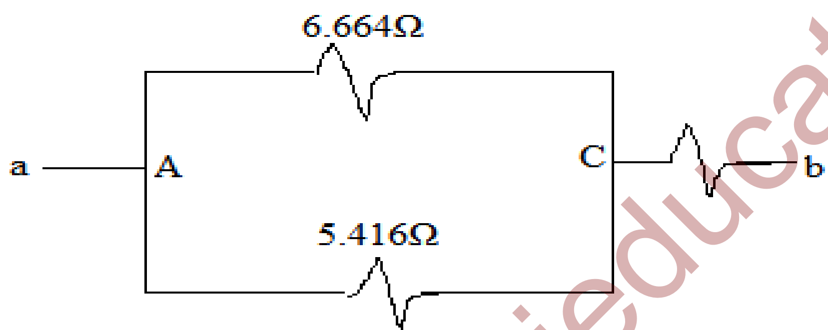
$$R_{BO} = \frac{R_{BC}*R_{DB}}{R_{BC}+R_{CD}+R_{DB}} = \frac{20.66*4.86}{27.398} = 3.664\ \Omega$$

$$R_{CO} = \frac{R_{BC}*R_{CD}}{R_{BC}+R_{CD}+R_{DB}} = \frac{4.86*1.878}{27.398} = 0.333\Omega$$

$$R_{DO} = \frac{R_{CD}*R_{DB}}{R_{BC}+R_{CD}+R_{DB}} = \frac{1.878*20.66}{27.398} = 1.416\ \Omega$$

The circuit can be drawn as





$\frac{6.664*5.416}{6.664+5.416} = \frac{36.09}{12.08} = 2.987\ \Omega$

$2.987 + 0.333 = 3.32\ \Omega$

Therefore, $R_{ab} = 3.32\ \Omega$