NETWORK THEOREMS -1

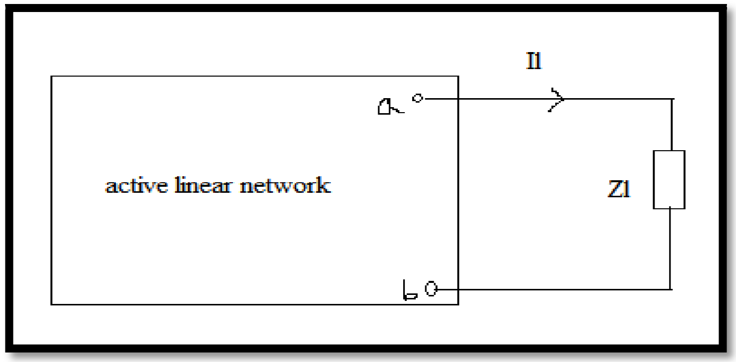
The loop current and node voltage methods are employed to solve all network problems. It is possible to determine all the branch currents and voltages. Sometimes, we may be interested in determine a particular branch current or voltage only. In such cases, network theorems are useful and which simplify the method of solution.

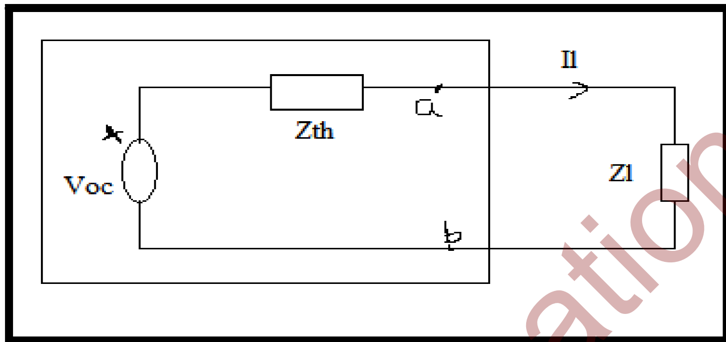
Thevenin's Theorem:

This theorem helps in reducing the given network to simple equivalent network with respect to a given pair of terminals. If a network is to be solved for the current through different load of impedances to be connected across two specified points in the network, we can employ either loop current or nodal method, but the network is to be solved as much number of times as there are different load of impedances. In such cases, it will be convenient to reduce the network to an equivalent network with respect to load terminals by applying thevenin's theorem and calculate the current for each load of impedance.

Statement:

The current in any load impedance connected to two terminals of a linear active network is the same as if this load impedance was connected to an equivalent consisting of a constant voltage source whose voltage is equal to the open circuit voltage at two terminals and whose internal impedance the impedance of the network looking back into the terminals with all source within the active network replaced by impedance equal to their internal impedances.





The fig shows an active linear network with two terminals a and b to which a load impedance Zl is connected. The venin's theorem says that so far as its external behavior is concerned, the network can replaced by an equivalent consisting of a voltage source Voc in series with an impedance Zth as indicated in fig, such that the load current remains the same in the original and equivalent circuit

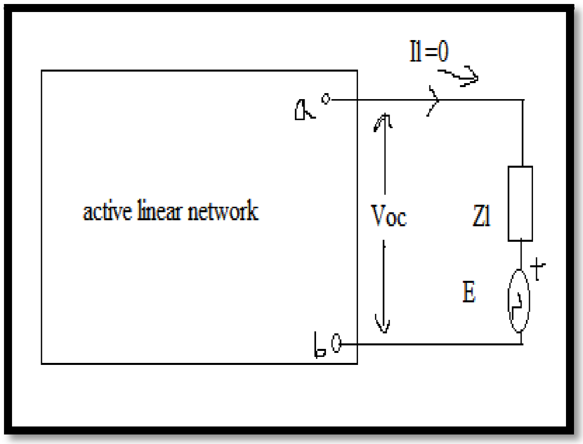
$$I_l = V_{oc} / (Z_{th} + Z_l)$$

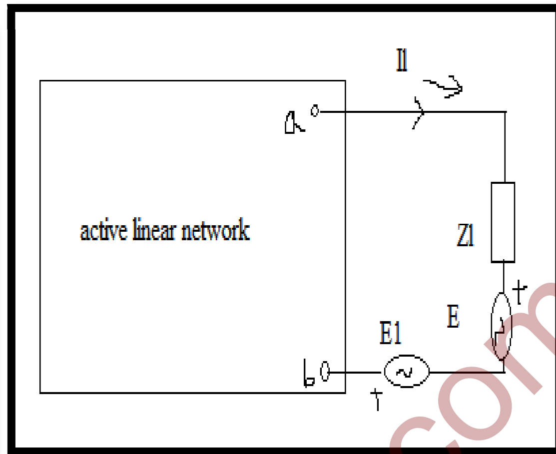
Where Voc is open circuit voltage between the terminals a and b

And Zth is thevenin's impedance and it is measured across the terminals a and b.

Proof: the proof makes use of superposition theorem

step1: Let us suppose that in the load circuit of fig is introduced a voltage source E of such a magnitude and direction that the load current reduced to zero.





Evidently the magnitude of the source E should be equal to the open circuit voltage, Voc of the active network at the load terminals and its direction must be opposite to that of Voc as indicated in fig.

Step2: let us suppose that yet another voltage source E1equal and opposite to the source E is introduced into the load circuit. The network is modified as that shown in fig. the introduction of the new source E1 cause a current I1to flow in the external circuit. Since there was no current in the external circuit before introducing E1, it would mean that I1 should be the current that would flow with E1 alone acting in the network with all other sources reduced to zero.

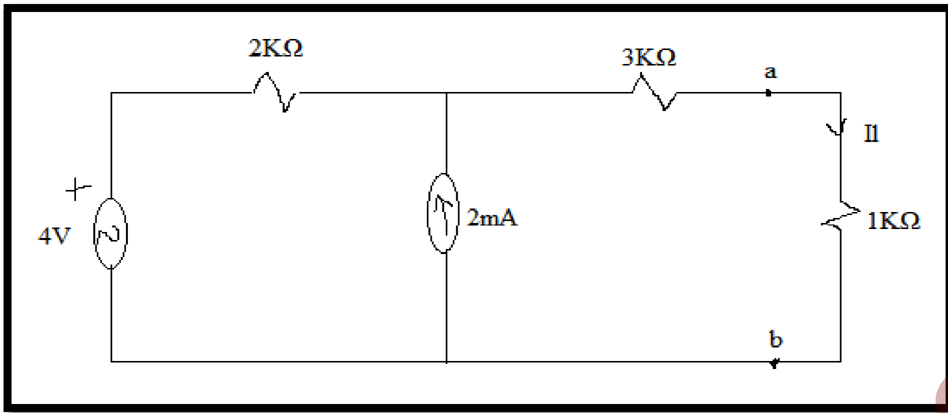
Therefore the current $I1 = E1 / (Zth + Zl)$

Step3: since E and E1 are equal and opposite, the resultant voltage across them is zero. The removal of the two sources E and E1 together does not affect the current I1 in the load circuit which is therefore the same as Il. The network reverts to its original form as in fig.

$$Il = I1 = Voc / (Zth + Zl)$$

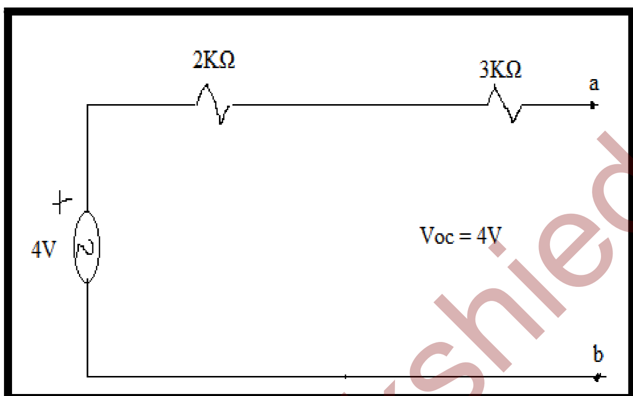
This is the current that flows in the load impedance Zl connected to an constant voltage source Voc of internal impedance Zth. The network in fig denotes the thevenin's equivalent of the original network in fig.

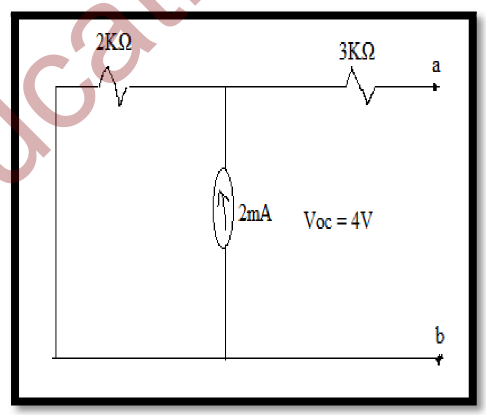
Example 1: determine Il using thevenin's theorem



To get thevenin's equivalent w. r. t a and b terminals

Thevenin's voltage Vth or Voc: we will use superposition theorem to calculate Voc

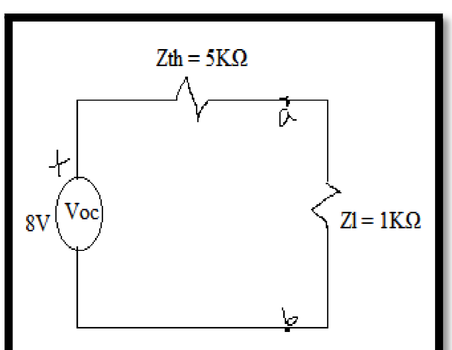




With 4V only, Voc = 4V

With 2mA only, Voc = 4V

From Voc simultaneously acting with two sources, Voc = 8V

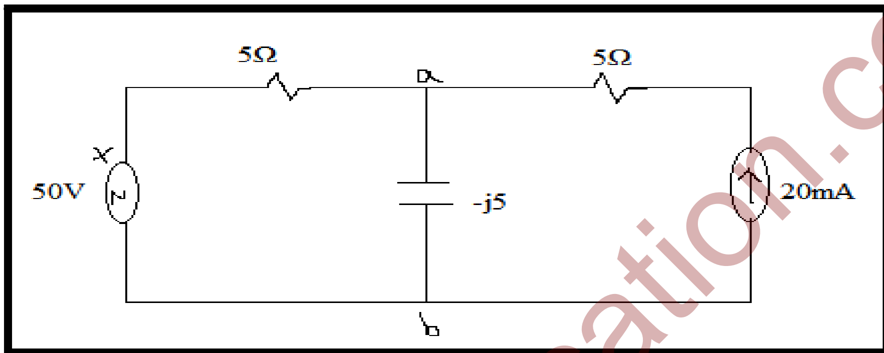


Thevenin's impedance: replace all the sources by their internal impedance and determine the impedance between a and b, Zth = 5KΩ

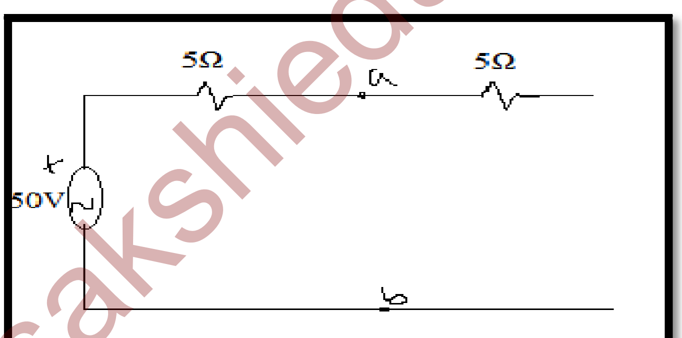
From the thevenin's equivalent shown in fig,

Il = Voc / (Zth + Zl) = 8 / (6 * 10^3) = 4/3mA

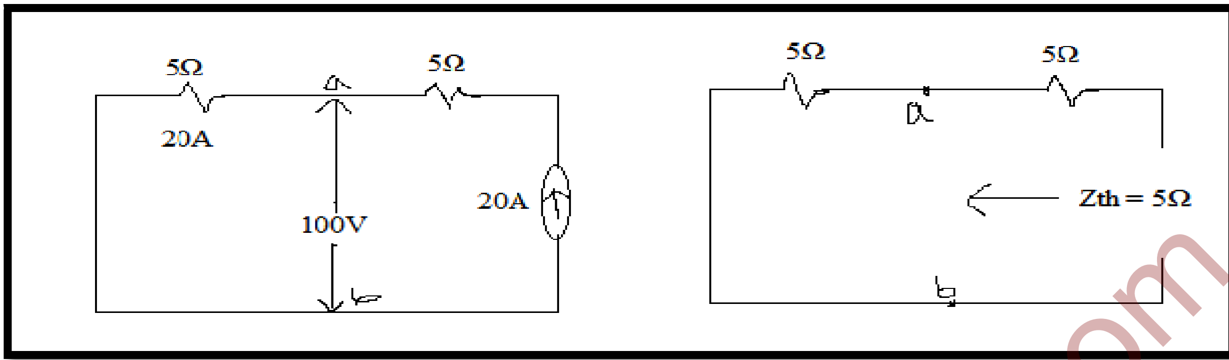
Example 2: Determine the current through the capacitor using Thevenin's theorem



Apply thevenin's theorem between terminals a and b



Thevenin's voltage: we will employ superposition theorem to calculate V^{th} or Voc



With only 50V source, Voc = 50V

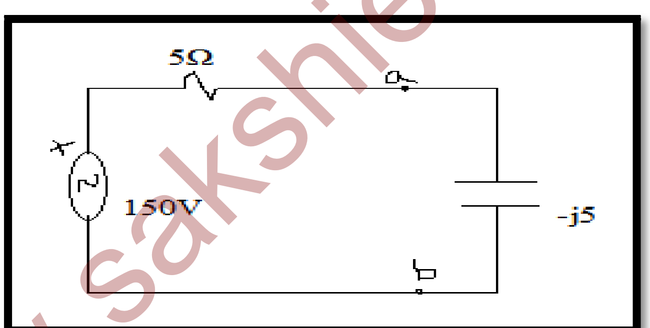
With only 20A source, Voc = 5 * 20 = 100V

When both sources are acting simultaneously, Voc = 50 + 100 = 150V

Thevenin's impedance:

$$Zth = Zab = 5\Omega$$

The thevenin's equivalent w. r. t to a and b terminals is



Current through capacitor Ic = Voc / (Zth – j5) = 150 / (5 – j5) = 15 + j 15 A

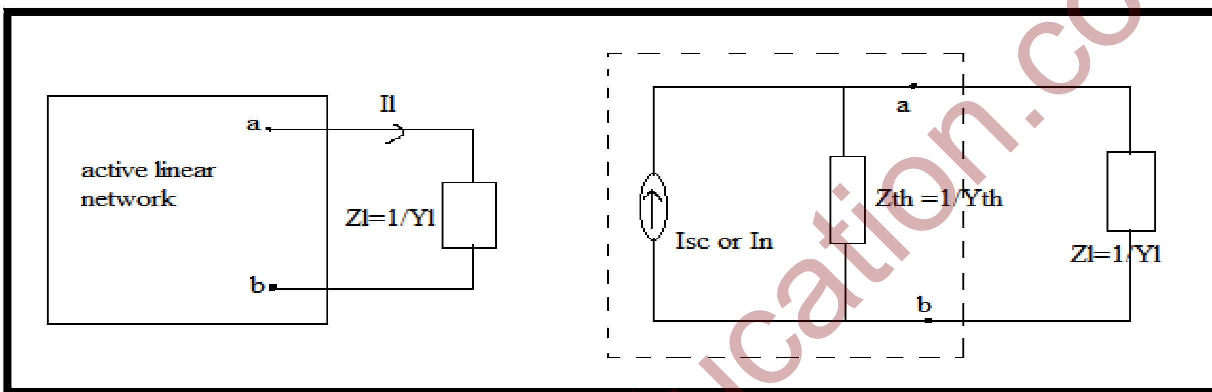
Norton's Theorem:

It is the dual of Thevenin's Theorem.

Statement:

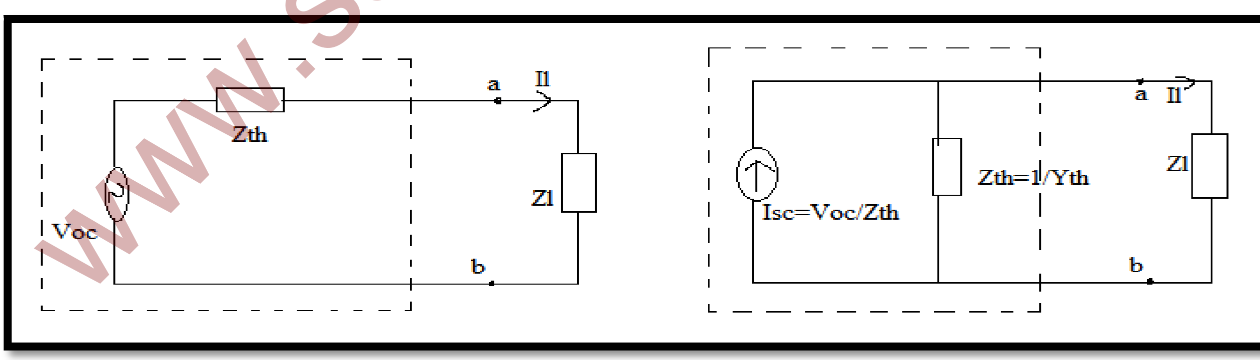
The current in any load impedance connected to two terminals of an active linear network is the same as if this load impudence was connected to a constant current generator whose current is equal to the current which flows through the two terminals when they are short circuited, and being in parallel with an admittance equal to the admittance of the network looking back into the terminals with all sources replaced by their internal admittances.

The fig shows an active network which can be replaced with respect to terminals a and b by norton's theorem by an equivalent shown in fig.



The equivalent circuit consists of a current source Isc in parallel with an admittance Yth (Admittance looking back into the network from terminals a and b when all the sources replaced by their internal admittances).

Proof: The proof of the theorem fallows from that of thevenin's theorem. By thevenin's theorem the active network reduced to an equivalent network consisting of a constant voltage (Voc or Vth) and series impedance Zth.



Replace the constant voltage generator Voc in series with the impedance Zth by an equivalent current source, Isc = Voc / Zth in shunt with impedance Zth or admittance Yth = 1 / Zth as shown in fig. Norton's theorem is hence proved. An independent proof also be given for Norton's theorem.

By Norton's theorem Il = Isc * (Zth / Zth + Zl)

Where Isc = Voc / Zth

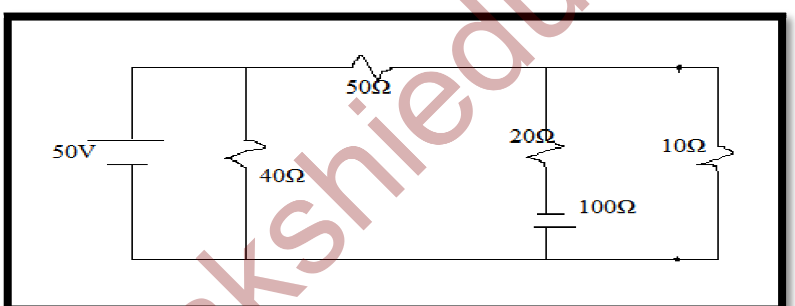
Hence Il = Voc / Zth + Zl

Voltage drop across the terminals = Il Zl = (Zth * Zl/ (Zth + Zl))

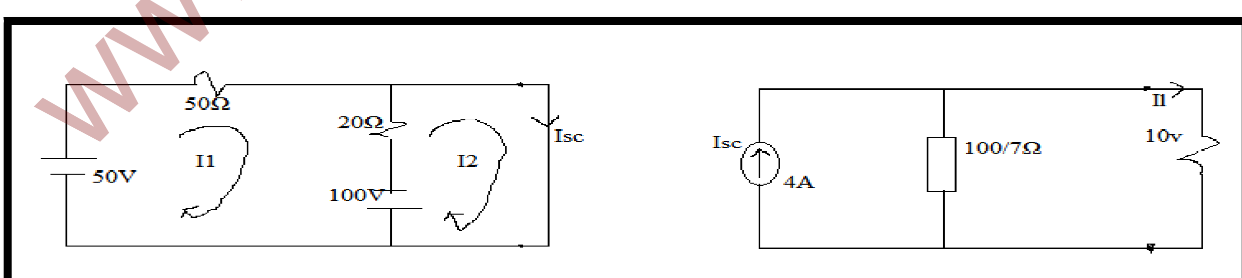
Vab = Isc * (1/(Yth+Yl))

The duality between thevenin's and norton's theorem is evident from the equations for Il based on theveni's theorem and the equation for Vab based on norton's theorem.

Example 1: Find the voltage across the 10Ω resistance in the network shown in fig. using Norton's theorem.



A 40 Ω resistor in parallel with a 50 V source is ineffective. Short circuit 10 Ω and determine Isc



Calculation of Isc:

50 I1 + 20 (I1 – I2) = 150

70 I1 – 20 I2 = 150

7 I1 – 2 I2 = 15 ---------- (1)

20 (I2 – I1) = 15

I1 – I2 = 5 --------------(2)

Solve the equations (1) & (2) we get, I1 = 1A and I2 = -4A

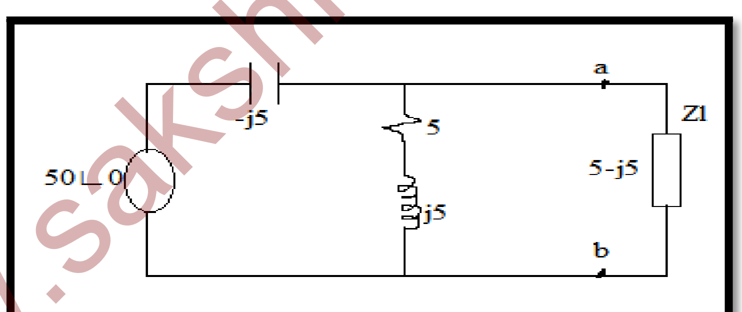
Isc = 4A in the up word direction.

Rth = 50 * 20 / 70 = 100/7 Ω

Current through 10 Ω, = (4 * 100/7) / (4 + (100/7)) = 40/17A

Voltage across 10 Ω, = 10 * (40 / 17) = 400 /17V

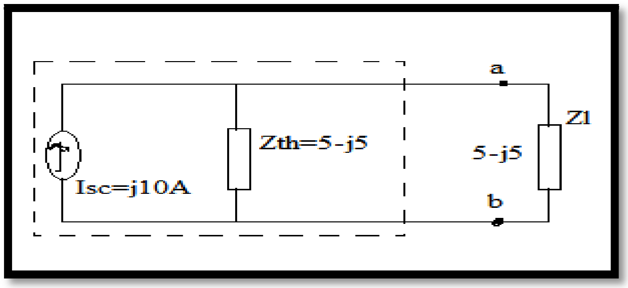
Example 2: Using the norton's theorem determine the current through load impudence Zl =5-j5 of the network shown in fig. find the power consumed by load.



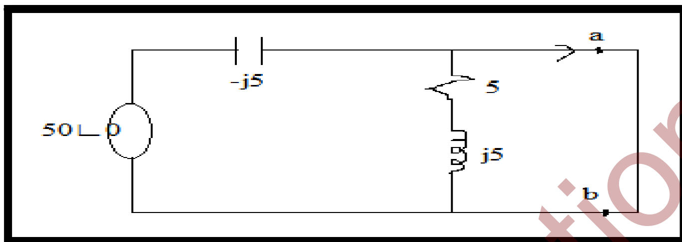
Get the norton's equivalent with respect to terminals a and b

Calculation of Isc:

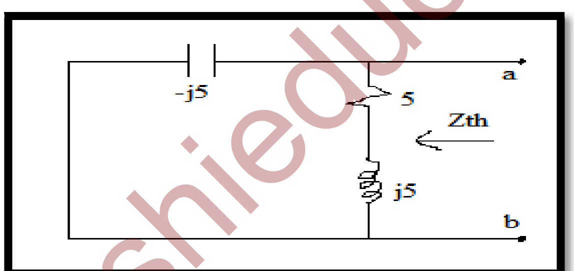
From the circuit, Isc = 50∟0 /50∟-90 = 10∟90 = j10A



Calculation of Zth:



From the circuit, Zth = (5 + j5)(-j5)/5 = 5 –j5



From the Norton's equivalent, Il = Isc * (5-j5)/ (5-j5) + (5+j5) = Isc /2 = 5∟90 = j5A

Power consumed by the load, Pl = * Rl = 5^2 * 5 =125W

Reciprocity Theorem:

Statement:

If a voltage applied in one branch of a linear, bilateral, passive network produces a certain current in any other branch of the network, the same voltage applied in the second branch will produce the same current in the first branch also.

Proof: Consider a network consists of L loops. Let the network be excited by only one voltage source Es in the Sth loop. To determine the current response Ir in the Rth loop is given by as

$$Ir = (1/Dz)(A1r\ E1 + A2r\ E2 + \text{----} + Ajr\ Ej + \text{----} + Asr\ Es + Anr\ En + \text{-----})$$

Since there is only one source in the Sth loop in the network,

$$Ir = Asr * Es / Dz = Yrs\ .Es$$

Let us now interchange the positions of excitation and response (exception to Rth loop and response to Sth loop). We have voltage source in the Rth loop and the response is measured in the Sth loop as Is.

$$Is = Ars * Er / Dz = Yrs\ .\ Er$$

The ratio of excitation to response remains constant i.e.

$$Es / Ir = Dz / Asr \quad \text{and} \quad Er / Is = Dz / Ars$$

If they are to be equal, Asr = Ars.

For a linear bilateral network, the cofactors Ars = Asr. Since the loop impedence matrix is symmetrical.

Hence theorem is proved. Note the other parts of the network will not remain the same.

Maximum Power Transfer Theorem:

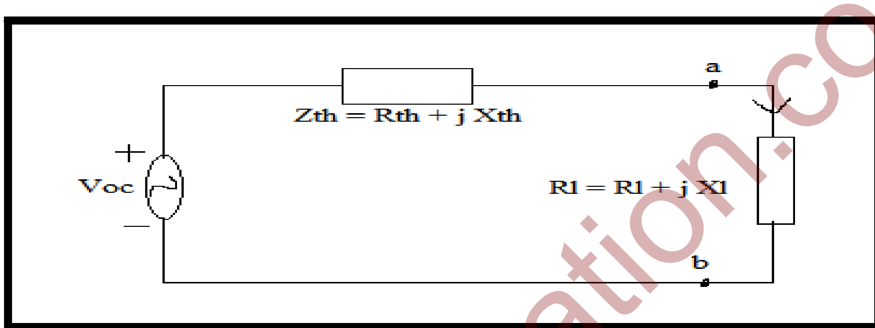
If load impedance is connected between two terminals of an active linear network, then the load impedance can be adjusted so that the power delivered to it is maximum. This process of adjusting the load impedance is known as matching the impedance to source power.

There are few theoretical aspects of matching based on the constraints placed on the parameters of the load impedance.

The general condition for maximum power transfer to the load is that the load impedance should be complex conjugate of the internal impedance of the active network.

There are 4 cases related to maximum power transfer theorem.

Case 1: The load impedance Zl = Rl + jXl in which both resistance and reactance can be varied, independently.



The active linear network to which the load is connected is thevenised into a single voltage source Voc in series with internal impedance Zth with respect to load terminals as shown in fig.

Then the power transferred to the load is Pl = Il^2 * Rl.

Therefore P ———————— --------------- (1)

Assuming first that only Xl is variable and Rl is fixed, for maximum power —— = 0.

—— = ———————————— = 0

Hence Xl = -Xth --------------- (2)

Now to maximize Pl further, let Rl also be varied so that, —— = 0

—— = . ————————————————

= 0 ------------ (3)

From equations (2) & (3) we get,

(Rth+Rl) (Rth –Rl) = 0 => Rl = Rth --------- (4)

Therefore for the power transferred to the load is maximum, which Rl and Xl are both variable.

Zl = Rth – j Xth = conjugate of Zth => Zl = Zth * ------- (5)

The maximum power transferred to the load under this condition is

Pmax = $\frac{(Voc)^2.Rth}{(2Rth)^2} = \frac{Voc^2}{4\,Rth}$ this is independent of Xth. ----- (6)

Case 2: In the load impedance Zl only resistance is variable while reactance Xl is fixed. For maximum power the condition as given by equation (3)

$$(Rth + Rl)^2 + (Xth + Xl)^2 - 2Rl\left((Rth + Rl)\right) = 0$$

$$(Rth + Rl)(Rth + Rl) + (Xth + Xl)^2 = 0$$

$$(Rth)^2 - (Rl)^2 + (Xth + Xl)^2 = 0$$

$$(Rl)^2 = (Rth)^2 + (Xth + Xl)^2$$

$$Rl = \sqrt{(Rth)^2 + (Xth + Xl)^2}$$

|Rl| = | Zth + j Xl| -------------- (7)

Case 3: When Rl is variable and load is purely resistive i.e., Xl = 0. For maximum power from equation (7) , |Rl| = |Zth| -------------------- (8)

i.e., the load resistance should be equal to absolute value of the thevenin's impedance of the active network.

Case 4: When the magnitude of load impedance is varied but the impedance angle of the load i.e.

$\theta = (tan)^{-1}\,\frac{XL}{Rl}$ remains constant.

Let Zl = Rl + jXl = Zl (cosθ + j sinθ)

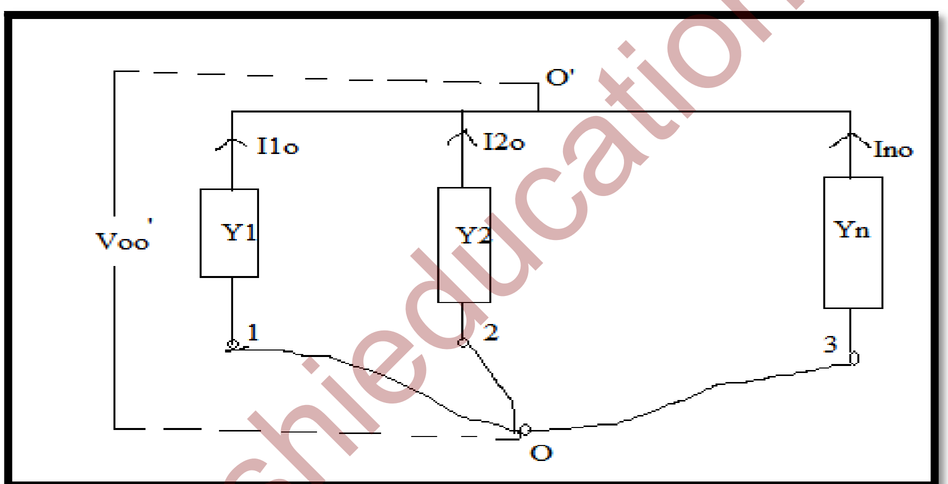
Pl = ____________________

For Pl is to be maximum Zl only, —— = 0

From that we can solve |Zl| = = |Zth|

From the above, it is seen that only when magnitude of load impedance is varied keeping impedance angle constant, max power transfer will occur when |Zl| = |Zth| of the active n/w.

Milliman's Theorem:



STATEMENT: This theorem states that the voltage drop from point 'O' to point 'O' junction of number of impedances in a network of the form shown in fig.

= ______________________________

= ——— where k=1 to n

Where Vo1, Vo2, ----, Von are the voltage drops '0' to 1,2, ---,n and Y1, Y2, ----, Yn are the admittance between nodes 1,2,----,n to common point 0. In applying this theorem it is not necessary to know interconnection between 0 and any other point n across which voltage Von exists.

Proof: the proof of this theorem is simple and straight forward.

The potential drop across Y1 = $(V1o)^1$ = $(V1o)^1$ + $(Voo)^1$ = $(Voo)^1$ - Vo1

Current, Y1 = Y1 $(V1o)^1$ = Y1 $((Voo)^1 - Vo1)$

Similarly current through Y2 = Y2 $(V2o)^1$ = Y2$((Voo)^1 - V02)$

Finally current through Yn = Yn $(Vno)^1$ = Yn $((Voo)^1 - V0n)$

Applying KCL at node $(O)^1$

$(I10)^1 + (I20)^1 + - - - - - - + (In0)^1 = 0$

$((Voo)^1 - Vo1)$ Y1 + $((Voo)^1 - Vo2)$ Y2 + - - - - - + $((Voo)^1 - Von)$ Yn = 0

$(Voo)^1$ (Y1 + Y2 + - - - - - + Yn) = Vo1 Y1 + Vo2 Y2 + - - - - - + Von Yn

$$(Voo)^1 = \frac{\text{Vo1 Y1 + Vo2 Y2} + - - - - - - + \text{Von Yn}}{(\text{ Y1 + Y2} + - - - - - - + \text{Yn })}$$

$$(Voo)^1 = \frac{\sum_{k=1}^{n} \text{Vok Yk}}{\sum_{k=1}^{n} \text{Yk}}$$

In this theorem, voltage drop is taken as positive.

And also this method is very useful in solving 3-phase unbalanced star connected load using neutral displacement method.