Networks Theorem -2

Tellegen's Theorem:

This theorem is valid for both linear and non linear networks.

Statement:

For a given network with e elements and n nodes, if a set of currents i_1,i_2, -----, i_e are specified so that KCL is satisfied at every node and another set of elements of voltages v_1,v_2, -----, v_e are specified so that KVL is satisfied for every loop of the network, then the theorem states that,

$\sum_{k=1}^{e} v_k \; i_k \; = 0$ where v_k and i_k are the voltage and current through k^{th} element.

Proof:

Let k^{th} element is connected between i^{th} and j^{th} nodes. Then voltage across the element

$$v_k = V_i - V_j$$

Multiplying i_k on both sides we get

$$v_k \; i_k = (V_i - V_j)\, i_k = V_i \; i_k - V_j \, i_k$$

$v_k \; i_k = V_i$ (current leaving i^{th} node through k^{th} element) + V_j (current entering j^{th} node from k^{th} element)

Hence, $\sum_{k=1}^{e} v_k \; i_k = V_1$ (algebraic sum of the currents at node 1) + V_2 (algebraic sum of the currents at node 1) + ----- + V_n (algebraic sum of the currents at node 1)

By KCL algebraic sum of the currents at any node is zero, $\sum_{k=1}^{e} v_k \; i_k = 0$ ------- (1)

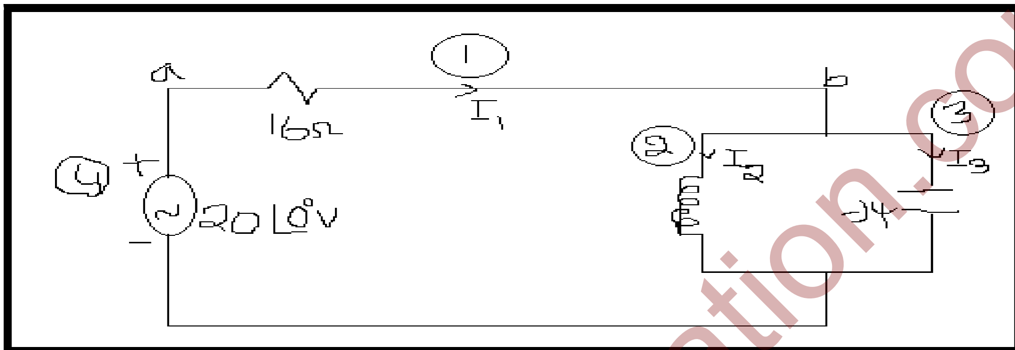
This theorem is valid for any instant (t) i.e., $\sum_{k=1}^{e} v_k\,(t)\, i_k\,(t)$ = 0 ---------- (2)

The set of voltages and set of currents could be taken at distinct instants t_1 and t_2 (t_1 =t_2)

= 0 --------- (3)

This is possible because the set of voltages corresponding to any instant to satisfy KVL for all loops, similarly set of currents at any other instant satisfy KCL at all nodes of the network.

Example 1: For the network shown in below verify the Tellegen's theorem.



Current through the elements:

= —— = ——

= ——— = = 0.8 – j0.6 A

Voltage across element1, = (0.8 – j 0.6) * 16 = 12.8 – j 9.6 V

Current through element2, = * – = = 3.2 – j2.4 A

Voltage across element2, = * = = 7.2 + j 9.6 V

Current through element3, = * — = = -2.4 + j1.8 A

Voltage across element3, = 7.2 + j 9.6 V

Current through element4, = 0.8 – j0.6 A

Voltage across element4, = = 20 V

According to Tellegen's theorem, = 0

$v_1\, i_1 + v_2\, i_2\; + v_3\, i_3\; + v_4\, i_4$ = (0.8 – j0.6)(12.8 – j 9.6) + (3.2 – j2.4)(7.2 + j 9.6) + (-2.4 + j1.8)(7.2 + j 9.6) + (0.8 – j0.6)(20) = 0

Superposition Theorem:

This theorem is very useful to find response at particular point which is excited by more than one independent source. The principle of superposition is applied to determine the current in the elements or voltages across branches which are linearly related to sources. Power cannot be determined by superposition, since the relation between the power and current or voltage is not linear.

Statement:

In a linear, bilateral network, containing several independent sources, the voltage across or current through any branch is the algebraic sum of all individual voltages or currents caused by each independent sources acting alone, with all other remaining independent sources reduced to zero. The particular sources are replaced by their internal impendence's.

If dependent sources are present, they are acting in every step. A linear circuit is a circuit composed of independent sources, linear dependent sources and linear elements.

A dependent source is said to be linear, who's current or voltage is proportional only to the first power of some other current or voltage variable in the circuit or to the sum of such linear quantities.

Proof:

A rigorous proof of the theorem may be given by considering linear bilateral network contain n loops. The equilibrium equations by the loop current method of analysis are

$$\begin{bmatrix} Z11\ Z12 - - - - - Z1n \\ Z21\ Z22 - - - - - Z2n \\ | \\ | \\ Zn1\ Zn2 - - - - - Znn \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ | \\ | \\ In \end{bmatrix} = \begin{bmatrix} E1 \\ E2 \\ | \\ | \\ En \end{bmatrix}$$

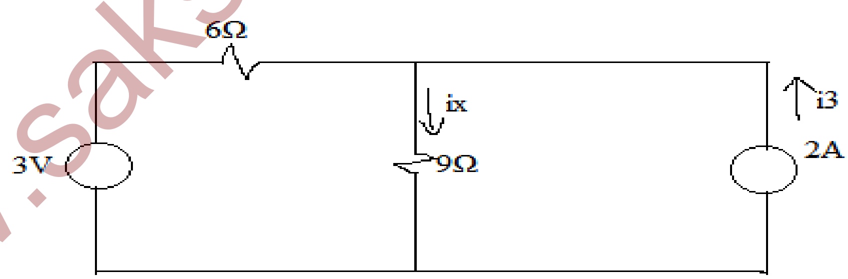
By Cramer's rule the current in sth loop is given by

Is = Ds/Dz = (E1.A1s/Dz) + (E2.A2s/Dz) + (Es.Ass/Dz) + - - - - - + (En.Ans/Dz)

Is = Ys1 E1 + Ys2 E2 + Yss Es + - - - - - + Ysn En

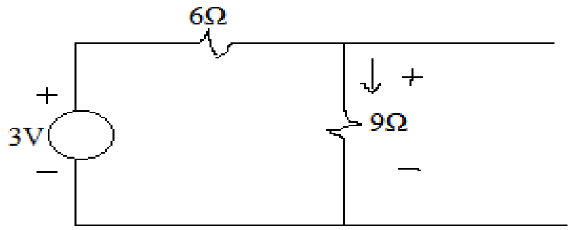
Assume that all the sources exist in the uncoupled branches of the various loops. That is no source appears in more than one loop. Then an inspection of equation at once reveals that each term on the right hand side denotes the contribution of the sth loop by one source alone acting and the remaining sources replaced by their internal impendence's. Thus the theorem is proved. This theorem may also be proved for current sources by the nodal method of analysis.

Example 1: In the network shown in fig determine ix by superposition theorem. Verify your result by nodal method.



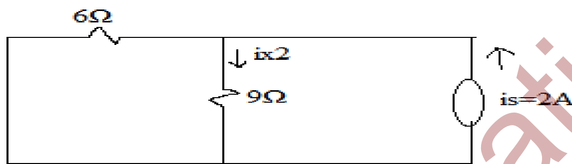
In the above network, there are two independent sources i.e. 3V voltage source and 2A current source.

Step1: assuming 3V source is only present and 2A is open circuited



ix1 = 3 / 15 = 0.2A

step2: assuming 2A current source is alone acting and 3V voltage source is short circuited, by superposition theorem, when both sources are acting simultaneously:

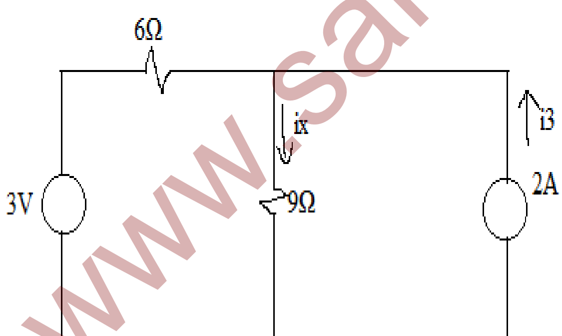


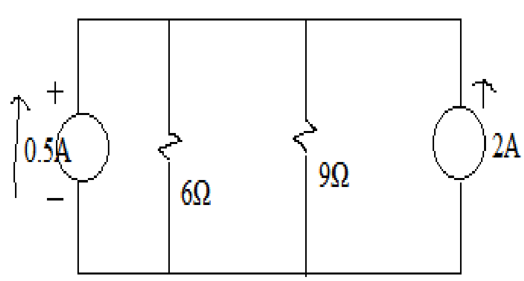
ix2 = 2 * 6 / 15 = 0.8A

The total current ix = ix1 + ix2 = 0.2 + 0.8 = 1A

Verification by Nodal Method:

Replacing the voltage source of 3V by equivalent current source





Taking V1 as the node voltage of node (1)

$$V1/6 + V1/9 = 2.5$$

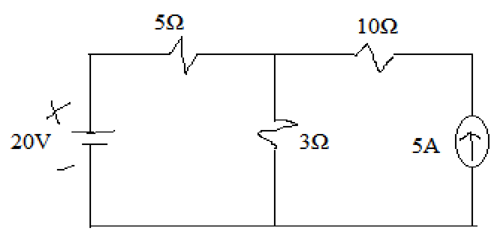
$$V1\ (1/6 + 1/9) = 5/2$$

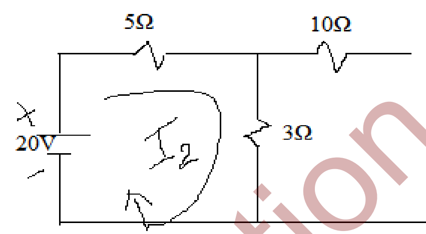
$$V1 = 9\ V$$

From this, ix = 9/9 = 1A

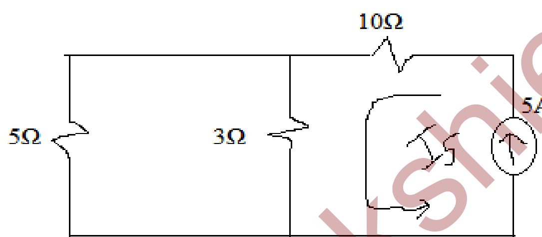
Above this superposition theorem was verified by nodal method.

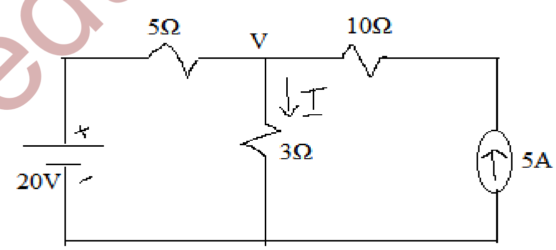
Example 2: From the below fig, find the current through 3Ω resistor and also prove that superposition not applicable for power responses.





According to the superposition theorem, the current I2 due to the 20V voltage source with 5A source open circuited = 20/(5 + 3) = 2.5A





The current I5 due to 5A source with 20V source is short circuited

$$I5 = 5 * 5 / (3 + 5) = 3.125A$$

So, total current passing through 3Ω resistor is, 2.5 + 3.125 = 5.625A

Let now examine the power responses,

Power dissipated in the 3Ω resistor due to voltage source acting alone,

$$P20 = (I20)^2 R = (2.5 * 2.5)\ 3 = 18.75\ W$$

Power dissipated in the 3Ω resistor due to current source acting alone,

$$P5 = (I5)^2 R = (3.125 * 3.125)\ 3 = 29.29\ W$$

Power dissipated in the 3Ω resistor due to both source acting alone simultaneously,

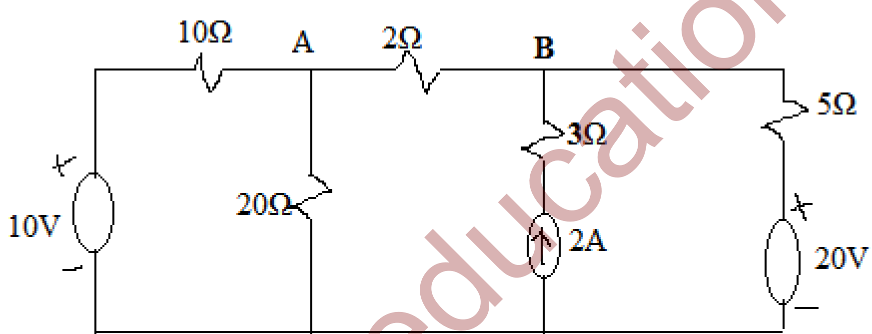
$$P = (5.625 * 5.625) * 3 = 94.92 \text{ W}$$

From the above results of superposition P20 + P5 = 48.04 W

So, therefore P20 + P5 ≠ P

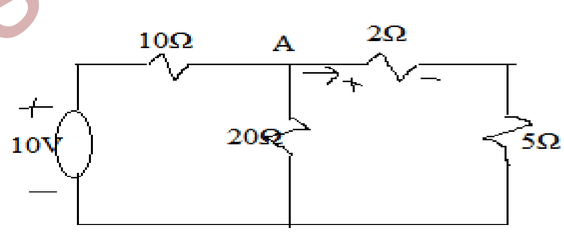
From this superposition theorem is not applicable for power responses, hence proved.

Example 3: find the voltage across the 2Ω resistor using below fig by superposition theorem.



Let us find the voltage across the 2V resistor due to individual sources. The algebraic sum of these voltages gives the total voltage across the 2Ω resistor.

Step1: voltage across 2Ω due to the 10V source and other sources equal to zero



Assuming a voltage V at node A as shown, the current equation is,

$$(V\text{-}10/10) + V/20 + V/7 = 0$$

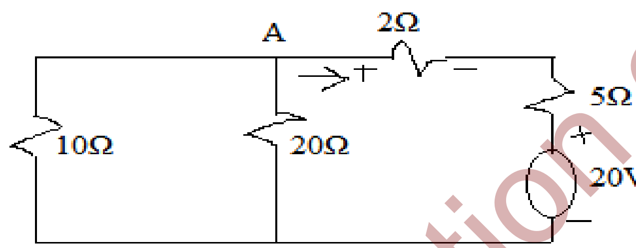
$$V(0.1 + 0.05 + 0.143) = 1$$

$$V = 3.41V$$

Or

The voltage across the 2Ω resistor due to 10 V source is, V2= V * 2/7 = 0.97 V

Step2: voltage across the 2 Ω due to 20V source and while the other sources set to zero



Assuming voltage V at node A as shown , the current equation is

$$(V-20/7) + V/20 + V/7 = 0$$

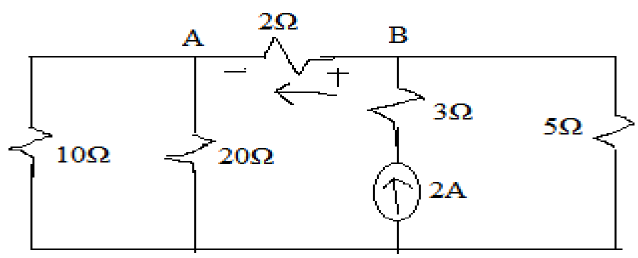
$$V(0.143 + 0.05 + 0.1) = 2.86$$

$$V = 9.76V$$

Or

The voltage across the 2 Ω resistor is due to 20V source is, V2= (V-20/7) * 2 = -2.92V

Step3: voltage across the 2 Ω resistor is due to the 2A current source while the other sources equal to zero,



The current in the 2 Ω resistor = 2 * (5/5+8.67) = 10/13.67 = 0.73A

The voltage across 2 Ω resistor = 0.73 * 2 = 1.46 V

From the superposition theorem, the voltage across 2 Ω resistor is algebraic sum of individual responses in the network

V = 0.97 – 2.92 – 1.46 = -3.41 V (voltage at A is negative)

Reciprocity Theorem:

Statement:

If a voltage applied in one branch of a linear, bilateral, passive network produces a certain current in any other branch of the network, the same voltage applied in the second branch will produces the same current in the first branch also.

In networks consisting of linear bilateral elements, the ratio of excitation to response when only one excitation is applied is constant when positions of excitation and response are interchanged. The excitation could be voltage or current source and the corresponding response is a current or voltage.

Proof: Consider a network consisting of L loops. Let the network be excited by only one voltage source E_s in the S^{th} loop. To determine the current response I_r in the r^{th} loop is given by

$$I_r = \frac{1}{D_Z}[A_{1r}\, E_1 + A_{2r}\, E_2 + \text{--------} + A_{jr}\, E_j + \text{- - - - -} + A_{sr}\, E_s + A_{nr}\, E_n + \text{--------}\ ]$$

There is only one source with S^{th} loop in the network

$$I_r = \frac{1}{D_Z} A_{sr}\, E_s = Y_{rs}\, E_s$$

Let us now interchange the positions of excitations and response (now response r^{th} loop to S^{th} loop). We have voltage source in the r^{th} loop and the response is measured in S^{th} loop as I_s .

$$I_s = \frac{1}{D_Z} A_{rs}\, E_r = Y_{sr}\, E_r$$

The ratio of excitation to response remains constant i.e.

$$\frac{E_s}{I_r} = \frac{D_Z}{A_{sr}} \quad \text{and} \quad \frac{E_r}{I_s} = \frac{D_Z}{A_{rs}}$$

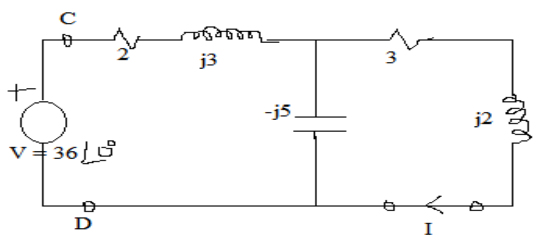
If they are to be equal means, $A_{rs} = A_{sr}$

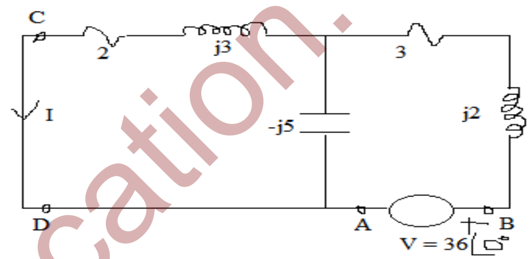
For a linear bilateral network, the cofactors $A_{rs} = A_{sr}$. Since the loop impedance matrix is symmetrical.(note that the other parts of the network will not remain the same).

Hence the theorem is proved.

Example 1: For the circuit shown in fig determine the current in branch AB when a voltage of

V = 36 ∟0°V is applied to branch CD. Verify the reciprocity theorem.





Before interchange:

Driving point impedance Z_{CD}= (2 + j3) + $\frac{(-j5)(3+j2)}{(3-j3)}$

$$= 2 + j3 + \frac{10-j\,15}{(3-j3)} = 6.537 \angle 19.36\ \Omega$$

Total current $I_T = \frac{V}{Z_{CD}} = \frac{36 \angle 0°}{6.537 \angle 19.36} = 5.507 \angle -19.36°$ A

$$I_{ab} = 5.507 \angle -19.36° * \frac{(-j5)}{(3-j3)} = 6.49 \angle -64.36°\ \text{A}$$

After interchange:

Driving point impedance Z_{AB}= (3 + j2) + $\frac{(-j5)(j3+2)}{(2-j2)}$

$$= 3 + j2 + \frac{15-j\,10}{(2-j2)} = 9.804 \angle 19.36°\ \Omega$$

Total current $I_{AB} = \frac{V}{Z_{AB}} = \frac{36 \angle 0°}{9.804 \angle 19.36°} = 3.672 \angle -19.36°$ A

= 3.672 - * —— = 6.49 - A

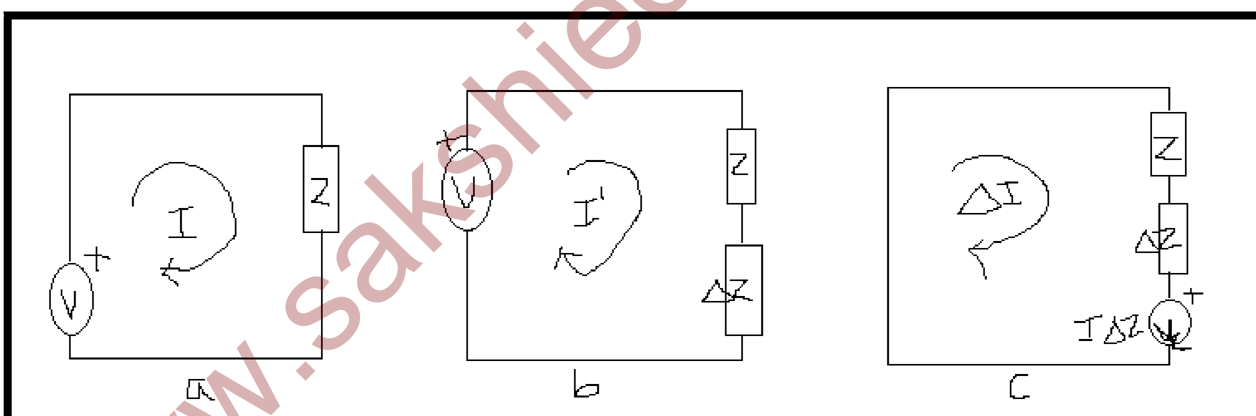
Therefore before and after interchange both are satisfied. So, reciprocity theorem satisfied.

Compensation Theorem:

If current in a branch of a linear bilateral network is I and the impedance of that branch is then increased any amount of ΔZ, the increment of current and voltage in each branch of the network is the current or voltage that would be produced by an opposing emf equal to IΔZ introduced into the modified branch after modification.

By this theorem, a change in impedance is substituted by an emf and hence theorem is called substitution theorem. This emf is called compensation emf. This theorem is useful in determining unbalanced current in a bridge network when there is a small change in branch impedance from balanced condition.

Consider the network shown in fig in which the impedance Z is carrying current I. the current I = V/ Z.



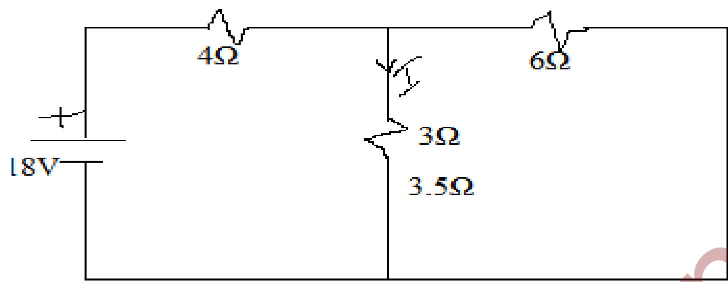
The impedance Z is changed to Z + ΔZ and the new current is —— as shown in fig.

Now a compensating IΔZ is introduced in the modified branch with setting V = 0 results in a current ΔI as shown in fig. ΔI is the change in current due to change in impedance ΔZ i.e.

ΔI = - I.

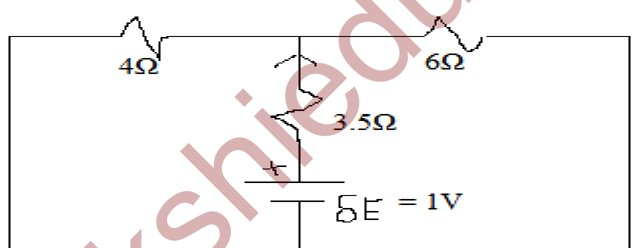
$$\Delta I = \frac{I\Delta Z}{Z + \Delta Z}$$

Example 1: In the circuit shown in fig resistance 3Ω changed to 3.5Ω. Determine the change in current 6Ω by using compensation theorem.



Original current in 3Ω = $\frac{18}{4+\frac{3*6}{9}} * \frac{6}{9}$ = 2A

Current in 6Ω = 1A.



Compensating emf δE = 2 * δ Z = 1V

Total resistance = 3.5 + $\frac{4*6}{10}$ = 5.9Ω

Change in current through 6Ω = $\frac{1}{5.9} * \frac{4}{10}$ = 0.067A