# MAGNETIC CIRCUITS

Consider an iron ring on which a coil is wound. Most rings are made like anchor rings in that their cross selection is circular. Such a ring is called a toroid. Whenever a current is flowing through the coil there will be magnetic flux produced and the path fallowed by the magnetic flux is known as magnetic circuit. The various terms associated with magnetic circuit are explained as fallows.

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circular cross
section area(sqm)
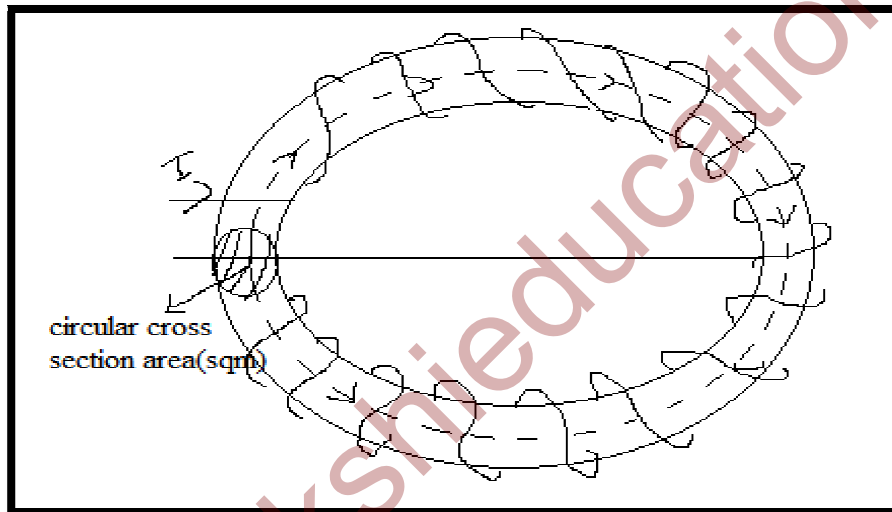
Magneto motive force (MMF): MMF is caused by current flowing through a coil having number of turns. The value of MMF is proportional to the current and to the number of turns and is expressed in ampere turns.

MMF = current in amps (I) * number of turns (N) = NI amp.turns

Magnetic flux (

The magnetic flux that is established in a magnetic circuit is proportional to the MMF. Magnetic flux(MMF.

Flux (= MMF/ Reluctance = F/ Reluctance (S)

Reluctance (S):

Reluctance of magnetic circuit is defined as the ratio of magneto motive force (MMF) to the flux established.

Reluctance = MMF/ Flux = AT/ wb

Reluctance of the magnetic circuit also depends on the physical dimensions of the magnetic path.

1) Directly proportional to the length of the magnetic circuit.
2) Inversely proportional to the area of cross section
3) Depends on the material of ring or specimen

$$S = \frac{1}{a\mu_o\mu_r}$$

Where μ_o= absolute permeability of the material

μ_r= absolute permeability of free space or vacuum = $4\pi * 10^{-7}$

For air, vacuum or non – magnetic material, $\mu_r = 1$.

Flux ($\emptyset$) = MMF/ Reluctance = F/ Reluctance (S) = NI * a * $\mu_o\mu_r$/ 1

This is called analogy called OHM's law for the magnetic circuit.

Magnetic field strength (H):

If the magnetic circuit is homogeneous, and of uniform cross section area, the magnetic field strength (H) is defined as the magneto motive force per unit length of the magnetic circuit.

H = MMF / length of the magnetic circuit = NI / l

Magnetic flux density (B):

Magnetic flux density in any material is defined as the magnetic flux established per unit area of cross section.

Magnetic flux density (B) = total flux/ area of cross section = $\emptyset$ / A

Unit: wb/ or tesla.

COMPARISION OF ELECTRIC AND MAGNETIC CIRCUITS:

Electric circuit		Magnetic circuit	
quantity	unit	Quantity	Unit
EMF	Volts	Magntic field strength MMF	Ampere turns AT /m
Current	Amperes	Magnetic flux	webers
Current density	A/	Magnetic flux density	Wb/ or Tesla
resistance	ohms	Reluctance	At/wb
conductivity		permiability	

FARADAYS LAWS OF ELECTRO MAGNETIC INDUCTION:

In 1831, Micheal Faraday formulated two laws on the bases of experiments. These laws are called Faraday's laws of electromagnetic induction.
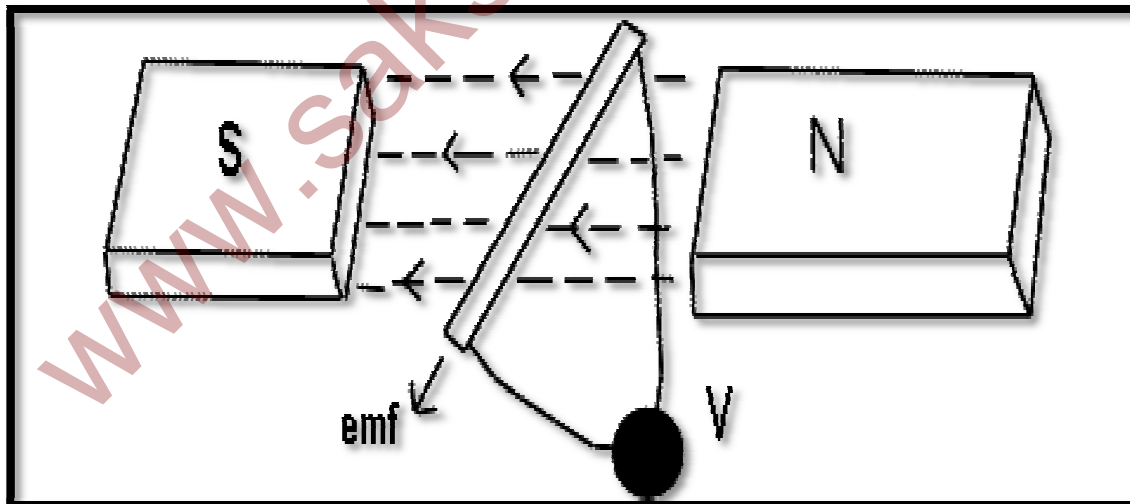
FIRST LAW:

First Law of Faraday's Electromagnetic Induction state that whenever a conductor are placed in a varying magnetic field emf are induced which is called induced emf, if the conductor circuit are closed current are also induced which is called induced current.
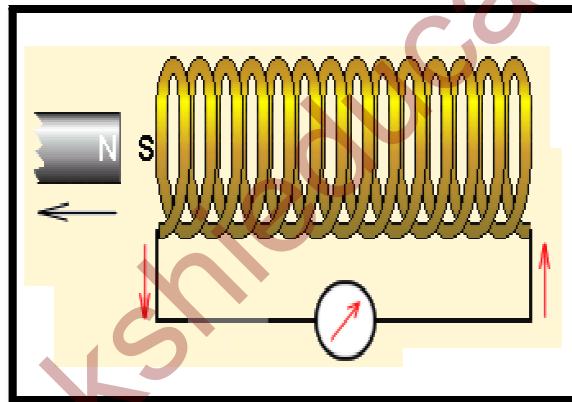
(Or)

Whenever a current carrying conductor is rotated in magnetic field it will induces an emf.

SECOND LAW:

Second Law of Faraday's Electromagnetic Induction state that the induced emf is equal to the rate of change of flux linkages (flux linkages is the product of turns, n of the coil and the flux associated with it).

Explanation:

Consider a magnet approaching towards a coil. Here we consider two instants of time T_1 and time T_2.

Flux linkage with the coil at time, $T_1 = N\Phi_1$ Weber (wb)

Flux linkage with the coil at time, $T_2 = N\Phi_2$ wb

Change in flux linkage = $N(\Phi_2 - \Phi_1)$

Let this change in flux linkage be, $\Phi = \Phi_2 - \Phi_1$

So, the Change in flux linkage = $N\Phi$

Now the rate of change of flux linkage = $N\Phi / t$

Take derivative on right hand side we will get

The rate of change of flux linkage = $Nd\Phi/dt$

But according to Faraday's law of electromagnetic induction, the rate of change of flux linkage is equal to induced emf.

Therefore, $E = N\dfrac{d\Phi}{dt}$

According to lenz's law,

$E = -N\dfrac{d\Phi}{dt}$

Where Φ in weber, $\Phi = B * A$

B = magnetic field strength

A = area of the coil

From this experiment, Faraday concluded that whenever there is relative motion between conductor and a magnetic field, the flux linkage with a coil changes and this change in flux induces a voltage across a coil.

How to increase emf induced in a coil:

By increasing the number of turns in the coil i.e. N- From the formulae derived above it is easily seen that if number of turns of coil is increased, the induced emf also gets increased.

By increasing magnetic field strength i.e. B surrounding the coil- Mathematically if magnetic field increases, flux increases and if flux increases emf induced will also get increased. Theoretically, if the coil is passed through a stronger magnetic field, there will be more lines of force for coil to cut and hence there will be more emf induced.

By increasing the speed of the relative motion between the coil and the magnet – If the relative speed between the coil and magnet is increased from its previous value, the coil will cut the lines of flux at a faster rate, so more induced emf would be produced.

Applications:

This law is the most important and basic laws of electro magnetism. This law finds its applications in electrical machines, industries, medical fields etc.

1) **Electrical Transformers:** It is a static ac device which is used to either step up or step down voltage or current. It is used in generating station,

transmission and distribution system. The transformer works on Faraday's law.

2) **Electrical Generators**: The basic working principle of electrical generator is Faraday's law of mutual induction. Electric generator is used to convert mechanical energy into electrical energy.

3) **Induction Cookers**: The Induction cooker is a fastest way of cooking. It also works on principle of mutual induction. When current flows through the coil of copper wire placed below a cooking container, it produces a changing magnetic field. This alternating or changing magnetic field induces an emf and hence the current in the conductive container, and we know that flow of current always produces heat in it.

4) **Electro Magnetic Flow Meters**: It is used to measure velocity of blood and certain fluids. When a magnetic field is applied to electrically insulated pipe in which conducting fluids are flowing, then according to Faraday's law, an electromotive force is induced in it. This induced emf is proportional to velocity of fluid flowing.

5) **Musical Instruments:** It is also used in musical instruments like electric guitar, electric violin etc.

Self Induction:

Inductance is the property of the circuit element which will oppose any change of current through it. By Faraday's laws of electromagnetic induction, it fallows that whenever there is change of flux linking with a coil with time, and then there will be an induced emf in the coil. The induced emf is proportional to the rate of change flux linkages of the coil.

$$\text{e} \propto \frac{d\varphi}{dx} \propto \text{N} \frac{d\Phi}{dt} \text{ ------ (1)}$$

Where N is the number of turns in the coil and Φ is the flux in weber in the coil.

$$\text{e} = \text{- N} \frac{d\Phi}{dt}$$

The negative sign indicates that the direction of induced emf is such that it opposes the every cause which is producing it, also known as LENZ'S law. Since the flux

in the coil is directly proportional to current flowing in it, the emf induced is proportional to the rate of change of current.

$$e \quad —$$

The constant proportional is called self inductance of the coil, e $\quad —$ ------- (2)

If the current I and flux linkages refer to the same physical system, than the parameter L is called self inductance. It is measured in HENRYS.

Mutual Inductance:

Let us consider that there are two coils which are placed on the same magnetic core such that the flux produced by current flowing through one coil completely links with the other coils also. Let the coil1 is connected to AC supply and coil2 is open circuit.
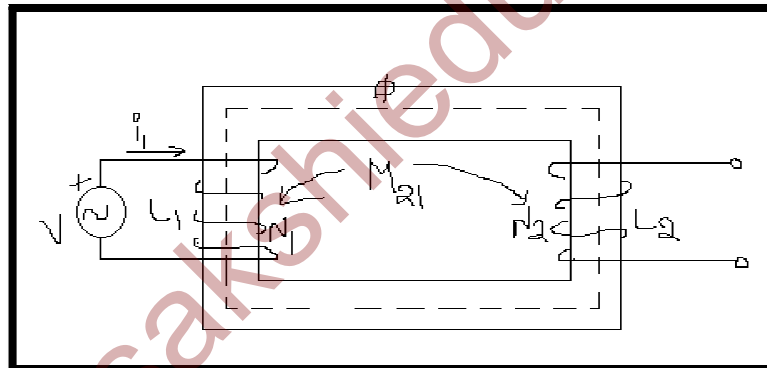
A current flowing in the first coil produces a flux as shown in fig. the direction of time varying flux is given by right hand thumb rule. The flux produced by current not only links with the coil1 but also links with coil2. The emf induced in coil1 is called self induced emf.

$$— = \quad — \text{------- (3)}$$

The flux linking with second coil also changes and hence there will be an induced emf in the second coil. Because of a change of current in the first coil there induced emf in the second coil and this is called as mutual induced emf.

$$e_2 = N_2 \frac{d\Phi_1}{dt} = M_{21} \frac{di_1}{dt} \quad \text{------- (4)}$$

The proportionality constant M_{21} between induced emf in the second coil and rate of change of current in the first coil is called mutual inductance. Any two such coils are said to be magnetically coupled.

The unit of mutual inductance is HENRY. The mutual inductance between two coils is said to be 1 Henry when a change of current of 1 Amp/Sec in one coil produces a mutual induced emf of 1 volt in the other coil.
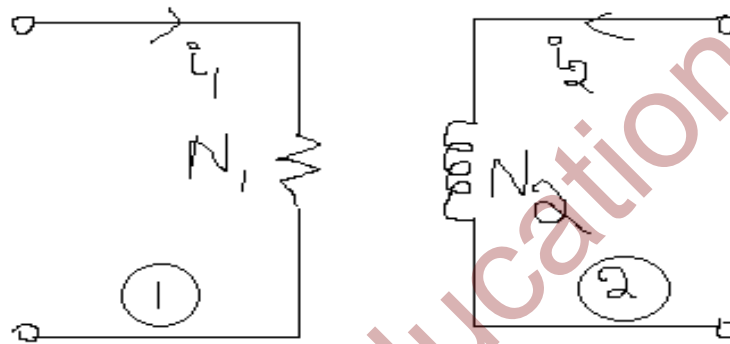
Self induced emf in coil1, $e_1 = L_{11} \frac{di_1}{dt}$

Mutual induced emf in the coil2, $e_2 = M_{21} \frac{di_1}{dt} \quad \text{----------- (5)}$

Let us assume that second coil also carries a current of i_2 as shown in fig, which in turn produces a self induced emf in coil2 and a mutual induced emf in coil1.

Self induced emf in coil2= $L_{22} \frac{di_2}{dt}$

Mutual induced emf in the coil1= $M_{21} \frac{di_2}{dt} \quad \text{----------- (6)}$

In practice all the flux produced by current in one coil may not completely link with the other coil. Depending on the position and orientation of the two coils, only a fraction of the flux may be linking with the other coil. Then the two circuits are said to be loosely coupled and if all the flux is linking with the other coil, then they are said to tightly coupled.

If Φ_1 is the total flux produced by i_1 and only Φ_{21} is common and linking with second coil, then the fraction of the flux linking with coil2 is $\frac{\Phi_{21}}{\Phi_1}$. Similarly Φ_2 is the total flux produced by i_2 and only Φ_{12} is common and linking with first coil, then the fraction of the flux linking with coil1 is $\frac{\Phi_{12}}{\Phi_2}$. These fractions indicate the degree of coupling between the two coils. If the two coils are very close to one another and properly oriented then these fractions approaches to unity.

Coefficient of Coupling:

It is the factor which indicates the degree of coupling between the couple coils given by

$$K = \sqrt{\frac{\Phi_{21}}{\Phi_1} * \frac{\Phi_{12}}{\Phi_2}} \quad \text{---------- (1)}$$

Expressing (1) in terms of self and mutual inductances,

$$L_1 = \frac{N_1 \ \Phi_1}{i_1} \quad \text{-------------- (2)}$$

$$M_{21} = \frac{N_2 \ \Phi_{21}}{i_1} \quad \text{-------------- (3)}$$

$$L_2 = \frac{N_1 \ \Phi_{12}}{i_2} \quad \text{-------------- (4)}$$

$$M_{12} = \frac{N_1 \ \Phi_{12}}{i_2} \quad \text{-------------- (5)}$$

Substitute (2), (3), (4) & (5) in equation (1) we get,

$$K = \sqrt{\left(\frac{M_{21} i_1 N_1}{N_2 L_1 i_1} \frac{M_{12} i_2 N_2}{N_1 L_2 i_2}\right)} = \sqrt{\frac{M_{21}}{L_1} \frac{N_1}{N_2} \frac{M_{12}}{L_2} \frac{N_2}{N_1}}$$

If $M_{21} = M_{12} = $ M then we get,

$$K = \sqrt{\frac{M^2}{L_1 L_2}} \quad \text{-------------- (7)}$$

This will be equal to 1 if coils are coupled tightly

$$M = K \sqrt{L_1 L_2} \text{ --------- (8)}$$

Coefficient of coupling is also defined as the ratio of mutual flux to total flux. It is always less than one ($K \leq 1$) (this is the principle used in transformer).

Example 1: Coil1 of a pair of coupled coils has a continuous current 5A and the corresponding fluxes Φ_{11} and Φ_{12} are 0.2 and 0.4 mwb respectively. If the turns are $N_1 = 500$ and $N_2 = 1500$. Find L_1, L_2, M and K.

Total flux, $\emptyset = \Phi_{11} + \Phi_{12} = 0.2 + 0.4 = 0.6$ mwb

$$L_1 = \frac{N_1 \ \Phi_1}{i_1} = \frac{500 * 0.6 * 10^{-3}}{5} = 0.6 \text{ H}$$

$$M = \frac{N_2 \ \Phi_{12}}{i_2} = \frac{1500 * 0.4 * 10^{-3}}{5} = 0.12 \text{ H}$$

$$M = K \sqrt{L_1} \ L_2$$

$$0.12 = 0.667 \sqrt{0.06 * L_2}$$

$$L_2 = 0.54 \text{ H}$$

Example 2: Two coupled coils of $L_1 = 0.8H$ & $L_2 = 0.2H$ have a coupling coefficient K = 0.9. find the mutual inductance M and the turns ratio $\frac{N_1}{N_2}$.

$$M = K \sqrt{L_1} \ L_2$$

$$= 0.9 \sqrt{0.8 * 0.2} = 0.36 \text{ H}$$

$$M = \frac{N_2 \ \Phi_{12}}{i_1} \qquad \text{but } \Phi_{12} = K \ \Phi_1$$

$$= \frac{N_2 \ K \Phi_1}{i_1}$$

Multiply by N_1,

$$M = \frac{N_2 \ K \Phi_1}{i_1} * \frac{N_1}{N_1} = K \ (\frac{N_2}{N_1})(\frac{N_1 \ \Phi_1}{i_1})$$

$$M = K \ \frac{N_2}{N_1} L_1$$

$$0.36 = 0.9 * \frac{N_2}{N_1} * 0.8$$

$$\frac{N_1}{N_2} = 2$$

Example 3: An iron ring has a mean diameter of 25 cms, and a cross sectional area of 4 cm^2. It is wound with a coil of 1200 turns. An iron gap of 1.5mm width is cut in the ring. Determine the current required in the coil to produce a flux of 0.48 mwb in the air gap. The relative permeability of iron under this condition is 800. Neglect leakage.

Flux = $0.48 * 10^{-3}$ wb

Area of cross section = 4 cm^2 = $4 * 10^{-4}$ m^2

B = $\frac{\emptyset}{A} = \frac{0.48 * 10^{-3}}{4 * 10^{-4}} = 1.2$ wb/m^2

Ampere turn for air gap, H = S = $\frac{B}{\mu_o} = \frac{1.2}{4\pi*10^{-7}}$ AT/m

L= $1.5 * 10^{-3} m$

AT required for air gap, = $\frac{1.2}{4\pi*10^{-7}} * 1.5 * 10^{-3} = 1430$ AT

AT for iron ring:

H for iron = $\frac{B}{\mu_o \mu_r} = \frac{1.2}{4\pi*10^{-7}*800}$

Length = $\pi * 0.25$ m

AT for iron portion, = $\frac{1.2}{4\pi*10^{-7}*800} * \pi * 0.25 = 937$ AT

Total AT required = 1430 + 937 = 2367 AT

Current required = $\frac{2367}{1200} = 1.975$ A.

Example 4: in the above problem, if the magnetic leakage is not negligible and leakage factor is 1.2, calculate the current required in the coil.

AT for air gap will remain the same i.e., 1430 AT

AT for iron:

Flux in the iron portion = B * A

Area $= 4 * 10^{-4} \, m^2$

Flux density = 1.44 wb/ m^2

Corresponding H for iron $= \frac{1.44}{4\pi*10^{-7}*800}$

AT for iron $= \frac{1.44}{4\pi*10^{-7}*800} * \pi * 0.25 = 1125$ AT

Total AT = 2555 AT

Current = 2555 / 1430 = 2.13 A