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UNIT-III
CHAPTER-VI
PRINCIPLES OF QUANTUM MECHANICS

INTRODUCTION:-

Classical Mechanics is a branch of physics, which deals with the motion of the objects, which are directly observable or with the help of the instruments like microscope. These objects are known as macroscopic particles.

Quantum Mechanics is a branch of physics, which deals with the motion of the objects, which are not observable or even with the help of the instruments like microscope. These objects are known as Microscopic particles.

Classical Mechanics failed to explain the following

i) Stability of atom
ii) Black body of radiation
iii) Spectrum of Hydrogen
iv) Photo Electric Effect
v) Compton Effect
vi) Specific heat of Solids…..etc

In 1904 to explain all these, the quantum Mechanics was introduced.

According to Classical theory, the energy of radiation takes place continuously. But according to Planck's idea the energy take s place only discontinuously and discreetly i.e, energy releases in the form of energy Packets. Each Packet is called "Quanta".

WAVES AND PARTICLES:-

A wave is spread out over a relatively large region of space and it cannot be said to located just here and there. Actually a wave is nothing but rather a spread out disturbance. A wave is specified by its Frequency, Wavelength, Phase, amplitude or Intensity etc..

According to Classical mechanics the radiation behave as Waves in experiments Interference, Diffraction etc…

According to Quantum mechanics the Radiations behave as Particles in experiments Photo Electric Effect, Compton Effect.

DE-BROGLIE HYPOTHESIS:-

The energy exhibits wave particle duality. i.e., sometimes behave as a wave and at some time as a particle.

According to De-Broglie Electromagnetic waves behave like particles and particles like Electrons will behave like Waves Matter Waves.

The wavelength of the Matter Wave is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad \text{----- (1)}$$

Where 'm' is Mass of the particle and 'v' is Velocity and 'p' is Momentum.

DE-BROGLIE WAVELENGTH FOR DIFFERENT PARTICLES:-

According to Planck's theory of radiation the energy of radiation is given by

$$E = h\nu = \frac{h}{\lambda} \qquad [\nu = \frac{c}{\lambda}]$$

Where 'c' is the Velocity of light and 'λ' is the Wavelength/

But According to Einstein's Energy-Mass relation $E = mc^2$

$$\text{So } \frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc} \quad \text{------ (2)}$$

If we consider the case of material particle of mass 'm' and moving with a velocity 'v' then the wavelength of that particle is given as follows.

According to De-Broglie

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad \text{----- (A)}$$

The Kinetic energy of that particle is $E = \frac{1}{2}mv^2$

Multiply and divide with Mass 'm'

$$E = \frac{1}{2}\frac{m^2v^2}{m}$$

$$E = \frac{p^2}{2m} \qquad [\because \text{p=mv}]$$

$$p = \sqrt{2mE}$$

Substitute ‘p’ value in equation (A) we gets

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \text{-----} \quad (B)$$

If a charged particle having a charge ‘q’ is accelerates by a Potential difference ‘V’ volts, then its energy is $E = qv$

Hence, the De-Broglie wavelength associated with this particle is given by

$$\lambda = \frac{h}{\sqrt{2mqv}} \quad \text{------} \quad (4)$$

When a material particle is in Thermal Equilibrium at a temperature T. Then $E = \frac{3}{2}kT$

K is Boltzmann’s constant = $1.38 \times 10^{-23}\, Joule/k$

So, the De-Broglie Wavelength of a material particle at temperature T is given by

Equation 3 $\Rightarrow \lambda = \frac{h}{\sqrt{2m\frac{3}{2}kT}}$

$$\lambda = \frac{h}{\sqrt{3mkT}} \quad \text{-----} \quad (5)$$

DE-BROGLIE WAVELENGTH ASSOCIATED WITH ELECTRONS:-

Let us consider the case of electron of Mass ‘m’ and Charge ‘e’ which is accelerated by a potential ‘V’ volts from rest to Velocity :v’ then

Kinetic Energy $= \frac{1}{2}mv^2$

Energy of Electron = eV

$$\frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2ev}{m}}$$

But $\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2ev}{m}}}$

$$\lambda = \frac{h}{\sqrt{2mev}}$$

Where $h = 6.625 \times 10^{-34}\ joule - \sec$

$e = 1.6 \times 10^{-19} c$

$m = 9.1 \times 10^{-31} kg$

By substituting all the values in above equation we get $\lambda = \frac{12.26}{\sqrt{V}} A^0$

If V=100 Volts then $\lambda = 1.226 A^0$

This shows that the Wavelengths associated with an electron accelerated to 100 Volts is $1.226 A^0$.

PROPERTIES OF MATTERWAVES:-

1) Lighter is the Particle, greater is the Wavelength associated with it i.e. $\lambda \propto \frac{1}{m}$
2) Smaller is the Velocity of the particle, greater is the wavelength associated with it i.e., $\lambda \propto \frac{1}{v}$
3) When $v = 0$ then $\lambda = \infty$, i.e., waves becomes in determine and if $v = \infty$ then $\lambda = 0$
4) The waves are produced whether the particles are charged or uncharged, but electromagnetic waves are produced only by the motion of charged particles. So, in this case new kind of matter waves are produced, These waves are called Matter Waves.
5) The velocity of matter Waves always greater than velocity of light.

Proof:-

Kinetic Energy of particle is $= \frac{1}{2} mv^2$

According to Einstein relation = mc^2

$$\frac{1}{2} mv^2 = mc^2$$

$$v^2 = 2c^2$$

$$v = \sqrt{2} c$$

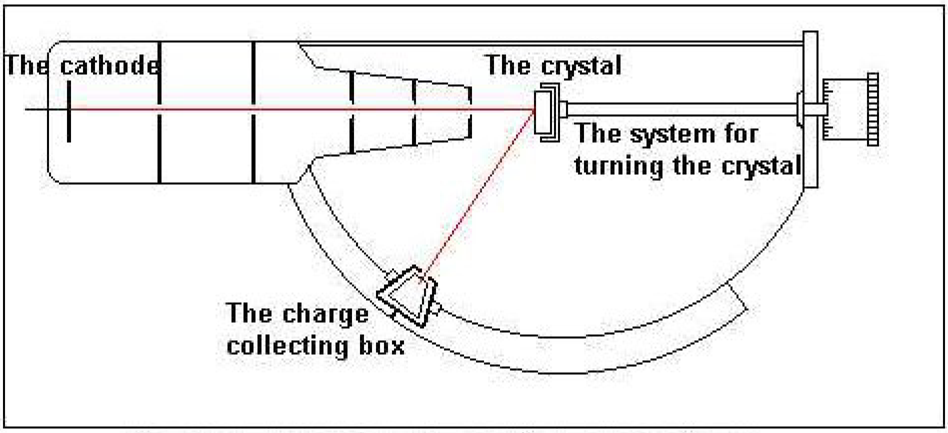
So $v > c$

EXPERIMENTAL STUDY OF MATTER WAVES:-

Several years after Debroglie's work, Davision and Germer and G.P Thomson independently demonstrated that streams of electrons are diffracted when they are scattered from crystals.

Davision and Germer's Experiment:-

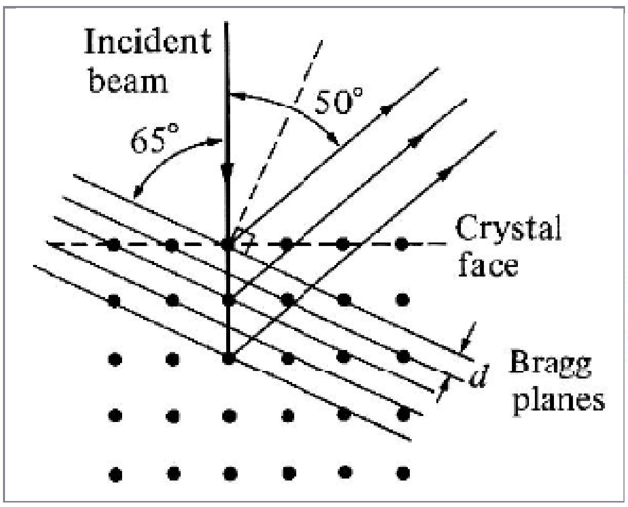
Principle:- Based on the concept of wave nature of matter fast moving electrons behave like waves. Hence accelerated electron beam can be used for diffraction studies in crystals.

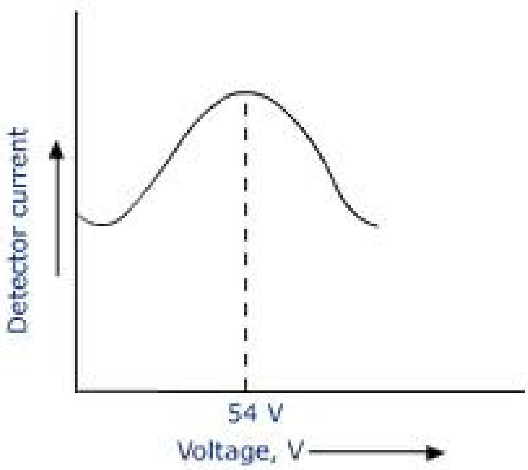


The diagram for the Davisson-Germer experiment.

An electron gun which consists of a tungsten filament F heated by a low battery B1, produces electrons. These electrons are accelerated to a desired velocity by applying suitable potential from high tension source B2. The accelerated electrons are collimated into a fine beam by allowing them to pass through a system of pin holes provided in the cylinder 'C'.

The fast moving electrons is made to strike the target (Ni crystal) capable of rotating about an axis perpendicular to the plane of the diagram i.e. incident ray direction. The electrons are now scattered in all directions by the atomic planes of the crystal. The intensity of the electron beam scattered in direction can be measured by the electron collector which can be rotated about the same axis as the target. The collector is connected to a sensitive Galvanometer whose deflection is proportional to the intensity of the electron beam entering the collector. The instrument is kept in an evacuated chamber.





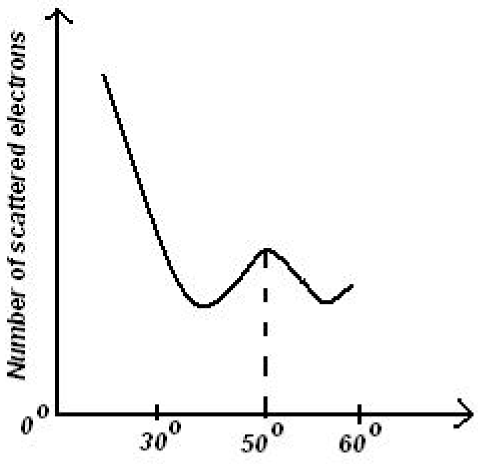


Figure (2) : Angle of Direction of Scattering in Degrees

In an investigation, the electron beam accelerated by 54V was directed to strike the given Nickel crystal are a sharp maximum electron distribution occurred at a angle of 50^0 with the incident beam. The incident and diffracted beam in the experiment make an angle of 65^0 with the Bragg's planes. The spacing of planes in this Bragg's planes by X-Ray diffraction is 0.91 nm.

New according to Bragg's law $2d\sin\theta = n\lambda$

$$2\times0.91\times10^{-10}\times\sin 65^0 = \lambda\times1 \quad (\because n=1)$$

$$\lambda = 1.64\times10^{-1}\times10^{-9}m$$

$$\lambda = \frac{1.64}{10}nm$$

$$\lambda = 0.164nm$$

For 54V electron the de Broglie wavelength associated with the electron is given by

$$\lambda = \frac{12.25}{\sqrt{54}}A^0$$

$$\lambda = 1.66\times10^{-10}m$$

$$\lambda = 0.166nm$$

This is excellent agreement with experimental value. The Davission-Germer experiment provides a verification of de-Broglie hypothesis of the nature of moving particle.

G.P THOMSON EXPERIMENT:-

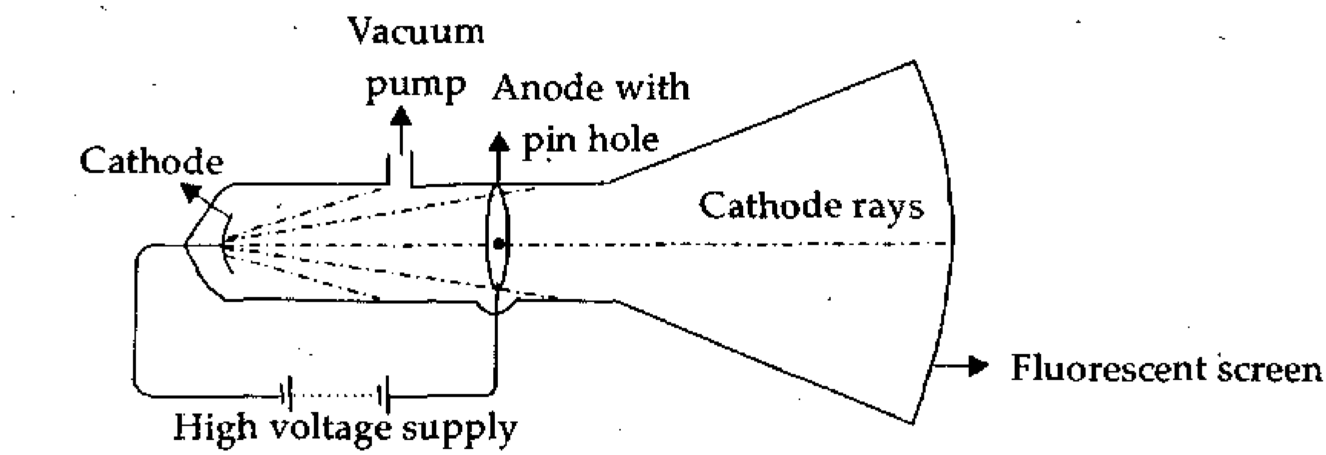
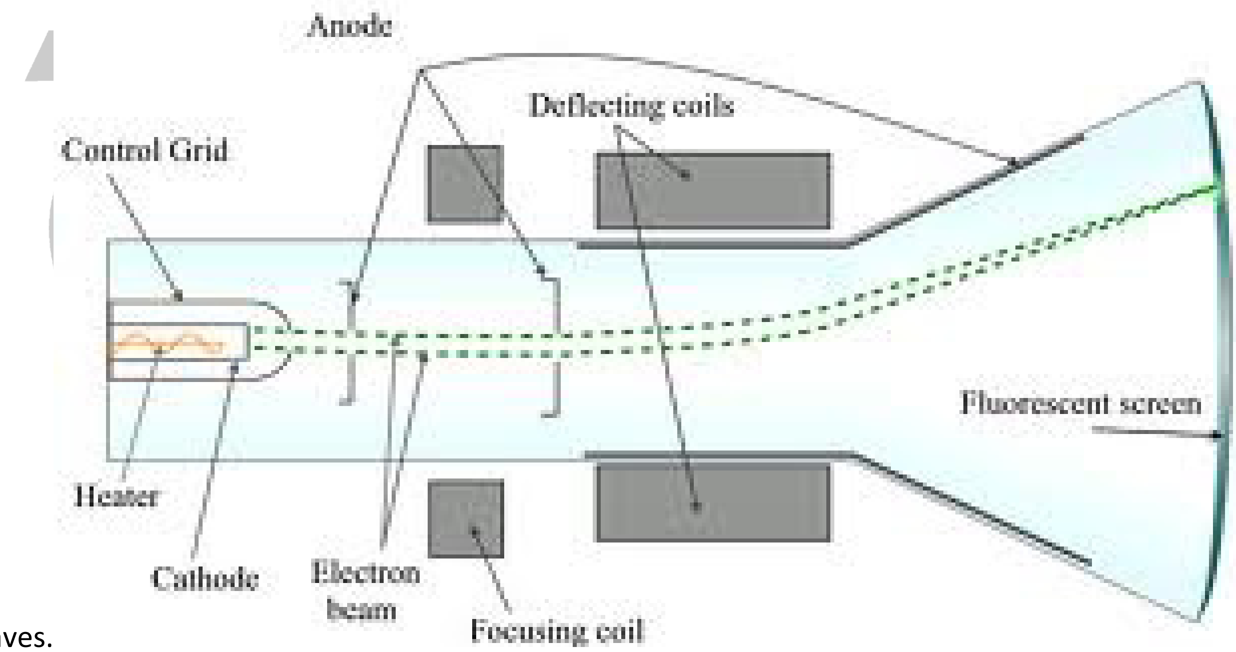


Fig. 1. The discharge tube and the cathode rays

G.P Thomson performed experiments in which electrons are accelerated from 10,000 to 50,000 volts. In these experiments the generation of electrons are considered analogous to X-Ray obtained by diffraction pattern. The diffraction pattern is obtained by only when wave is associated with particle. Hence Thomson explains the concept of matter



waves.

The electrons are emitted from the filament and only some accelerated electrons are passing through cathode 'C'. Next these electrons are passed through two slits S1 and S2 and a thin pencil beam of electrons is obtained. This electrons beam allowed to fall on a thin foil 'G' of gold or Aluminium of order $10^{-6} cm$. The photograph of electron beam from the foil is recorded on the photographic plate 'P'. Hence a pattern consists of concentric rings. The complete apparatus is kept in high vacuum chamber so that the electrons may not lose their energy y colliding with molecules of air or any inside the tube.

To conclude that, this pattern is due to the electrons and not due the X-Rays. The cathode rays inside the tube are affected by the magnetic fields. The beam shifting considerably along the field is observed. Hence we can conclude that the pattern obtained is due to electrons only since x-Rays are not affected by electric and magnetic fields.

HEISENBERG'S UNCERTAINITY PRINCIPLE:-

According to Heisenberg's it is impossible to measure position and momentum of a particle simultaneously and accurately.

If two physical variable of a particle are considered as measurable quantities the uncertainties or errors will be exerting in which the product of two uncertainties will be greater than or equal to order of $\frac{\lambda}{4\Pi}$.

Where 'h' is planks' constant.

If we consider uncertainties with respect to position and momentum then

$$\Delta P.\Delta X \geq \frac{\lambda}{4\Pi}$$

Where ΔP is the uncertainty or error of momentum of the particle.

ΔX is the uncertainty or error of position of the particle.

$$\Delta E.\Delta t \geq \frac{h}{4\pi}$$

Where ΔE is the uncertainty or error with respect to energy of the particle.

Δt is the uncertainty or error with respect to time of the particle.

$$\Delta J.\Delta \theta \geq \frac{h}{4\pi}$$

Where ΔJ is the uncertainty or error with respect to angular momentum of the particle.

$\Delta \theta$ is the uncertainty or error with respect to angle of the particle.

Equation of motion of Matter Waves:-

According to de-broglie theory, a material particle associated with a wave . So a mathematical reformation using a wave function associated with matter waves needed such a mathematical formation known as wave mechanics or quantum mechanics was developed in 1926 by Schrodinger. Schrodinger described the amplitude of matter waves by a complex quantity $\psi(x, y, z, t)$ known as wave function or state of the system. It describes the particular dynamical system under observation.

Schredinger time independent wave equation:-

According to de-Broglie theory, a particle of mass 'm' is always associated with a whose wavelength is given by $\lambda = \frac{h}{mV}$

If the particle has wave properties, it is expected that there should be some sort of wave equation which describes the behaviour of particle. Consider a system of stationary waves associated with a particle. Let x,y,z be the coordinates of the particle and Ψ, the displacements for the de-Broglie at any time 't'. Ψ is called as wave function. It is assumed that Ψ is finite.

The classical differential equation of a wave motion is given by,

$$\frac{\partial^2\Psi}{\partial t^2} = V^2 \left[\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2} \right] = V^2 \nabla^2\Psi \rightarrow (1)$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ [∵ ∇^2= Laplacian Operator]

And 'V' is velocity of wave.

The solution of equation (1) is,

$$\Psi = \Psi_0 \sin \omega t = \sin(2\pi v)t \rightarrow (2)$$

Where 'v' is the frequency of stationary wave associated with the particle.

Differentiating equation (2) we get

$$\frac{\partial\Psi}{\partial t} = \Psi_0 (2\pi v) \cos 2\pi v t$$

$$\frac{\partial^2\Psi}{\partial t^2} = - \Psi_0 (2\pi v)^2 \sin 2\pi v t$$

But $\Psi_0 \sin(2\pi v)t$ = Ψ, so

$$\frac{\partial^2\Psi}{\partial t^2} = -4\pi^2 v^2 \Psi. \qquad \lambda^2$$

So, $$\frac{\partial^2\Psi}{\partial t^2} = -4\pi^2 \frac{v^2}{\lambda^2} \Psi \rightarrow (3)$$

Substitute equation (3) in equation (1) we get.

$$V^2 \nabla^2\Psi = -4\pi^2 \frac{v^2}{\lambda^2} \Psi$$

$$\nabla^2\Psi + \frac{4\pi^2}{\lambda^2} \Psi = 0 \rightarrow (4)$$

Now from de-Broglie relation,

$$\lambda = \frac{h}{mV}$$

$$\nabla^2\Psi + \frac{4\pi^2}{\lambda^2} m^2 v^2 \Psi = 0 \rightarrow (5)$$

If E and v be the total energy and potential energies of the particles respectively, then its kinetic energy ½ mv 2 is given by

Total energy = P.E+K.E

$\frac{1}{2} mV^2 = E - V$

$m^2 V^2 = 2m(E - V) \rightarrow (6)$

∴ substitute equation (6) in equation (5) we get

$$\nabla^2\Psi + \frac{4\pi^2}{h^2} 2m (E - V)\Psi = 0$$

$$\nabla^2\Psi + \frac{8\pi^2 m}{h^2} (E - V)\Psi = 0 \rightarrow (7)$$

Equation (7) is known as Schrodinger time independent wave equation

Substitute $\hbar = \frac{h}{2\pi}$ in equation (7) we get

$$(7) \Rightarrow \quad \nabla^2\Psi + \frac{2m}{(\frac{h}{2\pi})^2} [E-V]\Psi = 0$$

$$\nabla^2\Psi + \frac{2m}{\hbar^2} [E-V]\Psi = 0 \rightarrow (8)$$

For free particle v=0 hence Schrodinger wave equation for free particle is

$$\nabla^2\Psi + \frac{2mE}{\hbar^2} [E-V]\Psi = 0 \rightarrow (9)$$

Schrodinger time dependent wave equation :-

The Schrodinger time dependent wave equation may be obtained from Schrodinger time dependent wave equation by eliminating E.

If order to derive the time dependent wave equation, Schrodinger introduced a mathematical function Ψ which is a variable quantity associated with a moving particle. This is a complex of space considering of the particle and time. The function Ψ is called as wave function as it characterises de-Broglie waves associated with particle.

The differential equation representing a one dimensional wave motion is,

$$\frac{\partial^2 \Psi}{\partial t^2} = V^2 \nabla^2\Psi \rightarrow (1)$$

Consider Ψ to be a complex function of a space coordinates of the particle and time, the general solution of equation (1) is given by,

Ψ_1 (x,y,.z) = Ψ_0 (x,y,z) e^{-iwt}

$$\Psi = \Psi_0 .\ e^{-iwt} \rightarrow (2)$$

Differentiating equation (2) w.r.to time ‘t’ we get

$$\frac{\partial\Psi}{\partial t} = \Psi_0.(-iw)\ e^{-iwt}$$

$$= \Psi_0.(-i.\ 2\pi v)\ e^{-iwt}$$

But $\Psi_0 .\ e^{-iwt} = \Psi$, so

$$\frac{\partial\Psi}{\partial t} = -2\pi i v\Psi$$

$$= -2\pi i(\tfrac{E}{h})\Psi$$

E= h/ v , v = E/h

$$= -i\frac{E}{(\frac{h}{2\pi})}\Psi$$

$\because \hbar = \frac{h}{2\pi}$

$$\frac{\partial\Psi}{\partial t} = \frac{-i}{\hbar}.E\Psi$$

$$E\Psi = \frac{\hbar}{-i}\frac{\partial\Psi}{\partial t}$$

$$E\Psi = i^2\frac{\hbar}{i}\frac{\partial\Psi}{\partial t} \qquad [\because i^2 = -1]$$

$$E\Psi = i.\hbar\frac{\partial\Psi}{\partial t} \dashrightarrow (3)$$

Substitute this value in Schrodinger time independent equation we get ,

First Schrodinger time independent equation is,

(8) => $\nabla^2\Psi + \frac{2m}{\hbar^2}[E-V]\Psi = 0$

$$\nabla^2\Psi + \frac{2m}{\hbar^2}[E\Psi - V\Psi] = 0$$

$$\nabla^2\Psi + \frac{2m}{\hbar^2}[i.\hbar \frac{\partial\Psi}{\partial t} - V\Psi] = 0$$

$$\nabla^2\Psi - \frac{2m}{\hbar^2}[i.\hbar \frac{\partial\Psi}{\partial t} - V\Psi] = 0$$

$$\frac{-\hbar^2}{2m}\nabla^2\Psi + V\Psi = i.\hbar \frac{\partial\Psi}{\partial t} \rightarrow (4)$$

This is known as Schrodinger time dependent equation,

$$[\frac{-\hbar^2}{2m}\nabla^2 + V]\Psi = i.\hbar \frac{\partial}{\partial t}\Psi$$

$$\Psi \hat{H} = \hat{E}\,\Psi \rightarrow (5)$$

Where $\hat{H}$ is Hamiltonian operator.

$$\hat{H} = [\frac{-\hbar^2}{2m}\nabla^2 + V]$$

And $\hat{E}$ IS Energy operator,

$$\hat{E} = i.\hbar \frac{\partial}{\partial t}.$$

Physical Significance of wave function 'Ψ' :-

→ The wave function Ψ measures the variations of the matter wave. These ,it converts the particle and its associated wave. It is the complex amplitude of the matter wave.

→ The wave function complex displacement Ψ is a complex quantity and we cannot measure it.

→ The wave function is used to identify the state of a particle in an atomic structure.

→ If can tell the probability of the of the particle at a time, but cannot predict the exact location of the particle at that time. These it lets us where the particle is likely to be not where it is.

→ We can say that the wave equation as probability amplitude since it is used to find the location of the particle.

→ $\int |\psi|^2 d\tau = 1$ where the particles presence is certain in the space.

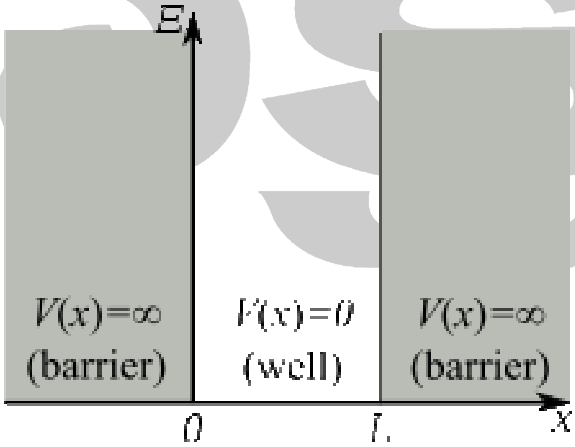
Application of Schrodinger wave equation:-

Particle in a one dimensional box :-

Consider an electron which is placed in an one dimension infinity by potential box width 'a'. We assume that the momentum of the electron is restricted by the sides of the wave and the electron is moving only in the x-direction. When it collides with the walls. There is no loss of energy of the electron and so the collisions are perfectly elastic.

Since the electron is moving freely inside the box its potential energy V of the electron is infinity high on the both and outside the box also. Due to that the electron cannot escape from the box through the sides.

Boundary Condition :-



1. Since the potential outside the box is infinity high, the probability of finding the particle outside must be zero.

i.e., $|\psi|^2 = 0 \quad 0 > x > a$

Therefore Ψ=0 at x=0 and x=a.

2. Inside the box the wave function is finite

i.e., $|\psi|^2 \neq 0 \quad 0 > x > a$

The one dimensional Schrodinger wave equation is given by

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}[E-V]\Psi = 0.$$

Here V=0 and E is simply equal to kinetic energy of electron.

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{2m}{\hbar^2} E\Psi = 0$$

$$\frac{\partial^2 \Psi}{\partial x^2} + k^2\Psi = 0$$

Where $k^2 = \frac{2mE}{\hbar^2} = \frac{2m}{\hbar^2} \frac{P^2}{2m} = \frac{4\pi^2}{\lambda^2}$

$\left[\because \lambda = h/p \Rightarrow p = h/\lambda \text{ and } \hbar = \frac{h}{2\pi}, \frac{h^2}{\lambda^2} \times \frac{4\pi^2}{h^2} = \frac{4\pi^2}{\lambda^2} \right]$

i.e., is called wave vector or wave number, $|k| = 2\pi/\lambda$

The above equation is similar to the equation of harmonic motion and so the solution can be written as

Ψ = A sin kx + B cos kx → (1)

To evaluate the constants A and B we must apply the boundary conditions namely

Ψ = 0 at x = 0 and x = a.

When x=0

Ψ = 0 = A sin (0) + B cos (0)

∴ B = 0.

When x = a

Ψ = 0 = A sin ka

Ka = nπ

$$K = \frac{n\Pi}{a}$$

But $k^2 = \frac{2mE}{\hbar^2}$

$$\frac{2mE}{\hbar^2} = \frac{n^2\Pi^2}{a^2}$$

$$E = \frac{n^2\Pi^2\hbar^2}{2ma^2} = \frac{n^2h^2}{8ma^2} \rightarrow (2)$$

$$\Psi_n = A \sin K = \frac{n\Pi x}{a} \rightarrow (3)$$

Let us find the value of A ' Ψ_n'

$$\int_0^a |\Psi_n|^2\, dx = 1$$

Since the electron should exist within the box substituting the value of ' Ψ_n' we get

$$\int_0^a A^2 \sin^2\left(\frac{n\Pi x}{a}\right) dx = 1$$

$$A^2 \int_0^a \left[\frac{1-\cos\left(\frac{2\Pi n x}{a}\right)}{2}\right] dx = 1$$

$$\frac{A^2}{2}\left[X - \frac{a}{2\Pi n} \sin\left(\frac{2\Pi n x}{a}\right)\right]_0^a = 1$$

$$\frac{A^2 a}{2} = 1$$

$$A = \sqrt{\frac{2}{a}}$$

$$\Psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\Pi}{a}\right)x \rightarrow (4)$$

Equation (2) the energy values of the electrons are so, the energy will be in any one of the above states Ψ_n at a given time. The lowest energy state is called the ground state.

Lowest energy level $E_1 = \frac{h^2}{8ma^2}$

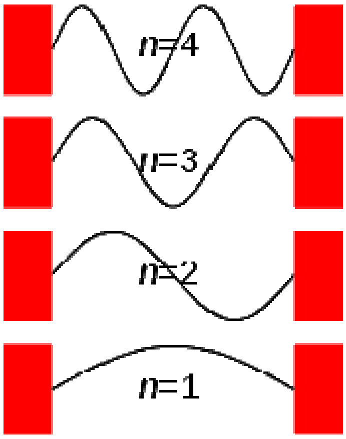
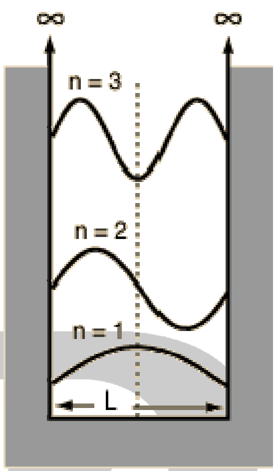


Fig: Initial wave functions for the first four states in a one-dimensional particle in a box



Results:-

1. The energy is quantised and so it cannot vary continuously.
2. For, the same value of quantum no 'n', the energy is increasely proportional to the mass of electron and to the square of the width of the well.

The energies of an electron confined in a well 1 A^0 width are,

$$E_n = \frac{n^2 \times (6.63\times10^{-34})^2}{8\times9.1\times10^{-31}\times10^{-20}} \text{ Joules.}$$

$= 38\, n^2$ eV.

$\therefore E_1 = 38$ eV , $E_2 = 152$ eV , $E_2 = 342$ eV.

The probability of finding the electron in the first energy level is maximum at the centres of the well. But in second energy level it is zero at the centre of the well. Thus in each energy level, the location of finding the electron is different.

FILL IN THE BLANKS:

1. If an electron is confined to one dimensional potential box of length L, the allowed energy values are given by E_n= __________

   Ans: $n^2 \frac{\pi^2 h^2}{2mL^2}$

2. According to de Broglie's hypothesis, a moving particle has _____________ properties associated with it.

   Ans: wave

3. The wave length λ associated with any moving material particle of momentum p is ______

   Ans: $\frac{h}{p}$

4. The waves associated with material particles are called matter waves or ________

   Ans: de Broglie waves

5. ______ possesses dual nature.

   Ans: Light

6. The state of an electron is denoted by the set of four quantum members ______

   Ans: $n, l, m_{l,} m_s$

7. The total number of available electron states per unit volume called energy density is _____

   Ans: $n = \frac{N}{V} = \frac{\pi}{3}\left[\frac{8n}{h^2}\right]^{\frac{3}{2}}$

8. ______ theory tell us how to obtain the wave function associated with a particle.

   Ans: Schrodinger's

9. Laws of classical physics can not explain the motion of _____ particles.

   Ans: Micro

10. $\lambda = \frac{h}{p}$ is called ______ wave length.

    Ans: de Broglie

11. $\Delta p_{x.} \Delta x \geq$ ______

    Ans: h

12. In German eigen means ______

    Ans: Proper

13. The potential experienced by an electron in passing through a crystal is taken as perfectly ______ with the period of lattice.

    Ans: Periodic

14. The wave function for the motion of the particle in a one dimensional potential box of lengths a is given by of $\psi_n = A\sin\left(\frac{n\pi x}{a}\right)$. Where A is the normalization constant. The value of A is ______

    Ans: $\sqrt{\frac{2}{a}}$

15. The energy of lowest energy state in a one dimensional potential box of length a is ______

    Ans: $\frac{h^2}{8ma^2}$

16. The spacing between the n^{th} energy level and the next higher level in a one dimensional potential box increases by ______

    Ans: $(2n+1)$

17. Energy E of the photon of wave length λ is ______

    Ans: $\frac{hc}{\lambda}$

18. The equation that relates particle and wave aspects of matter _______

    Ans: $\lambda = \frac{h}{p}$

19. Heisenberg uncertainty relation holds good for ______

    Ans: De-Brogliewave

20. The expression $|\psi(x,t)|^2$ stands for _____

    Ans: Normalisation

21. What is Heisenberg uncertainty principle ______

    Ans: $\Delta x \Delta p \geq \frac{h}{4\pi}$