# MATHEMATICS-I

# DIFFERENTIAL EQUATIONS-I

I YEAR B.TECH



**By**

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UNIT-5

## Differential Equations-I

# DIFFERENTIAL EQUATIONS

Differentiation: The rate of change of a variable w.r.t the other variable is called as a Differentiation.

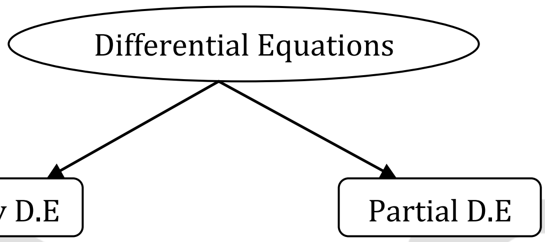
In this case, changing variable is called Dependent variable and other variable is called as an Independent variable.

***Example:*** $\frac{dy}{dx}$ is a Differentiation, Here $y$ is dependent variable and $x$ is Independent variable.

DIFFERENTIAL EQUATION: An equation which contains differential coefficients is called as a D.E.

***Examples:*** 1) $\frac{dy}{dx} + 1 = 0$ 2) $\frac{\partial^2 y}{\partial x^2} + \frac{\partial y}{\partial z} + 1 = 0$.

Differential Equations are separated into two types




Ordinary D.E: In a D.E if there exists single Independent variable, it is called as Ordinary D.E

***Example:*** 1) $\frac{dy}{dx} + 2y = 0$ is a Ordinary D.E 2) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + 1 = 0$ is a Ordinary D.E

Partial D.E: In a D.E if there exists more than one Independent variables then it is called as Partial D.E

***Example***: 1) $\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial z^2} + 1 = 0$ is a Partial D.E, since $x, t$ are two Independent variables.

2) $\frac{\partial^2 y}{\partial x\, \partial z} + 1 = 0$ is a Partial D.E, since $x, z$ are two Independent variables

ORDER OF D.E: The highest derivative in the D.E is called as Order of the D.E

***Example:*** 1) Order of $\frac{d^2y}{dx^2} + 2y = 0$ is one.

2) Order of $\frac{d^5y}{dx^5} + \left[\frac{d^3y}{dx^3}\right]^8 + 3y = 0$ is Five.

DEGREE OF D.E: The Integral power of highest derivative in the D.E is called as degree of the D.E

***Example:*** 1) The degree of $\left[\frac{d^2y}{dx^2}\right]^1 + 2\frac{dy}{dx} + 1 = 0$ is One.

2) The degree of $x\left[\frac{d^2y}{dx^2}\right]^8 + \left[\frac{dy}{dx}\right]^{11} + \left[\frac{d^3y}{dx^3}\right]^2 = 0$ is Two.

**NOTE:** Degree of the D.E does not exist when the Differential Co-efficient Involving with exponential functions, logarithmic functions, and Trigonometric functions.

***Example:*** 1) There is no degree for the D.E $e^{\frac{dy}{dx}} + 1 = 0$

2) There is no degree for the D.E $log\left(\frac{d^2y}{dx^2}\right) + 1 = 0$

3) There is no degree for the D.E $sin\left(\frac{d^3y}{dx^3}\right) + 1 = 0$ .

**NOTE:** 1) The degree of the D.E is always a +ve Integer, but it never be a negative (or) zero (or) fraction.

2) Dependent variable should not include fraction powers. It should be perfectly Linear.

Ex: For the D.E $\frac{d^2y}{dx^2} + \sqrt{y} = 0$ Degree does not exist.

## FORMATION OF DIFFERNTIAL EQUATION

A D.E can be formed by eliminating arbitrary constants from the given D.E by using Differentiation Concept. If the given equation contains 'n' arbitrary constants then differentiating it 'n' times successively and eliminating 'n' arbitrary constants we get the corresponding D.E.

**NOTE:** If the given D.E contains 'n' arbitrary constants , then the order of its corresponding D.E is 'n' .

**NOTE:** For $y = c_1e^{\alpha_1 x} + c_2e^{-\alpha_2 x}$ then the corresponding D.E is given by $(D - \alpha_1)(D - \alpha_2)y = 0$

In general, if $y = c_1e^{\alpha_1 x} + c_2e^{\alpha_2 x} + - - - + c_ne^{\alpha_n x}$ then its D.E is given by

$$(D - \alpha_1)(D - \alpha_2) - - - (D - \alpha_n)y = 0, \text{ where } D = \frac{d}{dx}$$

**Special Cases:** If $y = (c_1 + c_2x)e^{\alpha x}$ then D.E is given by $(D - \alpha)^2y = 0$ , where $D = \frac{d}{dx}$.

In general, if $y = (c_1 + c_2x + c_3x^2 + - - - + c_kx^{k-1})e^{\alpha x}$ then the corresponding D.E is given by

$$(D - \alpha)^k y = 0.$$

**NOTE:** For $y = e^{\alpha x}(c_1 cos\beta x + c_2 sin\beta x)$ then D.E is $[D^2 - 2\alpha D + (\alpha^2 + \beta^2)]y = 0.$

**WRANSKIAN METHOD**

Let $y = Ax + Bx^2$ be the given equation then its corresponding D.E is given by

$$\begin{vmatrix} y & x & x^2 \\ y^1 & 1 & 2x \\ y^2 & 0 & 2 \end{vmatrix} = 0$$

This method is applicable when there are two arbitrary constants only.

## DIFFERENTIAL EQUATIONS OF FIRST ORDER AND FIRST DEGREE

A D.E of the form $\frac{dy}{dx} = f(x, y)$ is called as a First Order and First Degree D.E in terms of dependent variable $y$ and independent variable $x$.

In order to solve above type of Equation's, following methods exists.

1) Variable Separable Method.
2) Homogeneous D.E and Equations reducible to Homogeneous.
3) Exact D.E and Equations made to exact.
4) Linear D.E and Bernoulli's Equations.

## Method-I: VARIABLE SEPERABLE METHOD

***First Form***: Let us consider given D.E $\frac{dy}{dx} = f(x, y)$

If $f(x,y) = \frac{f(x,y)}{g(x,y)}$ then proceed as follows

$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)} \Rightarrow g(y)dy = f(x)dx$

$\Rightarrow \int g(y)dy = \int f(x)dx + C$ is the required general solution.

***Second Form:*** If $\frac{dy}{dx} = f(ax + by + c)$ then proceed as follows

Let $ax + by = c = z \Rightarrow \frac{dy}{dx} = \frac{1}{b}\left[\frac{dz}{dx} - a\right]$

$\Rightarrow \frac{dz}{dx} = bf(z) + a$

By using variable separable method we can find its general solution.

Let it be $\emptyset(z, x, c_1) = 0$ . But $z = ax + by + c$

$\Rightarrow \emptyset(ax + by + c, x, c_1) = 0$ .

## Method-2: HOMOGENEOUS DIFFERENTIAL EQUATION METHOD

Homogeneous Function: A Function $f(x, y)$ is said to be homogeneous function of degree 'n' if

$$f(kx, ky) = k^n f(x, y)$$

***Example:*** 1) If $f(x,y) = \frac{x^3+y^3}{x^3+y^3}$ is a homogeneous function of degree '0'

2) If $f(x,y) = \frac{xy+y}{x^2+y^2}$ is not a homogeneous function.

Homogeneous D.E: A D.E of the form $\frac{dy}{dx} = f(x, y)$ is said to be Homogeneous D.E of first order and first degree in terms of dependent variable 'y' and independent variable 'x' if $f(x, y)$ is a homogeneous function of degree '0'.

**Ex:** 1) $\frac{dy}{dx} = \frac{x+y}{x-y}$ is a homogeneous D.E

2) $\frac{dy}{dx} = \frac{x^2+y^2}{x^2-y^2}$ is a homogeneous D.E

3) $\frac{dy}{dx} = \frac{xy+y^2}{x^2+y^2}$ is a homogeneous D.E

4) $\frac{dy}{dx} = \frac{xy+y}{x^2+y^2}$ is not a homogeneous D.E

5) $\frac{dy}{dx} = \frac{x^3+y^3}{x+y}$ is not a homogeneous D.E

**Working Rule:** Let us consider given homogeneous D.E $\frac{dy}{dx} = f(x, y)$

Substituting $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ we get

$\Rightarrow v + x\frac{dv}{dx} = f(x, vx)$

$\Rightarrow x\frac{dv}{dx} = f(x, vx) - v$

By using variable separable method we can find the General solution of it

Let it be $\emptyset(v, x, c) = 0$ . But $v = \frac{y}{x}$

$\emptyset\left(\frac{y}{x}, x, c\right) = 0$ be the required general solution.

## NON-HOMOGENEOUS DIFFERENTIAL EQUATION

A D.E of the form $\frac{dy}{dx} = \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$ is called as a Non-Homogeneous D.E in terms of independent variable $x$ and dependent variable $y$, where $a_1, b_1, c_1, and\ a_2, b_2, c_2$ are real constants.

Case (I): If $a_1b_2 - a_2b_1 \neq 0$ then procedure is as follows

Let us choose constants h & k in such a way that $\left.\begin{aligned} a_1h + b_1k + c_1 = 0 \\ a_2h + b_2k + c_2 = 0 \end{aligned}\right\} \Longrightarrow$ **I**

Let $x = X + h$ , $y = Y + k$ and also $\frac{dy}{dx} = \frac{dY}{dX}$ then above relation becomes

$$\frac{dY}{dX} = \frac{(a_1X+b_1Y)+(a_1h+b_1k+c_1)}{(a_2X+b_2Y)+(a_2h+b_2k+c_2)}$$

$$\Longrightarrow \frac{dY}{dX} = \frac{a_1X+b_2Y}{a_2X+b_2Y} \qquad \text{(From I)}$$

Which is a Homogeneous D.E of first order and of first degree in terms of $X$ and $Y$.

By using Homogeneous method, we can find the General solution of it. Let it be $\emptyset(Y, X, C) = 0$ .

But $x = X + h$ , $y = Y + k$

$\Longrightarrow \emptyset(y-k, x-h, c) = 0$ is the required General Solution of the given equation.

Case (II): If $a_1b_2 - a_2b_1 = 0$ , then By Using Second form of Variable Seperable method we can find the General Solution of the given equation.

## Method-3: EXACT DIFFERENTIAL EQUATION

A D.E of the form $Mdx + Ndy = 0$ is said to be exact D.E if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Its general solution is given by $\int^x Mdx + \int(terms\ independent\ of\ \ in\ N)dy = C$

$$(OR)$$

$$\int^x Mdx + \int(free\ from\ x\ terms\ in\ N)dy = C$$

$$(OR)$$

$$\int^x M\,dx + \int(terms\ not\ containing\ x\ in\ N)\,dy = C$$

## NON EXACT DIFFERENTIAL EQUATION

A D.E of the form $Mdx + Ndy = 0$ is said to be Non-Exact D.E if $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

In order to make above D.E to be Exact we have to multiply with $\mu(x, y) \neq 0$ which is known as an

Integrating Factor.

To Solve such a type of problems, we have following methods.

1) Inspection Method
2) Method to find Integrating Factor I.F $\frac{1}{Mx+Ny}$
3) Method to find Integrating Factor I.F $\frac{1}{Mx-Ny}$
4) Method to find Integrating Factor I.F $e^{\int f(x)\,dx}$
5) Method to find Integrating Factor I.F $e^{\int g(y)dy}$

## Method 1: INSPECTION METHOD

This Method is used for both exact and non-exact D.E

Some Formulae:

- $d\left(\frac{x^2+y^2}{2}\right) = xdx + ydy$
- $d(xy) = xdy + ydx$
- $d\left(\frac{x}{y}\right) = \frac{ydx-xdy}{y^2}$
- $d\left(\frac{e^y}{x}\right) = \frac{xe^y dy - e^y dx}{x^2}$
- $d(xe^y) = xe^y dy + e^y dx$
- $d\left(log\frac{y}{x}\right) = \frac{xdy-ydx}{xy}$
- $d\left(log\frac{x}{y}\right) = \frac{ydx-xdy}{xy}$
- $d\left(Tan^{-1}\frac{y}{x}\right) = \frac{xdy-ydx}{x^2+y^2}$
- $d\left(Tan^{-1}\frac{x}{y}\right) = \frac{ydx-xdy}{x^2+y^2}$

**Hints while solving the problems using Inspection Method**

- If in a problem $e^{\blacksquare}$ term is there then select another $e^{\blacksquare}$ term.
- Always take $\boldsymbol{ydx}$ combination with $\boldsymbol{xdy}$ .

99% of the problems can be solved using Inspection method

## Method-2: Method to find Integrating Factor $\frac{1}{Mx+Ny}$

If given D.E is $Mdx + Ndy = 0$ is Non-Exact and it is Homogeneous and also $Mx + Ny \neq 0$

Then $\frac{1}{Mx+Ny}$ is the Integrating Factor (I.F).

## Method-3: Method to find Integrating Factor $\frac{1}{Mx-Ny}$

Let the given D.E is $Mdx + Ndy = 0$ is Non-Exact, and if given D.E can be expressed as

$yf(x,y)dx + xg(x,y)dy = 0$ And also $Mx - Ny \neq 0$ then $\frac{1}{Mx-Ny}$ is an I.F

**Note:** Here in $f(x,y), g(x,y)$, there should be only $xy$ combination (with constants also)

i.e. With same powers $x^3y^3$, $x^ny^n$ , $(1 + xy + x^2y^2)$ $etc.$

## Method-4: Method to find the I.F $e^{\int f(x)dx}$

Let $Mdx + Ndy = 0$ be the Non-Exact D.E. If $\frac{\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)}{N} = f(x)$

Where $f(x)$= Function of x-alone or constant then I.F is $e^{\int f(x)dx}$

**NOTE:** In this case number of terms in M is greater than or equal to number of terms in N i.e. M≥N

## Method-5: Method to find the I.F $e^{\int g(y)dy}$

Let $Mdx + Ndy = 0$ be the Non-Exact D.E. If $\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M} = g(y)$

Where $g(y)$=Function of y-alone or constant then I.F is $e^{\int g(y)dy}$

**NOTE:** In this case number of terms in N is greater than or equal to number of items in M i.e. N≥M

## LINEAR DIFFERENTIAL EQUATION

A D.E of the form $\frac{dy}{dx} + P(x)y = Q(x)$ is called as a First order and First degree D.E in terms of dependent variable $y$ and independent variable $x$ where $P(x), Q(x)$ functions of x-alone (or) constant.

**Working Rule:** Given that $\frac{dy}{dx} + P(x)y = Q(x)$ -------- (1)

I.F is given by $e^{\int Pdx}$

Multiplying with this I.F to (1), it becomes $e^{\int Pdx}\left[\frac{dy}{dx} + P(x)y\right] = Qe^{\int Pdx}$

$\Rightarrow d\left[ye^{\int Pdx}\right] = Qe^{\int Pdx}$ Now Integrating both sides we get

$\Rightarrow ye^{\int Pdx} = \int Qe^{\int Pdx}dx + C$ is the required General Solution.

**ANOTHER FORM**

A D.E of the form $\frac{dx}{dy} + P(y)x = Q(y)$ is also called as a Linear D.E where $P(y)$, $Q(y)$ functions of y-alone. Now I.F in this case is given by I.F=$e^{\int Pdy}$ and General Solution is given by

$$xe^{\int Pdy} = \int Qe^{\int Pdy}dy + C$$

## Equations Reducible to Linear Form

An Equation of the form $f^{I}(y)\frac{dy}{dx} + P(x)f(y) = Q(x)$ is called as an Equation Reducible to Linear Form

**Working Rule:**

Given that $f^I(y)\frac{dy}{dx} + P(x)f(y) = Q(x) \longrightarrow (\text{I})$

Let $f(y) = z \implies f^I(y)\frac{dy}{dx} = \frac{dz}{dx}$

$(\text{I}) \implies \frac{dz}{dx} + P(x)z = Q(x)$ which is Linear D.E in terms of $z, x$

**Hint:** First make $\frac{dy}{dx}$ coefficient as 1, and then make R.H.S term purely function of $x$ alone

By using Linear Method we can find its General Solution.

Let it be $\emptyset(z, x, c) = 0$ But $f(y) = z$

$\implies \emptyset(f(y), x, c) = 0$ is the required solution

## BERNOULLIS EQUATION

A D.E of the form $\frac{dy}{dx} + P(x)y = Qy^n$ is called as Bernoulli's equation in terms of dependent variable y and independent variable x. where P and Q are functions of x-alone (or) constant.

**Working Rule:**

Given that $\frac{dy}{dx} + P(x)y = Q(x)y^n \longrightarrow 1$

$\implies y^{-n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x) \longrightarrow 2$

Let $y^{1-n} = z$

Differentiating with respect to x, we get

$\implies (1-n)y^{-n}\frac{dy}{dx} = \frac{dz}{dx}$

$\implies \frac{1}{1-n}\frac{dz}{dx} + P(x)z = (1-n)Q$ (From 2), which is a Linear D.E in terms of $z, x$

By using Linear Method we can find general solution of it.

Let it be $\emptyset(z, x, c) = 0 \implies \emptyset(y^{-n}, x, c) = 0$ which is general solution of the given equation.

## Orthogonal Trajectories

Trajectory: A Curve which cuts given family of curves according to some special law is called as a Trajectory.

Orthogonal Trajectory: A Curve which cuts every member of given family of curves at $90^o$ is called as an Orthogonal Trajectory.

## Orthogonal Trajectory in Cartesian Co-ordinates

Let $f(x, y, c) = 0$ be given family of curves in Cartesian Co-ordinates.

Differentiating it w.r.t $x$, we get $f\left(x, y, \frac{dy}{dx}\right) = 0$.

Substituting $\frac{dy}{dx} = -\frac{dx}{dy}$, we get $f\left(x, y, -\frac{dx}{dy}\right) = 0$ . By using previous methods we can find general solution of it. Let it be $g(x, y, c) = 0$, which is Orthogonal Trajectory of the given family of curves.

For O.T, $\frac{dy}{dx} = -\frac{dx}{dy}$ because, two lines are $\perp^r$ if product of $slopes = -1$

$$\Rightarrow \frac{dy}{dx} \cdot \frac{dy}{dx} = -1$$

$$\Rightarrow \frac{dy}{dx} = -\frac{dx}{dy}$$

## Orthogonal Trajectory in Polar Co-ordinates

Let $f(r, \theta, c) = 0$ be given family of curves in Polar Co-ordinates.

Differentiating it w.r.t $\theta$, we get $f\left(r, \theta, \frac{dr}{d\theta}\right) = 0$.

Substituting $\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$, we get $f\left(r, \theta, -r^2 \frac{d\theta}{dr}\right) = 0$. By using previous methods we can find general solution of it. Let it be $g(\theta, r, c) = 0$, which is Orthogonal Trajectory of the given family of curves.

$$\left(\frac{1}{r}\frac{dr}{d\theta}\right) \cdot \left(\frac{1}{r}\frac{dr}{d\theta}\right) = -1$$

$$\Rightarrow \frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$$

Self Orthogonal: If the Orthogonal Trajectory of given family of Curves is family of curves itself then it is called as Self Orthogonal.

Mutual Orthogonal: Given family of curves $f(x, y, c) = 0$ & $g(x, y, c) = 0$ are said to be Mutually Orthogonal if Orthogonal Trajectory of one given family of curves is other given family of curves.

## NEWTON'S LAW OF COOLING

**Statement:** The rate of the temperature of a body is proportional to the difference of the temperature of the body and that of the surrounding medium.

Let θ be the temperature of the body at the time $t$ and θ be the temperature of its surrounding medium (air). By the Newton's Law of cooling, we have

$$\frac{d\theta}{dt} \propto (\theta - \theta_o)$$

$$\Rightarrow \frac{d\theta}{dt} = -k(\theta - \theta_o)$$ , where $k$ is a positive constant

$$\therefore \frac{d\theta}{(\theta - \theta_o)} = -k\, dt$$

Integrating, we get $\int \frac{d\theta}{(\theta-\theta_o)} = -k \int dt$

$\Longrightarrow$ $\boxed{\theta = \theta_o + ce^{-kt}}$

## Problem

A body is originally at 80°C and cools down to 60°C in 20 minutes. If the temperature of the air is 40°C, find the temperature of the body after 40 minutes.

***Sol:*** Let θ be the temperature of the body at a time $t$

| $t$ | $\theta$ |
|---|---|
| 0 | 80 |
| 20 | 60 |
| 40 | ? |

We know that from Newton's Law of cooling

$$\frac{d\theta}{dt} \propto (\theta - \theta_o)$$

$\Longrightarrow \frac{d\theta}{dt} = -k(\theta - \theta_o)$ , where $k$ is a positive constant

Given temperature of the air $\theta_o = 40^oC$

$$\therefore \frac{d\theta}{(\theta-40)} = -k\, dt$$

Integrating, we get $\int \frac{d\theta}{(\theta-40)} = -k \int dt$

$$\Longrightarrow \theta = 40 + ce^{-kt} \longrightarrow \text{(I)}$$

Now, given at $t = 0,\ \theta = 80^oC$

(I) $\Longrightarrow 80 = 40 + c$

$\Longrightarrow c = 40$

Substituting this value of $c$ in (I) , we get

$\Longrightarrow \theta = 40 + 40e^{-kt} \longrightarrow \text{(II)}$

Again, given at $t = 20,\ \theta = 60^oC$

(II) $\Longrightarrow 60 = 40 + 40\, e^{-20k}$

$\Longrightarrow k = \frac{1}{20}\log 2$

Substituting this value of $k$ in (II) we get

$\Longrightarrow \theta = 40 + 40e^{-\left(\frac{1}{20}\log 2\right)t}$

$\Longrightarrow \theta = 40 + 40e^{-\left(\frac{t}{20}\log 2\right)} \longrightarrow \text{(III)}$

Again, when $t = 40, \theta = ?$

(III) $\Rightarrow \theta = 40 + 40e^{-\left(\frac{40}{20}\log 2\right)}$

$\Rightarrow$ At $t = 40, \theta = 50^{o}C$

## LAW OF NATURAL GROWTH (Or) DECAY

If $x(t)$ be the amount of substance at time $t$ , then the rate of change of amount $x(t)$ of a chemically changing substance is proportional to the amount of the substance available at that time.

$$\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = -kx$$

where $k$ is a proportionality constant.

Note: If as $t$ increases, $x$ increases we can take $\frac{dx}{dt} = kx \ \ (k > 0)$, and if as $t$ increases, $x$ decreases we can take $\frac{dx}{dt} = -kx \ \ (k > 0)$

## RATE OF DECAY OF RADIOACTIVE MATERIALS

If $u$ is the amount of the material at any time $t$, then $\frac{du}{dt} = -ku$, where $k$ is any constant.

* * *