

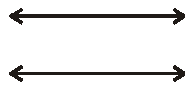
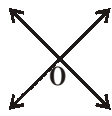
CHAPTER

9

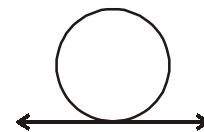
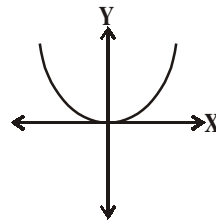
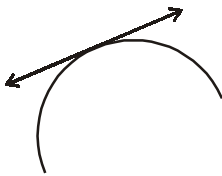
Tangents and Secants to a Circle

9.1 INTRODUCTION

We have seen two lines mostly intersect at a point or do not intersect in a plane. In some situations they coincide with each other.



Similarly, what happens when a curve and a line is given in a plane? You know a curve may be a parabola as you have seen in polynomials or a simple closed curve “circle” which is a collection of all those points on a plane that are at a constant distance from a fixed point.



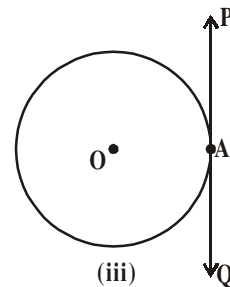
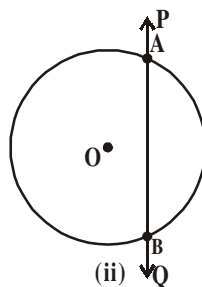
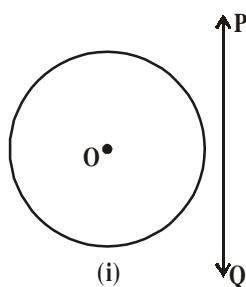
You might have seen circular objects rolling on a plane creating a path. For example; riding a bicycle, wheels of train on the track etc., where it seems to be a circle and a line. Does there a relation exist between them?

Let us see what happens, if a circle and a line are given in a plane.

9.1.1 A LINE AND A CIRCLE

You are given a circle and a line drawn on a paper. Salman argues that there can only be 3 possible ways of presenting them.

Consider a circle ‘O’ and a line PQ, the three possibilities are given in figure below:



In Fig.(i), the line PQ and the circle have no common point. In this case PQ is a non-intersecting line with respect to the circle.

In Fig.(ii), the line PQ intersects the circle at two points A and B. It forms a **chord** on the circle AB with two common points. In this case the line PQ is a **secant** of the circle.

In Fig.(iii), There is only one point A, common to the line PQ and the circle. This line is called a **tangent** to the circle.

You can see that there cannot be any other position of the line with respect to the circle. We will study the existence of tangents to a circle and also study their properties and constructions.

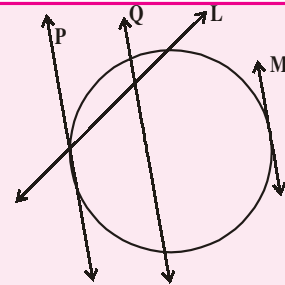
Do you know?

The word ‘tangent’ comes from the latin word ‘tangere’, which means to touch and was introduced by Danish mathematician Thomas Fineke in 1583.



Do THIS

- Draw a circle with any radius. Draw four tangents at different points. How many tangents can you draw to this circle ?
- How many tangents you can draw to circle from a point away from it.
- Which of the following are tangents to the circles



9.2 TANGENTS OF A CIRCLE

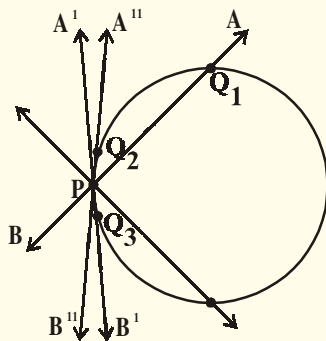
We can see that tangent can be drawn at any point lying on the circle. Can you say how many tangents can be drawn at any point on the surface of the circle.

To understand this let us consider the following activity.



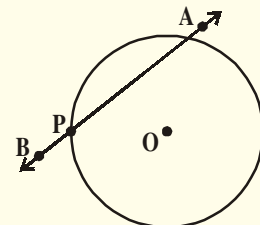
ACTIVITY

Take a circular wire and attach a straight wire AB at a point P of the circular wire, so that the system rotate about the point P in a plane.



The circular wire represents a circle and the straight wire AB represents a line intersects the circle at point P.

Put the system on a table and gently rotate the wire AB about the point P to get different positions of the straight wire as shown in the figure. The wire intersects the circular wire at P and at one more point through the points Q_1, Q_2 or Q_3 etc. So while it generally intersects circular wire at two points one of which is P in one particular position, it intersects the circle only at the



point P (See position $A'B'$ of AB). This is the position of a tangent at the point P of the circle. You can check that in all other positions of AB it will intersect the circle at P and at another point, $A'B'$ is a tangent to the circle at P.

We see that there is only one tangent to the circle at point P.

Moving wire AB in either direction from this position makes it cut the circular wire in two points. All these are therefore secants. Tangent is a special case of a secant where the two points of intersection of a line with a circle coincide.



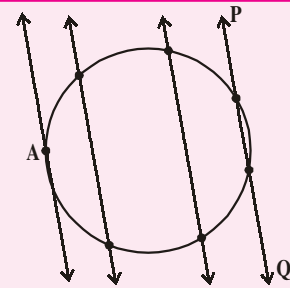
Do This

Draw a circle and a secant PQ of the circle on a paper as shown below. Draw various lines parallel to the secant on both sides of it.

What happens to the length of chord coming closer and closer to the centre of the circle?

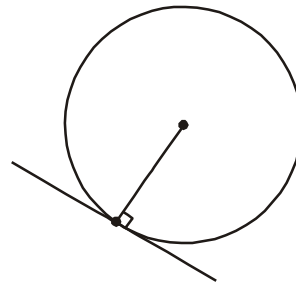
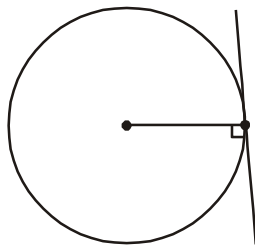
What is the longest chord?

How many tangents can you draw to a circle, which are parallel to each other?



The common point of the tangent and the circle is called the point of contact and the tangent is said to touch the circle at the common point.

Observe the tangents to the circle in the figures given below:



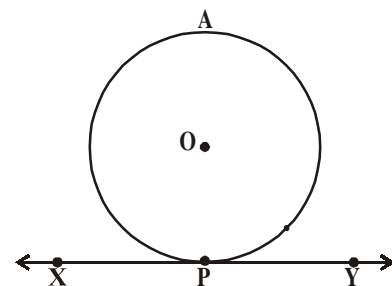
How many tangents can you draw to a circle at a point? How many tangents can you obtain to the circle in all? See the points of contact. Draw radii from the points of contact. Do you see anything special about the angle between the tangents and the radii at the points of contact. All appear to be perpendicular to the corresponding tangents. We can also prove it. Let us see how.

Theorem-9.1 : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given : A circle with centre 'O' and a tangent XY to the circle at a point P.

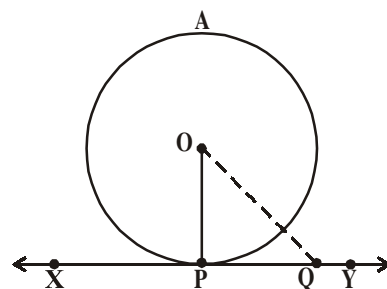
To prove : OP is perpendicular to XY. (i.e $OP \perp XY$)

Proof : Here, we will use the method that assumes that the



statement is wrong and shows that such an assumption leads to a fallacy. So we will suppose OP is not perpendicular to XY . Take a point Q on XY other than P and join OQ .

The point Q must lie outside the circle (why?) (Note that if Q lies inside the circle, XY becomes a secant and not a tangent to the circle)



Therefore, OQ is longer than the radius OP of the circle [Why?]

i.e., $OQ > OP$.

This must happen for all points on the line XY . It is therefore true that OP is the shortest of all the distances of the point O to the points of XY .

So our assumption that OP is not perpendicular to XY is false. Therefore, OP is perpendicular to XY .

Hence proved.

Note : The line containing the radius through the point of contact is also called the ‘normal to the circle at the point’.



TRY THIS

How can you prove the converse of the above theorem.

“If a line in the plane of a circle is perpendicular to the radius at its endpoint on the circle, then the line is tangent to the circle”.

We can find some more results using the above theorem

- (i) Since there can be only one perpendicular OP at the point P , it follows that one and only one tangent can be drawn to a circle at a given point on the circumference.
- (ii) Since there can be only one perpendicular to XY at the point P , it follows that the perpendicular to a tangent at its point of contact passes through the centre.

Think about these. Discuss these among your friends and with your teachers.

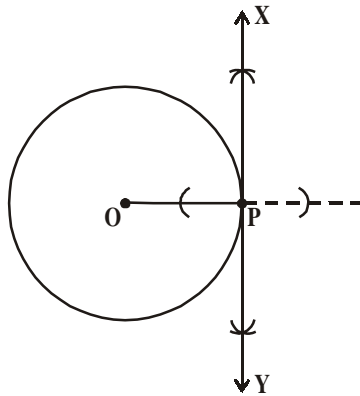
9.2.1 CONSTRUCTION OF TANGENT TO A CIRCLE

How can we construct a line that would be tangent to a circle at a given point on it? We use what we just found the tangent has to be perpendicular to the radius at the point of contact. To draw a tangent through the point of contact we need to draw a line perpendicular to the radius at that point. To draw this radius we need to know the center of the circle. Let us see the steps for this construction.

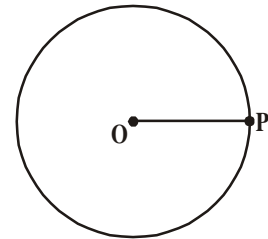
Construction : Construct a tangent to a circle at a given point when the centre of the circle is known.

We have a circle with centre 'O' and a point P anywhere on its circumference. Then we have to construct a tangent through P.

Steps of Construction :



1. Draw a circle with centre 'O' and mark a point 'P' anywhere on it. Join OP.



2. Draw a perpendicular line through the point P and name it as XY, as shown in the figure.

3. XY is the required tangent to the given circle passing through P.

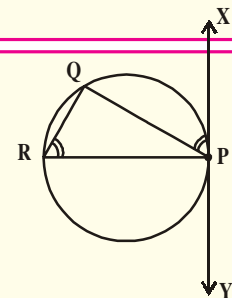
Can you draw one more tangent through P? give reason.



TRY THIS

How can you draw the tangent to a circle at a given point when the centre of the circle is not known?

Hint : Draw equal angles $\angle QPX$ and $\angle PRQ$. Explain the construction.



9.2.2 FINDING LENGTH OF THE TANGENT

Can we find the length of the tangent to a circle from a given point? Is the length of tangents from a given point to the circle the same? Let us examine this.

Example : Find the length of the tangent to a circle with centre 'O' and radius = 6 cm. from a point P such that $OP = 10$ cm.

Solution : Tangent is perpendicular to the radius at the point of contact (Theorem 9.1)

Here PA is tangent segment and OA is radius of circle

$$\therefore OA \perp PA \Rightarrow \angle OAP = 90^\circ$$

Now in $\triangle OAP$, $OP^2 = OA^2 + PA^2$ (pythagoras theorem)

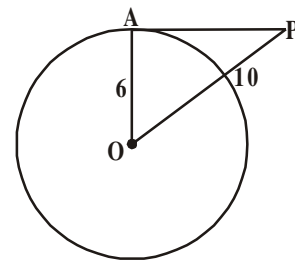
$$10^2 = 6^2 + PA^2$$

$$100 = 36 + PA^2$$

$$PA^2 = 100 - 36$$

$$= 64$$

$$\therefore PA = \sqrt{64} = 8 \text{ cm.}$$





EXERCISE - 9.1

- Fill in the blanks
 - A tangent to a circle intersects it in point (s).
 - A line intersecting a circle in two points is called a
 - A circle can have parallel tangents at the most.
 - The common point of a tangent to a circle and the circle is called
 - We can draw tangents to a given circle.
- A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that $OQ = 12$ cm. Find length of PQ.
- Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.
- Calculate the length of tangent from a point 15 cm. away from the centre of a circle of radius 9 cm.
- Prove that the tangents to a circle at the end points of a diameter are parallel.

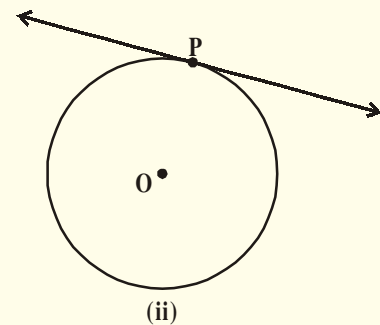
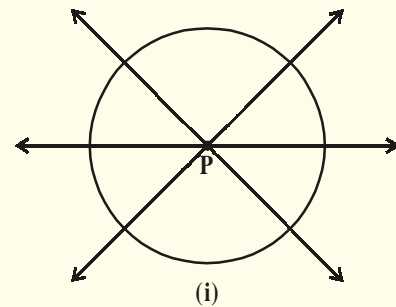
9.3 NUMBER OF TANGENT TO A CIRCLE FROM ANY POINT

To get an idea of the number of tangents from a point on a circle, Let us perform the following activity.

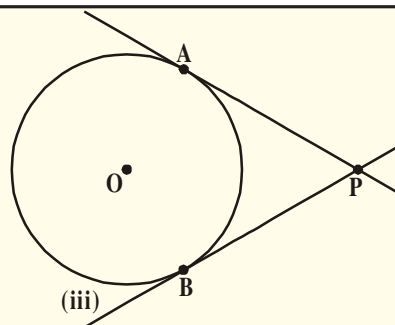


ACTIVITY

- Draw a circle on a paper. Take a point P inside it. Can you draw a tangent to the circle through this point? You will find that all the lines through this point intersect the circle in two points. What are these? These are all secants of a circle. So, it is not possible to draw any tangent to a circle through a point inside it. (See the adjacent figure)
- Next, take a point P on the circle and draw tangents through this point. You have observed that there is only one tangent to the circle at a such a point. (See the adjacent figure)



(iii) Now, take a point P outside the circle and try to draw tangents to the circle from this point. What do you observe? You will find that you can draw exactly two tangents to the circle through this point (See the adjacent figure)



Now, we can summarise these facts as follows :

Case (i) : There is no tangent to a circle passing through a point lying inside the circle.

Case(ii) : There is one and only one tangent to a circle passing through a point lying on the circle.

Case(iii) : There are exactly two tangents to a circle through a point lying outside the circle in this case, A and B are the points of contacts of the tangents PA and PB respectively.

The length of the segment from the external point P and the point of contact with the circle is called the length of the tangent from the point P to the circle.

Note that in the above figure (iii), PA and PB are the length of the tangents from P to the circle. What is the relation between lengths PA and PB.

Theorem-9.2 : The lengths of tangents drawn from an external point to a circle are equal.

Given : A circle with centre O, P is a point lying outside the circle and PA and PB are two tangents to the circle from P. (See figure)

To prove : PA = PB

Proof : Join OA, OB and OP.

$$\angle OAP = \angle OBP = 90^\circ \quad (\text{Angle between radii and tangents})$$

Now in the two right triangles according to theorem 9.1)

$\triangle OAP$ and $\triangle OBP$,

OA = OB (radii of same circle)

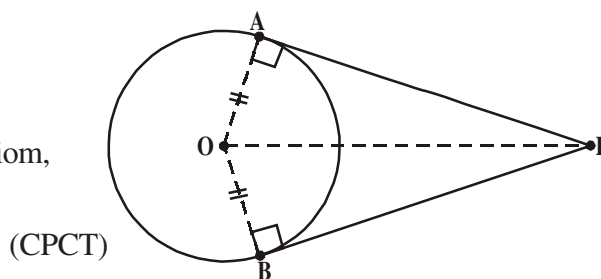
OP = OP (Common)

Therefore, By R.H.S. Congruency axiom,

$\triangle OAP \cong \triangle OBP$

This gives PA = PB

Hence Proved.



TRY THIS

Use pythagoras theorem and write proof of above theorem.

9.3.1. CONSTRUCTION OF TANGENTS TO A CIRCLE FROM AN EXTERNAL POINT

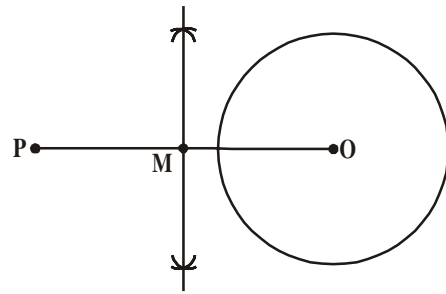
You saw that if a point lies outside the circle, there will be exactly two tangents to the circle from this point. We shall now see how to draw these tangents.

Construction : To construct the tangents to a circle from a point outside it.

Given : We are given a circle with centre 'O' and a point P outside it. We have to construct two tangents from P to the circle.

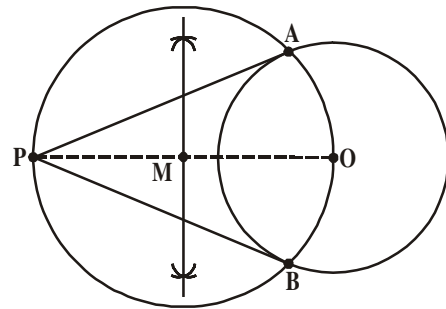
Steps of construction :

Step(i) : Join PO and draw a perpendicular bisector of it. Let M be the midpoint of PO.



Step (ii) : Taking M as centre and PM or MO as radius, draw a circle. Let it intersect the given circle at the points A and B.

Step (iii) : Join PA and PB. Then PA and PB are the required two tangents.



Proof : Now, Let us see how this construction is justified.

Join OA. Then $\angle PAO$ is an angle in the semicircle and,

therefore, $\angle PAO = 90^\circ$.

Can we say that $PA \perp OA$?

Since, OA is a radius of the given circle, PA has to be a tangent to the circle (By converse theorem of 9.1)

Similarly, PB is also a tangent to the circle.

Hence proved.

Some interesting statements about tangents and secants and their proof:

Statement-1 : The centre of a circle lies on the bisector of the angle between two tangents drawn from a point outside it. Can you think how we can prove it?

Proof : Let PQ and PR be two tangents drawn from a point P outside of the circle with centre O

Join OQ and OR, triangles OQP and ORP are congruent because we know that,

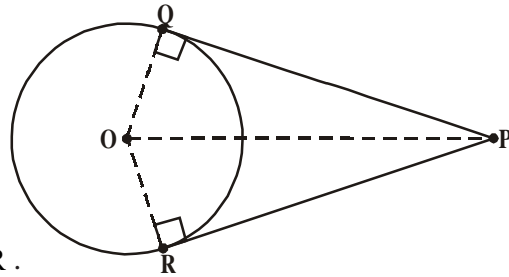
$$\angle OQP = \angle ORP = 90^\circ \text{ (Theorem 9.1)}$$

$$OQ = OR \text{ (Radii)}$$

OP is common.

This means $\angle OPQ = \angle OPR$ (CPCT)

Therefore, OP is the angle bisector of $\angle QPR$.



Hence, the centre lies on the bisector of the angle between the two tangents.

Statement-2 : In two concentric circles, such that a chord of the bigger circle, that touches the smaller circle is bisected at the point of contact with the smaller circle.

Can you see how is this?

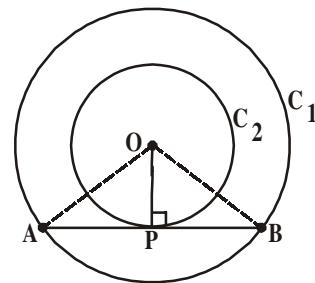
Proof : We are given two concentric circles C_1 and C_2 with centre O and a chord AB of the larger circle C_1 , touching the smaller circle C_2 at the point P (See figure) we need to prove that $AP = PB$.

Join OP.

Then AB is a tangent to the circle C_2 at P and OP is its radius.

Therefore, by Theorem 9.1

$$OP \perp AB$$



Now, $\triangle OAP$ and $\triangle OBP$ are congruent. (Why?) This means $AP = PB$. Therefore, OP is the bisector of the chord AB, as the perpendicular from the centre bisects the chord.

Statement-3 : If two tangents AP and AQ are drawn to a circle with centre O from an external point A then $\angle PAQ = 2\angle OPQ = 2\angle OQP$.

Can you see?

Proof : We are given a circle with centre O, an external point A and two tangents AP and AQ to the circle, where P, Q are the points of contact (See figure).

We need to prove that

$$\angle PAQ = 2\angle OPQ$$

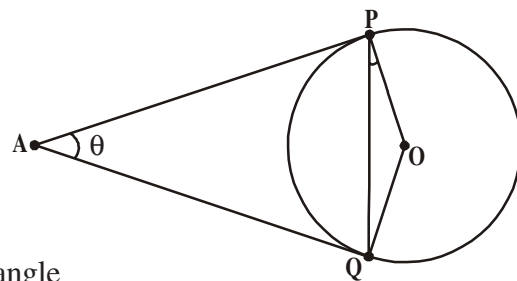
$$\text{Let } \angle PAQ = \theta$$

Now, by Theorem 9.2,

$AP = AQ$, So $\triangle APQ$ is an isosceles triangle

Therefore, $\angle APQ + \angle AQP + \angle PAQ = 180^\circ$ (Sum of three angles)

$$\angle APQ = \angle AQP = \frac{1}{2}(180^\circ - \theta)$$



$$= 90^\circ - \frac{1}{2}\theta$$

Also, by Theorem 9.1,

$$\angle OPA = 90^\circ$$

So, $\angle OPQ = \angle OPA - \angle APQ$

$$= 90^\circ - \left[90^\circ - \frac{1}{2}\theta \right] = \frac{1}{2}\theta = \frac{1}{2}\angle PAQ$$

This gives $\angle OPQ = \frac{1}{2}\angle PAQ$.

$\therefore \angle PAQ = 2\angle OPQ$. Similarly $\angle PAQ = 2\angle OQP$

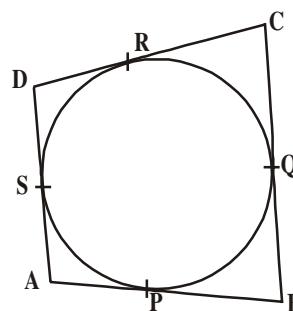


Statement-4 : If a circle touches all the four sides of a quadrilateral ABCD at points PQRS. Then $AB+CD = BC + DA$

Can you think how do we proceed? AB, CD, BC, DA are all chords to a circle.

For the circle to touch all the four sides of the quadrilateral at points P, Q, R, S, it has to be inside the quadrilateral. (See figure)

How do we proceed further?



Proof : The circle touched the sides AB, BC, CD and DA of Quadrilateral ABCD at the points P, Q, R and S respectively as shown

Since by theorem 9.2, the two tangents to a circle drawn from a point outside it, are equal,

$$AP = AS$$

$$BP = BQ$$

$$DR = DS$$

and $CR = CQ$

On adding, We get

$$AP + BP + DR + CR = AS + BQ + DS + CQ$$

$$\text{or } (AP + PB) + (CR + DR) = (BQ + QC) + (DS + SA)$$

$$\text{or } AB + CD = BC + DA.$$

Let us do an example of analysing a situation and know how we would construct something.

Example-1. Draw a pair of tangents to a circle of radius 5cm which are inclined to each other at an angle 60° .

Solution : To draw the circle and the two tangents we need to see how we proceed. We only have the radius of the circle and the angle between the tangents. We do not know the distance of the point from where the tangents are drawn to the circle and we do not know the length of the tangents either. We know only the angle between the tangents. Using this, we need to find out the distance of the point outside the circle from which we have to draw the tangents.

To begin let us consider a circle with centre 'O' and radius 5cm. Let PA and PB are two tangents drawn from a point 'P' outside the circle and the angle between them is 60° . In this $\angle APB = 60^\circ$. Join OP.

As we know,

OP is the bisector of $\angle APB$,

$$\angle OAP = \angle OPB = \frac{60^\circ}{2} = 30^\circ \quad (\because \triangle OAP \cong \triangle OBP)$$

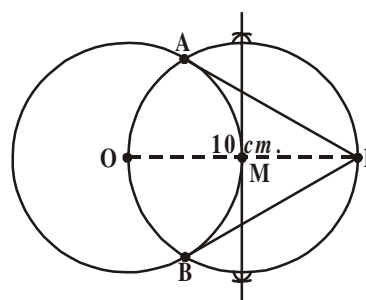
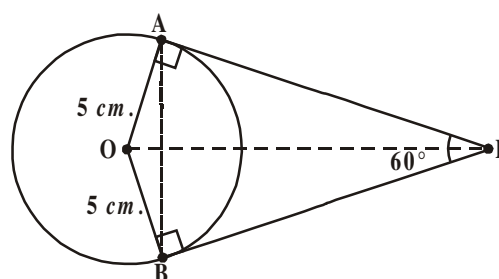
Now in $\triangle OAP$,

$$\sin 30^\circ = \frac{\text{Opp. side}}{\text{Hyp}} = \frac{OA}{OP}$$

$$\frac{1}{2} = \frac{5}{OP} \quad (\text{From trigonometric ratio})$$

$$OP = 10 \text{ cm.}$$

Now we can draw a circle of radius 5 cm with centre 'O'. We then mark a point at a distance of 10 cm from the centre of the circle. Join OP and complete the construction as given in construction 9.2. Hence PA and PB are the required pair of tangents to the given circle.



You can also try this construction without using trigonometric ratio.



TRY THIS

Draw a pair of radii OA and OB such that $\angle BOA = 120^\circ$. Draw the bisector of $\angle BOA$ and draw lines perpendicular to OA and OB at A and B. These lines meet on the bisector of $\angle BOA$ at a point which is the external point and the perpendicular lines are the required tangents. Construct and Justify.



EXERCISE - 9.2

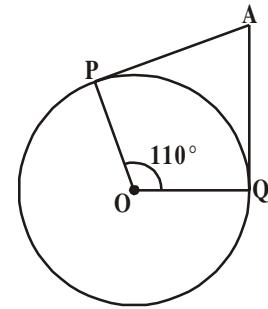
- Choose the correct answer and give justification for each.
 - The angle between a tangent to a circle and the radius drawn at the point of contact is
 - 60°
 - 30°
 - 45°
 - 90°

(ii) From a point Q, the length of the tangent to a circle is 24 cm. and the distance of Q from the centre is 25 cm. The radius of the circle is

- (a) 7cm (b) 12 cm (c) 15cm (d) 24.5cm

(iii) If AP and AQ are the two tangents a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PAQ$ is equal to

- (a) 60° (b) 70°
(c) 80° (d) 90°

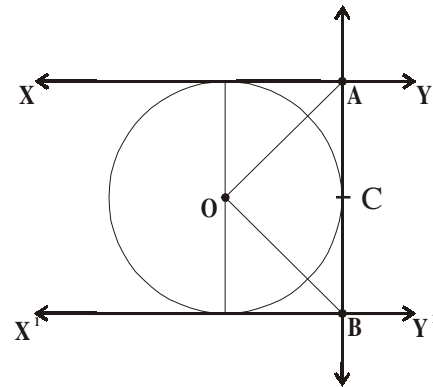


(iv) If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80° , then $\angle POA$ is equal to

- (a) 50° (b) 60° (c) 70° (d) 80°

(v) In the figure XY and X^1Y^1 are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X^1Y^1 at B then $\angle AOB =$

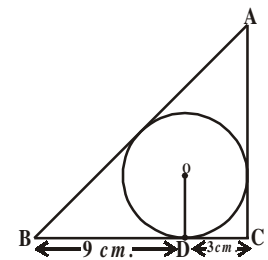
- (a) 80° (b) 100°
(c) 90° (d) 60°



2. Two concentric circles are radii 5 cm and 3cm are drawn. Find the length of the chord of the larger circle which touches the smaller circle.

3. Prove that the parallelogram circumscribing a circle is a rhombus.

4. A triangle ABC is drawn to circumscribe a circle of radius 3 cm. such that the segments BD and DC into which BC is divided by the point of contact D are of length 9 cm. and 3 cm. respectively (See adjacent figure). Find the sides AB and AC.

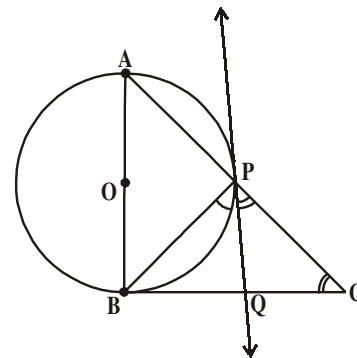


5. Draw a circle of radius 6cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. Verify by using Pythagoras Theorem.

6. Construct a tangent to a circle of radius 4cm from a point on the concentric circle of radius 6cm and measure its length. Also verify the measurement by actual calculation.

7. Draw a circle with the help of a bangle, Take a point outside the circle. Construct the pair of tangents from this point to the circle measure them. Write conclusion.

8. In a right triangle ABC , a circle with a side AB as diameter is drawn to intersect the hypotenuse AC in P . Prove that the tangent to the circle at P bisects the side BC .
9. Draw a tangent to a given circle with center O from a point 'R' outside the circle. How many tangents can be drawn to the circle from that point?

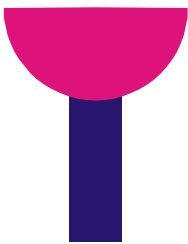


Hint : The distance of two points to the point of contact is the same.

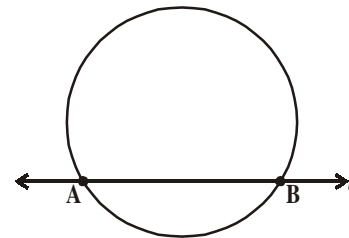
9.4 SEGMENT OF A CIRCLE FORMED BY A SECANT

We have seen a line and a circle. When a line meets a circle in only one point, it is a tangent. A secant is a line which intersects the circle at two distinct points represented in the chord.

Here 'l' is the secant and AB is the chord.

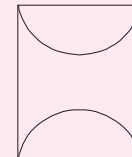
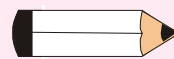
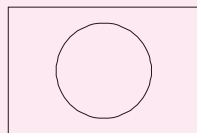
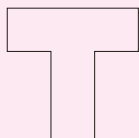


Shankar is making a picture by sticking pink and blue paper. He makes many pictures. One picture he makes is of washbasin. How much paper does he need to make this picture? This picture can be seen in two parts. A rectangle is there, but what is the remaining part? It is the segment of the circle. We know how to find the area of rectangle. How do we find the area of the segment? In the following discussion we will try to find this area.



Do This

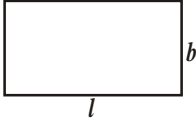

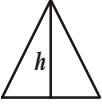
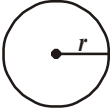
Shankar made the following pictures also along with washbasin.



What shapes can they be broken into that we can find area easily?

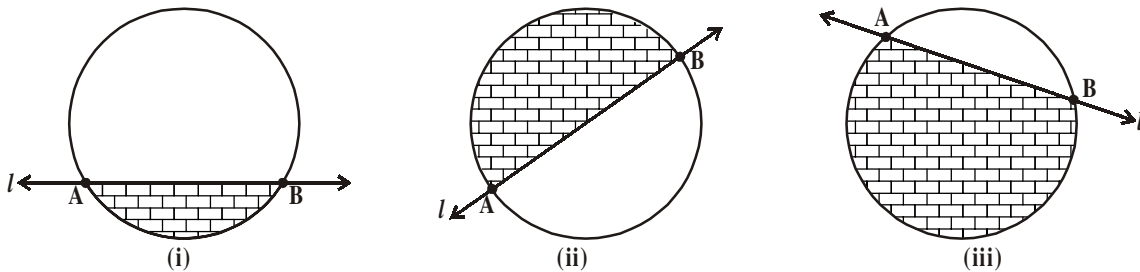
Make some more pictures and think of the shapes they can be divided into different parts.


Lets us recall how to find the area of the following geometrical figures as given in the table.

S.No.	Figure	Dimintions	Area
1.		length = l breadth = b	$A = lb$
2.		Side = s	$A = s^2$
3.		base = b	$A = \frac{1}{2}bh$
4.		radius = r	$A = \pi r^2$

9.4.1. FINDING THE AREA OF SEGMENT OF A CIRCLE

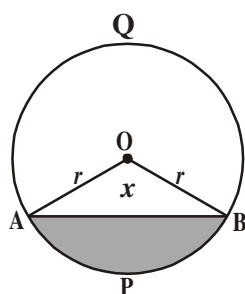
To estimate the area of segment of a circle, Swetha made the segments by drawing secants to the circle.



As you know a segment is a region, bounded by the arc and a chord, we can see the area that is shaded () in fig.(i) is a minor segment, semicircle in fig.(ii) and major segment in fig.(iii).

How do we find the area of the segment? Do the following activity.

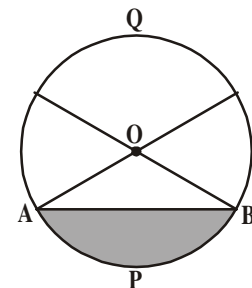
Take a circular shaped paper and fold it along with a chord less than the diameter and shade the smaller part as shown in in the figure. What do we call this smaller part? It is a minor segment (APB). What do



we call the unshaded portion of the circle? Obviously it is a major segment (AQB).

You have already come across the sector and segment in earlier classes. The portion of some unshaded part and shaded part (minor segment) is a sector which is the combination of a triangle and a segment.

Let OAPB be a sector of a circle with centre O and radius ' r ' as shown in the figure. Let the angle measure of $\angle AOB$ be ' x '.



You know that area of circle when the angle measure at the centre is 360° is πr^2 .

So, when the degree measure of the angle at the centre is 1° , then area of sector is

$$\frac{1^\circ}{360^\circ} \times \pi r^2.$$

Therefore, when the degree measure of the angle at the centre is x° , the area of sector is

$$\frac{x^\circ}{360^\circ} \times \pi r^2.$$

Now let us take the case of the area of the segment APB of a circle with centre 'O' and radius 'r', you can see that

$$\text{Area of the segment APB} = \text{Area of the sector OAPB} - \text{Area of } \Delta OAB$$

$$= \frac{x^\circ}{360^\circ} \times \pi r^2 - \text{area of } \Delta OAB$$



TRY THIS

How can you find the area of major segment using area of minor segment?



DO THIS

- Find the area of sector, whose radius is 7 cm. with the given angle:
 - 60°
 - 30°
 - 72°
 - 90°
 - 120°
- The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 10 minutes.

Now, we will see an example to find area of segment of a circle.

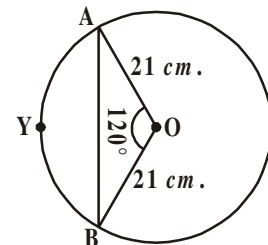
Example-1. Find the area of the segment AYB showing in the adjacent figure. If radius of the circle is 21 cm and $\angle AOB = 120^\circ$ (Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.732$)

Solution : Area of the segment AYB

$$= \text{Area of sector OAYB} - \text{Area of } \Delta OAB$$

$$\text{Now, area of the sector OAYB} = \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$$

$$= 462 \text{ cm}^2 \quad \dots(1)$$



For finding the area of ΔOAB , draw $OM \perp AB$ as shown in the figure:-

Note $OA = OB$. Therefore, by RHS congruence, $\Delta AMO \cong \Delta BMO$

So, M is the midpoint of AB and $\angle AOM = \angle BOM = \frac{1}{2} \times 120^\circ = 60^\circ$

Let, $OM = x$ cm

So, from $\triangle OMA$, $\frac{OM}{OA} = \cos 60^\circ$.

$$\text{or, } \frac{x}{21} = \frac{1}{2} \quad \left(\because \cos 60^\circ = \frac{1}{2} \right)$$

$$\text{or, } x = \frac{21}{2}$$

$$\text{So, } OM = \frac{21}{2} \text{ cm}$$

Also, $\frac{AM}{OA} = \sin 60^\circ$

$$\frac{AM}{21} = \frac{\sqrt{3}}{2} \quad \left(\because \sin 60^\circ = \frac{\sqrt{3}}{2} \right)$$

$$\text{So, } AM = \frac{21\sqrt{3}}{2} \text{ cm.}$$

$$\text{Therefore } AB = 2AM = \frac{2 \times 21\sqrt{3}}{2} \text{ cm.} = 21\sqrt{3} \text{ cm}$$

$$\text{So, Area of } \triangle OAB = \frac{1}{2} \times AB \times OM$$

$$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2} \text{ cm}^2.$$

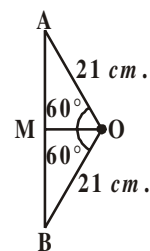
$$= \frac{441}{4} \sqrt{3} \text{ cm}^2. \quad \dots(2)$$

$$\text{Therefore, area of the segment AYB} = \left(462 - \frac{441}{4} \sqrt{3} \right) \text{ cm}^2.$$

(\because from (1), (2)]

$$= \frac{21}{4} (88 - 21\sqrt{3}) \text{ cm}^2$$

$$= 271.047 \text{ cm}^2$$



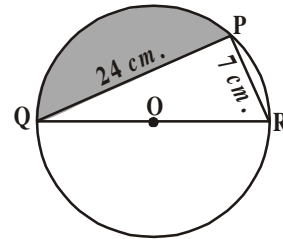
Example-2. Find the area of the segments shaded in figure, if $PQ = 24 \text{ cm.}$, $PR = 7 \text{ cm.}$ and QR is the diameter of the circle with centre O (Take $\pi = \frac{22}{7}$)

Solution : Area of the segments shaded = Area of sector $OQPR$ - Area of triangle PQR .

Since QR is diameter, $\angle QPR = 90^\circ$ (Angle in a semicircle)

So, using pythagoras Theorem

$$\begin{aligned} \text{In } \Delta QPR, \quad QR^2 &= PQ^2 + PR^2 \\ &= 24^2 + 7^2 \\ &= 576 + 49 \\ &= 625 \\ QR &= \sqrt{625} = 25 \text{ cm.} \end{aligned}$$



$$\begin{aligned} \text{Then radius of the circle} &= \frac{1}{2} QR \\ &= \frac{1}{2} (25) = \frac{25}{2} \text{ cm.} \end{aligned}$$

$$\begin{aligned} \text{Now, area of semicircle } OQPR &= \frac{1}{2} \pi r^2 \\ &= \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} \\ &= 327.38 \text{ cm}^2 \quad \dots (1) \end{aligned}$$



$$\begin{aligned} \text{Area of right angled triangle } QPR &= \frac{1}{2} \times PR \times PQ \\ &= \frac{1}{2} \times 7 \times 24 \\ &= 84 \text{ cm}^2 \quad \dots (2) \end{aligned}$$

From (1) and (2),

$$\begin{aligned} \text{Area of the shaded segments} &= 327.38 - 84 \\ &= 243.38 \text{ cm}^2 \end{aligned}$$

Example-3. A round table top has six equal designs as shown in the figure. If the radius of the table top is 14 cm. , find the cost of making the designs with paint at the rate of $\text{₹}5 \text{ per cm}^2$. (use $\sqrt{3} = 1.732$)

Solution : We know that the radius of circumscribing circle of a regular hexagon is equal to the length of its side.

\therefore Each side of regular hexagon = 14 cm.

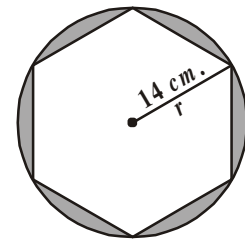
Therefore, Area of six design segments = Area of circle - Area of the regular hexagon.

$$\begin{aligned} \text{Now, Area of circle} &= \pi r^2 \\ &= \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2 \quad \dots (1) \end{aligned}$$

$$\begin{aligned} \text{Area of regular hexagon} &= 6 \times \frac{\sqrt{3}}{4} a^2 \\ &= 6 \times \frac{\sqrt{3}}{4} \times 14 \times 14 \\ &= 509.2 \text{ cm}^2 \quad \dots (2) \end{aligned}$$

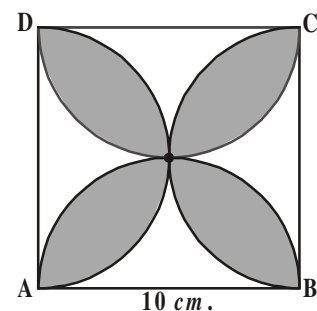
$$\begin{aligned} \text{Hence, area of six designs} &= 616 - 509.21 \text{ (from (1), (2))} \\ &= 106.79 \text{ cm}^2. \end{aligned}$$

$$\begin{aligned} \text{Therefore, cost of painting the design at the rate of ₹5 per cm}^2 & \\ &= ₹106.79 \times 5 \\ &= ₹533.95 \end{aligned}$$

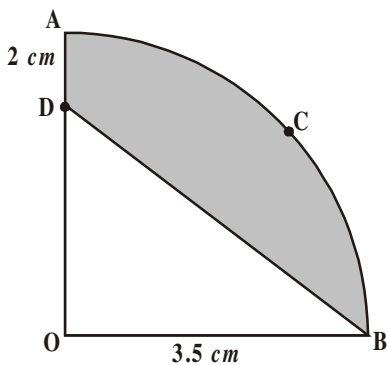


EXERCISE - 9.3

- A chord of a circle of radius 10 cm. subtends a right angle at the centre. Find the area of the corresponding: (use $\pi = 3.14$)
 - Minor segment
 - Major segment
- A chord of a circle of radius 12 cm. subtends an angle of 120° at the centre. Find the area of the corresponding minor segment of the circle (use $\pi = 3.14$ and $\sqrt{3} = 1.732$)
- A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm. sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades. (use $\pi = \frac{22}{7}$)
- Find the area of the shaded region in figure, where ABCD is a square of side 10 cm. and semicircles are drawn with each side of the square as diameter (use $\pi = 3.14$)



5. Find the area of the shaded region in figure, if ABCD is a square of side 7 cm. and APD and BPC are semicircles. (use

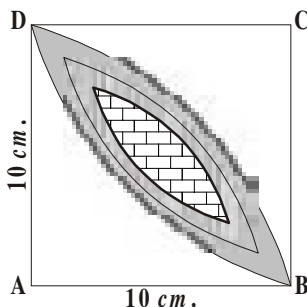
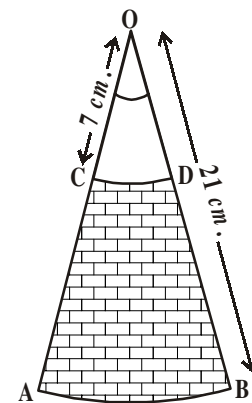
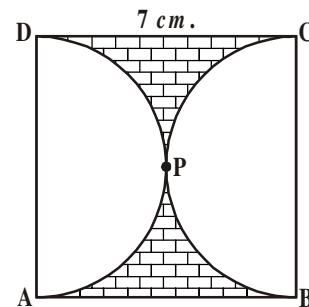


$$\pi = \frac{22}{7}$$

6. In figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm., find the area of the shaded region.

$$\text{(use } \pi = \frac{22}{7} \text{)}$$

7. AB and CD are respectively arcs of two concentric circles of radii 21 cm. and 7 cm. with centre O (See figure). If $\angle AOB = 30^\circ$, find the area of the shaded region. (use $\pi = \frac{22}{7}$)



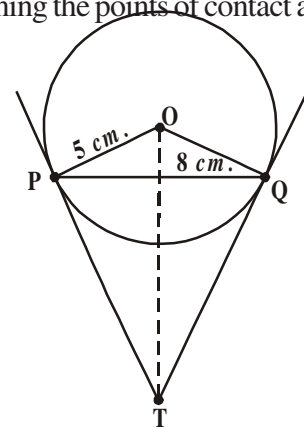
8. Calculate the area of the designed region in figure, common between the two quadrants of the circles of radius 10 cm. each. (use $\pi = 3.14$)



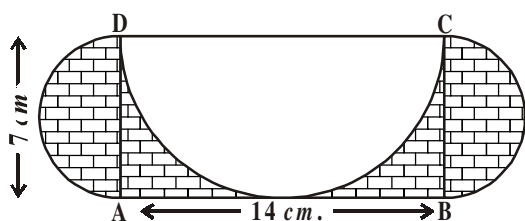
OPTIONAL EXERCISE

[This exercise is not meant for examination]

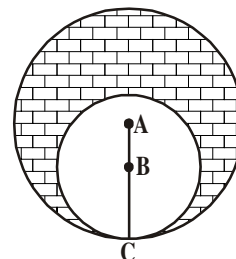
1. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line - segment joining the points of contact at the centre.
2. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T (See figure). Find the length of TP.
3. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
4. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.



5. Let ABC be a right triangle in which $AB = 6\text{ cm}$, $BC = 8\text{ cm}$ and $\angle B = 90^\circ$. BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.
6. Find the area of the shaded region in the figure, given in which two circles with centres A and B touch each other at the point C. If $AC = 8\text{ cm}$. and $AB = 3\text{ cm}$.



7. ABCD is a rectangle with $AB = 14\text{ cm}$. and $BC = 7\text{ cm}$. Taking DC, BC and AD as diameters, three semicircles are drawn as shown in the figure. Find the area of the shaded region.



WHAT WE HAVE DISCUSSED

In this chapter, we have studied the following points.

- The meaning of a tangent to a circle and a secant. We have used the idea of the chord of a circle.
- We used the ideas of different kinds of triangles particularly right angled triangles and isosceles triangles.
- We learn the following:
 - The tangents to a circle is perpendicular to the radius through the point of contact.
 - The lengths of the two tangents from an external point to a circle are equal.
- We learnt to do the following:
 - To construct a tangent to a circle at a given point when the centre of the circle is known.
 - To construct the pair of tangents from an external point to a circle.
- We learnt to understand how to prove some statements about circles and tangents. In this process learnt to use previous results and build on them logically to form new results.
- We have learnt

Area of segment of a circle = Area of the corresponding sector - Area of the corresponding triangle.