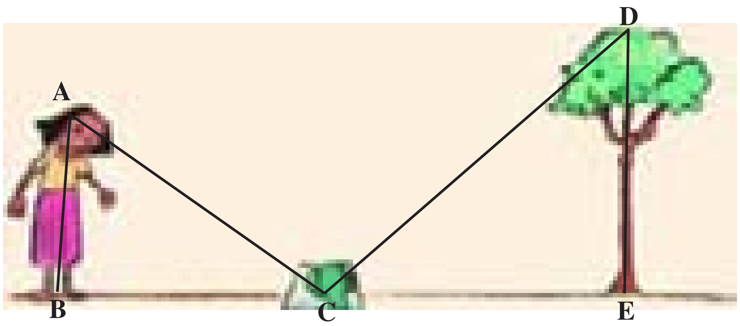
CHAPTER 8

Similar Triangles

8.1 Introduction

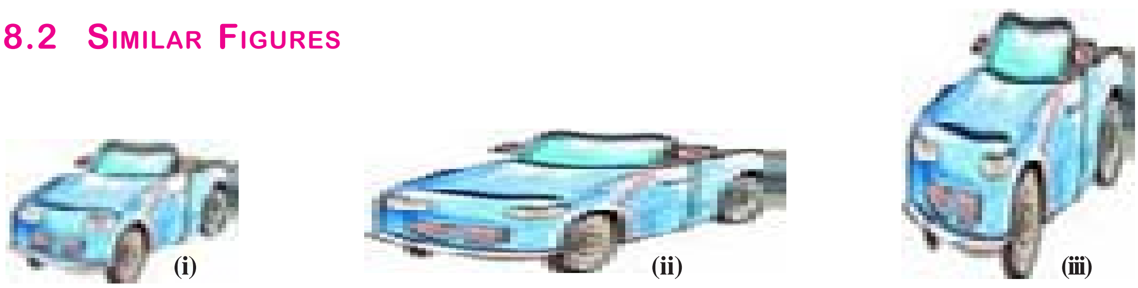
There is a tall tree in the backyard of Snigdha's house. She wants to find out the height of that tree but she is not sure about how to find it. Meanwhile, her uncle arrives at home. Snigdha requests her uncle to help her with the height. He thinks for a while and then ask her to bring a mirror. He places it on the ground at a certain distance from the base of the tree. He then asked Snigdha to stand on the otherside of the mirror at such a position from where she is able to see the top of the tree in that mirror.



When we draw the figure from (AB) girl to the mirror (C) and mirror to the tree (DE) as above, we observe triangles ABC and DEC. Now, what can you say about these two triangles? Are they congruent? No, because although they have the same shape but their sizes are different. Do you know what we call the geometrical figures which have the same shape, but are not necessarily of the same size? They are called **similar figures**.

Can you guess how the heights of trees, mountains or distances of far-away, objects such as the Sun have been found out? Do you think these can be measured directly with the help of a measuring tape? The fact is that all these heights and distances have been found out using the idea of indirect measurements which is based on the principle of similarity of figures.

8.2 Similar Figures



Observe the object (car) in the previous figure.

If its breadth is kept the same and the length is doubled, it appears as in fig.(ii).

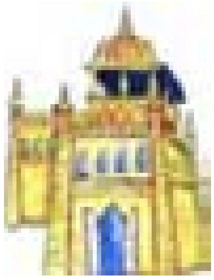
If the length in fig.(i) is kept the same and its breadth is doubled, it appears as in fig.(iii).

Now, what can you say about fig.(ii) and (iii)? Do they resemble fig.(i)? We find that the figure is distorted. Can you say that they are similar? No, they have same shape, yet they are not similar.

Think what a photographer does when she prints photographs of different sizes from the same film (negative) ? You might have heard about stamp size, passport size and post card size photographs. She generally takes a photograph on a small size film, say 35 *mm*., and then enlarges it into a bigger size, say 45 mm (or 55 mm). We observe that every line segment of the smaller photograph is enlarged in the ratio of 35 : 45 (or 35 : 55). Further, in the two photographs of different sizes, we can see that the angles are equal. So, the photographs are similar.

(i)

(ii)

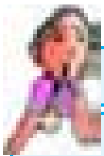


(iii)

Similarly in geometry, two polygons of the same number of sides are similar if their corresponding angles are equal and their corresponding sides are in the same ratio or proportion.

A polygon in which all sides and angles are equal is called a regular polygon.

The ratio of the corresponding sides is referred to as scale factor (or representative factor). In real life, blue prints for the construction of a building are prepared using a suitable scale factor.



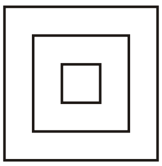
Think, Discuss and Write

Can you give some more examples from your daily life where scale factor is used.

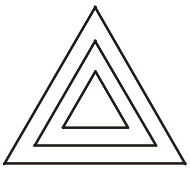
All regular polygons having the same number of sides are always similar. For example, all squares are similar, all equalateral triangles are similar and so on.

Circles with same radius are congruent and those with different radii are not congruent. But, as all circles have same shape, they are all similar.

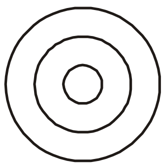
We can say that all congruent figures are similar but all similar figures need not be congruent.



Similar Squares



Similar equilateral triangles



Similar Circles

To understand the similarity of figures more clearly, let us perform the following activity.

Activity

Place a table directly under a lighted bulb, fitted in the ceiling in your classroom. Cut a polygon, say ABCD, from a plane cardboard and place it parallel to the ground between the bulb and the table. Then, a shadow of quadrilateral ABCD is cast on the table. Mark the outline of the shadow as quadrilateral $A'B'C'D'$.

Now this quadrilateral $A'B'C'D'$ is enlargement or magnification of quadrilateral ABCD. Further, A' lies on ray OA where 'O' is the bulb, B' on $\overrightarrow{OB}$, C' on $\overrightarrow{OC}$ and D' on $\overrightarrow{OD}$. Quadrilaterals ABCD and $A'B'C'D'$ are of the same shape but of different sizes.

A' corresponds to vertex A and we denote it symbotically as $A' \leftrightarrow A$. Similarly $B' \leftrightarrow B$, $C' \leftrightarrow C$ and $D' \leftrightarrow D$.

By actually measuring angles and sides, you can verify

(i) $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$, $\angle D = \angle D'$ and

(ii) $\dfrac{AB}{A'B'} = \dfrac{BC}{B'C'} = \dfrac{CD}{C'D'} = \dfrac{DA}{D'A'}$.

This emphasises that two polygons with the same number of sides are similar if

(i) All the corresponding angles are equal and

(ii) All the corresponding sides are in the same ratio (or in proportion)

Is a square similar to a rectangle? In both the figures, corresponding angles are equal but their corresponding sides are not in the same ratio. Hence, they are not similar. For similarity of polygons only one of the above two conditions is not sufficient, both have to be satisfied.

Think - Dissuss

Can you say that a square and a rhombus are similar? Discuss with your friends. Write why the conditions are not sufficient.

Do This

1. Fill in the blanks with similar / not similar.
   (i) All squares are ....
   (ii) All equilateral triangles are ....
   (iii) All isosceles triangles are ....
   (iv) Two polygons with same number of sides are .... if their corresponding angles are equal and corresponding sides are equal.
   (v) Reduced and Enlarged photographs of an object are ....
   (vi) Rhombus and squares are .... to each other.
2. Write the True / False for the following statements.
   (i) Any two similar figures are congruent.
   (ii) Any two congruent figures are similar.
   (iii) Two polygons are similar if their corresponding angles are equal.
3. Give two different examples of pair of
   (i) Similar fgures (ii) Non similar figures

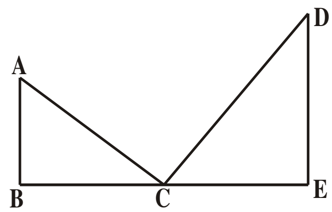
8.3 Similarity of Triangles

In the example we had drawn two triangles, those two triangles showed the property of similarity. We know that, two triangles are similar if their

(i) Corresponding Angles are equal and

(ii) Corresponding sides are in the same ratio (in proportion)

In ΔABC and ΔDEC in the introduction,



$\angle A = \angle D$, $\angle B = \angle E$, $\angle ACB = \angle DCE$

Also $\frac{DE}{AB} = \frac{EC}{BC} = \frac{DC}{AC} = K$ (scale factor)

then ΔABC is similar to ΔDEC

Symbolically we write $\Delta ABC \sim \Delta DEC$

(Symbol '~' is read as "Is similar to")

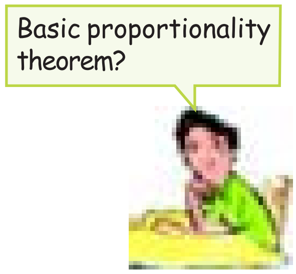
As we have stated K is a scale factor, So

if $K > 1$ we get enlarged figures,

$K = 1$ We get congruent figures and

$K < 1$ gives reduced (or diminished) figures

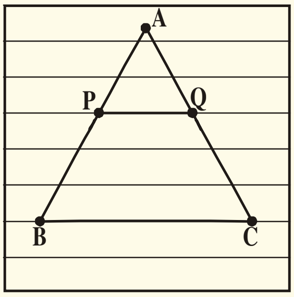
Further, in triangles ABC and DEC, corresponding angles are equal. So they are called equiangular triangles. The ratio of any two corresponding sides in two equiangular triangles is always the same. For proving this, Basic Proportionality theorem is used. This is also known as Thales Theorem.



To understand Basic proportionality theorem or Thales theorem, let us do the following activity.

Activity

Take any ruled paper and draw a triangle on that with base on one of the lines. Several lines will cut the triangle ABC. Select any one line among them and name the points where it meets the sides AB and AC as P and Q.



Find the ratio of $\frac{AP}{PB}$ and $\frac{AQ}{QC}$. What do you observe?

The ratios will be equal. Why ? Is it always true? Try for different lines intersecting the triangle. We know that all the lines on a ruled paper are parallel and we observe that every time the ratios are equal.

So in ΔABC, if $PQ \parallel BC$ then $\frac{AP}{PB} = \frac{AQ}{QC}$.

This is known as the result of basic proportionality theorem.

8.3.1 Basic Proportionality Theorem (Thales Theorem)

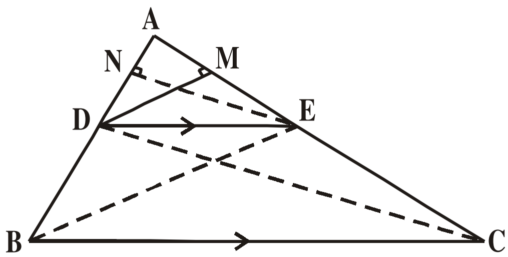
Theorem-8.1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Given : In ΔABC, $DE \parallel BC$ which intersects sides AB and AC at D and E respectively.

RTP: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join B, E and C, D and then draw

$DM \perp AC$ and $EN \perp AB$.



Proof : Area of $\Delta ADE = \frac{1}{2} \times AD \times EN$

Area of $\Delta BDE = \frac{1}{2} \times BD \times EN$

So $$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times BD \times EN} = \frac{AD}{BD} \quad ...(1)$$

Again Area of $\Delta ADE = \frac{1}{2} \times AE \times DM$

Area of $\Delta CDE = \frac{1}{2} \times EC \times DM$

$$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad ...(2)$$

Observe that ΔBDE and ΔCDE are on the same base DE and between same parallels BC and DE.

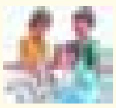
So $ar(\Delta BDE) = ar(\Delta CDE)$ ...(3)

From (1) (2) and (3), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved.

Is the converse of the above theorem also true? To examine this, let us perform the following activity.



ACTIVITY

Draw an angle XAY on your note book and on ray AX, mark points B_1, B_2, B_3, B_4 and B such that

$AB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B = 1\text{cm}$ (say)

Similarly on ray AY, mark points C_1, C_2, C_3, C_4 and C such that

$AC_1 = C_1C_2 = C_2C_3 = C_3C_4 = C_4C = 2$ cm (say)

Join B_1, C_1 and B, C.

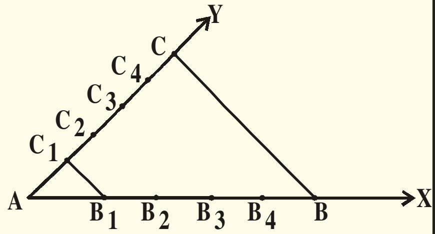
Observe that $\frac{AB_1}{B_1B} = \frac{AC_1}{C_1C} = \frac{1}{4}$ and $B_1C_1 \parallel BC$

Similarly, joining B_2C_2, B_3C_3 and B_4C_4, you see that

$$\frac{AB_2}{B_2B} = \frac{AC_2}{C_2C} = \frac{2}{3} \text{ and } B_2C_2 \parallel BC$$

$$\frac{AB_3}{B_3B} = \frac{AC_3}{C_3C} = \frac{3}{2} \text{ and } B_3C_3 \parallel BC$$

$$\frac{AB_4}{B_4B} = \frac{AC_4}{C_4C} = \frac{4}{1} \text{ and } B_4C_4 \parallel BC$$



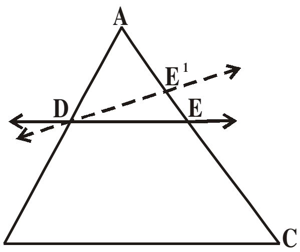
From this we obtain the following theorem called converse of the Thales theorem

Theorem-8.2 : If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.

Given : In ΔABC, a line DE is drawn such that $\frac{AD}{DB} = \frac{AE}{EC}$

RTP : DE ǁ BC

Proof : Assume that DE is not parallel to BC then draw the line DE^1 parallel to BC



So $\frac{AD}{DB} = \frac{AE'}{E'C}$ (why ?)

$\therefore$ $\frac{AE}{EC} = \frac{AE'}{E'C}$ (why ?)

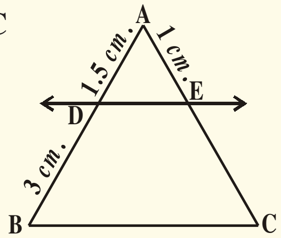
Adding 1 to both sides of the above, you can see that E and E′ must coincide (why ?)

Try This

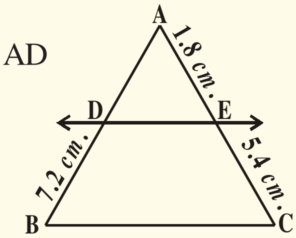
1. E and F are points on the sides PQ and PR respectively of ΔPQR. For each of the following, state whether. EF ǁQR or not?
   (i) PE = 3.9 cm EQ = 3 cm PF = 3.6 cm and FR = 2.4 cm
   (ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm.
   (iii) PQ = 1.28 cm PR = 2.56 cm PE = 1.8 cm and PF = 3.6 cm

2. In the following figures DE || BC.

(i) Find EC

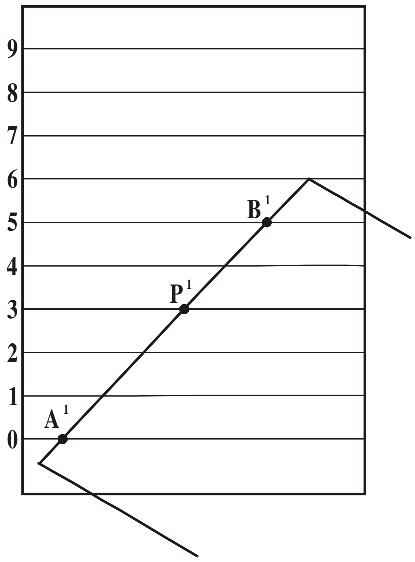


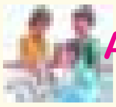
(ii) Find AD



Construction : Division of a line segment (using Thales theorem)

Madhuri drew a line segment. She wants to divide it in the ratio of 3 : 2. She measured it by using a scale and divided it in the required ratio. Meanwhile her elder sister came. She saw this and suggested Madhuri to divide the line segment in the given ratio without measuring it Madhuri was puzzled and asked her sister for help to do it. Then her sister explained. You may also do it by the following activity.

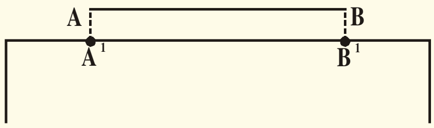




ACTIVITY

Take a sheet of paper from a lined note book. Number the lines by 1, 2, 3, ... starting with the bottom line numbered '0'.

Take a thick cardboard paper (or file card or chart strip) and place it against the given line segment AB and transfer its length to the card. Let A^1 and B^1 denote the points on the file card corresponding to A and B.



Now place A^1 on the zeroeth line of the lined paper and rotate the card about A^1 unitl point B^1 falls on the 5^{th} line (3 + 2).

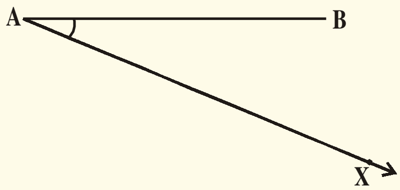
Mark the point where the third line touches the file card, by P^1.

Again place this card along the given line segment and transfer this point P^1 and denote it with 'P'.

So P is required point which divides the given line segment in the ratio 3:2.

Now let us learn how this construction can be done.

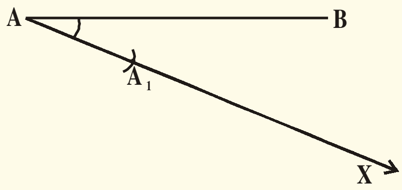
Given a line segment AB. We want to divide it in the ratio m : n where m and n are both positive integers. Let us take m = 3 and n = 2.

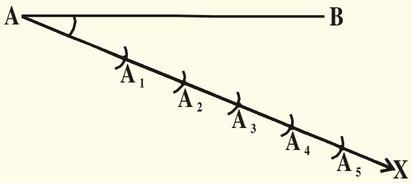


Steps :

1. Draw a ray AX through A making an acute angle with AB.

2. With 'A' as centre and with any length draw an arc on ray AX and label the point A_1.

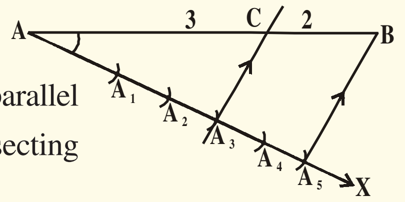




3. Using the same compass setting and with A_1 as centre draw another arc and locate A_2.

4. Like this locate 5 points $(= m + n)$ A_1, A_2, A_3, A_4, A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$

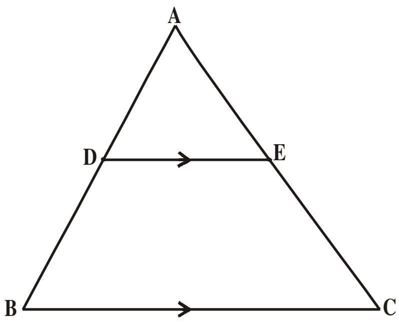
5. Join A_5B. Now through point $A_3 (m = 3)$ draw a line parallel to A_5B (by making an angle equal to $\angle A\ A_5\ B$) intersecting AB at C and obeserve that AC : CB = 3 : 2.



Now let us solve some examples on Thales theorem and its converse.

Example-1. In ΔABC, $DE \parallel BC$ and $\dfrac{AD}{DB} = \dfrac{3}{5}$.

AC = 5.6. Find AE.



Solution : In ΔABC, $DE \parallel BC$

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC} \text{ (by B.P.T)}$$

$$\text{but} \quad \frac{AD}{DB} = \frac{3}{5} \quad \text{So} \quad \frac{AE}{EC} = \frac{3}{5}$$

Given AC = 5.6 and AE : EC = 3 : 5.

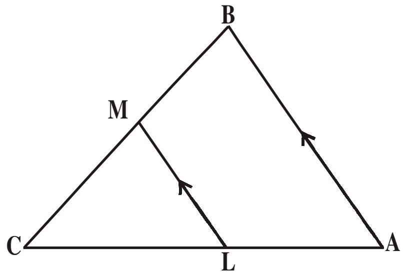
$$\frac{AE}{AC - AE} = \frac{3}{5}$$

$$\frac{AE}{5.6 - AE} \quad \frac{3}{5} \text{ (cross multiplication)}$$

$$5AE = (3 \times 5.6) - 3AE$$

$$8AE = 16.8$$

$$AE = \frac{16.8}{8} = 2.1\text{cm.}$$



Example-2. In the given figure LM || AB

AL = $x - 3$, AC = $2x$, BM = $x - 2$

and BC = $2x + 3$ find the value of x

Solution : In ΔABC, LM || AB

$$\Rightarrow \frac{AL}{LC} = \frac{BM}{MC} \text{ (by B.P.T)}$$

$$\frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)}$$

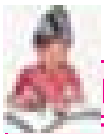
$$\frac{x-3}{x+3} \quad \frac{x-2}{x+5} \text{ (cross multiplication)}$$

$$(x - 3)(x + 5) = (x - 2)(x + 3)$$

$$x^2 + 2x - 15 = x^2 + x - 6$$

$$\Rightarrow \quad 2x - x = -6 + 15$$

$$x = 9$$

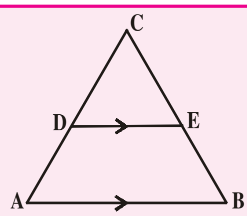


Do This

1. What value(s) of x will make DE || AB, in the given figure ?

   AD = $8x + 9$, CD = $x + 3$

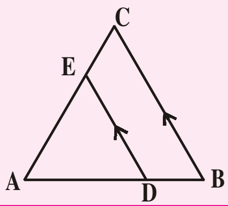
   BE = $3x + 4$, CE = x.



2. In ΔABC, DE || BC. AD = x, DB = $x - 2$,

   AE = $x + 2$ and EC = $x - 1$.

   Find the value of x.



Example-3. The diagonals of a quadrilateral ABCD intersect each other at point 'O' such that $\frac{AO}{BO} = \frac{CO}{DO}$. Prove that ABCD is a trapezium.

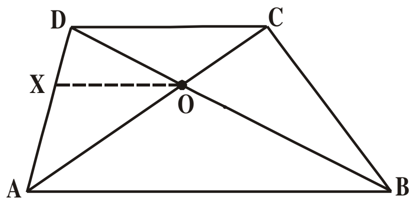
Solution : Given : In quadrilateral ABCD , $\frac{AO}{BO} = \frac{CO}{DO}$.

RTP : ABCD is a trapezium.

Construction : Through 'O' draw a line parallel to AB which meets DA at X.

Proof : In ΔDAB, XO || AB (by construction)

$$\Rightarrow \frac{DX}{XA} = \frac{DO}{OB} \qquad \text{(by basic proportionality theorem)}$$



$$\frac{AX}{XD} = \frac{BO}{OD} \qquad \text{..... (1)}$$

again $$\frac{AO}{BO} = \frac{CO}{DO} \qquad \text{(given)}$$

$$\frac{AO}{CO} = \frac{BO}{OD} \qquad \text{..... (2)}$$

From (1) and (2)

$$\frac{AX}{XD} = \frac{AO}{CO}$$

In ΔADC, XO is a line such that $\frac{AX}{XD} = \frac{AO}{OC}$

$\Rightarrow$ XO || DC (by converse of the basic the proportionality theorem)

$\Rightarrow$ AB || DC

In quadrilateral ABCD, AB || DC

$\Rightarrow$ ABCD is a trapezium (by definition)

Hence proved.

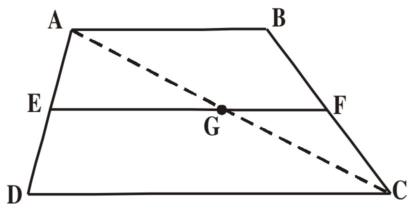
Example-4. In trapezium ABCD, AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF || AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$.

Solution : Let us join AC to intersect EF at G.

AB || DC and EF || AB (given)

$\Rightarrow$ EF || DC (Lines parallel to the same line are parallel to each other)

In ΔADC, EG || DC



So $\frac{AE}{ED} = \frac{AG}{GC}$ (by BPT) ...(1)

Similarly, In ΔCAB, GF || AB

$\frac{CG}{GA} = \frac{CF}{FB}$ (by BPT) i.e., $\frac{AG}{GC} = \frac{BF}{FC}$ ...(2)

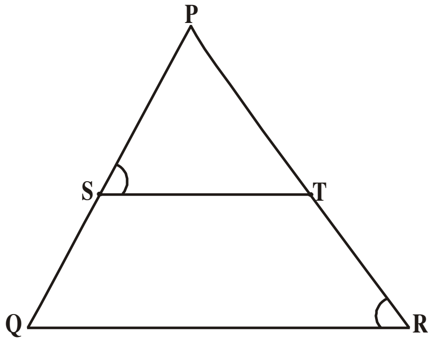
From (1) & (2) $\frac{AE}{ED} = \frac{BF}{FC}$.

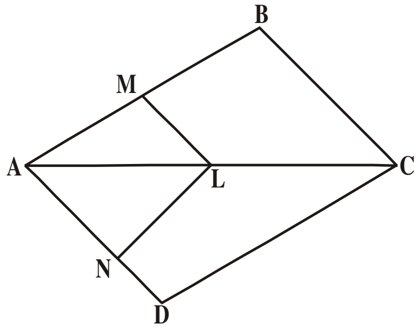
Exercise - 8.1

1. In ΔPQR, ST is a line such that $\frac{PS}{SQ} = \frac{PT}{TR}$ and

   also $\angle PST = \angle PRQ$.

   Prove that ΔPQR is an isosceles triangle.



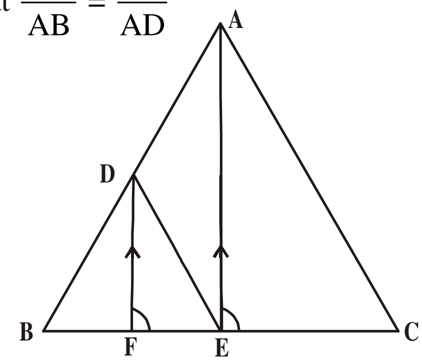


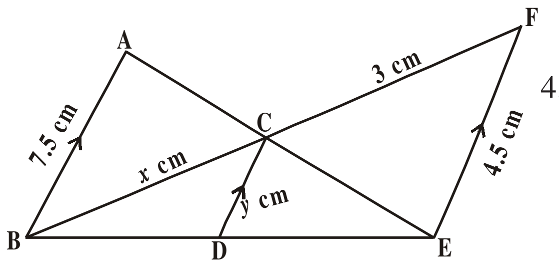
2. In the given figure, $LM \parallel CB$ and $LN \parallel CD$

   Prove that $\frac{AM}{AB} = \frac{AN}{AD}$

3. In the given figure, $DE \parallel AC$ and $DF \parallel AE$

   Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.





4. In the given figure, $AB \parallel CD \parallel EF$.

   given $AB = 7.5$ cm $DC = y$cm

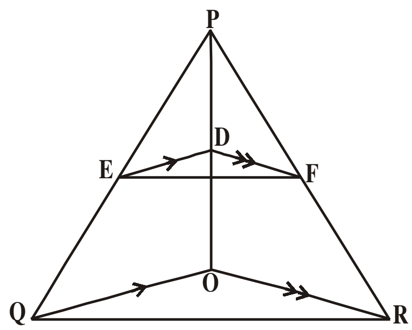
   $EF = 4.5$ cm, $BC = x$ cm.

   Calculate the values of x and y.

5. Prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side (Using basic proportionality theorem).

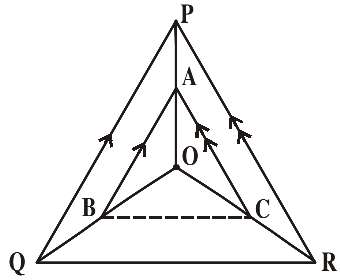
6. Prove that a line joining the midpoints of any two sides of a triangle is parallel to the third side. (Using converse of basic proportionality theorem)

7. In the given figure, $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.



8. In the adjacent figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC||PR.

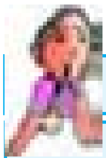
   Show that BC || QR.



9. ABCD is a trapezium in which AB||DC and its diagonals intersect each other at point 'O'.

   Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

10. Draw a line segment of length 7.2 cm and divide it in the ratio 5 : 3. Measure the two parts.



Think - Discuss and Write

Discuss with your friends that in what way similarity of triangles is different from similarity of other polygons?

8.4 Criteria for Similarity of Triangles

We know that two triangles are similar if corresponding angles are equal and corresponding sides are proportional. For checking the similarity of two triangles, we should check for the equality of corresponding angles and equality of ratios of their corresponding sides. Let us make an attempt to arrive at certain criteria for similarity of two triangles. Let us perform the following activity.

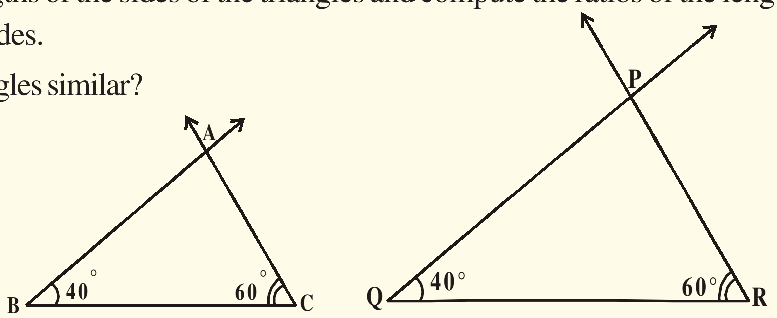
Activity

Use a protractor and ruler to draw two non congruent triangles so that each triangle has a 40° and 60° angle. Check the figures made by you by measuring the third angles of two triangles.

It should be each 80° (why?)

Measure the lengths of the sides of the triangles and compute the ratios of the lengths of the corresponding sides.

Are the triangles similar?



This activity leads us to the following criterion for similarity of two triangles.

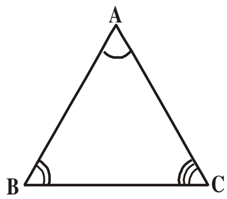
8.4.1 AAA Criterion for Similarity of Triangles

Theorem-8.3 : In two triangles, if the angles are equal, then the sides opposite to the equal angles are in the same ratio (or proportional) and hence the two triangles are similar.

Given : In triangles ABC and DEF,

$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

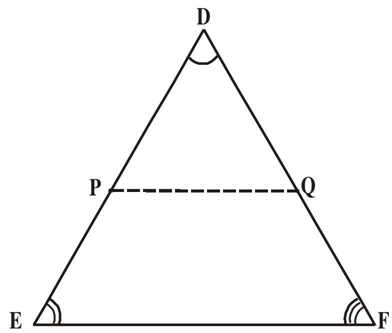
RTP : $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$



Construction : Locate points P and Q on DE and DF respectively,

such that AB = DP and AC = DQ. Join PQ.

Proof : $\Delta ABC \cong \Delta DPQ$ (why ?)



This gives $\angle B = \angle P = \angle E$ and PQ || EF (How ?)

$\therefore \dfrac{DP}{PE} = \dfrac{DQ}{QF}$ (why ?)

i.e., $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ (why ?)

Similarly $\dfrac{AB}{DE} = \dfrac{BC}{EF}$ and So $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$.

Hence proved.

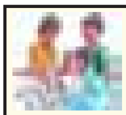
Note : If two angles of a triangle are respectively equal to the two angles of another triangle, then by the angle sum property of a triangle, third angles will also be equal.

So AA similarity criterion is stated as if two angles of one triangle are respectively equal to the two angles of anther triangle, then the two triangles are similar.

What about the converse of the above statement ?

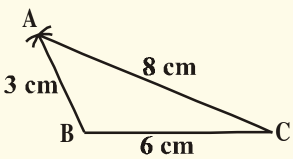
If the sides of a triangle are respectively proportional to the sides of another triangle, is it true that their corresponding angles are equal ?

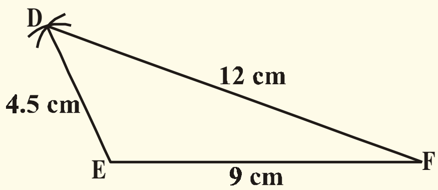
Let us exercise it through an activity.



Activity

Draw two triangles ABC and DEF such that AB = 3 cm, BC = 6 cm, CA = 8 cm, DE = 4.5 cm, EF = 9 cm and FD = 12 cm.





So you have $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2}{3}$.

Now measure the angles of both the triangles. What do you observe? What can you say about the corresponding angles? They are equal, so the triangles are similar. You can verify it for different triangles.

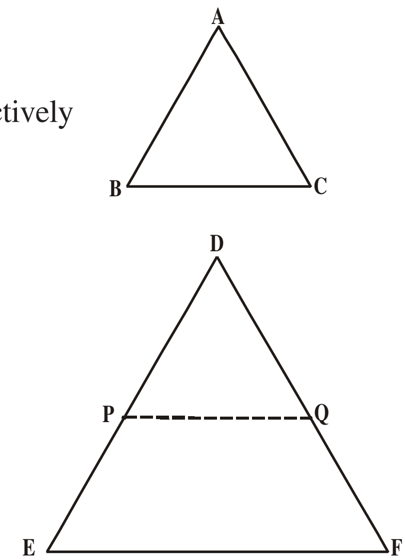
From the above activity, we can give the following criterion for similarity of two triangles.

8.4.2. SSS Criterion for Similarity of Triangles

Theorem-8.4 : If in two triangles, the sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the triangles are similar.

Given : ΔABC and ΔDEF are such that

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} \quad (< 1)$$



RTP : $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

Construction : Locate points P and Q on DE and DF respectively such that AB = DP and AC = DQ. Join PQ.

Proof : $\frac{DP}{PE} = \frac{DQ}{QF}$ and PQ || EF (why ?)

So $\angle P = \angle E$ and $\angle Q = \angle F$ (why ?)

$\therefore \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$

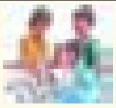
So $\frac{DP}{DE} = \frac{DQ}{DF} = \frac{BC}{EF}$ (why ?)

So BC = PQ (Why ?)

$\Delta ABC \cong \Delta DPQ$ (why ?)

So $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ (How ?)

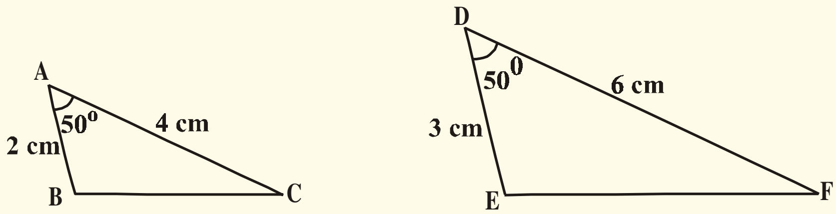
We studied that for similarity of two polygons any one condition is not sufficient. But for the similarity of triangles, there is no need for fulfillment of both the conditions as one automatically implies the other. Now let us look for SAS similarity criterion. For this, let us perform the following activity.



Activity

Draw two triangles ABC and DEF such that AB = 2 cm, $\angle A = 50^0$ AC = 4cm,

DE = 3 cm, $\angle D = 50^0$ and DF = 6cm.



Observe that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{2}{3}$ and $\angle A = \angle D = 50^0$.

Now measure $\angle B$, $\angle C$, $\angle E$, $\angle F$ also measure BC and EF.

Observe that $\angle B = \angle E$ and $\angle C = \angle F$ also $\frac{BC}{EF} = \frac{2}{3}$.

So, the two triangles are similar. Repeat the same for triangles with different measurements, which gives the following criterion for similarity of triangles.

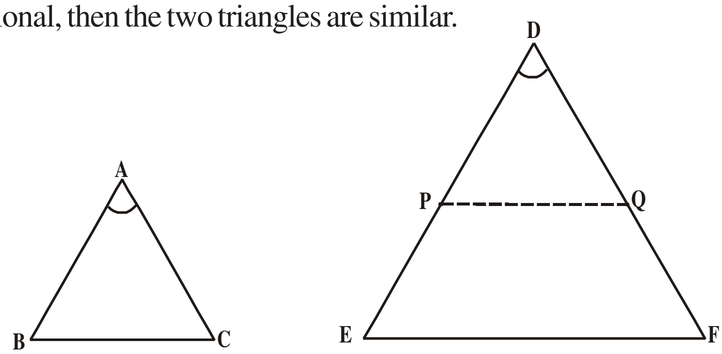
8.4.3 SAS Criterion for Similarity of Triangles

Theorem-8.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

Given : In ΔABC and ΔDEF

$\frac{AB}{DE} = \frac{AC}{DF}$ (<1) and

$\angle A = \angle D$



RTP : $\Delta ABC \sim \Delta DEF$

Construction : Locate points P and Q on DE and DF respectively such that AB = DP and AC = DQ. Join PQ.

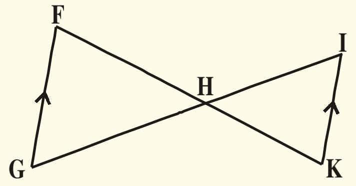
Proof : PQ || EF and $\Delta ABC \cong \Delta DPQ$ (How ?)

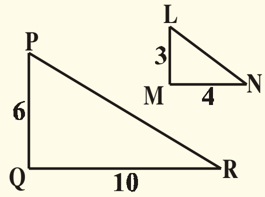
So $\angle A = \angle D$, $\angle B = \angle P$, $\angle C = \angle Q$

$\therefore$ $\Delta ABC \sim \Delta DEF$ (why ?)

TRY THIS

1. Are the triangles similar? If so, name the criterion of similarity. Write the similarity relation is symbolic form.

(i)


(ii)


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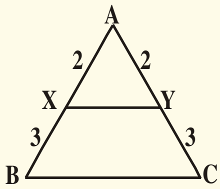

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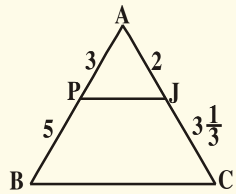

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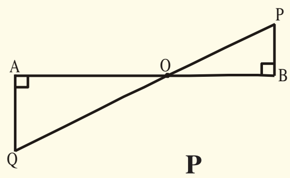

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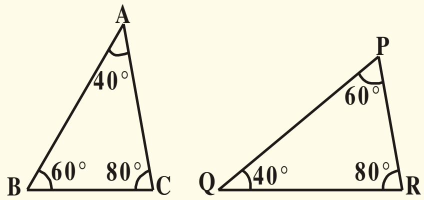

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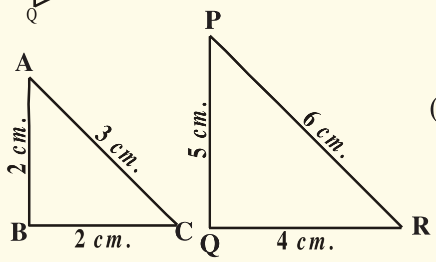

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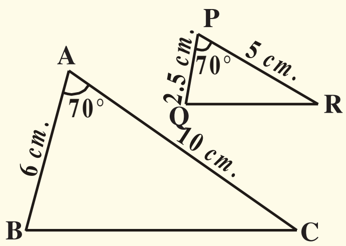
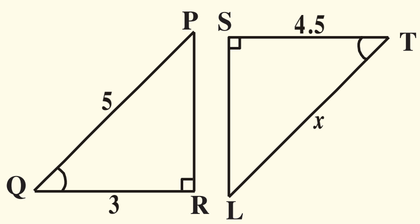
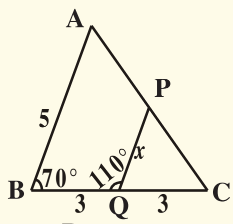
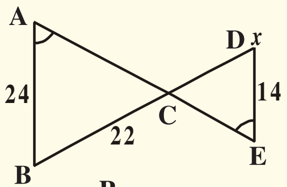
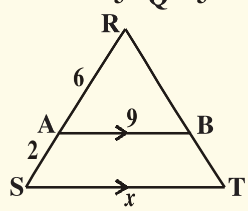


2. Explain why the triangles are similar and then find the value of x.

(i)


(ii)


(iii)


(iv)


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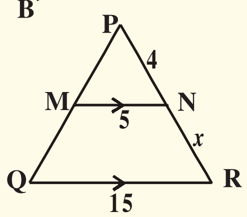

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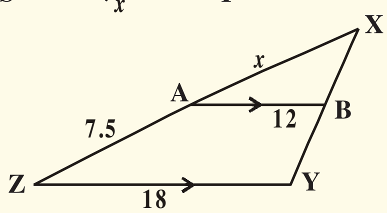
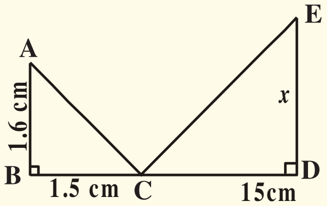
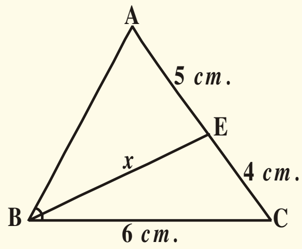
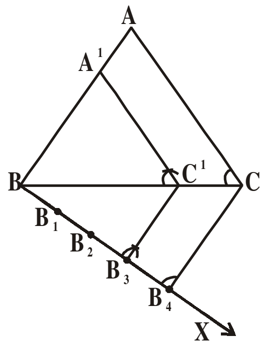


(vii) 

(viii) 

Construction : To construct a triangle similar to a given triangle as per given scale factor.

a) Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{3}{4}$ of corresponding sides of ΔABC (scale factor $\frac{3}{4}$)



Steps : 1. Draw a ray BX, making an acute angle with BC on the side opposite to vertex A.

2. Locate 4 points B_1, B_2, B_3 and B_4 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

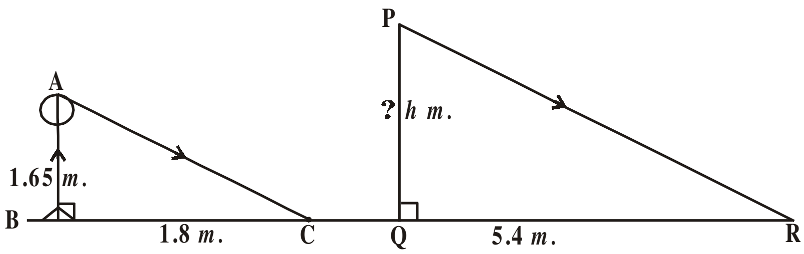
3. Join B_4C and draw a line through B_3 parallel to B_4C intersecting BC at C'.

4. Draw a line through C' parallel to CA to intersect AB at A'.

So $\Delta A'BC'$ is the required triangle.

Let us take some examples to illustrate the use of these criteria.

Example-5. A person 1.65m tall casts 1.8m shadow. At the same instance, a lamp-posts casts a shadow of 5.4 m. Find the height of the lamppost.



Solution: In ΔABC and ΔPQR

$\angle B = \angle Q = 90^0$.

$\angle C = \angle R$ (AC || PR, all sun's rays are parallel at any instance)

$\Delta ABC \sim \Delta PQR$ (by AA similarity)

$\frac{AB}{PQ} = \frac{BC}{QR}$ (cpst, corresponding parts of Similar triangles)

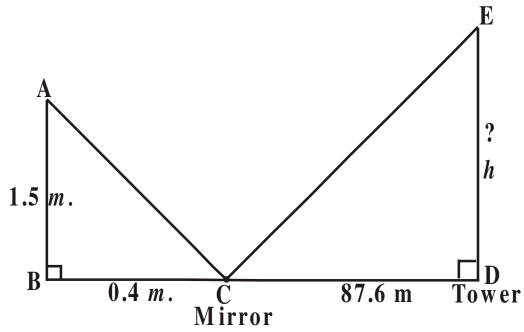
$$\frac{1.65}{PQ} = \frac{1.8}{5.4}$$

$$PQ = \frac{1.65 \times 5.4}{1.8} = 4.95m$$

The height of the lamp post is 4.95m.

Example-6. A man sees the top of a tower in a mirror which is at a distance of 87.6m from the tower. The mirror is on the ground facing upwards. The man is 0.4m away from the mirror and his height is 1.5m. How tall is the tower?

Solution : In ΔABC & ΔEDC



$\angle ABC = \angle EDC = 90°$

$\angle BCA = \angle DCE$ (angle of incidence and angle of reflection are same)

$\Delta ABC \sim \Delta EDC$ (by AA similarity)

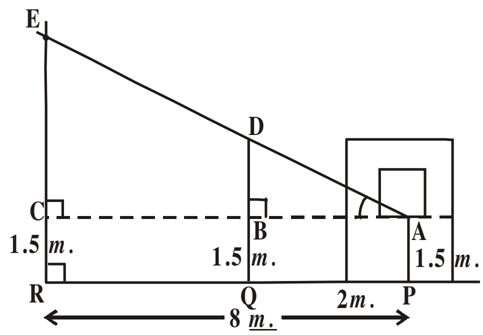
$$\frac{AB}{ED} = \frac{BC}{CD} \Rightarrow \frac{1.5}{h} = \frac{0.4}{87.6}$$

$$h = \frac{1.5 \times 87.6}{0.4} = 328.5m$$

Hence, the height of the towers is 328.5m.

Example7. Gopal is worrying that his neighbour can see into his living room from the top floor of his house. He has decided to build a fence that is high enough to block the view from their top floor window. What should be the height of the fence? The measurements are given in the figure.

Solution : In ΔABD & ΔACE



$\angle B = \angle C = 90°$

$\angle A = \angle A$ (common angle)

$\Delta ABD \sim \Delta ACE$ (by AA similarity)

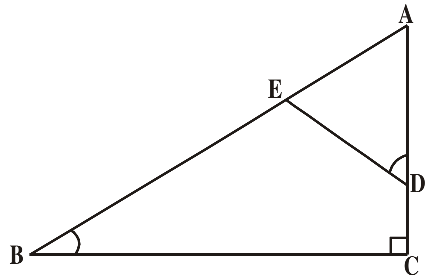
$$\frac{AB}{AC} = \frac{BD}{CE} \Rightarrow \frac{2}{8} = \frac{BD}{1.2}$$

$$BD = \frac{2 \times 1.2}{8} = \frac{2.4}{8} = 0.3m$$

Total height of the fence required is 1.5 m. + 0.3 m. = 1.8m to block the neightbour's view.

Exercise - 8.2

1. In the given figure, $\angle ADE = \angle B$



   (i) Show that $\Delta ABC \sim \Delta ADE$

   (ii) If AD = 3.8 cm, AE = 3.6cm

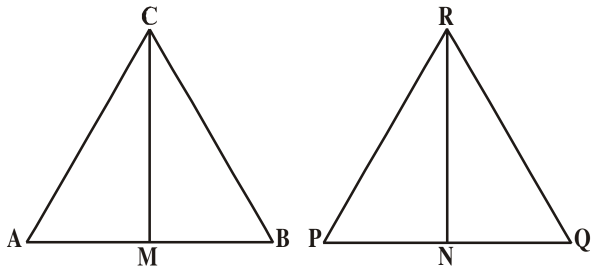
   BE = 2.1 cm   BC = 4.2 cm

   find DE.

2. The perimeters of two similar triangles are 30 cm and 20 cm respectively. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

3. A girl of height 90 cm is walking away from the base of a lamp post at a speed of 1.2 m/sec. If the lamp post is 3.6m above the ground, find the length of her shadow after 4 seconds.

4. CM and RN are respectively the medians of ΔABC and ΔPQR. Prove that



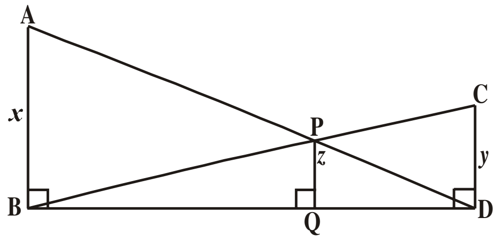
   (i) $\Delta AMC \sim \Delta PNR$

   (ii) $\dfrac{CM}{RN} = \dfrac{AB}{PQ}$

   (iii) $\Delta CMB \sim \Delta RNQ$

5. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point 'O'. Using the criterion of similarity for two triangles, show that $\dfrac{OA}{OC} = \dfrac{OB}{OD}$.

6. AB, CD, PQ are perpendicular to BD.



   AB = x, CD = y amd PQ = Z

   prove that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$.

7. A flag pole 4m tall casts a 6 *m*., shadow. At the same time, a nearby building casts a shadow of 24m. How tall is the building ?

8. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔFEG respectively. If $\Delta ABC \sim \Delta FEG$ then show thatG

   (i) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$ (ii) $\Delta DCB \sim \Delta HGE$ (iii) $\Delta DCA \sim \Delta HGF$

9. AX and DY are altitudes of two similar triangles ΔABC and ΔDEF. Prove that AX : DY = AB : DE.

10. Construct a triangle shadow similar to the given ΔABC, with its sides equal to $\frac{5}{3}$ of the corresponding sides of the triangle ABC.

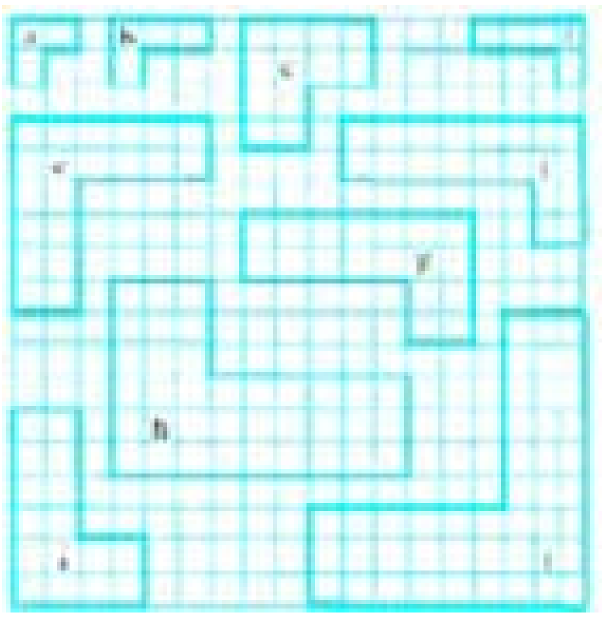
11. Construct a triangle of sides 4cm, 5 cm and 6 cm. Then, construct a triangle similar to it, whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

12. Construct an Isosceles triangle whose base is 8cm and altitude is 4 *cm*. Then, draw another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

8.5 Areas of Similar Triangles

For two similar triangles, ratio of their corresponding sides is the same. Do you think there is any relationship between the ratio of their areas and the ratio of their corresponding sides ? Let us do the following activity to understand this.

Activity



Make a list of pairs of similar polygons in this figure.

Find

(i) The ratio of similarity and

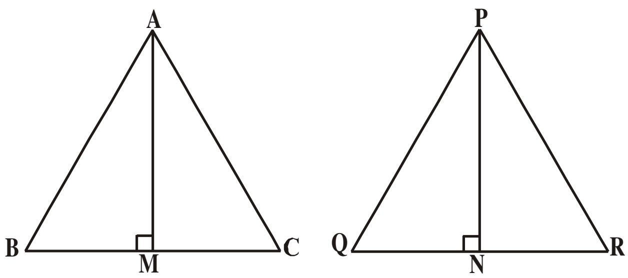
(ii) The ratio of areas.

You will observe that ratio of areas is the square of the ratoio of their corresponding sides.

Let us prove it like a theorem.

Theorem-8.6 : The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Given : ΔABC ~ ΔPQR



RTP : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$.

Construction : Draw $AM \perp BC$ and $PN \perp QR$.

Proof : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN}$ ...(1)

In ΔABM & ΔPQN

$\angle B = \angle Q$ ($\because$ $\Delta ABC \sim \Delta PQR$)

$\angle M = \angle N = 90^0$

$\therefore$ $\Delta ABM \sim \Delta PQN$ (by AA similarity)

$\frac{AM}{PN} = \frac{AB}{PQ}$ ...(2)

Also $\Delta ABC \sim \Delta PQR$ (given)

$\boxed{\frac{AB}{PQ} = \frac{BC}{QR}} = \frac{AC}{PR}$ ...(3)

$\therefore$ $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ from (1), (2) and (3)

$= \left(\frac{AB}{PQ}\right)^2$.

Now by using (3), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

Hence proved.

Now let us see some examples.

Example-8. Prove that if the areas of two similar triangles are equal, then they are congruent.

Solution : $\Delta ABC \sim \Delta PQR$

So $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{AC}{PR}\right)^2$

But $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1$ $(\because$ areas are equal)

$$\left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 = 1$$

So $AB^2 = PQ^2$

$BC^2 = QR^2$

$AC^2 = PR^2$

From which we get $AB = PQ$

$BC = QR$

$AC = PR$

$\therefore$ $\Delta ABC \cong \Delta PQR$ (by SSS congruency)

Example-9. $\Delta ABC \sim \Delta DEF$ and their areas are respectively $64cm^2$ and $121\ cm^2$.

If $EF = 15.4$ cm., then find BC.

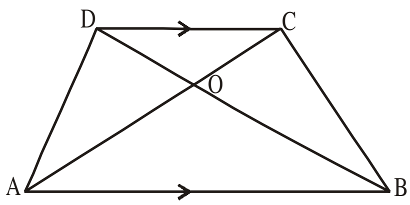
Solution : $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\dfrac{BC}{EF}\right)^2$

$$\frac{64}{121} = \left(\frac{BC}{15.4}\right)^2$$

$$\frac{8}{11} = \frac{BC}{15.4} \Rightarrow BC = \frac{8 \times 15.4}{11} = 11.2cm.$$

Example-10. Diagonals of a trapezium ABCD with AB || DC, intersect each other at the point 'O'. If AB = 2CD, find the ratio of areas of triangles AOB and COD.

Solution : In trapezium ABCD, AB || DC also AB = 2CD.



In ΔAOB and ΔCOD

$\angle AOB = \angle COD$ (vertically opposite angles)

$\angle OAB = \angle OCD$ (alternate interior angles)

$\Delta AOB \sim \Delta COD$ (by AA similarity)

$$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{DC^2}$$

$$= \frac{(2DC)^2}{(DC)^2} = \frac{4}{1}$$

$\therefore$ ar(ΔAOB) : ar(ΔCOD) = 4 : 1.

Exercise - 8.3

1. Equilateral triangles are drawn on the three sides of a right angled triangle. Show that the area of the triangle on the hypotenuse is equal to the sum of the areas of triangles on the other two sides.

2. Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangles described on its diagonal.

3. D, E, F are mid points of sides BC, CA, AB of ΔABC. Find the ratio of areas of ΔDEF and ΔABC.

4. In ΔABC, XY || AC and XY divides the triangle into two parts of equal area. Find the ratio of $\frac{AX}{XB}$.

5. Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

6. $\Delta ABC \sim \Delta DEF$. BC = 3cm EF = 4cm and area of ΔABC = 54 cm^2. Determine the area of ΔDEF.

7. ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm. and BP = 3cm., AQ = 1.5 cm., CQ = 4.5 cm.

   Prove that (area of ΔAPQ) = $\frac{1}{16}$(area of ΔABC).

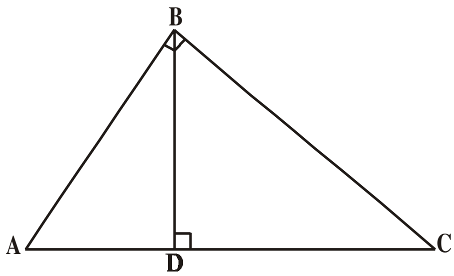
8. The areas of two similar triangles are 81cm^2 and 49 cm^2 respectively. If the attitude of the bigger triangle is 4.5 cm. Find the corresponding attitude fo the smaller triangle.

8.6 Pythagoras Theorem

You are familar with the Pythagoras theorem, you had verified this theorem through some activities. Now we shall prove this theorem using the concept of similarity of triangles. For this, we make use of the following result.

Theorem-8.7 : If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

Proof: ABC is a right triangle, right angled at B. Let BD be the perpendicular to hypotenuse AC.



In ΔADB and ΔABC

$\angle A = \angle A$

And $\angle ADB = \angle ABC$ (why ?)

So $\Delta ADB \sim \Delta ABC$ (How ?) ...(1)

Similarly, $\Delta BDC \sim \Delta ABC$ (How ?) ...(2)

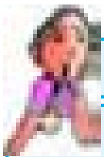
So from (1) and (2), triangles on both sides of the perpendicular BD are similar to the whole triangle ABC.

Also since $\Delta ADB \sim \Delta ABC$

$\Delta BDC \sim \Delta ABC$

So $\Delta ADB \sim \Delta BDC$

This leads to the following theorem.

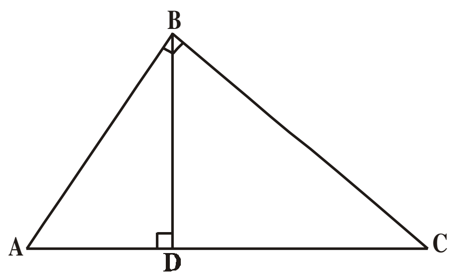


Think - Discuss

For a right angled triangle with integer sides atleast one of its measurements must be an even number. Why? Discuss this with your friends and teachers.

8.6.1 Pythagoras Theorem (Baudhayan Theorem)

Theorem-8.8 : In a right triangle, the square of hypotenuse is equal to the sum of the squares of the other two sides.



Given: ΔABC is a right triangle right angled at B.

RTP : $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$.

Proof : $\Delta ADB \sim \Delta ABC$

$\Rightarrow \dfrac{AD}{AB} = \dfrac{AB}{AC}$ (sides are proportional)

$AD \,.\, AC = AB^2$ ...(1)

Also, $\Delta BDC \sim \Delta ABC$

$$\Rightarrow \frac{CD}{BC} = \frac{BC}{AC}$$

$CD \,.\, AC = BC^2$ ...(2)

On adding (1) & (2)

$AD \,.\, AC + CD \,.\, AC = AB^2 + BC^2$

$AC\,(AD + CD) = AB^2 + BC^2$

$AC \,.\, AC = AB^2 + BC^2$

$$\boxed{AC^2 = AB^2 + BC^2}$$

The above theorem was earlier given by an ancient Indian mathematician **Baudhayan** (about 800 BC) in the following form.

"The diagonal of a rectangle produces by itself the same area as produced by its both sides (i.e. length and breadth)." So sometimes, this theorem is also referred to as the Baudhayan theorem.

What about the converse of the above theorem ?

We prove it like a theorem, as done earlier also.

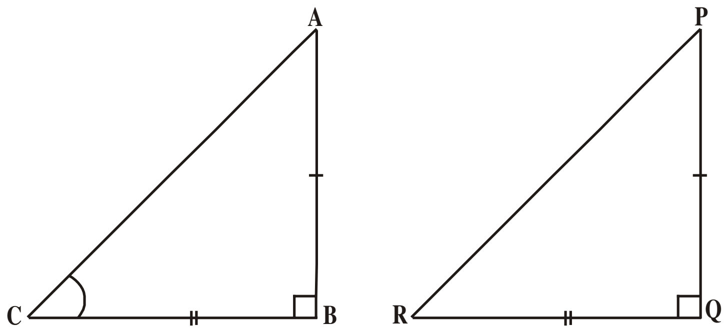
Theorem-8.9 : In a triangle if square of one side is equal to the sum of squares of the other two sides, then the angle opposite to the first side is a right angle and the triangle is a right angled triangle.

Given : In ΔABC,

$AC^2 = AB^2 + BC^2$

RTP : $\angle B = 90^0$.

Construction : Construct a right angled triangle ΔPQR right angled at Q such that PQ = AB and QR = BC.



Proof : In ΔPQR, $PR^2 = PQ^2 + QR^2$ (Pythagores theorem as $\angle Q = 90^0$)

$PR^2 = AB^2 + BC^2$ (by construction) ...(1)

but $AC^2 = AB^2 + BC^2$ (given) ...(2)

$\therefore$ AC = PR from (1) & (2)

Now In ΔABC and ΔPQR

AB = PQ (by construction)

BC = QE (by construction)

AC = PR (proved)

$\therefore \quad \Delta ABC \cong \Delta PQR$ (by SSS congruency)

$\therefore \quad \angle B = \angle Q$ (by cpct)

but $\angle Q = 90°$ (by construction)

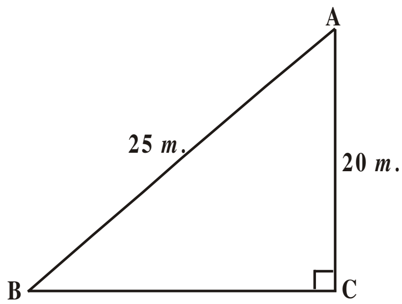
$\therefore \quad \angle B = 90°$.

Hence proved.

Now let us take some examples.

Example-11. A ladder 25m long reaches a window of building 20m above the ground. Determine the distance of the foot of the ladder from the building.

Solution : In ΔABC, $\angle C = 90°$



$\Rightarrow AB^2 = AC^2 + BC^2$ (by Pythogorous theorem)

$25^2 = 20^2 + BC^2$

$BC^2 = 625 - 400 = 225$

$BC = \sqrt{225} = 15m$

Hence, the foot of the ladder is at a distance of 15m from the building.

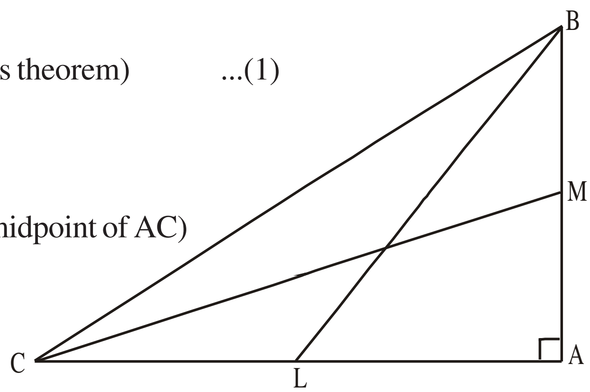
Example-12. BL and CM are medians of a triangle ABC right angled at A.

Prove that $4(BL^2 + CM^2) = 5BC^2$.

Solution : BL and CM are medians of ΔABC in which $\angle A = 90°$.

In ΔABC

$BC^2 = AB^2 + AC^2$ (Pythagores theorem) ...(1)



In ΔABL, $BL^2 = AL^2 + AB^2$

So $BL^2 = \left(\frac{AC}{2}\right)^2 + AB^2$ ($\because$ L is the midpoint of AC)

$BL^2 = \frac{AC^2}{4} + AB^2$

$\therefore \quad 4BL^2 = AC^2 + 4AB^2$ ...(2)

In ΔCMA, $CM^2 = AC^2 + AM^2$

$CM^2 = AC^2 + \left(\frac{AB}{2}\right)^2$ ($\because$ M is the mid point of AB)

$$CM^2 = AC^2 + \frac{AB^2}{4}$$

$$4CM^2 = 4AC^2 + AB^2 \quad ...(3)$$

On adding (2) and (3), we get

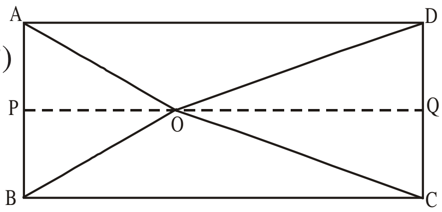
$$4(BL^2 + CM^2) = 5(AC^2 + AB^2)$$

$$\therefore\ 4(BL^2 + CM^2) = 5BC^2 \quad \text{from (1).}$$

Example-13. 'O' is any point inside a rectangle ABCD.

Prove that $OB^2 + OD^2 = OA^2 + OC^2$

Solution : Through 'O' draw PQ || BC so that P lies on AB and Q lies on DC.



Now PQ || BC

$\therefore$ PQ $\perp$ AB & PQ $\perp$ DC ($\because \angle B = \angle C = 90°$)

So, $\angle BPQ = 90°$ & $\angle CQP = 90°$

$\therefore$ BPQC and APQD are both rectangles.

Now from ΔOPB. $OB^2 = BP^2 + OP^2$ ...(1)

Similarly from ΔOQD, we have $OD^2 = OQ^2 + DQ^2$ ...(2)

From ΔOQC, we have $OC^2 = OQ^2 + CQ^2$ ...(3)

And from ΔOAP, $OA^2 = AP^2 + OP^2$

Adding (1) & (2)

$$OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2$$

$$= CQ^2 + OP^2 + OQ^2 + AP^2 \quad (\because BP = CQ \text{ and } DQ = AP)$$

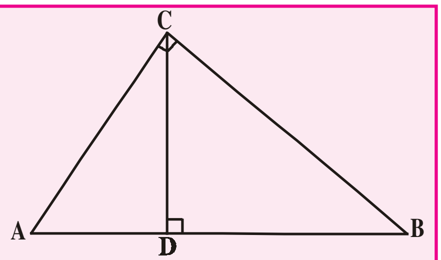
$$= CQ^2 + OQ^2 + OP^2 + AP^2$$

$$= OC^2 + OA^2 \text{ (from (3) \& (4))}$$

Do This

1. In ΔACB, $\angle C = 90^0$ and CD $\perp$ AB

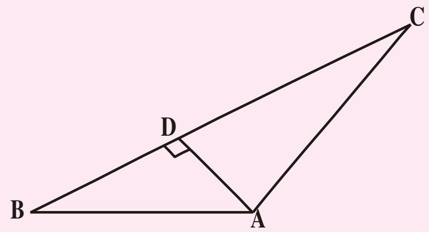
   Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$.



2. A ladder 15m long reaches a window which is 9 m above the ground on one side of a street. Keeping its foot at the same point, the ladder is turned to other side of the street to reach a window 12m high. Find the width of the street.

3. In the given fig. if $AD \perp BC$

Prove that $AB^2 + CD^2 = BD^2 + AC^2$.



Example-14. The hypotenuse of a right triangle is 6m more than twice of the shortest side. If the third side is 2 *m*., less than the hypotenuse, find the sides of the triangle.

Solution : Let the shortest side be x m.

Then htypotenuse = (2x + 6)m and third side = (2x + 4)m.

by Pythagores theorem, we have

$(2x + 6)^2 = x^2 + (2x + 4)^2$

$4x^2 + 24x + 36 = x^2 + 4x^2 + 16x + 16$

$x^2 - 8x - 20 = 0$

$(x - 10)(x + 2) = 0$

$x = 10$ or $x = -2$

but x can't be negative as side of a triangle.

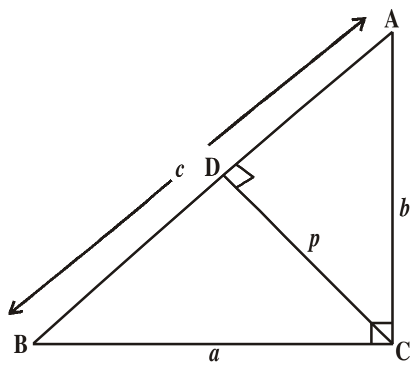
$\therefore\ x = 10$

Hence, the sides of the triangle are 10m, 26m and 24m.

Example-15. ABC is a right triangle right angled at C. Let BC = a, CA = b, AB = c and let p be the length of perpendicular from C on AB. Prove that (i) pc = ab (ii) $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$.

Solution :

(i) $CD \perp AB$ and CD = p.



Area of $\Delta ABC = \dfrac{1}{2} \times AB \times CD$

$= \dfrac{1}{2} cp$.

also area of $\Delta ABC = \dfrac{1}{2} \times BC \times AC$

$= \dfrac{1}{2} ab$

$$\frac{1}{2}cp = \frac{1}{2}ab$$

$$cp = ab \qquad ...(1)$$

(ii) Since ΔABC is a right triangle right angled at C.

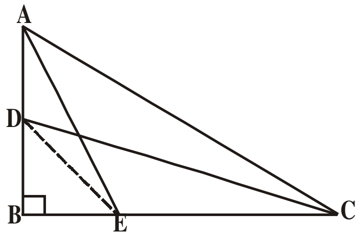
$$AB^2 = BC^2 + AC^2$$

$$c^2 = a^2 + b^2$$

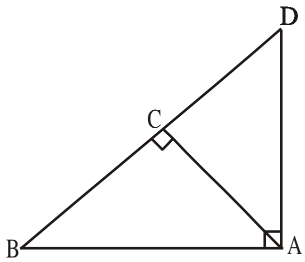
$$\left(\frac{ab}{p}\right)^2 = a^2 + b^2$$

$$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$

Exercise - 8.4



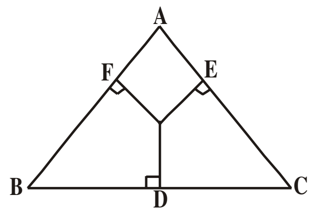
1. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.
2. ABC is a right triangle right angled at B. Let D and E be any points on AB and BC respectively.
   Prove that $AE^2 + CD^2 = AC^2 + DE^2$.
3. Prove that three times the square of any side of an equilateral triangle is equal to four times the square of the altitude.
4. PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$.
   Show that $PM^2 = QM \,.\, MR$.
5. ABD is a triangle right angled at A and $AC \perp BD$

   Show that (i) $AB^2 = BC.BD$.

   (ii) $AC^2 = BC.DC$

   (iii) $AD^2 = BD.CD$.
6. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.
7. 'O' is any point in the interior of a triangle ABC.
   $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$, show that

   (i) $OA^2+OB^2+OC^2-OD^2-OE^2-OF^2 = AF^2+BD^2+CE^2$

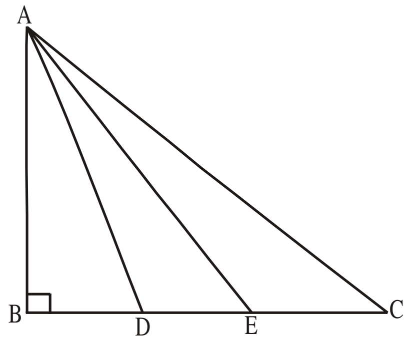
   (ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.

8. A wire attached to veritical pole of height 18m is 24m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

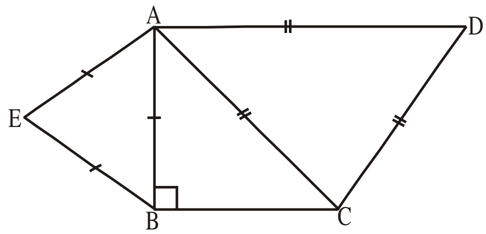
9. Two poles of heights 6m and 11m stand on a plane ground. If the distance between the feet of the poles is 12m find the distance between their tops.

10. In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that $9AD^2 = 7AB^2$.

11. In the given figure, ABC is a triangle right angled at B. D and E are ponts on BC trisect it. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



12. ABC is an isosceles triangle right angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and ΔACD.



8.7 Different forms of theoritical statements

1. Negation of a statement :

We have a statement and if we add "Not" after the statement, we will get a new statement; which is called negation of the statement.

For example take a statement "ΔABC is a equilateral". If we denote it by "*p*", we can write like this.

p : Triangle ABC is equilateral and its negation will be "Triangle ABC is not equilateral". Negation of statement *p* is denoted by ~*p*; and read as negotiation of *p*. the statement ~*p* negates the assertion that the statement *p* makes.

When we write the negation of the statements we would be careful that there should no confusion; in understanding the statement.

Observe this example carefully

P : All irrational numbers are real numbers. We can write negation of *p* like these ways.

i) $\sim p$: All irrational numbers are not real numbers.

ii) $\sim p$ Not all the irrational are real numbers.

How do we decide which negation is correct? We use the following criterion "Let p be a statement and $\sim p$ its negation. Then $\sim p$ is false whenever p is true and $\sim p$ is true whenever p is false.

For example s : $2 + 2 = 4$ is True

$\sim s$: $2 + 2 \neq 4$ is False

2. Converse of a statement :

A sentence which is either true or false is called a simple statement. If we combine two simple statements then we will get a compound statement. Connecting two simple statements with the use of the words "If and then" will give a compound statement which is called implication (or) conditional.

Combining two simple statements p & q using if and then, we get p implies q which can be denoted by $p \Rightarrow q$. In this $p \Rightarrow q$, suppose we interchange p and q we get $q \Rightarrow p$. This is called its converse.

Example : $p \Rightarrow q$: In ΔABC, if AB = AC then $\angle C = \angle B$

Converse $q \Rightarrow p$: In ΔABC, if $\angle C = \angle B$ then AB = AC

3. Proof by contradiction :

In this proof by contradiction, we assume the negation of the statement as true; which we have to prove. In the process of proving we get contradiction somewhere. Then, we realize that this contradiction occur because of our wrong assumption which is negation is true. Therefore we conclude that the original statement is true.

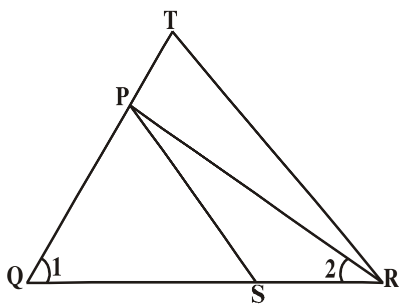
Optional Exercise

[This exercise is not meant for examination]

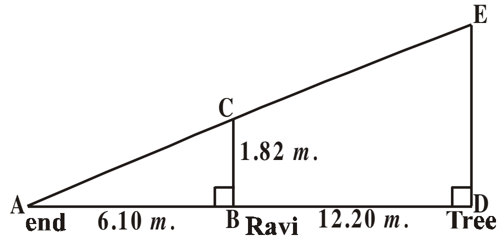
1. In the given figure,

$$\frac{QT}{PR} = \frac{QR}{QS} \text{ and } \angle 1 = \angle 2$$

prove that $\Delta PQS \sim \Delta TQR$.



2. Ravi is 1.82m tall. He wants to find the height of a tree in his backyard. From the tree's base he walked 12.20 *m.* along the tree's shadow to a position where the end of his shadow exactly overlaps the end of the tree's shadow. He is now 6.10m from the end of the shadow. How tall is the tree ?

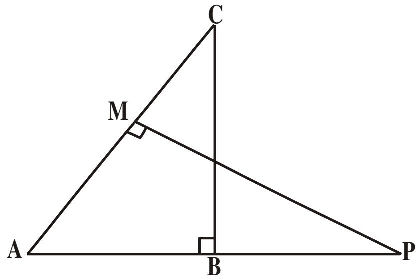


3. The diagonal AC of a parallelogram ABCD intersects DP at the point Q, where 'P' is any point on side AB. Prove that $CQ \times PQ = QA \times QD$.

4. ΔABC and ΔAMP are two right triangles right angled at B and M respectively.

   Prove that (i) $\Delta ABC \sim \Delta AMP$

   (ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$.



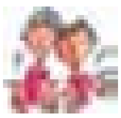
5. An aeroplane leeves an airport and flies due north at a speed of 1000 kmph. At the same time another aeroplane leeves the same airport and flies due west at a speed of 1200 kmph. How far apart will the two planes be after $1\frac{1}{2}$ hour?

6. In a right triangle ABC right angled at C. P and Q are points on sides AC and CB respectively which divide these sides in the ratio of 2 : 1.

   Prove that (i) $9AQ^2 = 9AC^2 + 4BC^2$

   (ii) $9BP^2 = 9BC^2 + 4AC^2$

   (iii) $9(AQ^2 + BP^2) = 13AB^2$



What We Have Discussed

1. Two figures having the same shape but not necessarily of the same size are called similar figures.
2. All the congruent figures are similar but the converse is not true.
3. Two polygons of the same number of sides are similar

   If(i) their corresponding angles are equal and

   (ii) Their corresponding sides are in the same ratio (ie propostion)

   For similarity of polygons either of the above two condition is not sufficient.
4. If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points then the other two sides are divided in the same ratio.

5. If a line divides any two sides of a triangle in the same ratio, then the line is parllel to the third side.
6. In in two triangles, angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (AAA similarity)
7. If two angles of a triangle are equal to the two angles of another triangle, then third angles of both triangles are equal by angle sum property of at triangle.
8. In in two triangles, corresponding sides are in the serve ratio, then their corresponding angles are equal and hence the triangles are similar. (SSS similar)
9. If one angle of triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio then the triangle are similar. (SAS similarity)
10. The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
11. If a perpendicular is drawn from the vertex of a right the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.
12. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Pythogores Theorem).
13. In a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Puzzle

Draw a triangle. Join the mid-point of the sides of the triangle. You get 4 triangles Again join the mid-points of these triangles. Repeat this process. All the triangles drawn are similar triangles. Why ? think and discuss with your friends.