CHAPTER 6

Progressions

6.1 INTRODUCTION

You must have observed that in nature, many things follow a certain pattern such as the petals of a sunflower, the holes of a honeycomb, the grains on a maize cob, the spirals on a pineapple and on a pine cone etc.

Can you see a pattern in each of the given example? We can see the natural patterns have a repetition which is not progressive. The identical petals of the sunflower are equidistantly grown. In a honeycomb identical hexagonal shaped holes are arranged symmetrically around each hexagon. Similarly, you can find out other natural patterns in spirals of pineapple....

You can look for some other patterns which occur in our day-to-day life. Some examples are:

(i) List of the last digits (digits in unit place) taken from the values of $4, 4^2, 4^3, 4^4, 4^5, 4^6$ ..... is

4, 6, 4, 6, 4, 6, ...

(ii) Mary is doing problems on patterns as part of preparing for a bank exam. One of them is "find the next two terms in the following pattern".

1, 2, 4, 8, 10, 20, 22 ....

(iii) Usha applied for a job and got selected. She has been offered a job with a starting monthly salary of ₹ 8000, with an annual increment of ₹500. Her salary (in rupees) for to 1st, 2nd, 3rd ... years will be 8000, 8500, 9000 ..... respectively.

(iv) The lengths of the rungs of a ladder decrease uniformly by 2 cm from bottom to top. The bottom rung is 45 cm in length. The lengths (in cm) of the 1st, 2nd, 3rd, ... 8th rung from the bottom to the top are 45, 43, 41, 39, 37, 35, 33, 31 respectively.

Can you see any relationship between the terms in the pattern of numbers written above?

Pattern given in example (i) has a relation of two numbers one after the other i.e. 4 and 6 are repeating alternatively.

Now try to find out pattern in exampled (ii). In examples (iii) and (iv), the relationship between the numbers in each list is constantly progressive. In the given list 8000, 8500, 9000, ... each succeeding term is obtained by adding 500 to the preceding term.

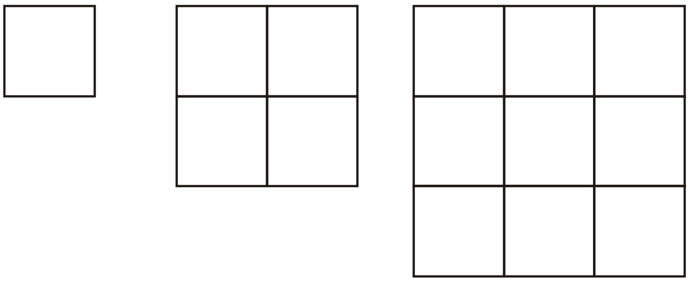
Where as in 45, 43, 41, ... each succeeding term is obtained by adding '-2' to each preceding term. Now we can see some more examples of progressive patterns.

(a) In a savings seheme, the amount becomes $\frac{5}{4}$ times of itself after 3 years.

The maturity amount (in Rupees) of an investment of ₹8000 after 3, 6, 9 and 12 years will be respectively. 10000, 12500, 15625, 19531.25.

(b) The number of unit squares in squares with sides 1, 2, 3, ... units are respectively.

$1^2, 2^2, 3^2, ...$

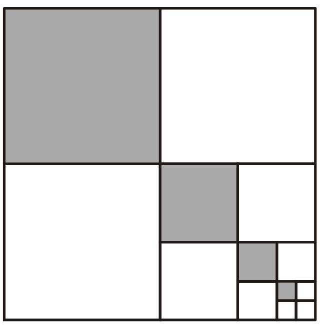


(c) Hema put Rs. 1000 into her daughter's money box when she was one year old and increased the amount by Rs. 500 every year. The amount of money (in Rs.) in the box on her 1st, 2nd, 3rd, 4th .... birthday would be.

1000, 1500, 2000, 2500, ... respectively.

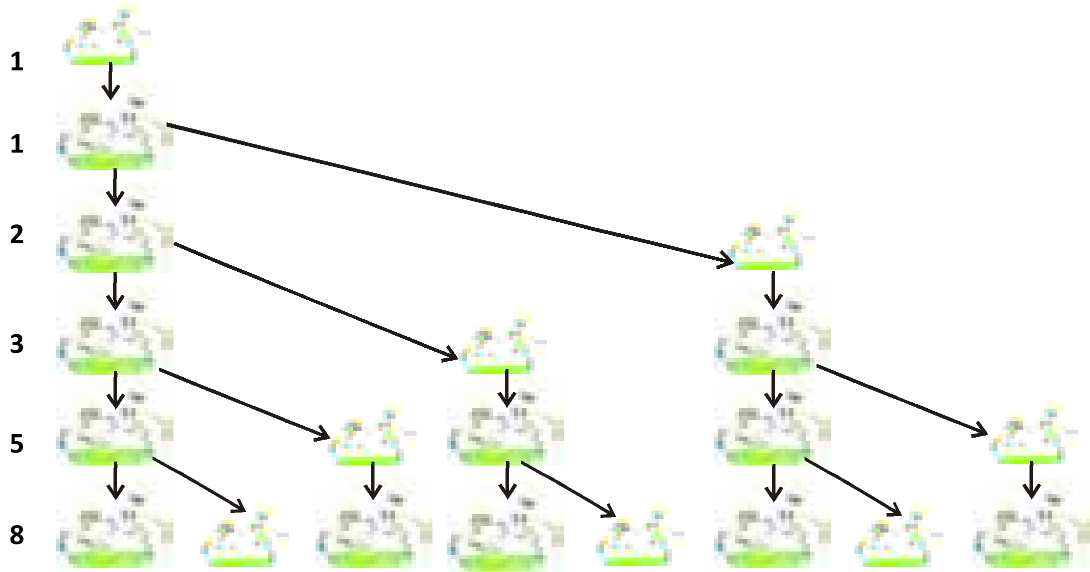
(d) The fraction of first, second, third ..... shaded regions of the squares in the following figure will be respectively.

$\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}, ...$



(e) A pair of rabbits are too young to produce in their first month. In the second, and every subsequent month, they produce a new pair. Each new pair of rabbits produce a new pair in their second month and in every subsequent month (see the figure below). Assuming no rabbit dies, the number of pairs of rabbits at the start of the 1st, 2nd, 3rd, ..., 6th month, respectively are :

1, 1, 2, 3, 5, 8



In the examples above, we observe some patterns. In some of them, we find that the succeeding terms are obtained by adding a fixed number or in other by multiplying with a fixed number or in another, we find that they are squares of consecutive numbers and so on.

In this chapter, we shall discuss some of these patterns in which succeeding terms are obtained by adding a fixed number to the preceding terms or multiplying preceding terms by a fixed number. We shall also see how to find their n^{th} term and the sum of n consecutive terms, and use this knowledge in solving some daily life problems.

History : Evidence is found that Babylonians some 400 years ago, knew of Arithmetic and geometric progressions. According to Boethins (570 AD), these progressions were known to early Greek writers. Among the Indian mathematicians, Aryabhata (470 AD) was the first to give formula for the sum of squares and cubes of natural number in his famous work Aryabhatiyam written around 499 A.D. He also gave the formula for finding the sum of n terms of an Arithmetic Progression starting with p^{th} term. Indian mathematician Brahmagupta (598 AD), Mahavira (850 AD) and Bhaskara (1114-1185 AD) also considered the sums of squares and cubes.

6.2 Arithmetic Progressions

Consider the following lists of numbers :

(i) 1, 2, 3, 4, . . .

(ii) 100, 70, 40, 10, . . .

(iii) – 3, –2, –1, 0, . . .

(iv) 3, 3, 3, 3, . . .

(v) –1.0, –1.5, –2.0, –2.5, . . .

Each of the numbers in the list is called a **term**.

Given a term, can you write the next term in each of the lists above? If so, how will you write it? Perhaps by following a pattern or rule, let us observe and write the rule.

In (i), each term is 1 more than the term preceding it.

In (ii), each term is 30 less than the term preceding it.

In (iii), each term is obtained by adding 1 to the term preceding it.

In (iv), all the terms in the list are 3 , i.e., each term is obtained by adding (or subtracting) 0 to the term preceding it.

In (v), each term is obtained by adding – 0.5 to (i.e., subtracting 0.5 from) the term preceding it.

In all the lists above, we can observe that successive terms are obtained by adding or substracting a fixed number to the preceding terms. Such list of numbers is said to form an **Arithmetic Progression (AP)**.

Try This

(i) Which of these are Arithmetic Progressions and why?

(a) 2, 3, 5, 7, 8, 10, 15, ...

(b) 2, 5, 7, 10, 12, 15, ...

(c) -1, -3, -5, -7, ...

(ii) Write 3 more Arithmetic Progressions.

6.2.1 What is an Arithmetic Progression?

We observe that an arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

This fixed number is called the **common difference** of the AP. Remember that it **can be positive, negative or zero**.

Let us denote the first term of an AP by a_1, second term by a_2, . . ., nth term by a_n and the common difference by d. Then the AP becomes $a_1, a_2, a_3, \ldots, a_n$.

So, $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$.

Let us see some more examples of AP :

(a) Heights (in cm) of some students of a school standing in a queue in the morning assembly are 147 , 148, 149, . . ., 157.

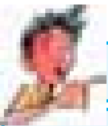
(b) Minimum temperatures (in degree celsius) recorded for a week, in the month of January in a city, arranged in ascending order are

$$-3.1, -3.0, -2.9, -2.8, -2.7, -2.6, -2.5$$

(c) The balance money (in ₹) after paying 5% of the total loan of ₹1000 every month is 950, 900, 850, 800, . . ., 50.

(d) Cash prizes (in ₹) given by a school to the toppers of Classes I to XII are 200, 250, 300, 350, . . ., 750 respectively.

(e) Total savings (in ₹) after every month for 10 months when Rs 50 are saved each month are 50, 100, 150, 200, 250, 300, 350, 400, 450, 500.



Think - Discuss

1. Think how each of the list given above form an AP Discuss with your friends.
2. Find the common difference of each of the above lists? Think when is it positive?
3. Make a positive Arithmetic progression in which the common difference is a small positive quantity.
4. Make an AP in which the common difference is big(large) positive quantity.
5. Make an AP in which the common difference is negative.

General form of AP : Can you see that all AP's can be written as.

$$a, a + d, a + 2d, a + 3d, . . .$$

This is called general form of an A.P where 'a' is the first term and 'd' is the common difference

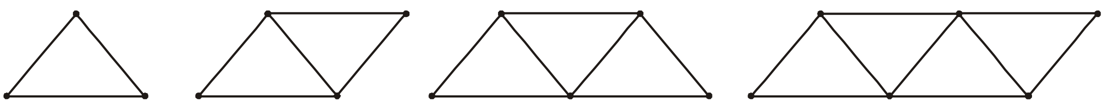
For example in 1, 2, 3, 4, 5, ...

The first terms is 1 and the common difference is also 1.

In 2, 4, 6, 8, 10 ..... What is the first term and common difference?

Activity

(i) Make the following figures with match sticks



(iii) Write down the number of match sticks required for each figure.

(iv) Can you find a common difference in members of the list?

(v) Does the list of these numbers form an AP?

6.2.2 Parameters of a Arithmetic Progressions

Note that in examples (a) to (e) above, in section 6.2.1 there are only a finite number of terms. Such an AP is called a **finite AP.** Also note that each of these Arithmetic Progressions (APs) has a last term. The APs in examples (i) to (v) in the section 6.2, are not finite APs and so they are called **infinite Arithmetic Progressions**. Such APs are never ending and do not have a last term.

Do this

Write three examples for finite AP and three for infinite AP.

Now, to know about an AP, what is the minimum information that you need? Is it enough to know the first term? Or, is it enough to know only the common difference?

We can see that we will need to know both – the first term a and the common difference d. These two parameters are sufficient for us to complete the Arithmetic Progression.

For instance, if the first term a is 6 and the common difference d is 3, then the AP is

$$6, 9, 12, 15, \ldots$$

and if a is 6 and d is – 3, then the AP is

$$6, 3, 0, -3, \ldots$$

Similarly, when

$a = -7$, $d = -2$, the AP is $-7, -9, -11, -13, \ldots$

$a = 1.0$, $d = 0.1$, the AP is $1.0, 1.1, 1.2, 1.3, \ldots$

$a = 0$, $d = 1\frac{1}{2}$, the AP is $0, 1\frac{1}{2}, 3, 4\frac{1}{2}, 6, \ldots$

$a = 2$, $d = 0$, the AP is $2, 2, 2, 2, \ldots$

So, if you know what a and d are, you can list the AP.

Let us try other way. If you are given a list of numbers, how can you say whether it is an A.P. or not?

For example, for any list of numbers :

$$6, 9, 12, 15, \ldots,$$

We check the difference of the succeeding terms. In the given list we have $a_2 - a_1 = 9 - 6 = 3$,

$$a_3 - a_2 = 12 - 9 = 3,$$
$$a_4 - a_3 = 15 - 12 = 3$$

We see that $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 \ldots = 3$

Here the difference of any two consecutive terms in each case is 3. So, the given list is an AP whose first term a is 6 and common difference d is 3.

For the list of numbers : 6, 3, 0, – 3, . . .,

$$a_2 - a_1 = 3 - 6 = -3,$$
$$a_3 - a_2 = 0 - 3 = -3$$
$$a4 - a_3 = -3 - 0 = -3$$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = -3$$

Similarly, this is also an AP whose first term is 6 and the common difference is –3.

So, we see that if the difference between succeeding terms is constant then it is an Arithmetic Progression.

In general, for an AP $a_1, a_2, \ldots, a_n$, we can say

$$d = a_{k+1} - a_k \quad \text{where } k \in \mathrm{N}; k \geq 1$$

where a_{k+1} and a_k are the $(k + 1)$th and the kth terms respectively.

Consider the list of numbers 1, 1, 2, 3, 5, ... By looking at it, you can tell that the difference between any two consecutive terms is not the same. So, this is not an AP.

Note : To find d in the AP : 6, 3, 0, – 3, . . ., we have subtracted 6 from 3 and not 3 from 6. We have to subtract the k^{th} term from the $(k+1)$ th term even if the $(k+1)^{th}$ term is smaller and to find 'd' in a given A.P. we need not find all of $a_2 - a_1, a_1 - a_2 \ldots$. It is enough to find only one of them

Do This

1. Take any Arithmetic Progression.
2. Add a fixed number to each and every term of AP. Write the resulting numbers as a list.
3. Similarly subtract a fixed number from each and every term of AP. Write the resulting numbers as a list.
4. Multiply and divide each term of AP by a fixed number and write the resulting numbers as a list.
5. Check whether the resulting lists are AP in each case.

6. What is your conclusion?

Let us consider some examples

Example-1. For the AP : $\frac{1}{4}, \frac{-1}{4}, \frac{-3}{4}, \frac{-5}{4}$ ...., write the first term a and the common difference d. And find the 7th term

Solution : Here, $a = \frac{1}{4}$; $d = \frac{-1}{4} - \frac{1}{4} = \frac{-1}{2}$

Remember that we can find d using any two consecutive terms, once we know that the numbers are in AP.

The seventh term would be $\frac{-5}{4} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = \frac{-11}{4}$

Example-2. Which of the following forms an AP? If they form AP then write next two terms?

(i) 4, 10, 16, 22, . . . (ii) 1, – 1, – 3, – 5, . . . (iii) – 2, 2, – 2, 2, – 2, . . .

(iv) 1, 1, 1, 2, 2, 2, 3, 3, 3, . . . (v) $x, 2x, 3x, 4x$ ......

Solution : (i) We have $a_2 - a_1 = 10 - 4 = 6$

$$a_3 - a_2 = 16 - 10 = 6$$

$$a_4 - a_3 = 22 - 16 = 6$$

i.e., $a_{k+1} - a_k$ is same every time.

So, the given list of numbers forms an AP with the common difference $d = 6$.

The next two terms are: 22 + 6 = 28 and 28 + 6 = 34.

(ii) $a_2 - a_1 = -1 - 1 = -2$

$$a_3 - a_2 = -3 - (-1) = -3 + 1 = -2$$

$$a_4 - a_3 = -5 - (-3) = -5 + 3 = -2$$

i.e., $a_{k+1} - a_k$ is same every time.

So, the given list of numbers forms an AP with the common difference $d = -2$.

The next two terms are:

$$-5 + (-2) = -7 \text{ and } -7 + (-2) = -9$$

(iii) $a_2 - a_1 = 2 - (-2) = 2 + 2 = 4$

$a_3 - a_2 = -2 - 2 = -4$

As $a_2 - a_1 \neq a_3 - a_2$, the given list of numbers do not form an AP.

(iv) $a_2 - a_1 = 1 - 1 = 0$

$a_3 - a_2 = 1 - 1 = 0$

$a_4 - a_3 = 2 - 1 = 1$

Here, $a_2 - a_1 = a_3 - a_2 \neq a_4 - a_3$.

So, the given list of numbers do not form an AP.

(v) We have $a_2 - a_1 = 2x - x = x$

$a_3 - a_2 = 3x - 2x = x$

$a_4 - a_3 = 4x - 3x = x$

i.e., $a_{k+1} - a_k$ is same every time.

$\therefore$ So, the given list form an AP.

The next two terms are $4x + x = 5x$ and $5x + x = 6x$.

Exercise - 6.1

1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

   (i) The taxi fare after each km when the fare is ₹ 20 for the first km and rises by ₹ 8 for each additional km.

   (ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

   (iii) The cost of digging a well, after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.

   (iv) The amount of money in the account every year, when ₹10000 is deposited at compound interest at 8 % per annum.

2. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) $a = 10, d = 10$ (ii) $a = -2, d = 0$

(iii) $a = 4, d = -3$ (iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

3. For the following APs, write the first term and the common difference:

(i) 3, 1, – 1, – 3, . . .

(ii) – 5, – 1, 3, 7, . . .

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$

(iv) 0.6, 1.7, 2.8, 3.9, . . .

4. Which of the following are APs ? If they form an AP, find the common difference *d* and write three more terms.

(i) 2, 4, 8, 16, . . .

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, ...$

(iii) – 1.2, – 3.2, – 5.2, – 7.2, . . .

(iv) – 10, – 6, – 2, 2, . . .

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, ...$

(vi) 0.2, 0.22, 0.222, 0.2222, . . .

(vii) 0, – 4, – 8, –12, . . .

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, ...$

(ix) 1, 3, 9, 27, . . .

(x) $a, 2a, 3a, 4a, \ldots$

(xi) $a, a^2, a^3, a^4, \ldots$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$

6.3 n^{th} Term of an Arithmetic Progression

Let us consider the offer to Usha who applied for a job and got selected. She has been offered a starting monthly salary of ₹ 8000, with an annual increment of ₹ 500. What would be her monthly salary of the fifth year?

To answer this, let us first see what her monthly salary for the second year would be.

It would be ₹ (8000 + 500) = ₹ 8500.

In the same way, we can find the monthly salary for the 3rd, 4th and 5th year by adding ₹ 500 to the salary of the previous year.

So, the salary for the 3rd year = ₹(8500 + 500)

= ₹ (8000 + 500 + 500)

= ₹ (8000 + 2 × 500)

= ₹ [8000 + **(3 – 1)** × 500] (for the **3rd year**)

= ₹ 9000

Salary for the 4th year = ₹ (9000 + 500)

= ₹ (8000 + 500 + 500 + 500)

$= ₹\ (8000 + 3 \times 500)$

$= ₹\ [8000 + \mathbf{(4 - 1)} \times 500]$ (for the $\mathbf{4^{th}}$ **year**)

$= ₹\ 9500$

Salary for the 5^{th} year $= ₹\ (9500 + 500)$

$= ₹\ (8000+500+500+500 + 500)$

$= ₹\ (8000 + 4 \times 500)$

$= ₹\ [8000 + \mathbf{(5 - 1)} \times 500]$ (for the $\mathbf{5^{th}}$ **year**)

$= ₹\ 10000$

Observe that we are getting a list of numbers

8000, 8500, 9000, 9500, 10000, . . .

These numbers are in Arithmetic Progression.

Looking at the pattern above, can we find her monthly salary in the 6^{th} year? The 15^{th} year? And, assuming that she is still working in the same job, what would be her monthly salary in the 25^{th} year? Here we can calculate the salary of the present year by adding ₹ 500 to the salary of previous year. Can we make this process shorter? Let us see. You may have already got some idea from the way we have obtained the salaries above.

Salary for the 15^{th} year = Salary for the 14^{th} year + ₹ 500

$$= ₹\left[8000 + \underbrace{500 + 500 + 500 + \ldots + 500}_{13\text{ times}}\right] + ₹500$$

$= ₹\ [8000 + 14 \times 500]$

$= ₹\ [8000 + \mathbf{(15 - 1)} \times 500] = ₹\ 15000$

i.e., First salary + (15 – 1) × Annual increment.

In the same way, her monthly salary for the 25^{th} year would be

$₹\ [8000 + \mathbf{(25 - 1)} \times 500] = ₹\ 20000$

= First salary + **(25 – 1)** × Annual increment

This example has given us an idea about how to write the 15^{th} term, or the 25^{th} term. By using the same idea, now let us find the n^{th} term of an AP.

Let $a_1, a_2, a_3, \ldots$ be an AP whose first term a_1 is a and the common difference is d.

Then,

the **second term** $a_2 = a + d = a + \mathbf{(2 - 1)}\ d$

the **third** term $a_3 = a_2 + d = (a + d) + d = a + 2d = a + \mathbf{(3 - 1)}\, d$

the **fourth** term $a_4 = a3 + d = (a + 2d) + d = a + 3d = a + \mathbf{(4 - 1)}\, d$

. . . .

. . . .

Looking at the pattern, we can say that the ***n*th** term $a_n = a + (n - 1)\, d$.

So, **the n^{th} term *an* of the AP with first term *a* and common difference *d* is** given by $a_n = a + (n - 1)\, d$.

a_n is also called the **general term of the AP**.

If there are m terms in the AP, then a_m represents the **last term which is sometimes also denoted by *l***.

Finding terms of an AP : Using the above formula we can find different terms of an arithemetic progression.

Let us consider some examples.

Example-3. Find the 10th term of the AP : 5, 1, –3, –7 . . .

Solution : Here, $a = 5$, $d = 1 - 5 = -4$ and $n = 10$.

We have $a_n = a + (n - 1)\, d$

So, $a_{10} = 5 + (10 - 1)(-4) = 5 - 36 = -31$

Therefore, the 10th term of the given AP is –31.

Example-4. Which term of the AP : 21, 18, 15, . . . is – 81?

Is there any term 0? Give reason for your answer.

Solution : Here, $a = 21$, $d = 18 - 21 = -3$ and if $a_n = -81$, we have to find n.

As $\quad a_n = a + (n - 1)\, d,$

we have $\quad -81 = 21 + (n - 1)(-3)$

$$-81 = 24 - 3n$$

$$-105 = -3n$$

So, $\quad n = 35$

Therefore, the 35th term of the given AP is – 81.

Next, we want to know if there is any n for which $a_n = 0$. If such an n is there, then

$$21 + (n - 1)(-3) = 0,$$

i.e., $\quad 3(n - 1) = 21$

i.e., $\quad n = 8$

So, the eighth term is 0.

Example-5. Determine the AP whose 3rd term is 5 and the 7th term is 9.

Solution : We have

$$a_3 = a + (3 - 1)\, d = a + 2d = 5 \qquad (1)$$

and $$a_7 = a + (7 - 1)\, d = a + 6d = 9 \qquad (2)$$

Solving the pair of linear equations (1) and (2), we get

$$a = 3,\ d = 1$$

Hence, the required AP is 3, 4, 5, 6, 7, . . .

Example-6. Check whether 301 is a term of the list of numbers 5, 11, 17, 23, . . .

Solution : We have :

$a_2 - a_1 = 11 - 5 = 6$, $a_3 - a_2 = 17 - 11 = 6$, $a_4 - a_3 = 23 - 17 = 6$

As $(a_{k+1} - a_k)$ is the same for $k = 1, 2, 3$, etc., the given list of numbers is an AP.

Now, for this AP we have $a = 5$ and $d = 6$.

We choose to begin with the assumption that 301 is a term, say, the nth term of the this AP. We will see if an 'n' exists for which $a_n = 301$.

We know

$$a_n = a + (n - 1)\, d$$

So, for 301 to be a term we must have

$$301 = 5 + (n - 1) \times 6$$

or $$301 = 6n - 1$$

So, $$n = \frac{302}{6} = \frac{151}{3}$$

But n should be a positive integer (Why?).

So, 301 is not a term of the given list of numbers.

Example-7. How many two-digit numbers are divisible by 3?

Solution : The list of two-digit numbers divisible by 3 is :

$$12,\ 15,\ 18,\ . . . ,\ 99$$

Is this an AP? Yes it is. Here, $a = 12$, $d = 3$, $a_n = 99$.

As $$a_n = a + (n - 1)\, d,$$

we have $99 = 12 + (n - 1) \times 3$

i.e., $87 = (n - 1) \times 3$

i.e., $n - 1 = \frac{87}{3} = 29$

i.e., $n = 29 + 1 = 30$

So, there are 30 two-digit numbers divisible by 3.

Example-8. Find the 11^{th} term from the last of the the AP series given below :

AP : 10, 7, 4, . . ., – 62.

Solution : Here, $a = 10$, $d = 7 - 10 = -3$, $l = -62$,

where $l = a + (n - 1)\, d$

To find the 11th term from the last term, we will find the total number of terms in the AP.

So, $-62 = 10 + (n - 1)(-3)$

i.e., $-72 = (n - 1)(-3)$

i.e., $n - 1 = 24$

or $n = 25$

So, there are 25 terms in the given AP.

The 11^{th} term from the last will be the 15^{th} term of the series. (Note that it will not be the 14^{th} term. Why?)

So, $a_{15} = 10 + (15 - 1)(-3) = 10 - 42 = -32$

i.e., the 11^{th} term from the end is – 32.

Note : The 11^{th} term from the last is also equal to 11^{th} term of the AP with first term – 62 and the common difference 3.

Example-9. A sum of ₹ 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years.

Solution : We know that the formula to calculate simple interest is given by

$$\text{Simple Interest} = \frac{P \times R \times T}{100}$$

So, the interest at the end of the 1^{st} year = ₹ $\frac{1000 \times 8 \times 1}{100}$ = ₹ 80

The interest at the end of the 2^{nd} year = ₹ $\frac{1000 \times 8 \times 2}{100}$ = ₹ 160

The interest at the end of the 3rd year = $\dfrac{1000 \times 8 \times 3}{100}$ = ₹ 240

Similarly, we can obtain the interest at the end of the 4th year, 5th year, and so on. So, the interest (in Rs) at the end of the 1st, 2nd, 3rd, . . . years, respectively are

$$80, 160, 240, \ldots$$

It is an AP as the difference between the consecutive terms in the list is 80,

i.e., $d = 80$. Also, $a = 80$.

So, to find the interest at the end of 30 years, we shall find a_{30}.

Now, $a_{30} = a + (30 - 1)\, d = 80 + 29 \times 80 = 2400$

So, the interest at the end of 30 years will be ₹ 2400.

Example-10. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Solution : The number of rose plants in the 1st, 2nd, 3rd, . . ., rows are :

$$23, 21, 19, \ldots, 5$$

It forms an AP (Why?).

Let the number of rows in the flower bed be n.

Then $a = 23$, $d = 21 - 23 = -2$, $a_n = 5$

As, $a_n = a + (n - 1)\, d$

We have, $5 = 23 + (n - 1)(-2)$

i.e., $-18 = (n - 1)(-2)$

i.e., $n = 10$

So, there are 10 rows in the flower bed.

Exercise - 6.2

1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the nth term of the AP:

S. No.	a	d	n	a_n
(i)	7	3	8	. . .
(ii)	– 18	. . .	10	0

(iii)	. . .	– 3	18	– 5
(iv)	– 18.9	2.5	. . .	3.6
(v)	3.5	0	105	. . .

2. Find the

   (i) 30^{th} term of the A.P. 10, 7, 4 ......

   (ii) 11^{th} term of the A.P. : $-3, \frac{-1}{2}, 2, \ldots$

3. Find the respective terms for the following APs.

   (i) $a_1 = 2;\ a_3 = 26$ find a_2 (ii) $a_2 = 13;\ a_4 = 3$ find a_1, a_3

   (iii) $a_1 = 5;\ a_4 = 9\frac{1}{2}$ find a_2, a_3 (iv) $a_1 = -4;\ a_6 = 6$ find a_2, a_3, a_4, a_5

   (v) $a_2 = 38;\ a_6 = -22$ find a_1, a_3, a_4, a_5

4. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

5. Find the number of terms in each of the following APs :

   (i) 7, 13, 19, . . . , 205 (ii) $18, 15\frac{1}{2}, 13, \ldots, -47$

6. Check whether, –150 is a term of the AP : 11, 8, 5, 2 . . .

7. Find the 31^{st} term of an AP whose 11^{th} term is 38 and the 16^{th} term is 73.

8. If the 3^{rd} and the 9^{th} terms of an AP are 4 and – 8 respectively, which term of this AP is zero?

9. The 17^{th} term of an AP exceeds its 10^{th} term by 7. Find the common difference.

10. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

11. How many three-digit numbers are divisible by 7?

12. How many multiples of 4 lie between 10 and 250?

13. For what value of n, are the nth terms of two APs: 63, 65, 67, ... and 3, 10, 17, . . . equal?

14. Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

15. Find the 20^{th} term from the end of the AP : 3, 8, 13, . . ., 253.

16. The sum of the 4^{th} and 8^{th} terms of an AP is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the AP.

17. Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?

6.4 Sum of First *n* Terms in Arithmetic Progression

Let us consider the situation again given in Section 6.1 in which Hema put ₹ 1000 money box when her daughter was one year old, ₹ 1500 on her second birthday, ₹ 2000 on her third birthday and will continue in the same way. How much money will be collected in the money box by the time her daughter is 21 years old?

Here, the amount of money (in Rupees) put in the money box on her first, second, third, fourth . . . birthday were respectively 1000, 1500, 2000, 2500, . . . till her 21st birthday. To find the total amount in the money box on her 21st birthday, we will have to write each of the 21 numbers in the list above and then add them up. Don't you think it would be a tedious and time consuming process? Can we make the process shorter?

This would be possible if we can find a method for getting this sum. Let us see.

6.4.1 How 'Gauss' find the sum of terms

We consider the problem given to Gauss, to solve when he was just 10 years old. He was asked to find the sum of the positive integers from 1 to 100. He immediately replied that the sum is 5050. Can you guess how can he do? He wrote :

Carl Fredrich Gauss (1777-1855) is a great German Mathematician

$$S = 1 + 2 + 3 + . . . + 99 + 100$$

And then, reversed the numbers to write

$$S = 100 + 99 + . . . + 3 + 2 + 1$$

When he added these two he got 25 as both the sums have to be equal. So he work,

$$2S = (100 + 1) + (99 + 2) + . . . + (3 + 98) + (2 + 99) + (1 + 100)$$

$$= 101 + 101 + . . . + 101 + 101 \text{ (100 times) (check this out and discuss)}$$

So, $S = \dfrac{100\times101}{2} = 5050,$ i.e., the sum = 5050.

6.4.2 Sum of n terms of an AP.

We will now use the same technique that was used by transs to find the sum of the first n terms of an AP :

$$a, a + d, a + 2d, \ldots$$

The nth term of this AP is $a + (n - 1)\, d$.

Let S_n denote the sum of the first n terms of the A.P. Whose n^{th} term is

$$a_n = a + (n - 1)\, d$$

$$\therefore \quad S_n = a + (a + a) + (a + 2d) + \ldots + a + (n-1)d$$

$$S_n = (a + (n-1)d) + (a + (n-2)d + \ldots + a$$

Adding $2S_n = (2a + (n - 1)d) + (2a + (n - 1)d) + \ldots. + (2a + (n - 1)d)$ (n times)

$$= n(2a + (n - 1)d)$$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + a + (n-1)d] = \frac{n}{2}\ [\text{first term} + \text{nth term}] = \frac{n}{2}(a + a_n)$$

If the first and last terms of an A.P. are given and the common difference is not given then

$S_n = \frac{n}{2}(a + a_n)$ is very useful to find S_n.

Money for Hema's daughter

Now we return to the question that was posed to us in the beginning. The amount of money (in Rs) in the money box of Hema's daughter on 1^{st}, 2^{nd}, 3^{rd}, 4th birthday, . . ., were 1000, 1500, 2000, 2500, . . ., respectively.

This is an AP. We have to find the total money collected on her 21st birthday, i.e., the sum of the first 21 terms of this AP.

Here, $a = 1000$, $d = 500$ and $n = 21$. Using the formula :

$$S_n = \frac{n}{2}[2a + (n-1)d],$$

we have

$$S = \frac{21}{2}[2 \times 1000 + (21-1) \times 500]$$

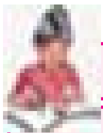
$$= \frac{21}{2}[2000 + 10000]$$

$$= \frac{21}{2}[12000] = 126000$$

So, the amount of money collected on her 21st birthday is ₹ 12600.

We use S_n in place of S to denote the sum of first n terms of the AP so that we know how many terms we have added. We write S_{20} to denote the sum of the first 20 terms of an AP. The formula for the sum of the first n terms involves four quantities S_n, a, d and n. If we know any three of them, we can find the fourth.

Remark : The n^{th} term of an AP is the difference of the sum to first n terms and the sum to first $(n-1)$ terms of it, i.e., $a_n = S_n - S_{n-1}$.



Do This

Find the sum of indicated number of terms in each of the following A.P.s

(i) 16, 11, 6 .....; 23 terms

(ii) –0.5, –1.0, –1.5, ...; 10 terms

(iii) $-1, \frac{1}{4}, \frac{3}{2}$....., 10 terms

Let us consider some examples.

Example-11. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.

Solution : Here, $S_n = 1050$; $n = 14$, $a = 10$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$1050 = \frac{14}{2}[2a+13d] = 140+91d$$

$$910 = 91d$$

$$\therefore \quad d = 10$$

$$\therefore \quad a_{20} = 10 + (20 - 1)\ 10 = 200$$

Example-12. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

Solution : Here, $a = 24$, $d = 21 - 24 = -3$, $Sn = 78$. We need to find n.

We know that $S_n = \frac{n}{2}[2a+(n-1)d]$

So, $78 = \frac{n}{2}[48+(n-1(-3)] = \frac{n}{2}[51-3n]$

or $3n^2 - 51n + 156 = 0$

or $n^2 - 17n + 52 = 0$

or $(n - 4)(n - 13) = 0$

or $n = 4$ or 13

Both values of n are admissible. So, the number of terms is either 4 or 13.

Remarks :

1. In this case, the sum of the first 4 terms = the sum of the first 13 terms = 78.
2. Two answers are possible because the sum of the terms from 5^{th} to 13^{th} will be zero. This is because a is positive and d is negative, so that some terms are positive and some are negative, and will cancel out each other.

Example-13. Find the sum of :

(i) the first 1000 positive integers (ii) the first n positive integers

Solution :

(i) Let S = 1 + 2 + 3 + . . . + 1000

Using the formula $S_n = \frac{n}{2}(a+l)$ for the sum of the first n terms of an AP, we have

$$S_{1000} = \frac{1000}{2}(1+1000) = 500 \times 1001 = 500500$$

So, the sum of the first 1000 positive integers is 500500.

(ii) Let $S_n = 1 + 2 + 3 + . . . + n$

Here $a = 1$ and the last term l is n.

Therefore, $S_n = \frac{n(1+n)}{2}$ (or) $S_n = \frac{n(n+1)}{2}$

So, **the sum of first n positive integers is given by**

$$\mathbf{S_n = \frac{n(n+1)}{2}}$$

Example-14. Find the sum of first 24 terms of the list of numbers whose nth term is given by

$$a_n = 3 + 2n$$

Solution : As $a_n = 3 + 2n,$

so, $a_1 = 3 + 2 = 5$

$a_2 = 3 + 2 \times 2 = 7$

$a_3 = 3 + 2 \times 3 = 9$

$\vdots$

List of numbers becomes 5, 7, 9, 11, . . .

Here, $7 - 5 = 9 - 7 = 11 - 9 = 2$ and so on.

So, it forms an AP with common difference $d = 2$.

To find S_{24}, we have $n = 24$, $a = 5$, $d = 2$.

Therefore, $S_{24} = \frac{24}{2}[2 \times 5 + (24 - 1) \times 2] = 12(10 + 46) = 672$

So, sum of first 24 terms of the list of numbers is 672.

Example-15. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find :

(i) the production in the 1^{st} year (ii) the production in the 10^{th} year

(iii) the total production in first 7 years

Solution : (i) Since the production increases uniformly by a fixed number every year, the number of TV sets manufactured in 1^{st}, 2^{nd}, 3^{rd}, . . ., years will form an AP.

Let us denote the number of TV sets manufactured in the nth year by a_n.

Then, $a_3 = 600$ and $a_7 = 700$

or, $a + 2d = 600$

and $a + 6d = 700$

Solving these equations, we get $d = 25$ and $a = 550$.

Therefore, production of TV sets in the first year is 550.

(ii) Now $a_{10} = a + 9d = 550 + 9 \times 25 = 775$

So, production of TV sets in the 10^{th} year is 775.

(iii) Also, $S_7 = \frac{7}{2}[2 \times 550 + (7 - 1) \times 25]$

$= \frac{7}{2}[1100 + 150] = 4375$

Thus, the total production of TV sets in first 7 years is 4375.

EXERCISE - 6.3

1. Find the sum of the following APs:

   (i) 2, 7, 12, . . ., to 10 terms.

   (ii) –37, –33, –29, . . ., to 12 terms.

   (iii) 0.6, 1.7, 2.8, . . ., to 100 terms.

   (iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ...,to 11terms.

2. Find the sums given below :

   (i) $7 + 10\frac{1}{2} + 14 + ... + 84$

   (ii) 34 + 32 + 30 + . . . + 10

   (iii) –5 + (–8) + (–11) + . . . + (–230)

3. In an AP:

   (i) given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

   (ii) given $a = 7$, $a_{13} = 35$, find d and S_{13}.

   (iii) given $a_{12} = 37$, $d = 3$, find a and S_{12}.

   (iv) given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

   (v) given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

   (vi) given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.

   (vii) given $l = 28$, $S = 144$, and there are total 9 terms. Find a.

4. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

5. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

6. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

7. Show that $a_1, a_2, . . ., a_n, . . .$ form an AP where a_n is defined as below :

   (i) $a_n = 3 + 4n$

   (ii) $a_n = 9 - 5n$

   Also find the sum of the first 15 terms in each case.

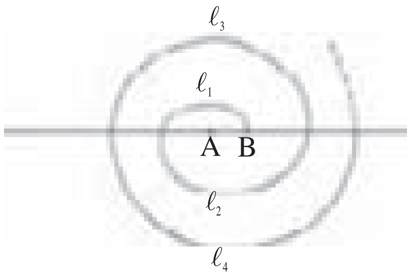
8. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (remember the first term is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

9. Find the sum of the first 40 positive integers divisible by 6.

10. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

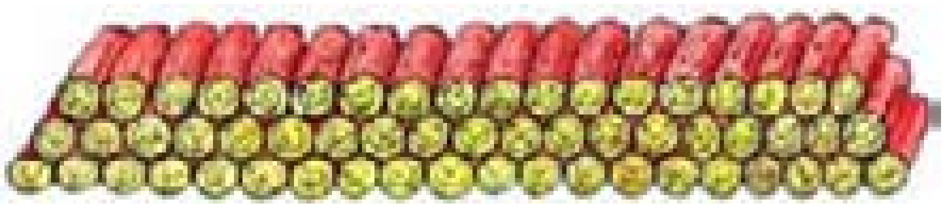
11. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

12. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)



[**Hint :** Length of successive semicircles is $l_1, l_2, l_3, l_4, \ldots$ with centres at A, B, A, B, . . ., respectively.]

13. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how may rows are the 200 logs placed and how many logs are in the top row?



14. In a bucket and ball race, a bucket is placed at the starting point, which is 5 m from the first ball, and the other balls are placed 3 m apart in a straight line. There are ten balls in the line.

A competitor starts from the bucket, picks up the nearest ball, runs back with it, drops it in the bucket, runs back to pick up the next ball, runs to the bucket to drop it in, and she continues in the same way until all the balls are in the bucket. What is the total distance the competitor has to run?

[**Hint :** To pick up the first ball and the second ball, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

6.5 Geometric Progressions

Consider the lists

(i) 30, 90, 270, 810 ..... (ii) $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}.....$

(iii) 30, 24, 19.2, 15.36, 12.288

Given a term, can we write the next term in each of the lists above ?

in (i) each term is obtained by multiplying the preceeding term by 3.

in (ii) each term is obtained by multiplying the preceeding term by $\frac{1}{4}$.

in (iii) each term is obtained by multiplying the preceeding term by 0.8.

In all the lists above, we see that successive terms are obtained by multiplying the preceeding term by a fixed number. Such a list of numbers is said to form **Geometric Progression (GP).** This fixed number is called the common ration 'r' of GP. So in the above example (i), (ii), (iii) the common ratios are 3, $\frac{1}{4}$, 0.8 respectively.

Let us denote the first term of a GP by a and common ratio r. To get the second term according to the rule of Geometric Progression, we have to multiply the first term by the common ratio r.

$\therefore$ The second term $= ar$

Third term $= ar \cdot r = ar^2$

$\therefore$ a, ar, ar^2 ..... is called the general form of a GP.

in the above GP the ratio between any term (except first term) and its preceding term is 'r'

i.e., $\frac{ar}{a} = \frac{ar^2}{ar} = ... = r$

If we denote the first term of GP by a_1, second term by a_2 ..... nth term by an

then $\frac{a_2}{a_1} = \frac{a_3}{a_2} = ... = \frac{a_n}{a_{n-1}} = r$

$\therefore$ A list of numbers $a_1, a_2, a_3 .... a_n ...$ is called a geometric progression (GP), if each term is non zero and

$$\frac{a_n}{a_{n-1}} = r$$

Where n is a natural number and $n \geq 2$.

Do This

Find which of the following are not G.P.

1. 6, 12, 24, 48, ...
2. 1, 4, 9, 16, ...
3. 1, –1, 1, –1, ...
4. –4, –20, –100, –500, ...

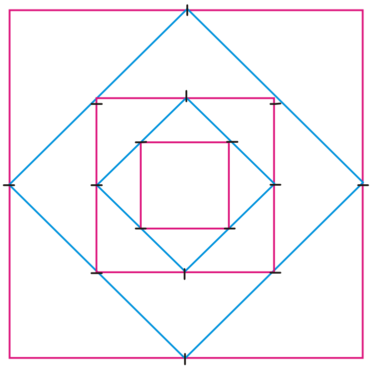
Some more example of GP are :

(i) A person writes a letter to four of his friends. He asks each one of them to copy the letter and give it to four different persons with same instructions so that they can move the chain ahead similarly. Assuming that the chain is not broken the number of letters at first, second, third ... stages are

1, 4, 16, 256 .... respectively.

(ii) The total amount at the end of first , second, third .... year if ₹ 500/- is deposited in the bank with annual rate 10% interest compounded annually is

550, 605, 665.5 ......



(iii) A square is drawn by joining the mid points of the sides of a given square. A third square is drawn inside the second square in the same way and this process continues indefinitely. If a side of the first square is 16cm then the area of first, second, third ..... square will be respectively.

256, 128, 64, 32, ...

(iv) Initially a pendulum swings through an arc of 18 cms. On each sucessive swing the length of the arc is 0.9 of the previous length. So the length of the arc at first, second, third....... swing will be resepectively.

18, 16.2, 14.58, 13.122......

Think - Discuss

1. Explain why each of the lists above is a G.P.
2. To know about a G.P. what is minimum information that we need ?

Now let us learn how to construct a GP. when the first term 'a' and common ratio 'r' are given. And also learn how to check whether the given list of numbers is a G.P.

Example-16. Write the GP. if the first term $a = 3$, and the common ratio $r = 2$.

Solution : Since 'a' is the first term it can easily be written

We know that in GP. every succeeding term is obtained by multiplying the preceding term with common ratio 'r'. So to get the second term we have to multiply the first term $a = 3$ by the common ratio $r = 2$.

$\therefore$ Second term $= ar = 3 \times 2 = 6$

Similarly the third term = second term $\times$ common ratio

$= 6 \times 2 = 12$

If we proceed in this way we get the following G.P.

3, 6, 12, 24,..... .

Example-17. Write GP. if $a = 256$, $r = \dfrac{-1}{2}$

Solution : General form of GP $= a, ar, ar^2, ar^3, \ldots\ldots$

$$= 256, 256\left(\frac{-1}{2}\right), 256\left(\frac{-1}{2}\right)^2, 256\left(\frac{-1}{2}\right)^3$$

$$= 256, -128, 64, -32 \ldots\ldots$$

Example-18. Find the common ratio of the GP $25, -5, 1, \dfrac{-1}{5}$.

Solution : We know that if the first, second, third .... terms of a GP are a_1, a_2, a_3 .... respectively

the common ratio $r = \dfrac{a_2}{a_1} = \dfrac{a_3}{a_2} = \ldots\ldots$

Here $a_1 = 25$, $a_2 = -5$, $a_3 = 1$.

So common ratio $r = \dfrac{-5}{25} = \dfrac{1}{-5} = \dfrac{-1}{5}$.

Example-19. Which of the following list of numbers form GP. ?

(i) 3, 6, 12, ...　　　(ii) 64, −32, 16,

(iii) $\dfrac{1}{64}, \dfrac{1}{32}, \dfrac{1}{8}, \ldots\ldots$

Solution : (i)We know that a list of numbers $a_1, a_2, a_3, ...a_n$ ..... is called a GP if each term is non zero and $\frac{a_2}{a_1} = \frac{a_3}{a_2} = ... \frac{a_n}{a_{n-1}} = r$

Here all the terms are non zero. Further

$$\frac{a_2}{a_1} = \frac{6}{3} = 2 \text{ and}$$

$$\frac{a_3}{a_2} = \frac{12}{6} = 2$$

i.e., $$\frac{a_2}{a_1} = \frac{a_3}{a_2} = 2$$

So, the given list of number form a G.P. which contain ratio 2.

(ii) All the terms are non zero.

$$\frac{a_2}{a_1} = \frac{-32}{64} = \frac{-1}{2}$$

and $$\frac{a_3}{a_1} = \frac{16}{-32} = \frac{-1}{2}$$

$$\therefore \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{-1}{2}$$

So, the given list of numbers form a GP with common ratio $\frac{-1}{2}$.

(iii) All the terms are non zero.

$$\frac{a_2}{a_1} = \frac{\frac{1}{32}}{\frac{1}{64}} = 2$$

$$\frac{a_3}{a_2} = \frac{\frac{1}{8}}{\frac{1}{32}} = 4$$

Here $\frac{a_2}{a_1} \neq \frac{a_3}{a_2}$

So, the given list of numbers does not form GP.

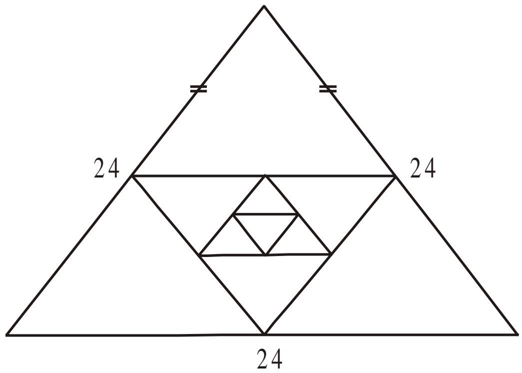
Exercise - 6.4

1. In which of the following situations, does the list of numbers involved in form a GP.?

   (i) Salary of Sharmila, when her salary is ₹ 5,00,000 for the first year and expected to receive yearly increase of 10% .

   (ii) Number of bricks needed to make each step, if the stair case has total 30 steps. Bottom step needs 100 bricks and each successive step needs 2 brick less than the previous step.

   (iii) Perimeter of the each triangle, when the mid points of sides of an equilateral triangle whose side is 24 cm are joined to form another triangle, whose mid points in turn are joined to form still another triangle and the process continues indefinitely.



2. Write three terms of the GP when the first term 'a' and the common ratio 'r' are given?

   (i) $a = 4; \quad r = 3$ (ii) $a = \sqrt{5}\,; \quad r = \frac{1}{5}$

   (iii) $a = 81; \quad r = \frac{-1}{3}$ (iv) $a = \frac{1}{64}\,; \quad r = 2$

3. Which of the following are GP? If they are GP. Write three more terms?

   (i) 4, 8, 16 ..... (ii) $\frac{1}{3}, \frac{-1}{6}, \frac{1}{12}$.....

   (iii) 5, 55, 555, ... (iv) $-2, -6, -18$ .....

   (v) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$..... (vi) $3, -3^2, 3^3$, ...

   (vii) $x, 1, \frac{1}{x}$,..... (viii) $\frac{1}{\sqrt{2}}, -2, \frac{8}{\sqrt{2}}$.....

   (ix) 0.4, 0.04, 0.004, ...

4. Find x so that $x, x+2, x+b$ are conseecitive terms of a geometric progression.

6.6 n^{th} TERM OF A GP

Let us examine a problem the number of bacteria in a certain culture triples every hour. If there were 30 bacteria present in the culture originally. Then, what would be number of bacteria in fourth hour?

To answer this let us first see what the number of bacteria in second hour would be.

Since for every hour it triples

No.of bacteria in Second hour $= 3 \times$ no.of bacteria in first hour

$$= 3 \times 30 = 30 \times 3^1$$

$$= 30 \times 3^{(2-1)}$$

$$= 90$$

No.of bacteria in third hour $= 3 \times$ no.of bacteria in second hour

$$= 3 \times 90 \qquad = 30 \times (3\times3)$$

$$= 30 \times 3^2 \qquad = 30 \times 3^{(3-1)}$$

$$= 270$$

No.of bacteria in fourth hour $= 3 \times$ no.of bacteria in third hour

$$= 3 \times 270 \qquad = 30 \times (3\times3\times3)$$

$$= 30 \times 3^3 \qquad = 30 \times 3^{(4-1)}$$

$$= 810$$

Observe that we are getting a list of numbers

30, 90, 270, 810, ...

These numbers are in GP (why ?)

Now looking at the pattern formed above, can you find number of bacteria in 20th hour?

You may have already got some idea from the way we have obtained the number of bacteria as above. By using the same pattern, we can compute that Number of bacteria in 20^{th} hour.

$$= 30\times\underbrace{(3\times3\times\ldots\times3)}_{19\text{ terms}}$$

$$= 30 \times 3^{19} \qquad = 30 \times 3^{(20-1)}$$

This example would have given you some idea about how to write the 25th term. 35^{th} term and more generally the nth term of the GP.

Let a_1, a_2, a_3 ..... be in GP whose 'first term' a_1 is a and the common ratio is $\boldsymbol{r}$

then the second term $a_2 = ar = ar^{(2-1)}$

the third term $a_3 = a_2 \times r = (ar) \times r = ar^2 = ar^{(3-1)}$

the fourth term $a_4 = a_3 \times r = ar^2 \times r = ar^3 = ar^{(4-1)}$

....

....

Looking at the pattern we can say that n[th] term $a_n = ar^{n-1}$

So n[th] term an of a GP with first term 'a' and common ratio 'r' is given by $a_n = ar^{n-1}$.

Let us consider some examples

Example-20. Find the 20[th] and n^{th} term of the GP.

$$\frac{5}{2}, \frac{5}{4}, \frac{5}{8} ....$$

Solution : Here $a = \frac{5}{2}$ and $r = \frac{\frac{5}{4}}{\frac{5}{2}} = \frac{1}{2}$

Then $$a_{20} = ar^{20-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{19} = \frac{5}{2^{10}}$$

and $$a_n = ar^{n-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{5}{2^n}$$

Example-21. Which term of the GP : 2, $2\sqrt{2}$, 4 ..... is 128 ?

Solution : Here $a = 2$ $r = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Let 128 be the n^{th} term of the GP.

Then $a_n = ar^{n-1} = 128$

$$2.(\sqrt{2})^{n-1} = 128$$

$$(\sqrt{2})^{n-1} = 64$$

$$(2)^{\frac{n-1}{2}} = 2^6$$

$$\Rightarrow \quad \frac{n-1}{2} = 6$$

$$\therefore \quad n = 13.$$

Hence 128 is the 13th term of the GP.

Example-22. In a GP the 3rd term is 24 and 65th term is 192. Find the 10th term.

Solution : Here $a_3 = ar^2 = 24$ ...(1)

$a_6 = ar^5 = 195$ ...(2)

Dividing (2) by (1) we get $\frac{ar^5}{ar^2} = \frac{195}{24}$

$$\Rightarrow \quad r^3 = 8 = 2^3$$

$$\Rightarrow \quad r = 2$$

Substituting $r = 2$ in (1) we get $a = 6$.

$\therefore \ a_{10} = ar^9 = 6(2)^9 = 3072.$

Exercise-6.5

1. For each geometric progression find the common ratio 'r', and then find a_n

   (i) $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}$......... (ii) $2, -6, 18, -54$

   (iii) $-1, -3, -9, -18$ .... (iv) $5, 2, \frac{4}{5}, \frac{8}{25}$.........

2. Find the 10th and nth term of GP. : 5, 25, 125, ...

3. Find the indicated term of each geometric Progression

   (i) $a_1 = 9;\ r = \frac{1}{3};$ find a_7 (ii) $a_1 = -12;\ r = \frac{1}{3};$ find a_6

4. Which term of the GP.

   (i) 2, 8, 32, ... is 512 ? (ii) $\sqrt{3}, 3, 3\sqrt{3}$ ...... is 729 ?

   (iii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$..... is $\frac{1}{2187}$?

5. Find the 12^{th} term of a GP. whose 8^{th} term is 192 and the common ratio is 2.

6. The 4th term of a geometric progression is $\frac{2}{3}$ and the seventh term is $\frac{16}{81}$. Find the geometric series.

7. If the geometric progressions 162, 54, 18 ..... and $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}$.... have their n^{th} term equal. Find the value of n.

Optional Exercise

[This exercise is not meant for examination]

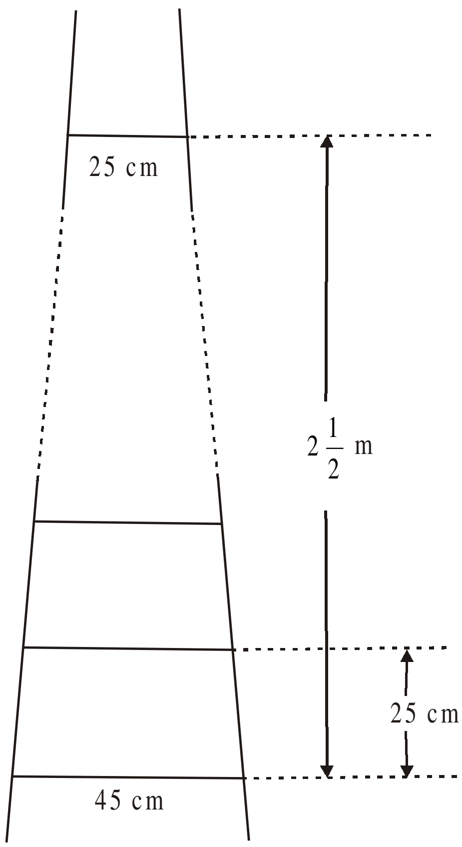
1. Which term of the AP : 121, 117, 113, . . ., is the first negative term?

   [**Hint :** Find n for $a_n < 0$]

2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

3. A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

   [**Hint :** Number of rungs = $\frac{250}{25} + 1$]



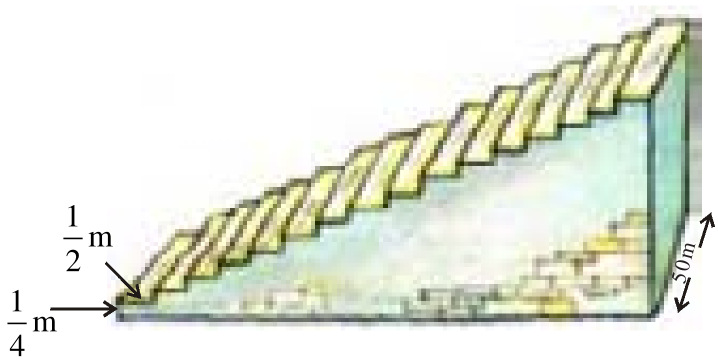
4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. And find this value of x.

   [**Hint :** $S_{x-1} = S_{49} - S_x$]

5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. (see Fig. 5.8). Calculate the total volume of concrete required to build the terrace.

[**Hint :** Volume of concrete required to build the first step $= \frac{1}{4} \times \frac{1}{2} \times 50\,\text{m}^3$]



6. 150 workers ware engaged to finish a piece of work in a certain number of days. Four workers dropped from the work in the second day. Four workers dropped in third day and so on. It took 8 more days to finish the work. Find the number of days in which the was and completed.

[let the no.of days to finish the work is 'x' then

$$150x = \frac{x+8}{2}[2\times150+(x+8-1)(-4)]$$

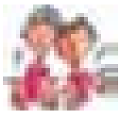
[Ans. $x = 17 \Rightarrow x + 8 = 17 + 8 = 25$]

7. A machine costs ₹ 5,00,000. If the value depreciates 15% in the first year, $13\frac{1}{2}$ % in the second year, 12% in the third year and so on. What will be its value at the end of 10 years, when all the percentages will be applied to the original cost?

[Total depreciation $= 15 + 13\frac{1}{2} + 12 + \ldots.10$ terms.

$$S_n = \frac{10}{2}[30-13.5] = 82.5\%$$

$\therefore$ after 10 year original cost $= 100 - 82.5 = 17.5$ i.e., 17.5% of 5,00,000



What We Have Discussed

In this chapter, you have studied the following points :

1. An **arithmetic progression** (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term. The fixed number d is called the **common difference**.

   The terms of AP are $a, a + d, a + 2d, a + 3d, \ldots$

2. A given list of numbers $a_1, a_2, a_3, \ldots$ is an AP, if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$, give the same value, i.e., if $a_{k+1} - a_k$ is the same for different values of k.

3. In an AP with first term a and common difference d, the nth term (or the general term) is given by $a_n = a + (n - 1) d$.

4. The sum of the first n terms of an AP is given by :

   $$S = \frac{n}{2}[2a + (n-1)d]$$

5. If l is the last term of the finite AP, say the nth term, then the sum of all terms of the AP is given by :

   $$S = \frac{n}{2}(a + l).$$

6. A Geometric Progression (GP) is a list of numbers in which each term is obtained by multiplying preceeding term with a fixed number 'r' except first term. This fixed number is called common ratio 'r'.

   The general form of GP is $a, ar, ar^2, ar^3 \ldots.$

7. If the first term and common ratio of a GP are a, r respectively then nth term $a_n = ar^{n-1}$.