CHAPTER 12

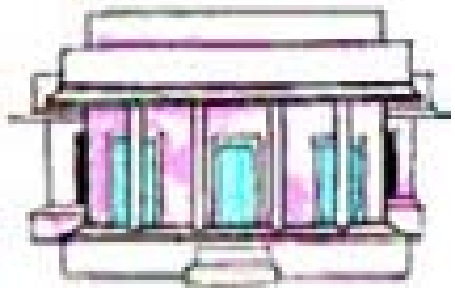
Applications of Trigonometry

12.1 INTRODUCTION

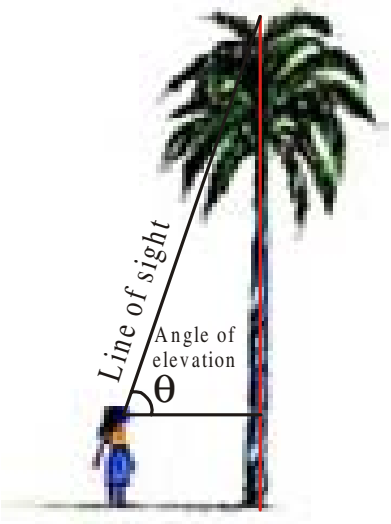
You have studied in social studies that the highest mountain peak in the world is Mount Everest and its height is 8848 meters.

Kuntala waterfall in Adilabad district is the highest natural waterfall in Andhra Pradesh. Its height is 147 feet.

How were these heights measured? Can you measure the height of your school building or the tallest tree in or around your school?



Let us understand through some examples. Vijaya wants to find the height of a palm tree. She tries to locate the top most point of the tree. She also imagines a line joining the top most point and her eye.



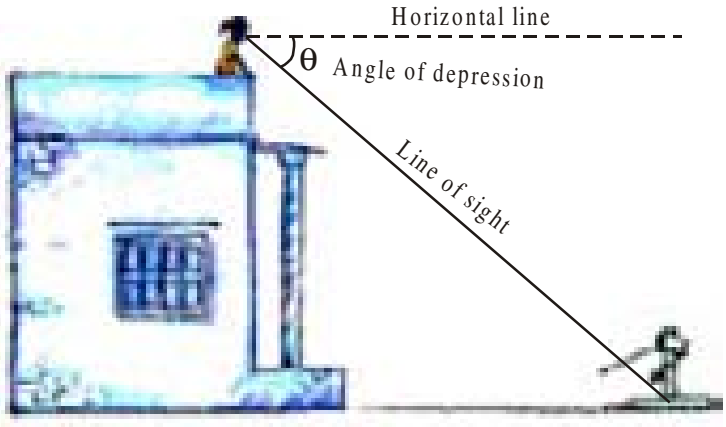
This line is called "line of sight". She also imagines a horizontal line, parallal to earth, from her eye to the tree.

Here, "the line of sight", "horizontal line" and "the tree" form a right angle triangle.

To find the height of the tree, she needs to find a side and an angle in this triangle.

"The line of sight is above the horizontal line and angle between the line of sight and the horizontal line is called **angle of elevation**".

Suppose you are standing on the top of your school building and you want to find the distance of borewell from the building on which you are standing. For that, you have to observe the base of the borewell.



Then, the line of sight from your eye to the base of borewell is below the horizontal line from your eye.

Here, "the angle between the line of sight and horizantal line is called **angle of depression**."

Trigonometry has been used by surveyors for centuries. They use Theodolites to measure angles of elevation or depression in the process of survey. In nineteenth century, two large Theodolites were built by British India for the surveying project "great trigonometric survey". During the survey in 1852, the highest mountain peak in the world was discovered in the Himalayas. From the distance of 160 km, the peak was observed from six different stations and the height of the peak was calculated. In 1856, this peak was named after Sir George Everest, who had commissioned and first used the giant Theodolites. Those theodolites are kept in the museum of the Survey of India in Dehradun for display.

12.2 Drawing Figures to Solve Problems

When we want to solve the problems of heights and distances, we should consider the following:

(i) All the objects such as towers, trees, buildings, ships, mountains etc. shall be considered as linear for mathematical convenience.

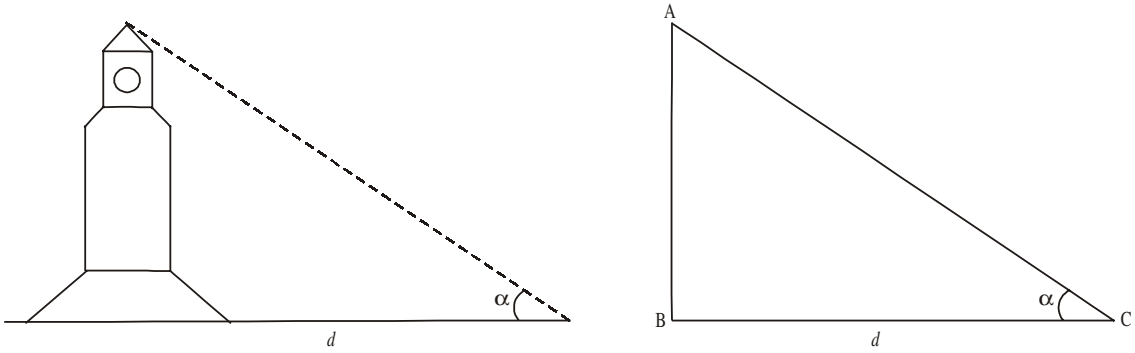
(ii) The angle of elevation or angle of depression is considered with reference to the horizontal line.

(iii) The height of the observer is neglected, if it is not given in the problem.

When we try to find heights and distances at an angle of elevation or depression, we need to visualise geometrically. To find heights and distances, we need to draw figures and with the help of these figures we can solve the problems. Let us see some examples.

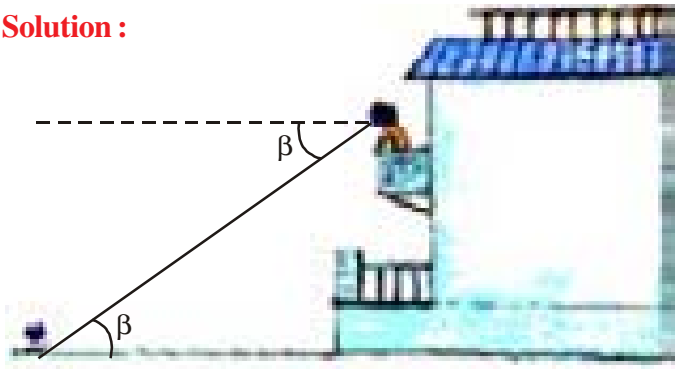
Example-1. The top of a clock tower is observed at angle of elevation of α° and the foot of the tower is at the distance of d meters from the observer. Draw the diagram for this data.

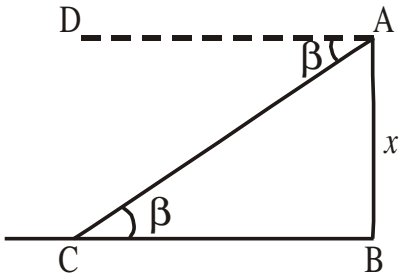
Solution : The diagrams are as shown below :



Example-2. Rinky observes a flower on the ground from the balcony of the first floor of a building at an angle of depression β^o. The height of the first floor of the building is x meters. Draw the diagram for this data.

Solution :



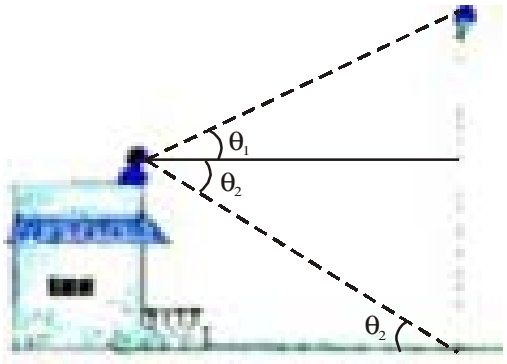


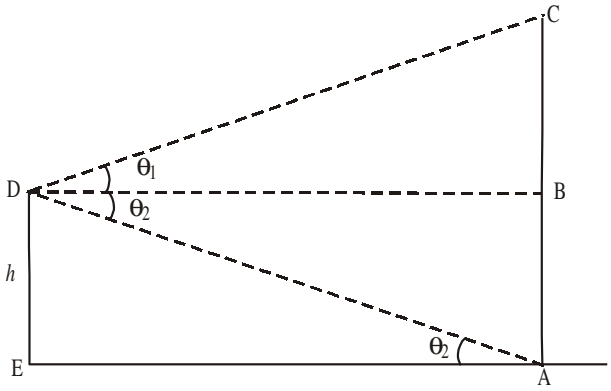
Here $\angle DAC = \angle ACB = \beta$ (why?)

Example-3. A large balloon has been tied with a rope and it is floating in the air. A person has observed the balloon from the top of a building at angle of elevation of θ_1 and foot of the rope at an angle of depression of θ_2. The height of the building is h feet. Draw the diagram for this data.

Solution : We can see that

$\angle BDA = \angle DAE$. (Why?)





Do This

1. Draw diagram for the following situations :
   (i) A person is flying a kite at an angle of elevation α and the length of thread from his hand to kite is 'ℓ'.
   (ii) A person observes two banks of a river at angles of depression θ_1 and θ_2 ($\theta_1 < \theta_2$) from the top of a tree of height h which is at a side of the river. The width of the river is 'd'.

Think - Discuss

1. You are observing top of your school building at an angle of elevation α from a point which is at d meter distance from foot of the building.
   Which trigonometric ratio would you like to consider to find the height of the building?
2. A ladder of length x meter is leaning against a wall making angle θ with the ground. Which trigonometric ratio would you like to consider to find the height of the point on the wall at which the ladder is touching?

Till now, we have discussed how to draw diagrams as per the situations given. Now, we shall discuss how to find heights and distances.

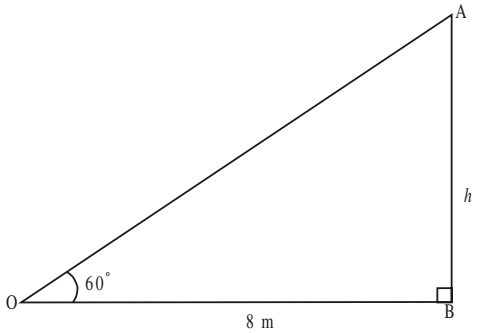
Example-4. A boy observed the top of an electric pole at an angle of elevation of 60° when the observation point is 8 meters away from the foot of the pole. Find the height of the pole.

Solution : From the figure, in triangle OAB

$OB = 8$ meters

$\angle AOB = 60^\circ$

Let, height of the pole = AB = h meters

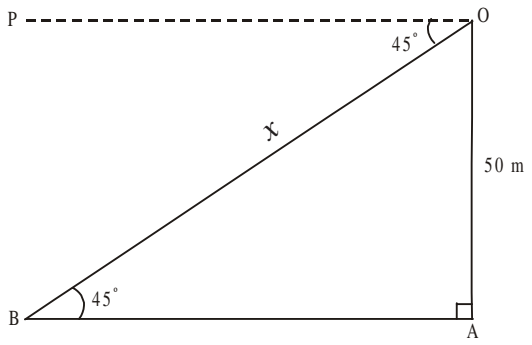


(we know the adjacent side and we need to find the opposite side of $\angle AOB$ in the triangle ΔOAB. Hence we need to consider the trigonometric ratio "tan" to solve the problem).

$$\tan 60^\circ = \frac{AB}{OB}$$

$$\sqrt{3} = \frac{h}{8} \qquad h = 8\sqrt{3}m.$$

Example-5. Rajender observes a person standing on the ground from a helicopter at an angle of depression 45°. If the helicopter flies at a height of 50 meters from the ground, what is the distance of the person from Rajender?



Solution : From the figure, in triangle OAB

OA = 50 meters

$\angle POB = \angle OAB = 45°$ (why ?)

OB = distance of the person from Rajender = x.

(we know the opposite side of $\angle OBA$ and we need to find hypotenuse OB in the triangle OAB. Hence, we need to consider the ratio "sin".)

$$\sin 45° = \frac{OA}{OB}$$

$$\frac{1}{\sqrt{2}} = \frac{50}{x}$$

$$x = 50\sqrt{2} \text{ meters}$$

(The distance from the person to Rajendar is $50\sqrt{2}$ m)

Exercise - 12.1

1. A tower stands vertically on the ground. From a point which is 15 meter away from the foot of the tower, the angle of elevation of the top of the tower is 45°. What is the height of the tower?
2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground by making 30° angle with the ground. The distance between the foot of the tree and the top of the tree on the ground is 6m. Find the height of the tree before falling down.
3. A contractor wants to set up a slide for the children to play in the park. He wants to set it up at the height of 2 m and by making an angle of 30° with the ground. What should be the length of the slide?
4. Length of the shadow of a 15 meter high pole is $5\sqrt{3}$ meters at 7 o'clock in the morning. Then, what is the angle of elevation of the Sun rays with the gound at the time?
5. You want to erect a pole of height 10 m with the support of three ropes. Each rope has to make an angle 30° with the pole. What should be the length of the rope?

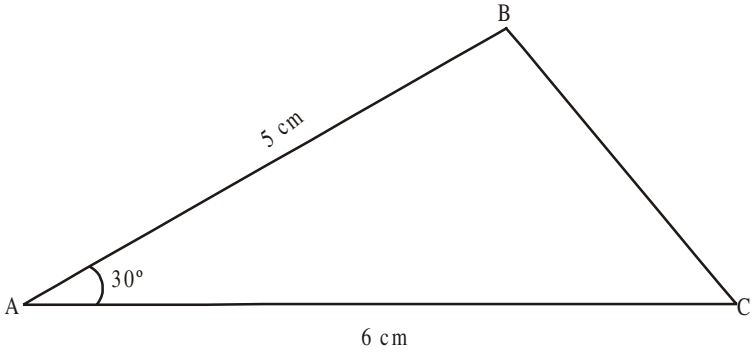
6. Suppose you are shooting an arrow from the top of a building at an height of 6 m to a target on the ground at an angle of depression of 60°. What is the distance between you and the object?

7. An electrician wants to repair an electric connection on a pole of height 9 m. He needs to reach 1.8 m below the top of the pole to do repair work. What should be the length of the ladder which he should use, when he climbs it at an angle of 60° with the ground? What will be the distance between foot of the ladder and foot of the pole?

8. A boat has to cross a river. It crosses the river by making an angle of 60° with the bank of the river due to the stream of the river and travels a distance of 600m to reach the another side of the river. What is the width of the river?

9. An observer of height 1.8 m is13.2 m away from a palm tree. The angle of elevation of the top of the tree from his eyes is 45°. What is the height of the palm tree?

10. In the adjacent figure, AC = 6 cm, AB = 5 cm and $\angle BAC = 30°$. Find the area of the triangle.

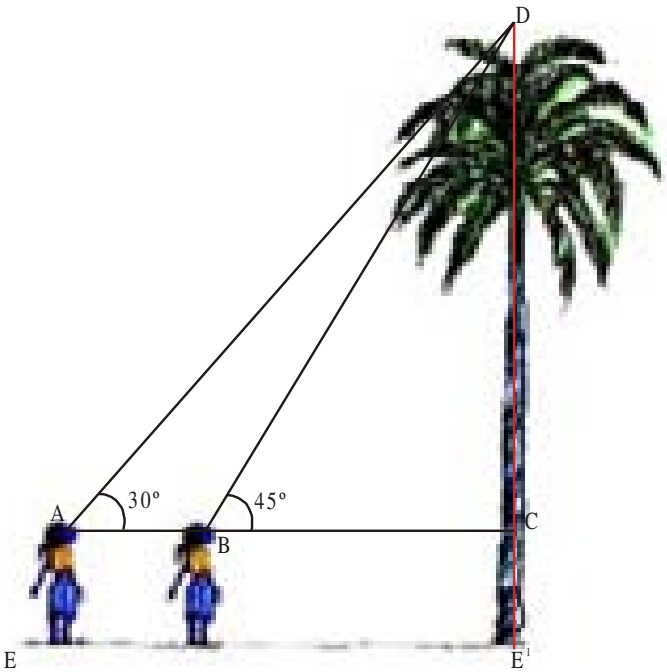


12.3 Solution for Two Triangles

We have discussed the solution of a one triangle problem. What will be the solution if there are two triangles?

Suppose you are standing on one side of a tree. You want to find the height of a tree and you want to observe the tree from different points of observations.

How can you do this? Suppose you are observing the top of the palm tree at an angle of elevation 45°. The angle of elevation changes to 30° when you move 11 m away from the tree.



Let us see how we can find height of the tree.

From figure, we have

AB = 11 m

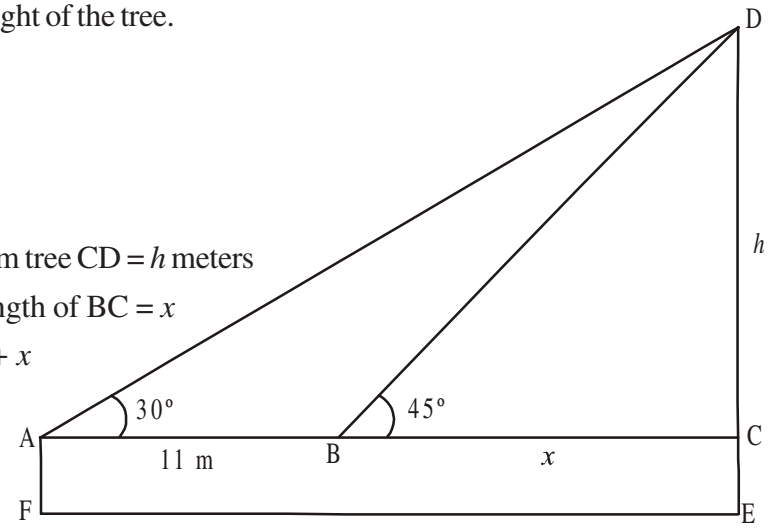
$\angle DAC = 30^\circ$

$\angle DBC = 45^\circ$

Let the height of the palm tree CD = h meters

and length of BC = x

AC = $11 + x$



from triangle BDC

$$\tan 45^\circ = \frac{DC}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = h\sqrt{3} \qquad ...(1)$$

from triangle ADC

$$\tan 30^\circ = \frac{DC}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{11+x}$$

$$h = \frac{11+x}{\sqrt{3}}$$

$$h = \frac{11}{\sqrt{3}} + \frac{h}{\sqrt{3}}$$

$$h - \frac{h}{\sqrt{3}} = \frac{11}{\sqrt{3}}$$

$$h\frac{\left(\sqrt{3}-1\right)}{\sqrt{3}} = \frac{11}{\sqrt{3}}$$

$$h = \frac{11}{\left(\sqrt{3}-1\right)} \text{ meters.}$$

Note : Total height of the palm tree is CD + CE where CE = AF, which is the height of the girl.

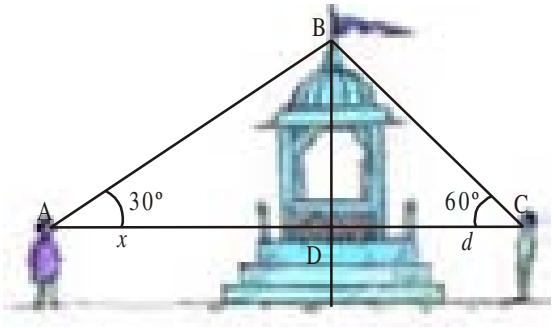
Example-6. Two men on either side of a temple of 30 meter height observe its top at the angles of elevation 30° and 60° respectively. Find the distance between the two men.

Solution : Height of the temple BD = 30 meter.

Angle of elevation of one person $\angle BAD = 30°$

Angle of elevation of another person $\angle BCD = 60°$

Let the distance between the first person and the temple, AD = x and distance between the second person and the temple, CD = d



From ΔBAD

$$\tan 30° = \frac{BD}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{30}{x}$$

$$x = 30\sqrt{3} \quad \ldots\ldots\ldots (1)$$

From ΔBCD

$$\tan 60° = \frac{BD}{d}$$

$$\sqrt{3} = \frac{30}{d}$$

$$d = \frac{30}{\sqrt{3}} \quad \ldots\ldots\ldots (2)$$

from (1) and (2) distance between the persons = BC + BA = $x + d$

$$= 30\sqrt{3} + \frac{30}{\sqrt{3}} = \frac{30 \times 4}{\sqrt{3}} = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ meter}$$

Example-7. A straight highway leads to the foot of a tower. Ramaiah standing at the top of the tower observes a car at an angle of depression 30°. The car is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Solution :

Let the distance travelled by the car in 6 seconds = AB = x meters

Heights of the tower CD = h meters

The remaining distance to be travelled by the car BC = d meters

and AC = AB + BC = $(x + d)$ meters

$\angle PDA = \angle DAP = 30°$ (why?)

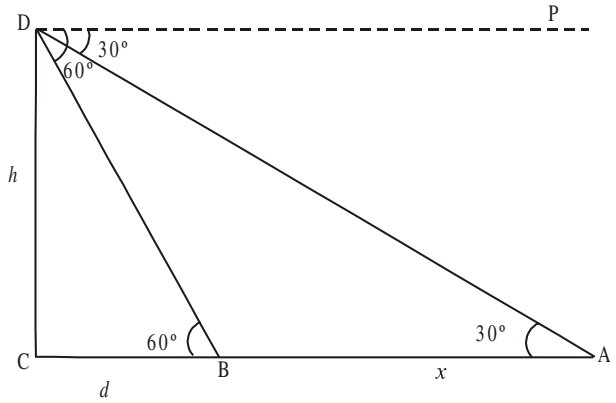
$\angle PDB = \angle DBP = 60°$ (why?)

From ΔBCD

$$\tan 60^\circ = \frac{CD}{BC}$$

$$\sqrt{3} = \frac{h}{d}$$

$$h = \sqrt{3}d \quad \ldots(1)$$



From ΔACD

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{(x+d)}$$

$$h = \frac{(x+d)}{\sqrt{3}} \quad \ldots(2)$$

From (1) & (2), we have

$$\frac{x+d}{\sqrt{3}} = \sqrt{3}d$$

$$x + d = 3d$$

$$x = 2d$$

$$d = \frac{x}{2}$$

Time taken to travel 'x' meters = 6 seconds.

Time taken to travel the distance of 'd' meters

i.e., $\frac{x}{2}$ meters = 3 seconds.

EXERCISE - 12.2

1. A TV tower stands vertically on the side of a road. From a point on the other side directly opposite to the tower, the angle of elevation of the top of tower is 60^0. From another point 10 m away from this point, on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the road.

2. A 1.5 m tall boy is looking at the top of a temple which is 30 meter in height from a point at certain distance. The angle of elevation from his eye to the top of the crown of the temple increases from 30º to 60º as he walks towards the temple. Find the distance he walked towards the temple.

3. A statue stands on the top of a 2m tall pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60º and from the same point, the angle of elevation of the top of the pedestal is 45º. Find the height of the statue.

4. From the top of a building, the angle of elevation of the top of a cell tower is 60º and the angle of depression to its foot is 45º. If distance of the building from the tower is 7m, then find the height of the tower.

5. A wire of length 18 m had been tied with electric pole at an angle of elevation 30º with the ground. Because it was convering a long distance, it was cut and tied at an angle of elevation 60º with the ground. How much length of the wire was cut?

6. The angle of elevation of the top of a building from the foot of the tower is 30º and the angle of elevation of the top of the tower from the foot of the building is 60º. If the tower is 30 m high, find the height of the building.

7. Two poles of equal heights are standing opposite to each other on either side of the road, which is 120 feet wide. From a point between them on the road, the angles of elevation of the top of the poles are 60º and 30º respectively. Find the height of the poles and the distances of the point from the poles.

8. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m, find the height of the tower from the base of the tower and in the same straight line with it are complementary.

9. The angle of elevation of a jet plane from a point A on the ground is 60º. After a flight of 15 seconds, the angle of elevation changes to 30º. If the jet plane is flying at a constant height of $1500\sqrt{3}$ meter, find the speed of the jet plane. $\left(\sqrt{3}=1.732\right)$

10. Clinky observes a tower PQ of height 'h' from a point A on the ground. She moves a distance 'd' towards the foot of the tower and finds that the angle of elevation has direction and finals that the angle of elevation is '3' times at A. Prove that $36h^2 = 35d^2$.

Optional Exercise

[This exercise his not meant for examination]

1. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60º. After some time, the angle of elevation reduces to 30º. Find the distance travelled by the balloon during the interval.

2. The angle of elevation of the top of a tower from the foot of the building is 30º and the angle of elevation of the top of the building from the foot of the tower is 60º. What is the ratio of heights of tower and building.

3. The angles of elevation of the top of a lighthouse from 3 boats A, B and C in a straight line of same side of the light house are a, $2a$, $3a$ respectively. If the distance between the boats A and B is x meters. Find the hight of light house?

4. Inner part of a cupboard is in the cuboidical shape with its length, breadth and height in the ratio $1 : \sqrt{2} : 1$. What is the angle made by the longest stick which can be inserted cupboard with its base inside.

5. An iron sperical ball of volume 232848 cm^3 has been melted and converted into a cone with the vertical angle of 120^o. What are its height and base?

What We Have Discussed

In this chapter, we have studied the following points :

1. (i) The line of sight is the line drawn from the eye of an observer to a point on the object being viewed by the observer.

   (ii) The angle of elevation of the object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

   (iii) The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.

2. The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.