Chapter – 3

Polynomials

Polynomial: Let x be a variable, n be a positive integer and $a_1, a_2, \ldots\ldots\ a_n$ be constants (real numbers).

Then

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1x + a_0$ is called a **polynomial** in variable x.

In the polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1x + a_0$

$a_nx^n, a_{n-1} x^{n-1}, \ldots\ldots a_1x$ and a_0 are known as the terms of the polynomial and $a_n, a_{n-1}, \ldots\ldots a_1, a_0$ are their coefficients.

Ex: $f(x) = 2x + 3$ is a polynomial in variable x.

$g(y) = 2y^2 - 7y + 4$ is a polynomial in variable y.

Note: The expressions like $2x^2 - 3\sqrt{x} + 5, \frac{1}{x^2 - 2x + 5}, 2x^3 - \frac{3}{x} + 4$ are not polynomials.

Degree of a Polynomial: The exponent of the highest degree term in a polynomial is known as its degree.

In other words, the highest power of x in a polynomial f(x) is called the degree of the polynomial f(x).

Ex: $f(x) = 5x^3 - 4x^2 + 3x - 4$ is a polynomial in the variable x of degree '3'.

Constant Polynomial: A polynomial of degree zero is called a Constant Polynomial.

Ex: $f(x) = 7, p(t) = 1$

Linear Polynomial: A polynomial of degree 1 is called a linear polynomial.

Ex: $p(x) = 4x - 3$; $f(t) = \sqrt{3}t + 5$

Quadratic Polynomial: Polynomial of degree 2 is called **Quadratic Polynomial**.

Ex: $f(x) = 2x^2 + 3x - \frac{1}{2}$

$g(x) = ax^2+bx+c$, $a \neq 0$

Note: A quadratic polynomial may be a monomial or a binomial or trinomial.

Ex: $f(x) = \frac{2}{3}x^2$ is a monomial, $g(x) = 5x^2 - 3$ is a binomial and $h(x) = 3x^2 - 2x + 5$ is a trinomial.

Cubic Polynomial: A polynomial of degree 3 is called a **cubic polynomial**.

Ex: $f(x) = \frac{2}{3}x^3 - \frac{1}{7}x^2 + \frac{4}{5}x + \frac{1}{4}$

Polynomial of n^th^ Degree: $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots\ldots + a_1 x + a_0$ is a polynomial of n^{th} degree, where $a_n, a_{n-1}, \ldots\ldots\ldots a_1, a_0$ are real coefficients and $a_n \neq 0$.

Value of a Polynomial: The value of a polynomial P(x) at x = k, where k is a real number, is denoted by P(k) and is obtained by putting k for x in the polynomial.

Ex: Value of the polynomial $f(x) = x^2 - 2x - 3$ at $x = 2$ is $f(2) = 2^2 - 2(2) - 3 = -3$.

Zeroes of a Polynomial: A real number k is said to be a zero of the polynomial f(x) if f(k) =0

Ex: Zeroes of a polynomial $f(x) = x^2 - x - 6$ are –2 and 3,

Because $f(-2) = (-2)^2 - (-2) - 6 = 0$ and $f(3) = 3^2 - 3 - 6 = 0$

Zero of the linear polynomial $ax + b$, $a \neq 0$ is $\frac{'-b'}{a}$

Graph of a Linear Polynomial:

i) Graph of a linear polynomial $ax + b, a \neq 0$ is a straight line.

ii) A linear polynomial $ax + b, a \neq 0$ has exactly one zero, namely X co-ordinate of the point where the graph of $y = ax + b$ intersects the X-axis.

iii) The line represented by $y = ax + b$ crosses the X-axis at exactly one point, namely $\left(-\frac{b}{a}, 0\right)$.

Graph of a Quadratic Polynomial:

For any quadratic polynomial $ax^2 + bx + c, a \neq 0$, the graph of the corresponding equation $y = ax^2 + bx + c$ either opens upwards like $\cup$ or opens downwards like $\cap$. This depends on whether $a > 0$ or $a < 0$. The shape of these curves are called **parabolas.**

The zeroes of a quadratic polynomial $ax^2 + bx + c, a \neq 0$ are precisely the X-coordinates of the points where the parabola representing $y = ax^2 + bx + c$ intersects the X-axis.

- A quadratic polynomial can have at most 2 zeroes.
- A cubic polynomial can have at most '3' zeroes.
- A constant polynomial has no zeroes.
- A polynomial f(x) of degree n, the graph of y = f(x) interacts the X-axis at most 'n' points.

Therefore, a polynomial f(x) of degree n has at most 'n' zeroes.

Relationship between Zeroes and Coefficients of a Polynomial:

i) The zero of the linear polynomial $ax + b, a \neq 0$ is $-\frac{b}{a}$.

ii) If α, β are the zeroes of the quadratic polynomial $ax^2 + bx + c, a \neq 0$ then

Sum of the zeroes = $\alpha+\beta=-\frac{b}{a}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of the zeroes = $\alpha\beta=\frac{c}{a}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

iii) If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, $a \neq 0$ then

$$\alpha+\beta+\gamma=-\frac{b}{a}=\frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}=\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha.\beta.\gamma=-\frac{d}{a}=\frac{-(\text{constant term})}{\text{coefficient of } x^3}$$

- A quadratic polynomial with zeroes α and β is given by

  $k\{x^2 - (\alpha + \beta) x + \alpha\beta\}$, where $k (\neq 0)$ is real.

- A cubic polynomial with zeroes α, β and γ is given by

  $k\{x^3 - (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma) x - \alpha\beta\gamma\}$ where $k (\neq 0)$ is real.

Division Algorithm for Polynomials: Let p(x) and g(x) be any two polynomials where $g(x) \neq 0$. Then on dividing p(x) by g(x), we can find two polynomials q(x) and r(x) such that

$p(x) = g(x) \times q(x) + r(x)$, where either $r(x) = 0$

Or degree of r(x) < degree of g(x).

This result is known as "Division Algorithm for polynomials".

Note: i) If r(x) = 0, then g(x) will be a factor of p(x).

ii) If a real number k is a zero of the polynomial p(x), then (x – k) will be a factor of p(x).

iii) If q(x) is linear polynomial then r(x) = Constant

iv) If p(x) is divided by (x – a), then the remainder is p(a).

v) If degree of q(x) = 1, then degree of p(x) = 1 + degree of g(x).

Essay Question (5 marks)

(1) Draw the graph of y = 2x – 5 and find the point of intersection on x – axis. Is the X – Coordinates of these points also the zero the polynomial.
(Visualization and Representation)

Solution: Y = 2x – 5

The following table lists the values of y corresponding to different values of x .

X	-2	-1	0	1	2	3	4
Y	-9	-7	-5	-3	-1	1	3

The points (-2, -9), (-1, -7), (0, -5), (1, -3), (2, -1), (3, 1) and (4, 3) are plotted on the graph paper on a suitable scale. A line is drawn passing through these points to obtain the graph of the given linear equation.

The graph cuts the x- axis at $p(\frac{5}{2}, 0)$

This is also the zero of the liner equation

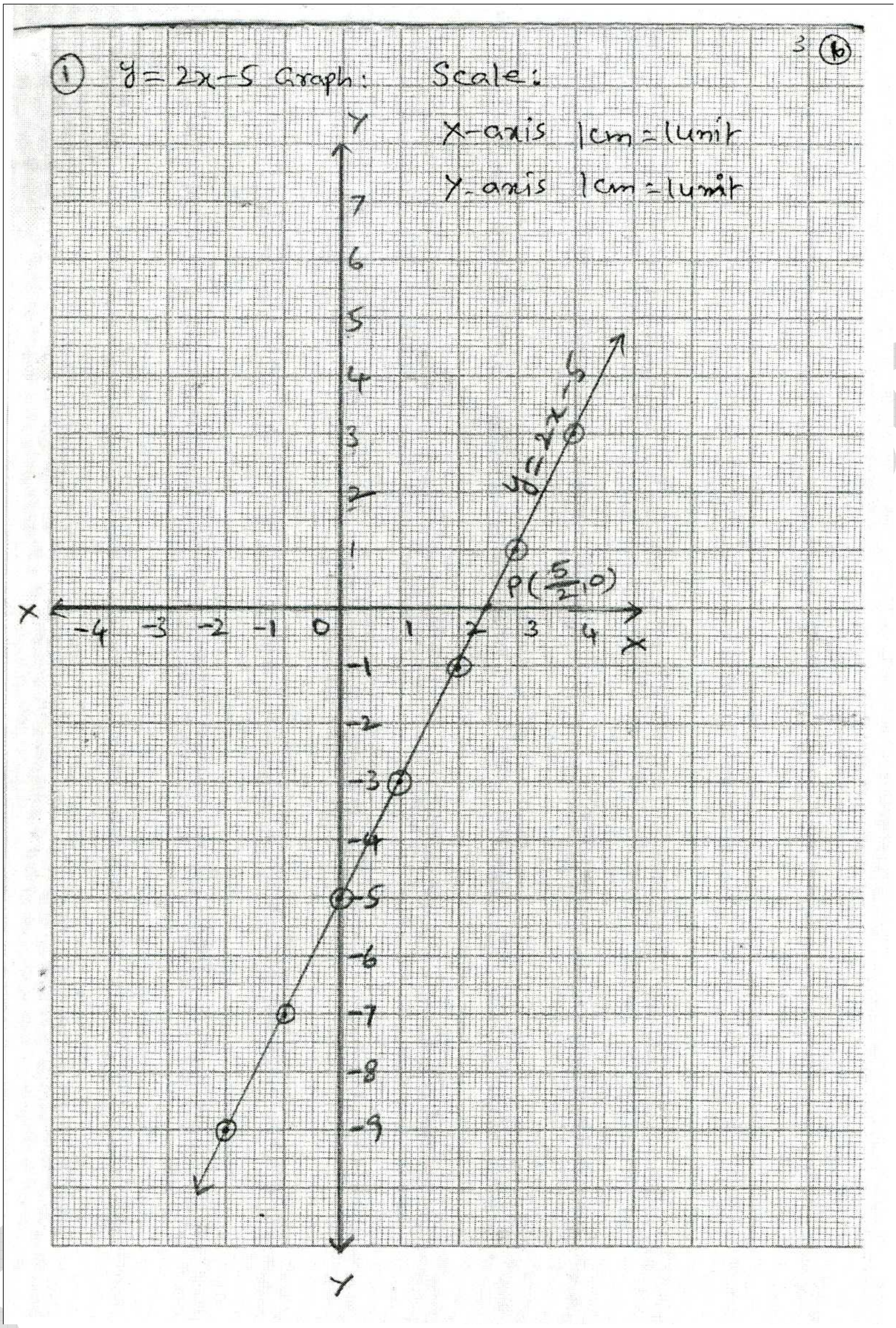
Y = 2x – 5

Because To find the zero of y = 2x -5 ,

$2x - 5 = 0 \Rightarrow 2x = 5 \Rightarrow X = 5/2$

$\therefore$ The zero of the liner equation is $5/2$

Model Question: Draw the graph of y = 2x+3.


3 (b)
① y = 2x − 5 Graph:
Scale:
X-axis 1cm = 1unit
Y-axis 1cm = 1unit
y = 2x − 5
P(5/2, 0)
X
Y

(2) Draw the graph of the polynomial f(x) = x^2-2x-8 and find zeroes. Verify the zeroes of the polynomial.

Solution: Let y = x^2-2x-8

The following table given the values of y for various values of x.

X	-3	-2	-1	0	1	2	3	4	5
Y= x^2-2x-8	7	0	-5	-8	-9	-8	-5	0	7
(x, y)	(-3,7)	(-2,0)	(-1,-5)	(0,-8)	(1, -9)	(2, -8)	(3, -5)	(4, 0)	(5, 7)

The Points (-3, 7), (-2, 0), (-1, -5), (0, -8), (1, -9), (2, -8), (3, -5), (4, 0) and (5, 7) are plotted on the graph paper on the suitable scale and drawn a smooth free hand curve passing through these points.

The curve thus obtained represents the graph of the polynomial y = x^2-2x-8. This is called a parabola.

The curve cuts the x – axis at (-2, 0) and (4, 0) .

The x – coordinates of these points are zeroes of the polynomial y = x^2-2x-8. Thus -2 and 4 are the zeroes.

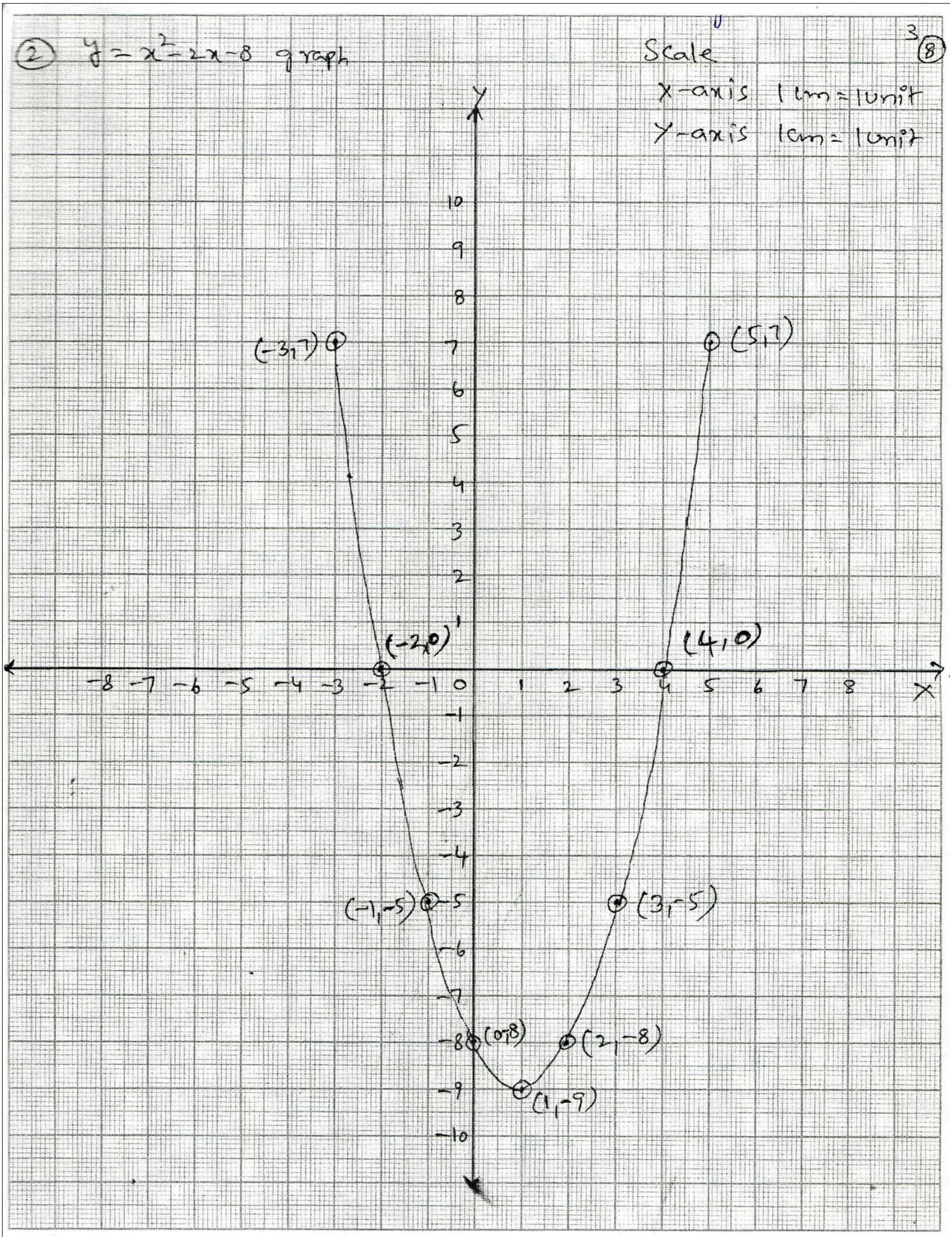
Verification: To find zeroes of x^2-2x-8

$x^2-2x-8 \Rightarrow x^2-4x+2x-8 = 0$

$x(x-4)+2(x-4) = 0$

$(x-4)(x+2) = 0$

$x - 4 = 0$ or $x+2 = 0 \Rightarrow x = 4$ or -2 are the zeroes.



② y = x² − 2x − 8 graph
Scale
X-axis 1cm = 1unit
Y-axis 1cm = 1unit
(−3,7)
(5,7)
(−2,0)
(4,0)
(−1,−5)
(3,−5)
(0,−8)
(2,−8)
(1,−9)
X
Y

(3) Draw the graph of f(x) = 3-2x-x² and find zeroes .Find zeroes. Verify the zeroes of the polynomial.

Solution: Let $y = 3-2x-x^2$

The following table given of values of y for various values of x.

X	-4	-3	-2	-1	0	1	2	3
$Y=3-2x-x^2$	-5	0	3	4	3	0	-5	-12
(x ,y)	(-4,-5)	(-3,0)	(-2,3)	(-1,4)	(0,3)	(1,0)	(2,-5)	(3,-12)

The points (-4,5), (-3,0), (-2,3), (-1,4), (0,3), (1,0), (2,-5) and (3,-12) are plotted on the graph paper on a suitable scale and drawn a smooth free hand curve passing through these points.

The curve thus obtained represent s the graph of the polynomial $y = 3-2x-x^2$. This called parabola opening downward.

The curve cuts the x- axis at (-3, 0) and (1,0) .

The x – coordinates of these points are zeroes of the polynomial. Thus the zeroes are -3, 1

Verification:

To find zeroes of $y = 3-2x-x^2$,

$$3-2x-x^2 = -x^2 -2x +3 = 0$$

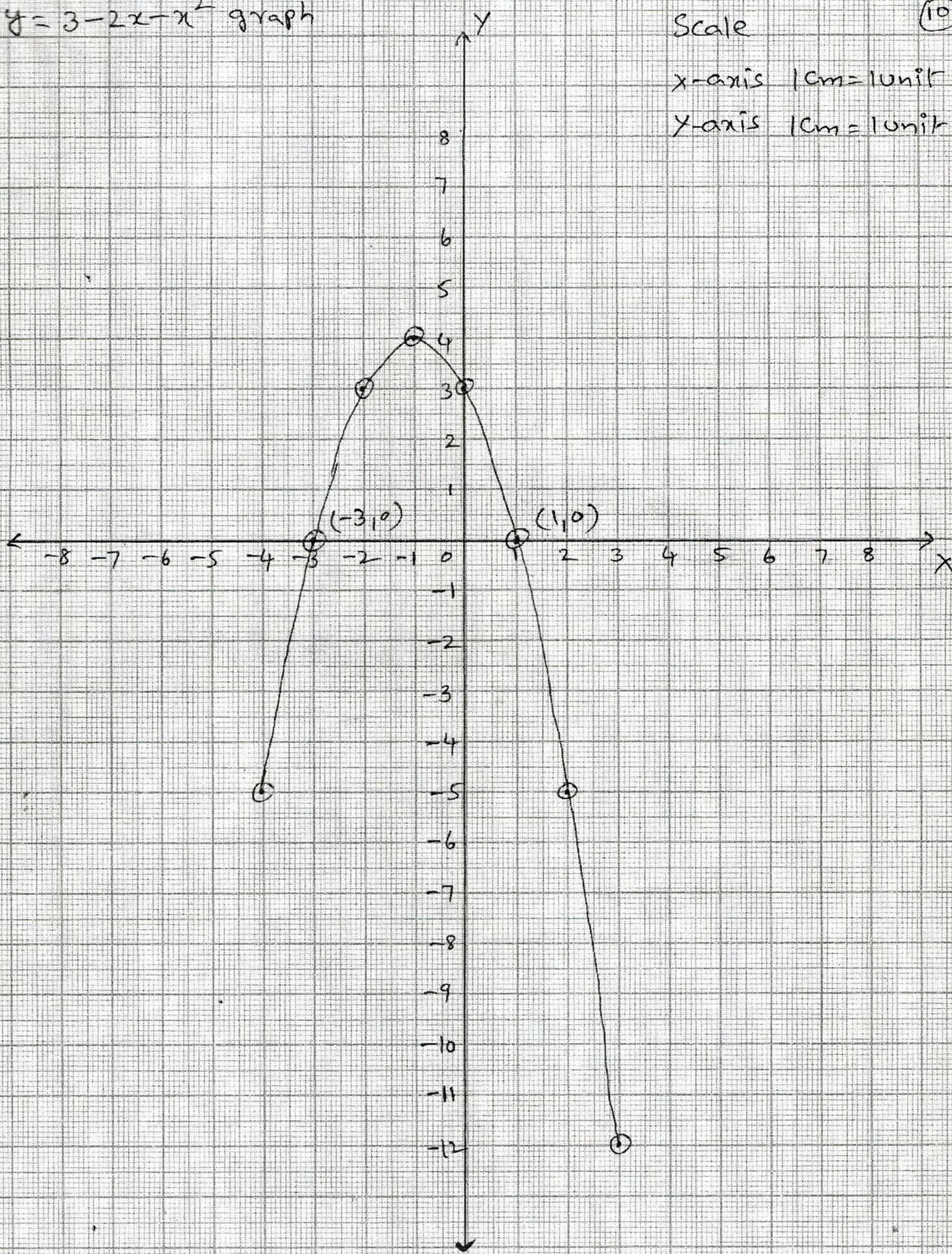
$$-x^2 -3x+x+3 = 0$$

$$-x(x+3)+1(x+3) =0$$

$$(x+3)(1-x) = 0$$

$$x+3 = 0 \text{ or } 1-x =0 \Rightarrow x = -3 \text{ or } 1 \text{ are the zeroes.}$$

③ $y = 3 - 2x - x^2$ graph



Scale
x-axis 1cm = 1unit
y-axis 1cm = 1unit

(4) Draw the graph of y =x^2-6x+9 and find zeroes verify the zeroes of the polynomial.

Solution: Let y =x^2-6x+9

The following table gives the values of y for various values of x

X	-2	-1	0	1	2	3	4	5	6
y =x^2-6x+9	25	16	9	4	1	0	1	4	9
(x, y)	(-2,25)	(-1,16)	(0,9)	(1,4)	(2,1)	(3,0)	(4,1)	(5,4)	(6,9)

The point (-2,25), (-1,16), (0,9), (1,4), (2,1), (3,0), (4,1), (5,4) and (6,9) are plotted on the graph paper on a suitable scale and drawn a smooth free hand curve passing through these points.

The curve thus obtained represents the graph of the polynomial y = x^2-6x+9.

The curve touches x-axis at one point (3,0) .The x- coordinate of this point is the zero of the polynomial y =x^2-6x+9. Thus the zero is 3 .

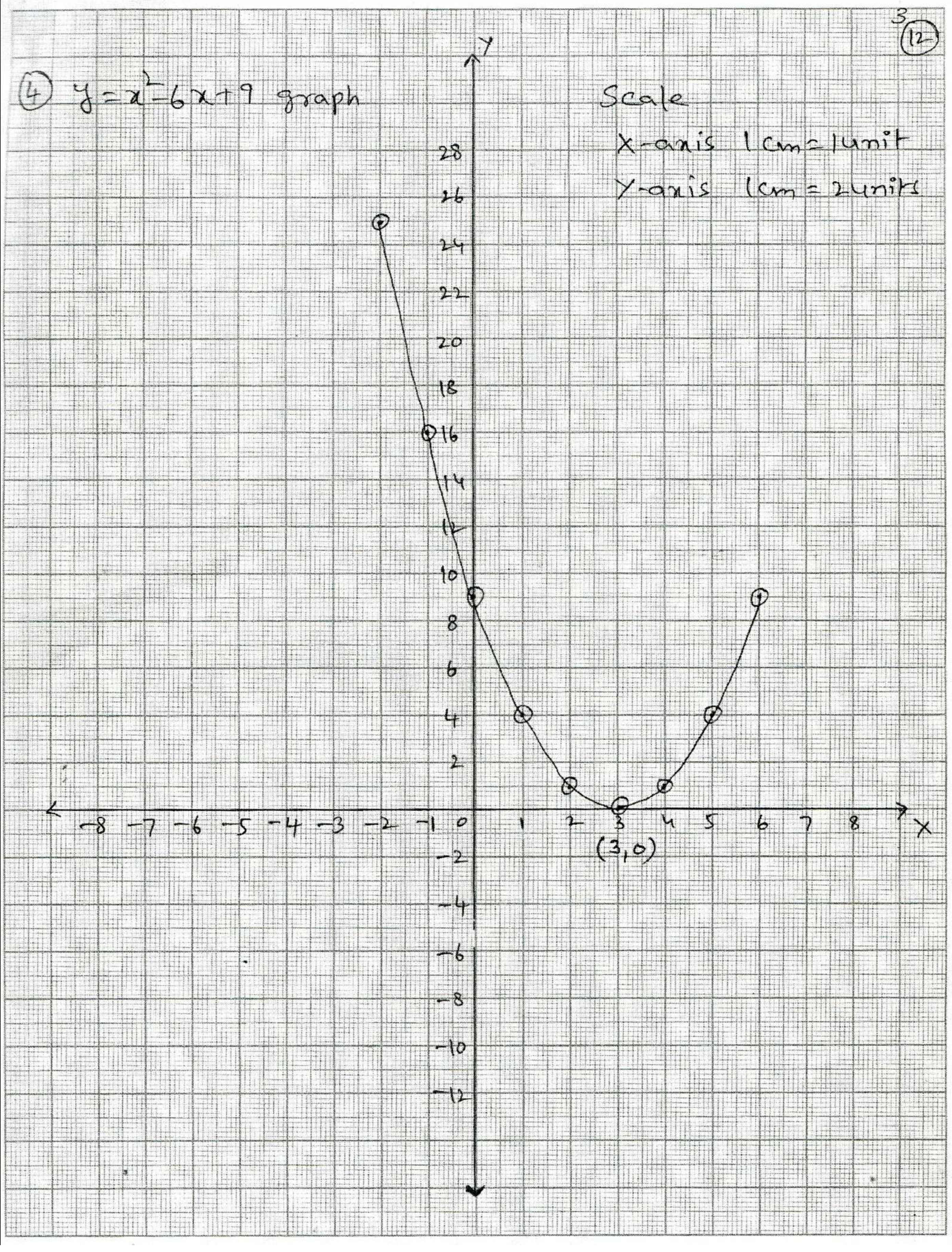
Verification:

To find zeros of x^2-6x+9

$x^2-6x+9=0 \Rightarrow (x-3)^2 = 0$

$x - 3 = 0$ or $x - 3 = 0$

$x = 3$ is the zero.


④ y = x² − 6x + 9 graph
Scale
X-axis 1 cm = 1 unit
Y-axis 1 cm = 2 units
(3, 0)
X
Y

(5) Draw the graph of the polynomial $y = x^2-4x+5$ and find zeroes . Verify the zeroes of the polynomial.

Solution: $y = x^2-4x+5$

The following table gives the values of y for various values of x.

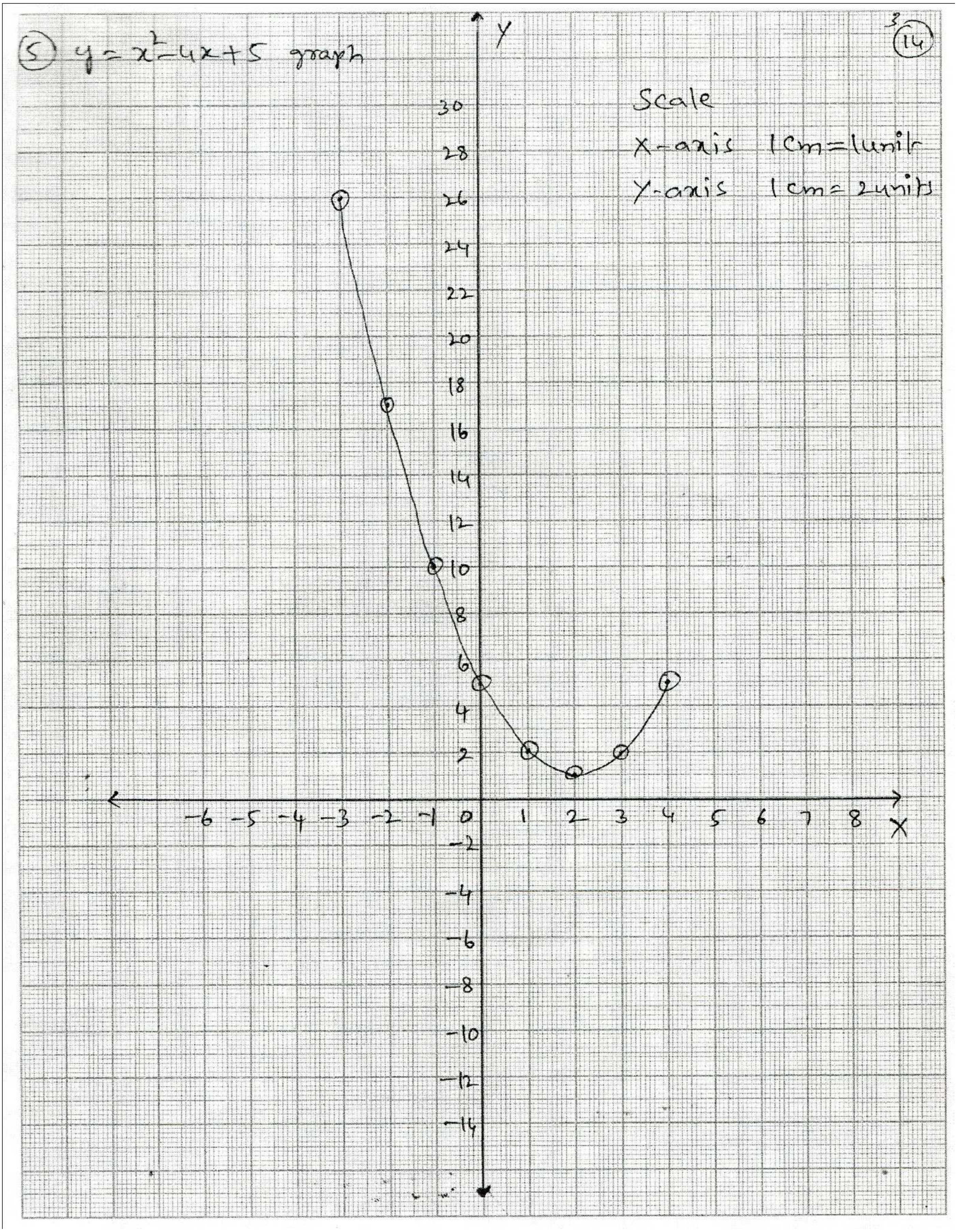
X	-3	-2	-1	0	1	2	3	4
$y = x^2-4x+5$	26	17	10	5	2	1	2	5
(x, y)	(-3,26)	(-2,17)	(-1,10)	(0,5)	(1,2)	(2,1)	(3,2)	(4,5)

The point (-3,26), (-2,17), (-1,10), (0,5), (1,2), (2,1), (3,2) and (4,5) are plotted on the graph paper on a suitable scale and drawn a smooth free hand curve passing through these points.

The curve thus obtained represents the graph of the polynomial $y = x^2-4x+5$

The curve does not intersect the x-axis.

∴ There are no zeroes of the polynomial $y = x^2-4x+5$



⑤ y = x² - 4x + 5 graph
14
Scale
X-axis 1cm = 1unit
Y-axis 1cm = 2units
Y
X
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
-2
-4
-6
-8
-10
-12
-14
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

(6) Draw the graph of the polynomial f(x) = x^3-4x and find zeroes. Verify the zeros Of the polynomial.

Solution: Let $y = x^3 - 4x$

The following table gives the values of y for various of x.

X	-3	-2	-1	0	1	2	3
$y = x^3 - 4x$	-15	0	3	0	-3	0	15
(x, y)	(-3, -15)	(-2,0)	(-1,3)	(0,0)	(1,-3)	(2,0)	(3,15)

The points (-3,15) , (-2,0), (-1,3), (0,0), (1,-3), (2,0) and (3,15) are plotted on the graph paper on a suitable scale and drawn a smooth free hand curve passing through these points.

The curve thus obtained represents the graph of the polynomial $y = x^3$-4x .

The curve touches x-axis at (-2,0), (0,0), (2,0) .The x- coordinate of this points are the zero of the polynomial y = x^3-4x. Thus -2, 0, 2, are the zeroes of the polynomial.

Verification:

To find zeroes of x^3-4x

$x^3-4x = 0 \Rightarrow x(x^2 - 4) = 0$

$\Rightarrow x(x-2)(x+2) = 0$

$\Rightarrow x = 0$ or $x - 2 = 0$ or $x + 2 = 0$

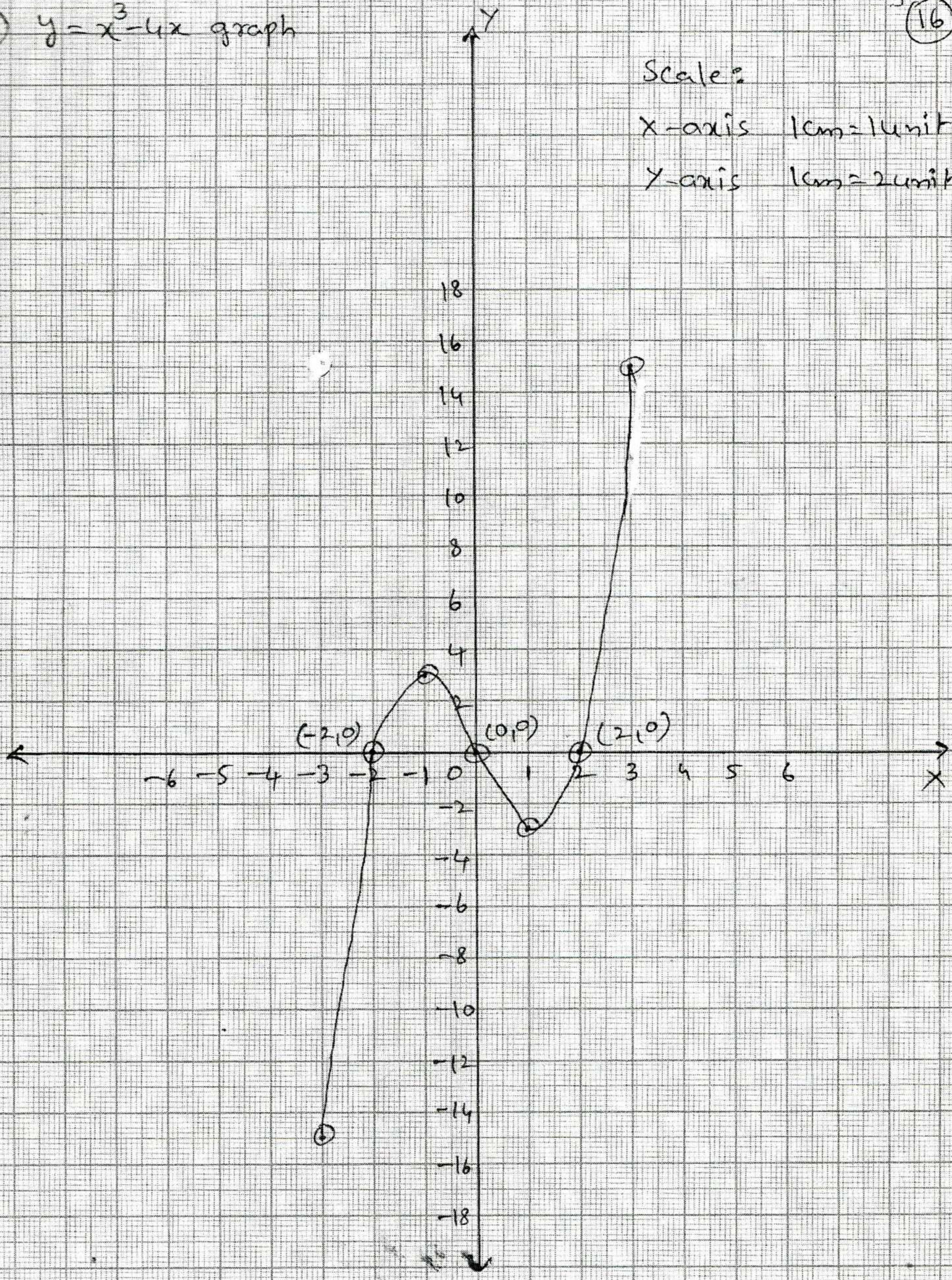
$\Rightarrow x = 0$ or 2 or -2 are the zeroes.

(6) $y = x^3 - 4x$ graph

Scale:

X-axis 1cm = 1unit

Y-axis 1cm = 2units



Essay Questions

(1) Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(I). x^2-2x-8 (ii). $6x^2-3-7x$

Solution: (I) Given polynomial x^2-2x-8

$= x^2-4x+2x-8$

$= x(x-4)+2(x-4)$

$= (x-4)(x+2)$

For zeroes of the polynomial, the value of $x^2-2x-8 = 0$

$(x-4)(x+2) = 0$

$x-4 = 0$ or $x+2 = 0$

$x = 4$ or $x = -2$

∴ The zeroes of x^2-2x-8 are -2 and 4.

We observe that

Sum of the zeroes = $-2+4 = 2 = -(-2)$

$$= \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

Product of the zeroes $= (-2)\times 4 = -8 = \frac{8}{1} = \frac{\text{cons tan t term}}{\text{coeffieient of } x^2}$

(ii) . Given polynomial $6x^2-3-7x$

$= 6x^2-7x-3$

$= 6x^2 -9x+2x-3$

$= 3x(2x-3)+1(2x-3)$

$= (2x-3)(3x+1)$

For zeroes of the polynomial , the value of $6x^2-3-7x = 0$ are

$(2x-3)(3x+1) = 0$

$2x - 3 = 0$ or $3x+1 = 0$

$x = 3/2$ or $x = -1/3$

$\therefore$ The zeroes of $6x^2-3-7x = 0$ are $3/2$ and $-1/3$

We observe that

Sum of the zeroes $= \frac{3}{2} + (-\frac{1}{3}) = \frac{9-2}{6} = \frac{7}{6}$

$= \frac{-(-7)}{6} = \frac{-(\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of the zeroes $= (\frac{3}{2})(-\frac{1}{3}) = -\frac{1}{2} = -\frac{3}{6} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

(2) **Verify that 1, -1 and -3 are the zeroes of the cubic polynomial x^3+3x^2-x-3 and verify the relationship between zeroes and the coefficients.**

Solution: Comparing the given polynomial with ax^3+bx^2+cx+d,

We get $a=1$, $b=3$, $c=-1$, $d=-3$

Let $p(x) = x^3+3x^2-x-3$

$P(1) = 1^3+3(1)^2-1-3 = 1+3-1+3 = 0$

$\therefore$ $P(1) = 0 \Rightarrow$ 1 is a zero of the polynomial p(x)

$P(-1) = (-1)^3+3(-1)^2+-(-1)-3 = -1+3+1-3 = 0$

$\therefore$ $p(-1) = 0 \Rightarrow$ -1 is a zero of the polynomial p(x)

$p(-3) = (-3)^3+3(-3)^2-(-3) = -27+27+3-3 = 0$

$\therefore$ $P(-3) = 0 \Rightarrow$ ' -3 ' is a zero of the polynomial p(x)

$\therefore$ 1 , − 1 , and -3 are the zeroes of x^3+3x^2-x-3 .

So , we take $\alpha = 1, \beta = -1 \quad \gamma = -3$

$$\alpha+\beta+\gamma = 1+(-1)+(-3) = -3 = \frac{-3}{1} = \frac{-b}{a} = \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = (1)(-1)+(-1)(-3)+(3)(1) = -1+3-3 = -1$$

$$= \frac{-1}{1} = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta\gamma = (1)(-1)(-3) = \frac{-(-3)}{1} = \frac{-(\text{constant term})}{\text{coefficient of } x^3}$$

(3) If the zeroes of the polynomial x^2+px+q are double in value to the zeroes of $2x^2-5x-3$, find the values of 'p' and 'q' .

Solution: Given polynomial $2x^2-5x-3$

To find the zeroes of the polynomial, we take

$$2x^2-5x-3 = 0$$

$$2x^2-6x+x-3 = 0$$

$$2x(x-3) + 1(x-3) = 0$$

$$(x-3)\ (2x+1) = 0$$

$$\Rightarrow x - 3 = 0 \quad \text{or} \quad 2x+1 = 0$$

$$\Rightarrow x - 3 = 0 \quad \text{or} \quad 2x+1 = 0$$

$$\Rightarrow x = 3 \quad \text{or} \quad x = {-1}/{2}$$

$\therefore$ The zeroes of $2x^2-5x-3$ are $3, {-1}/{2}$

$\therefore$ zeroes of the polynomial x^2+px+q are double in the value to the zeroes of $2x^2-5x-3$

i.e. . $2(3)$ and $2\ ({-1}/{2})$

$\Rightarrow$ 6 and -1

Sum of the zeroes $= 6+(-1) = 5$

$$\Rightarrow \frac{-p}{1}=5 \qquad \left(\because \text{sum of the zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} = \frac{-p}{1}\right)$$

$p = -5$

Product of the zeros = (6)(-1) = -6

$\frac{q}{1} = -6$ $\quad$ ($\because$ Product of the zeroes $= \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{q}{1}$

q = -6

$\therefore$ The values of p and q are -5, -6

(4). If α and β are the zeroes of the polynomial $6y^2$-7y+2, find a quadratic polynomial whose zeroes are $1/\alpha$ and $1/\beta$

Solution: The given polynomial is

$6y^2-7y+2$

Comparing with ay^2+by+c, we get a =6 , b= -7 , c =2

$\therefore$ Sum of zeroes = $\alpha + \beta = -\frac{b}{a} = \frac{7}{6}$

$\alpha + \beta = \frac{7}{6}$ ………………………..> (1)

and , a product of zeroes = $\alpha\beta = \frac{c}{a} = \frac{2}{6} = \frac{1}{3}$

$\alpha\beta = \frac{1}{3}$ ....>(2)

For a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Sum of zeroes = $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{7/6}{1/3}$ ($\because$ Form (1)&(2))

$= \frac{7}{2}$

Product of zeroes $= \frac{1}{\alpha}.\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{(\frac{1}{3})}$ $(\because \text{From}(2))$

$\therefore$ The required quadratic polynomial is

$$K\{x^2 - (\frac{1}{\alpha} + \frac{1}{\beta})x + (\frac{1}{\alpha}.\frac{1}{\beta})\}, \text{ where k is real.}$$

$$K(x^2 - \frac{7}{2}x + 3), \text{ K is real.}$$

(5) If α and β are the zeroes of a quadratic polynomial such that α + β = 24 and α – β = 8 , find quadratic polynomial having α and β as its zeroes. Verify the relationship between the zeroes and the coefficient of the polynomial.

Solution: α and β are the zeroes of a quadratic polynomial.

α + β = 24 ……….> (1)

α – β = 8 …………..> (2)

Adding (1) + (2) we get 2α =32 $\Rightarrow$ α = 16

Subtraction (1) & (2) we get 2β =16 $\Rightarrow$ β = 8

The quadratic polynomial having α and β as its zeroes is $k\{x^2-(\alpha+\beta)x+\alpha\beta\}$, where k is real.

$\Rightarrow$ $K\{ x^2 -(16+8)x + (16)(8)\}$, k is a real

$\Rightarrow$ $K\{x^2-24x+128)$, k is a real

$\Rightarrow$ $K x^2-24kx+128k$, k is real

Comparing with ax^2+bx+c , we get a = k ,b = -24k, c = 128k

Sum of the zeroes $= -\frac{b}{a} = \frac{24k}{k} = 24 = \alpha + \beta$

Product of the zeroes $= \frac{c}{a} = \frac{128k}{k} = 128 = \alpha\beta$

Hence, the relationship between the zeroes and the coefficients is verified.

(6) Find a cubic polynomial with the sum, sum of product of its zeroes taken two at a time , and product of its zeroes as 2, -7 , -14 respectively.

Solution: Let α, β and γ are zeroes of the cubic polynomial

Given $\alpha + \beta + \gamma = 2$

$\alpha\beta + \beta\gamma + \gamma\alpha = -7$

$\alpha\beta\gamma = -14$

Cubic polynomial whose zeroes are α, β and γ is

$\Rightarrow x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma$

$\Rightarrow x^3 - 2x^2 + (-7)x - (-14)$

$\Rightarrow x^3 - 2x^2 - 7x + 14$

$\therefore$ Required cubic polynomial is $x^3 - 2x^2 - 7x + 14$

(7) Divide x^4-$3x^2$+4x+5 , by x^2+1-x , and verify the division algorithm.

Solution: Dividend $= x^4-3x^2+4x+5$

$= x^4+0x^3-3x^2+4x+5$

Divisor $= x^2-x+1$

$$
\begin{array}{r|l}
x^2 - x + 1 \,) & x^4 + 0x^3 - 3x^2 + 4x + 5 \quad (\, x^2 + x - 3 \\
& x^4 - x^3 + x^2 \\
& (-)\ \ (+)\ \ (-) \\ \hline
& x^3 - 4x^2 + 4x \\
& x^3 - x^2 + x \\
& (-)\ \ (+)\ \ (-) \\ \hline
& -3x^2 + 3x + 5 \\
& -3x^2 + 3x - 3 \\
& (+)\ \ (-)\ \ (+) \\ \hline
& 8
\end{array}
$$

First term quotient

$$\frac{x^4}{x^2} = x^2$$

second term of quotient

$$\frac{x^3}{x^2} = x$$

third term of quotient

$$= \frac{-3x^2}{x^2} = -3$$

We stop here since degree of the remainder is less than the degree of (x^2+ x – 3) the divisor.

So , quotient $= x^2 + x - 3$, remainder $= 8$

Verification:

(Divisor x Quotient)+ Remainder

$= (x^2-x+1)(x^2+x-3)+8$

$= x^4+x^3-3x^2-x^3-x^2+3x+x^2+x-3+8$

$= x^4 - 3x^2+4x+5$ = dividend

∴ Dividend = (Divisor × quotient) + Remainder

∴ The division algorithm is verified.

(8) Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are

Solution: Since, two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Therefore,

$$(x-\sqrt{\frac{5}{3}})\ (x+\sqrt{\frac{5}{3}})\ = x^2-\frac{5}{3}$$ is a factor of the given polynomial,

Now, we apply the division algorithm to the given polynomial and $x^2-\frac{5}{3}$

$$x^2-\frac{5}{3}\ )\ 3x^4+6x^3-2x^2-10x-5\ (3x^2+6x+3$$

$$3x^4+0x^3-5x^2$$

$$(-)\qquad (+)$$

$$6x^3+3x^2-10x$$

$$6x^3+0x^2-10x$$

$$(-)\qquad (+)$$

$$3x^2\qquad -5$$

$$3x^2\qquad -5$$

$$(-)\qquad (+)$$

$$0$$

So , $3x^4+6x^3-2x^2-10x-5 = (x^2-\frac{5}{3})\ (3x^2+6x+3)$

Now $3x^2+6x+3 = 3(x^2+2x+1) = 3(x+1)^2$

So , its zeros are -1, and -1

∴ The other zeroes of the given fourth degree polynomial are -1 and -1 .

(9) On division x^3-$3x^2$+x+2 by a polynomial g(x), the quotient and remainder were x- 2 and -2x+4 , respectively. Find g(x).

Solution: Given

Dividend = x^3-3x^2+x+2

Divisor = g(x)

Quotient = x – 2

Remainder = -2x+4

By division algorithm

Dividend = ((Divisor × quotient) + Remainder

$$\text{Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$$

$$g(x) = \frac{(x^3 - 3x^2 + x + 2) - (-2x + 4)}{x - 2}$$

$$g(x) = \frac{x^3 - 3x^2 + 3x + 2}{x - 2} \quad \text{..................(1)}$$

$$\begin{array}{l} x - 2 \;)\; x^3 - 3x^2 + 3x - 2 \;(x^2 - x + 1 \\ \qquad\quad x^3 - 2x^2 \\ \qquad\quad (-)\;(+) \\ \hline \qquad\qquad -x^2 + 3x \\ \qquad\qquad -x^2 + 2x \\ \qquad\qquad (+)\quad(-) \\ \hline \qquad\qquad\qquad x - 2 \\ \qquad\qquad\qquad x - 2 \\ \hline \qquad\qquad\qquad 0 \end{array}$$

From equation (1)

$$g(x) \;= \frac{x^3 - 3x^2 + 3x - 2}{x - 2} = x^2 - x + 1$$

(10) Check by division whether x^2 -2 is a factor of x^4 +x^3 + x^2 - 2x – 3

Solution:

Dividend = $x^4 + x^3 + x^2 - 2x - 3$

Divisor = $x^2 - 2$

$$x^2 - 2 \;)\; x^4 + x^3 + x^2 - 2x - 3 \;(\, x^2 + x + 3$$
$$x^4 - 2x^2$$
$$(-) \quad (+)$$
$$x^3 + 3x^2 - 2x$$
$$x^3 - 2x$$
$$(-) \quad (+)$$
$$3x^2 - 3$$
$$3x^2 - 6$$
$$(-) \quad (+)$$
$$3$$

Since, remainder = 3 ($\neq 0$)

$\therefore$ $x^2 - 2$ is a not a factor of $x^4 + x^3 + x^2 - 2x - 3$

Short Answer Question

(1) If $P(t) = t^3 - 1$, find the value of P(1), P(-1), P(0), P(2) , p(-2)

Solution: $P(t) = t^3 - 1$

$P(1) = 1^3 - 1 = 1 - 1 = 0$

$P(-1) = (-1)^3 - 1 = -1 - 1 = -2$

$P(0) = 0^3 - 1 = -1$

$P(2) = 2^3 - 1 = 8 - 1 = 7$

$P(-2) = (-2)^3 - 1 = -8 - 1 = -9$

(2) Check whether 3 and -2 are the zeros of the polynomial P(x) when $p(x) = x^2 - x - 6$

Solution: Given $p(x) = x^2 - x - 6$

$P(x) = 3^2 - 3 - 6 = 9 - 3 - 6 = 0$

$P(x) = (-2)^2 - (-2) - 6$

$= 4 + 2 - 6$

$= 0$

Since p(3) = 0 , P(-2) = 0

3 and -2 are zeroes of $p(x) = x^2 - x - 6$

(3) Find the number of zeroes of the given polynomials. And also find their values

(i). $P(x) = 2x+1$ (ii) $q(x) = y^2 - 1$ (iii) $r(z) = z^3$

Solution:

(i). $P(x) = 2x+1$ is a linear polynomial. It has only one zero.

To find zeroes.

Let p(x) = 0

$2x + 1 = 0$

$$\therefore \; x = -\frac{1}{2}$$

The zero of the given polynomial is $-\frac{1}{2}$

(ii) $q(y) = y^2 - 1$ is a quadratic polynomial. It has at most two zeroes.

To find zeroes , Let q(y) = 0

$$y^2 - 1 = 0$$

$$(y+1)(y-1) = 0$$

$$y = -1 \quad \text{or} \quad y = 1$$

$\therefore$ The zeroes of the polynomial are -1 and 1

(iii) $r(z) = z^3$ is a cubic polynomial .It has at most there zeroes .

Let r(z) = 0

$$z^3 = 0$$

$$z = 0$$

$\therefore$ The zero of the polynomial is '0'.

(4). Find the quadratic polynomial , with the zeroes $\sqrt{3}$ and $-\sqrt{3}$

Solution: Given

The zeroes of polynomial $\alpha = \sqrt{3}$, $\beta = \sqrt{3}$

$$\alpha + \beta = -\sqrt{3} + \sqrt{3} = 0$$

$$\alpha\beta = (-\sqrt{3})(\sqrt{3}) = -3$$

The quadratic polynomial with zeroes α and β is given by

$$K\{x^2 - (\alpha + \beta)x + \alpha\beta\}, \quad K(\neq 0) \text{ is real}$$

$$K(x^2 - 0x - 3\} \qquad k\ (\neq 0) \text{ is real}$$

$K(x^2 - 3)$ K $(\neq 0)$ is real.

(5) If the Sum and product of the zeroes of the polynomial ax^2 -5x+c is equal to 10 each , find the values of 'a' and 'c' .

Solution: Given polynomial $ax^2 -5x+c$

Let the zeroes of the polynomial are α, β

Given $\alpha + \beta = 10$ ……………. (1)

And $\alpha\beta = 10$ …………….(2)

We know that

$$\alpha+\beta = -\frac{b}{a} = \frac{-(-5)}{a} = \frac{5}{a} = 10 \quad \because \text{(from (1))}$$

$$a = \frac{5}{10} = \frac{1}{2}$$

$$\alpha\beta = \frac{c}{a} \quad ....> \ 10 = \frac{c}{1/2}$$

$$C = 5$$

$$\therefore \quad a = 1/2 \ , \quad c = 5$$

(6) If the Sum of the zeroes of the polynomial P(x) = (a+1) x^2 + (2a+3)x+(3a+4), then find the product of its zeroes.

Solution: Given polynomial $P(x) = (a+1)x^2 + (2a+3)x+(3a+4)$

Compare with ax^2+bx+c,

we get a = a+ 1

b = 2a+3

c = 3a+4

$$\alpha + \beta = -\frac{b}{a}$$

$$-1 = \frac{-(2a+3)}{a+1}$$

$\Rightarrow$ - a – 1 = -2a -3

$\Rightarrow$ – a + 2a = - 3+1

$\Rightarrow$ a = -2

Product of the zeroes = $\alpha\beta = \frac{c}{a} = \frac{3a+4}{a+1}$

$$= \frac{3(-2)+4}{-2+1} = \frac{-2}{-1} = 2$$

(7) On dividing the polynomial $2x^3+4x^2+5x+7$ by a polynomial g(x) , the quotient and the remainder were 2x and 7 – 5x respectively . Find g(x)

Solution: Given

Dividend = $2x^3+4x^2+5x+7$

Divisor = g(x)

Quotient = 2x

Remainder = 7 -5x

By division algorithm

Dividend = (divisor × quotient) + remainder

$$\text{Divisor} = \frac{\text{dividend} - \text{remainder}}{\text{quotient}}$$

$$g(x) = \frac{(2x^3 + 4x^2 + 5x + 7) - (7 - 5x)}{2x}$$

$$= \frac{2x^3 + 4x^2 + 5x + 7 - 7 + 5x}{2x}$$

$$= \frac{2x^3 + 4x^2 + 10x}{2x}$$

$$= \frac{2x(x^2 + 2x + 5)}{2x}$$

$$g(x) = x^2 + 2x + 5$$

(8) If p(x) = $x^3 - 2x^2 + kx + 5$ is divided by (x -2) , the remainder is 11. Find K.

Solution:

$x - 2$) $x^3 - 2x^2 + kx + 5$ ($x^2 + k$

$x^3 - 2x^2$

(-) (+)

$kx + 5$

$kx - 2k$

(-) (+)

$2k + 5$

Remainder = 2k+5 = 11 (given)

$$k = \frac{11 - 5}{2} = 3$$

Very Short Answer Questions

(1) Write a quadratic and cubic polynomials in variable x in the general form.

Solution:

The general form of the a quadratic polynomial is ax^2+bx+c , $a \neq 0$

The general form of a cubic polynomial is ax^3+bx^2+cx+d , $a \neq 0$

(2) If $p(x) = 5x^7 - 6x^5+7x-6$,find (Problem solving)

(i) Co – efficient of x^5 (ii) degree of p(x)

Solution:

Given polynomial $p(x) = 5x^7 - 6x^5+7x-6$

(i) Co – efficient of x^5 is '-6'

(ii) Degree of p(x) is '7'

(3) Check whether – 2 and 2 are the zeroes of the polynomial x^4 - 16

(Reasoning proof)

Solution: $p(x) = x^4 - 16$

$P(2) = 2^4 -16 = 16 – 16 = 0$

$P(-2) = (-2)^4 -16 = 16 – 16 = 0$

Since P(2) = 0 and P(-2) = 0

∴ -2 , 2 are the zeroes of given polynomial

(4) Find the quadratic polynomial whose sum and product of its zeroes respectively $\sqrt{2}$, $\frac{1}{3}$ **(Communication)**

Solution: Given

Sum of the zeroes $\alpha + \beta = \sqrt{2}$ ....>(1)

Product of the zeroes $\alpha\beta = \frac{1}{3}$ ....> (2)

The quadratic polynomial with α and β as zeroes is $K\{x^2-(\alpha+\beta)x + \alpha\beta\}$, where $k(\neq 0)$ is a real number.

$K\{x^2 - \sqrt{2}x + \frac{1}{3}\}$, $K(\neq 0)$ is a real number (∵ From (1) & (2))

$k(\frac{3x^2 - 3\sqrt{2}\,x + 1}{3})$, $k(\neq 0)$ is real number

We can put different values of 'k'

∴ when k = 3 , we get $3x^2 - 3\sqrt{2}\,x + 1$

(5) If the sum of the zeroes of the quadratic polynomial $f(x) = kx^2-3x+5$ is 1 . Write the value of K .

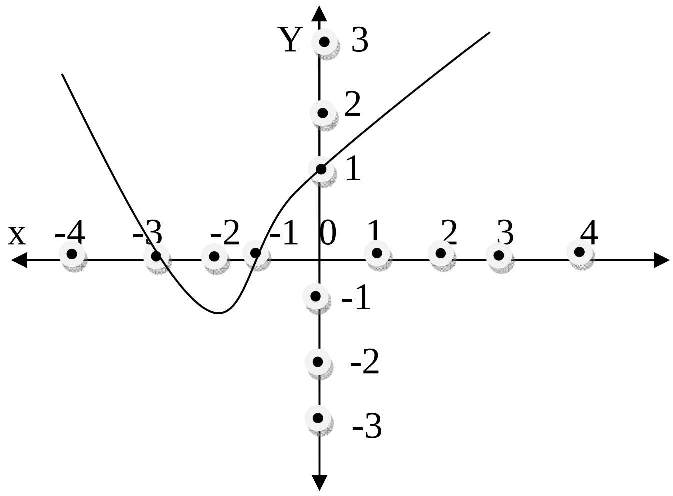
Solution: Given polynomial $f(x) = kx^2-3x+5$

Sum of the zeroes $\alpha + \beta = \frac{-b}{a}$

$1 = \frac{-(-3)}{k}$ (∵ Given $\alpha + \beta = 1$

K = 3

(6) From the graph find the zeroes of the polynomial.



Solution: The zeroes of the polynomial are precisely the x – co-ordinates of the point . Where the curve intersects the x- axis

∴ From the graph the zeroes are – 3 and -1.

(7) If a –b , a a + b are zeroes of the polynomial f(x) = $2x^3 - 6x^2+5x-7$, write the value of the a.

Solution: Let α, β, γ are the zeroes of cubic polynomial

ax^3+bx^2+cx+d then $\alpha + \beta + \gamma = \frac{-b}{a}$

$$a - b + a + a + b = \frac{-(-6)}{2}$$

$$3a = 3$$

$$a = 1$$

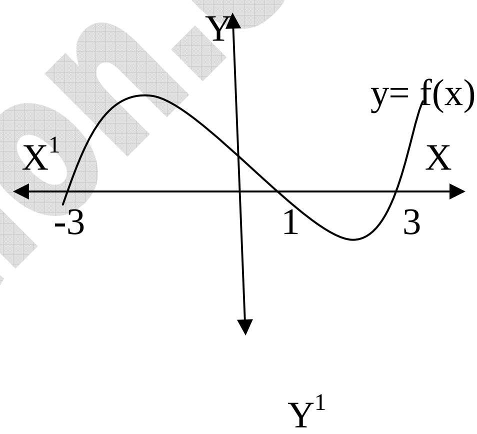
Objective Type Questions

(1) The graph of the polynomial f(x) = 3x – 7 is a straight line which intersects the x- axis at exactly one point namely …………. [ ]

(A) $(\frac{-7}{3}, 0)$ (B) $(0, \frac{-7}{3})$ (C) $(\frac{7}{3}, 0)$ (D) $(\frac{7}{3}, \frac{-7}{3})$

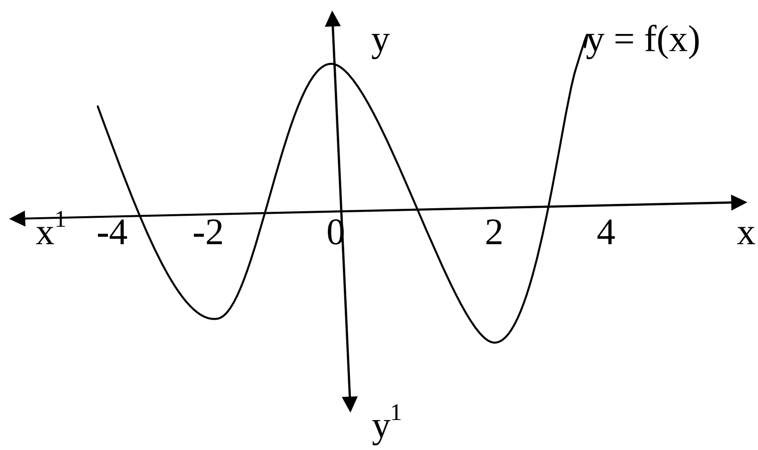
(2) In the given figure , the number of zeros of the polynomial f(x) are ………[ ]

(A) 1 (B) 2 (C) 3 (D) 4



(3) **The number of zeros lying between -2 and 2 of the polynomial f(x) whose graph is given figure is …………..** [ ]

(A) 2 (B) 3 (C) 4 (D) 1



(4) **Which of the following is not a quadratic polynomial** ………………….. [ ]

(A) X^2+3x+4 (B) x^2-3x+4

(C) $6+(x^2-4x)$ (D) $(x-3)(x+3)-(x^2+7x)$

(5) **The degree of the constant polynomial is …………** [ ]

(A) 0 (B) 1 (C) 2 (D) 3

(6) **The zero of p(x) = ax – b is …………** [ ]

(A) a (B) b (C) $\frac{-b}{a}$, (D) $\frac{b}{a}$

(7) Which of the following is not a zero of the polynomial $x^3-6x^2+11x-6$?.......[ ]

(A) 1 (B) 2 (C) 3 (D) 0

(8) If α and β are the zeroes of the polynomial $3x^2+5x+2$,then the value of $\alpha+\beta+\alpha\beta$ is [ ]

(A) - 1 (B) – 2 (C) 1 (D) 4

(9) **If the sum of the zeroes of the polynomial $p(x) = (k^2-14)x^2 -2x-12$ is 1, then k. takes the value(s) ………………….** [ ]

(A) $\sqrt{14}$ (B) -14 (C) 2 (D) ± 4

(10) If α, β are zeroes of $p(x) = x^2-5x+k$ and $\alpha - \beta = 1$ then the value of k is [ ]

(A) 4 (B) – 6 (C) 2 (D) 5

(11) If α, β, γ are the zeros of the polynomial ax^3+bx^2+cx+d , then the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ Is ………………… [ ]

(A) $\frac{c}{d}$ (B) $\frac{-c}{d}$ (C) $\frac{b}{d}$ (D) $\frac{-b}{d}$

(12) **If the product of the two zeros of the polynomial $x^3-6x^2+11x-6$ is 2 is then the third zero is …………** [ ]

(A) 1 (B) 2 (C) 3 (D) 4

(13) **The zeros of the polynomial is x^3-x^2 are** [ ]

(A) 0, 0, 1 (B) 0, 1, 1 (C) 1, 1, 1 (D) 0, 0, 0

(14) If the zeroes of the polynomial x^3-3x^2+x+1 are $\frac{a}{r}$, a and ar then the value of a is ………………….. [ ]

(A) 1 (B) -1 (C) 2 (D) - 3

(15) **If α and β are the zeroes of the quadratic polynomial $9x^2-1$, find the value of $\alpha^2+\beta^2$ ……….** [ ]

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

(16) If α, β, γ are the zeroes of the polynomial x^3+px^2+qx+r then find **[ ]**

$$\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}$$

(A) $\frac{p}{r}$ (b) $-\frac{p}{r}$ (C) $\frac{q}{r}$ (D) $\frac{-q}{r}$

(17) **The number to be added to the polynomial x^2-5x+4 , so that 3 is the zero of the polynomial is ………………..** [ ]

(a) 2 (B) -2 (C) 0 (D) 3

(18). If α, and β are zeroes of p(x) = $2x^2$-x-6 then the value of $\alpha^{-1}+\beta^{-1}$ is **[ ]**

(A) $\frac{1}{6}$ (B) $\frac{-1}{6}$ (C) $\frac{1}{2}$ (D) $\frac{-1}{3}$

(19). What is the coefficient of the first term of the quotient when $3x^3+x^2+2x+5$ is Divided by $1+2x+x^2$ ………… [ ]

(A) 1 (B) 2 (C) 3 (D) 5

(20) If the divisor is x^2 and quotient is x while the remainder 1, then the dividend is **[ ]**

(A) x^2 (B) x (C) x^3 (D) x^3+1

Key:

1.C **2.** C **3.** A **4.** D **5.** A **6.** D **7.** D **8.** **A** **9.** D **10.** C

11. B **12.** C **13.** A **14.** B **15.** B **16.** A **17.** A **18.** B **19.** C **20.** D

Fill in the Blanks

(1) The maximum number of zeroes that a polynomial of degree 3 can have is **3**

(2) The number of zeroes that the polynomial $f(x) = (x-2)^2 + 4$ can have is **2**

(3) The graph of the equation $y = ax^2 + bx + c$ is an upward parabola , If **$(a > 0)$**

(4) If the graph of a polynomial does not intersect the x – axis, then the number zeroes of the polynomial is **0**

(5) The degree of a biquadratic polynomial is **4**

(6) The degree of the polynomial $7\mu^6 - \frac{3}{2}\mu^4 + 4\mu + \mu - 8$ is **6**

(7) The values of $p(x) = x^3 - 3x - 4$ at $x = -1$ is **-2**

(8) The polynomial whose whose zeroes are -5 and 4 is **$x^2 + x - 20$**

(9) If – 1 is a zeroes of the polynomial $f(x) = x^2 - 7x - 8$ then other zero is **8**

(10) If the product of the zeroes of the polynomial $ax^3 - 6x^2 + 11x - 6$ is 6 , then the value of a is **1**

(11) A cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes are 2, -7 and -14 respectively, is **$x^3 - 2x^2 - 7x + 14$**

(12) For the polynomial $2x^3 - 5x^2 - 14x + 8$, find the sum of the products of zeroes , taken two at a time is **-7**

(13) If the zeroes of the quadratic polynomial ax^2+bx+c are reciprocal to each other,

Then the value of c is **a**

(14) What can be the degree of the remainder at most when a biquadrate polynominal is divided by a quadratic polynomial is **1**