

NATIONAL ENTRANCE SCREENING TEST (NEST – 2014)

SET A

Total Marks: 200

Time: 3 hours

General instructions

1. This question booklet contains five sections. Each section carries 50 marks.
2. Section 1 is the General section. Sections 2 to 5 are for the subjects Biology, Chemistry, Mathematics and Physics, respectively. Attempt the General section **and any three** of the four subject sections.
3. Read the instructions given at the beginning of each section carefully.
4. Calculators, cell phones, log tables, etc. are **NOT** permitted in the examination hall.
5. Answers to the questions are to be marked on the **OMR sheet provided**.
6. **Please make sure that the question booklet code (A or B or C or D) matches with the OMR sheet code (A or B or C or D).** In case of discrepancy, please inform the invigilator immediately.
7. Return the OMR sheet to the invigilator at the end of the examination.

Instructions for writing on OMR sheet

1. Read and follow the instructions given on the OMR sheet.
2. As far as possible, fill in the answers only after you are sure that you do not need to change them. In case you do have to change the answer after filling, erase the mark completely so that no black spot is left inside the bubble.
3. **Check that you are filling the correct answers for the correct section on the OMR sheet.**
4. **Your roll number (as given in the admit card) MUST BE ENTERED CORRECTLY. If entered incorrectly or not entered at all, the OMR sheet will be taken to be invalid and will not be graded.**

Some useful constants

Acceleration due to gravity on Earth	g	\approx	9.8 m s^{-2}
Planck constant	h	\approx	$6.63 \times 10^{-34} \text{ J s}$
Universal gas constant	R	\approx	$8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
Permittivity of free space	ϵ_0	\approx	$8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Magnitude of charge of an electron	e	\approx	$1.60 \times 10^{-19} \text{ C}$

Element	H	C	N	O	Na	P	S	Cl
Atomic Mass (amu)	1	12	14	16	23	31	32	35.5

Section 1: GENERAL

Marks for Section 1: 50

This section contains 18 questions.

*For each question, **only one** of the four options is the correct answer. For questions 1.1 to 1.14, a correct answer will earn 3 marks. For questions 1.15 to 1.18, a correct answer will earn 2 marks. For this **GENERAL** section, a wrong answer or an unattempted question will earn 0 marks.*

Read the following passage carefully and answer questions 1.1 to 1.3.

Darwin had given the study of natural animal behaviour a scientific turn. He had seen the central idea of ethology, that is animal behavioural routines are as important an aspect of the animal's adaptation to its environment as its anatomical structure or its physiological processes. Routines are elaborate, integrated chains of behaviour directed to the achievement of certain ends adaptive to the breeding success of a species.

As an example, consider the behavioural routine of gaping (wide opening of mouth) for food amongst young birds. The capacities to recognise the right stimulus are not always inherited. The young of many species of birds do not recognize conspecifics (birds of their own kind), if they have not been introduced to them at a definite period of their development.

The young of Godwits hatch at an advanced stage of development and soon after hatching can recognize the adult bird. The young birds immediately display appropriate behaviour in the presence of adults, say gaping for food. They flee from human beings without any special prompting or learning. For these birds and similar species, the capacity to recognize the object of a behavioural routine, as well as the capacity to perform the routine, must be innate or inherited.

On the other hand, the Greylag Geese do not have a prolonged period of hatching. They vividly display another pattern of development. When young geese are reared wholly by human beings it is towards humans that the young geese direct their behavioural routines. They seem to acquire as a prime object of interest whatever creature happens to be present at the right moment in their development. The first recorded observation of this phenomenon (now called 'imprinting') is due to Oskar Heinroth.

1.1 The word 'conspecifics' refers to

- (A) Birds with a specific period of development.
- (B) Birds displaying appropriate behaviour in the presence of adults.
- (C) Birds of the same species.**
- (D) Bird species not needing any special prompting or learning.

1.2 The pattern of development in Greylag Geese demonstrates that

- (A) **The hatching at an advanced stage is critical for imprinting.**
- (B) The capacity to recognise an adult of its own species is necessarily inherited.
- (C) Rearing by humans does not alter the object of the behavioural routine.
- (D) Geese reared by humans cannot be taught to respond to its own species.

1.3 Which of the following was an insight of Darwin?

- (A) **Behavioural animal routines need to be regarded as the animal's environmental adaptation.**
- (B) Behavioural animal routines are more significant than the anatomical structure in the animal's environmental adaptation.
- (C) Behavioural animal routines are not related to environmental adaptation.
- (D) Behavioural animal routines are always instinctive in their origin.

Read the following passage carefully and answer questions 1.4 to 1.6.

Cryptography is the practice and study of techniques for secure communication. Applications of cryptography include coded military messages, ATM cards and computer passwords. In cryptography, "encryption" is the process of converting ordinary messages (or plaintext) into unintelligible text (or ciphertext). "Decryption" is the reverse, i.e. converting the ciphertext back to plaintext. The method to encode and decode a message is called a "cipher". The cipher is controlled by a "key". The key can be in the form of a string of either characters or numbers and is the additional input necessary for encoding or decoding the message.

In modern cryptography, symmetric-key cryptography refers to encryption methods in which both the sender and the receiver share the same key. Symmetric key ciphers are implemented as either block ciphers or stream ciphers. In a stream cipher, every character of the plaintext is encoded by a single rule. A block cipher encrypts plaintext input in blocks of fixed length of characters. Thus, adjacent characters may get encoded by different rules. A major disadvantage of symmetric ciphers is that each distinct pair of communicating parties must share a unique, different key. Thus, very large number of keys are required in a big network of communicating parties. In public-key or asymmetric key cryptography, two different but mathematically related keys are generated secretly, by an intended recipient of the information. The public key, used for encryption may be freely distributed by this recipient, while its paired private key, used for decryption, must remain secret with him/her. Thus, in a public key encryption, senders of information will be able to encrypt the information, but only the recipient will be able to decrypt the same.

1.4 In the public key cryptography, number of keys required for secure communication between n parties will be,

- (A) $(n - 1)^2$
- (B) $2n$
- (C) $\frac{n(n + 1)}{2}$
- (D) $\frac{n(n - 1)}{2}$

1.5 In a particular cipher, each character of the message is conveyed to the other party as a string of numbers indicating a page number of a previously agreed book, a line number on that page and the position of a character in that line. Choose the correct statement.

- (A) The book in this example acts as a public key.
- (B) The string of numbers in this example is a symmetric key.
- (C) The text from the book is plaintext.
- (D) This is an example of a symmetric key stream cipher.**

1.6 In a particular cipher, 1st, 5th, 9th, ..., character in the text is replaced by the 4th letter of the alphabet after that character and 4th, 8th, 12th, ..., character is replaced the 3rd letter of the alphabet prior to that character. This is an example of,

- (A) Symmetric key stream cipher.
- (B) Symmetric key block cipher.**
- (C) Public key cipher.
- (D) Information not sufficient.

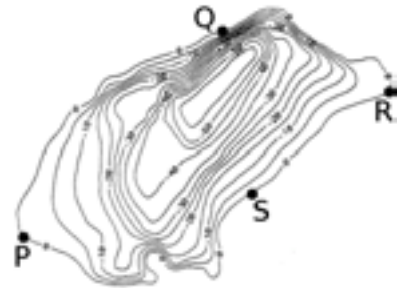
1.7 Given the statement “All the students in a class passed the examination” is false, consider the following four statements :

- I. All the students in the class failed the examination.
- II. There are some students in the class who failed the examination.
- III. No student in the class passed the examination.
- IV. Not all the students in the class passed the examination.

Which of the above statements are always true?

- (A) I and III only.**
- (B) I and II only.
- (C) II and IV only.
- (D) III and IV only.

1.8 Contour maps are used in Geography to show the heights at various locations. In a contour map, each line (contour) denotes a specific height. The map on the right is a contour map of a mountain. Four climbers (P, Q, R and S) reach four different points at the foot of the mountain. Which of these four is facing the steepest side of the mountain?



- (A) P
- (B) Q**
- (C) R
- (D) S

1.9 If a , b and c are three natural numbers, which of the following can never be true?

- (A) $a^2 - b^2 = c^2$
- (B) $a^2 + b^2 = c^3$
- (C) $a^3 + b^3 = c^3$**
- (D) $a^3 + b^3 = c^2$

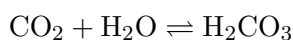
1.10 The practice of alternating pulses (like moong, peas, gram) with cereals (like rice, wheat) by farmers is an example of crop rotation. The primary reason for this practice is to

- (A) Replenish the soil microbes.**
- (B) Avoid competition among pollinating agents.
- (C) Protect themselves against price fluctuation of cereals.
- (D) Maintain the water table in the field.

1.11 *Lactobacillus sp.* and *Streptococcus sp.* are two bacterial species responsible for curdling of milk. One quantum of each of these species was introduced to a very large container of milk. One quantum of either species can curdle 10 ml of milk in 26 minutes, which is also the doubling time for *Streptococcus sp.* The doubling time for *Lactobacillus sp.*, however, is 78 minutes. What will be ratio of the total milk curdled by *Streptococcus sp.* to *Lactobacillus sp.* at the end of 156 minutes?

- (A) 3 (B) 3.5 (C) 5.25 (D) 7

1.12 In a closed bottle of a carbonated soft drink, one can create the following equilibrium



At the start of the reaction, 5 moles of CO_2 and 5 moles of water are present and there is no carbonic acid. At equilibrium, 2 moles of carbonic acid were formed. How many moles of water exist at equilibrium?

- (A) 1 (B) 2 (C) 3 (D) 4

1.13 A square with vertices at $O(0,0)$, $A(1,0)$, $B(1,1)$ and $C(0,1)$ is transformed into another quadrilateral with vertices at $O(0,0)$, $A'(2,0)$, $B'(3,2)$ and $C'(1,2)$. The transformations involved in the process are

- (A) Magnification and rotation. (B) Translation and shear.
(C) Translation and rotation. (D) **Magnification and shear.**

1.14 Which of the following sentences represents a “hypothesis”?

- (A) The bicycle tyre has a puncture.
(B) Bicycle tyres must be inflated for riding the bicycle.
(C) When I filled air in the bicycle tyre, the tyre did not inflate.
(D) **If a bicycle tyre is flat, then it must have a leak in it.**

For rest of the questions in this section, each correct answer will earn 2 marks.

1.15 Recently, Prof. C. N. R. Rao was awarded the “Bharat Ratna”. His primary research work is in the field of

- (A) **Material Science.** (B) Statistics.
(C) Space Science. (D) Particle Physics.

1.16 Toothpastes do **not** contain

- (A) Calcium carbonate. (B) **Methanol.**
(C) Titanium dioxide. (D) Sorbitol.

1.17 Which of the following substances has purely planar (2-dimensional) structure?

- (A) Diamond (B) Graphite (C) **Graphene** (D) Fullerene

1.18 The rocky Deccan plateau is a result of volcanic activity in the past. Which of the following is the resulting rock composition?

- (A) Marble (B) Gypsum (C) **Granite** (D) Lignite

Section 2: BIOLOGY

Marks for Section 2: 50

This section contains 14 questions.

For questions 2.1 to 2.10, only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

2.1 The antigen binding site of the antibody molecule is composed of

- (A) Light chains only. (B) Heavy chains only.
(C) The Fc fragment. (D) **Both light and heavy chains.**

2.2 Induced pluripotent stem cells are formed by

- (A) Isolating and growing cells from the early embryo.
(B) Fusing enucleated ovum with adult somatic cells.
(C) **Inserting a few relevant genes into adult somatic cells.**
(D) Fusing egg and sperm.

2.3 Evolutionary relationships between species, such as humans and the apes, can be studied by using immunological techniques to compare proteins. A rabbit when injected with human serum protein albumin, forms antibodies against it. These antibodies are extracted and purified. A fixed amount of antibody is reacted with a fixed amount of albumin from some of the ape species -gorilla, orangutan, chimpanzee and gibbon, to obtain respective precipitates. The amounts of precipitates obtained are given below.

Species	Gorilla	Orangutan	Gibbon	Chimpanzee
Amount of precipitate (units/ml)	82	70	56	96

Based on the given data, the evolutionary closeness of the above apes to humans is:

- (A) Orangutan > Gorilla (B) Gibbon > Orangutan
(C) **Chimpanzee > Gorilla** (D) Gibbon > Chimpanzee

2.4 Which of the following properties of cellular transport across cell membranes would be applicable for transport of molecules which have a significant hydrophilic moiety but do not undergo active transport?

- (A) **Involvement of special membrane proteins**
(B) Dependence on osmotic pressure
(C) Transport against a concentration gradient
(D) Utilization of ATP

2.5 Which of the following tissues of plant origin have the potential to introduce and transfer genetic variability to their progeny?

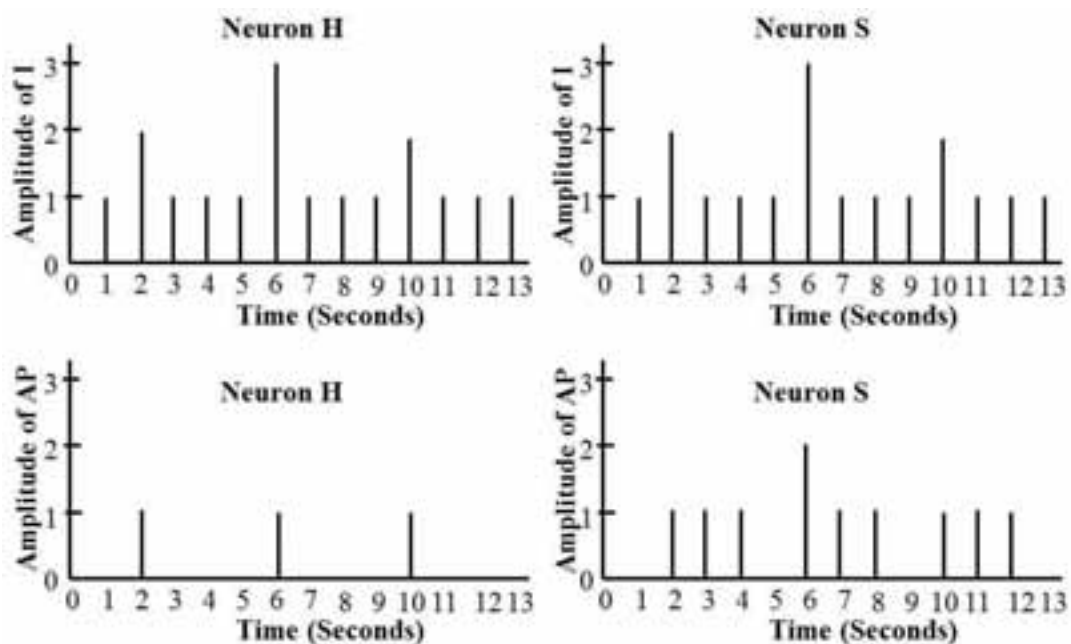
- (A) "Eyes" of potato tubers (B) **Seeds of mustard**
(C) Rhizome of ginger (D) Leaf buds of Bryophyllum

- (A) Cardiac contraction
 (C) Hair growth

- (B) Red blood corpuscle production
 (D) Intestinal absorption of fat

For questions 2.11 to 2.14, **one or more than one** of the 4 options may be correct. Your answer is regarded correct **only if you choose all the correct option(s) and no incorrect option(s)**. A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

2.11 Neuronal response to a stimulus can be probed by electrophysiology measurements and recorded as an action potential (AP) generated in response to an input signal (I). In an experiment, corresponding neurons obtained from a healthy rat (H) and from a hyper-sensitive rat (S) were probed after an input signal of varying amplitude was administered once every second. The results obtained are shown in the following graphs.



Which of the following statements about the experiment is/are correct?

- (A) Both H and S neurons initiate an AP when the input signal (I) is ≤ 1 unit.
 (B) Both H and S neurons initiate an AP only when the input signal (I) is ≥ 2 units.
 (C) Both H and S neurons initiate an AP when the input signal (I) is ≥ 2 units, but only S can sustain it for more than 1 second.
 (D) S, but not H, neurons always initiate an AP when the input signal (I) is ≤ 1 unit and is sustained.

2.12 In a plant breeding experiment, scientists crossed two homozygous parental lines (P1 and P2) to produce F1 hybrids, designated as F1- P1/P2. Another group of researchers produced a different set of F1 hybrids (F1-P3/P4) using homozygous parents P3 and P4. Unfortunately, the seeds of F1- P1/P2 hybrids got mixed with F1-P3/P4 hybrid seeds and with seeds of other parental plants not included in the above crosses. To identify the F1 progeny and their respective parental combinations, a DNA fingerprinting experiment was done using molecular markers on four confirmed parental lines (P1, P2, P3, P4) and six randomly selected, potential F1 individuals (designated F1-1, F1-2, F1-3, F1-4, F1-5 and F1-6). The DNA fingerprint (profile of bands) obtained for various plants is given below:

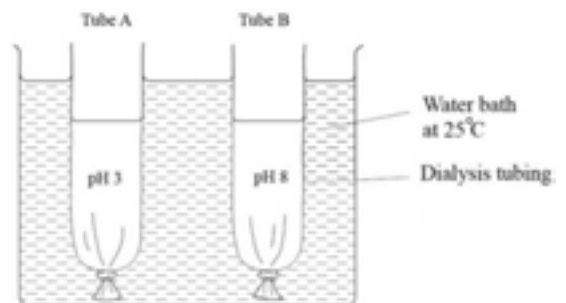
P1	P2	P3	P4	F1-1	F1-2	F1-3	F1-4	F1-5	F1-6
—			—	—			—		—
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Based on the above data, which of the following is/are correct combination(s) of parents and their corresponding F1 progeny?

- (A) P1 × P2 = F1-1
 (C) P3 × P4 = F1-6

- (B) P1 × P2 = F1-5
 (D) P3 × P4 = F1-4

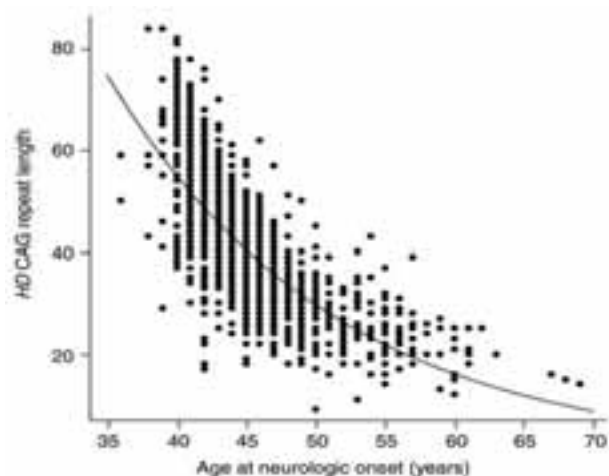
2.13 An experimental setup to study the effect of pH on the activity of the enzyme amylase is shown in the figure. The tubes A and B are made of semi-permeable dialysis tubing. At the beginning of the experiment, equal volumes of starch and amylase solution are added to each tube. Then buffer solution at pH 3 is added to tube A, and buffer solution at pH 8 is added to tube B. After 30 min, the height of the solutions in tubes A and B is measured and the solutions in the tubes tested using Benedict's reagent and heating. Which of the following statement/s would be correct after 30 min of the experiment?



- (A) The level of solution in tube A would be higher than that in tube B due to diffusion and Benedict's test would give a red-brown colour in tube A.
 (B) The level of solution in tube B would be higher than that in tube A due to diffusion and Benedict's test would give a red-brown colour in tube B.
 (C) **The level of the solution in tube B would be higher than that in tube A due to osmosis and Benedict's test would give a red-brown colour in tube B.**

- (D) The level of solution in tube A would be higher than that in tube B due to osmosis and Benedict's test would give a blue colour in tube A.

2.14 Huntington's disease is a genetic disorder that leads to defects in brain function and is always fatal. The affected gene consists of a sequence of DNA having multiple repeats of the nucleotide sequence CAG. A factor that determines whether an allele of this gene causes Huntington's disease or not is the number of CAG repeats. The graph on the right shows the age (in years) at which patients (each represented by a dot) first developed symptoms of Huntington's disease and the number of CAG repeats in the allele causing Huntington's disease in each patient. From the above graph, which of the following would be correct?



- (A) There could be factors other than number of CAG repeats that determine the age of onset.
- (B) There is a negative correlation between age of onset and number of CAG repeats.
- (C) Huntington's disease is fatal and is therefore not heritable.
- (D) Huntington's disease is heritable and can be passed onto offspring.

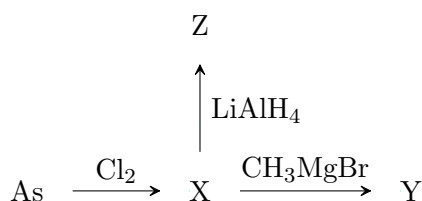
Section 3: Chemistry

Marks for Section 3: 50

This section contains 14 questions.

For questions 3.1 to 3.10, only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

3.1 Consider the following reaction scheme

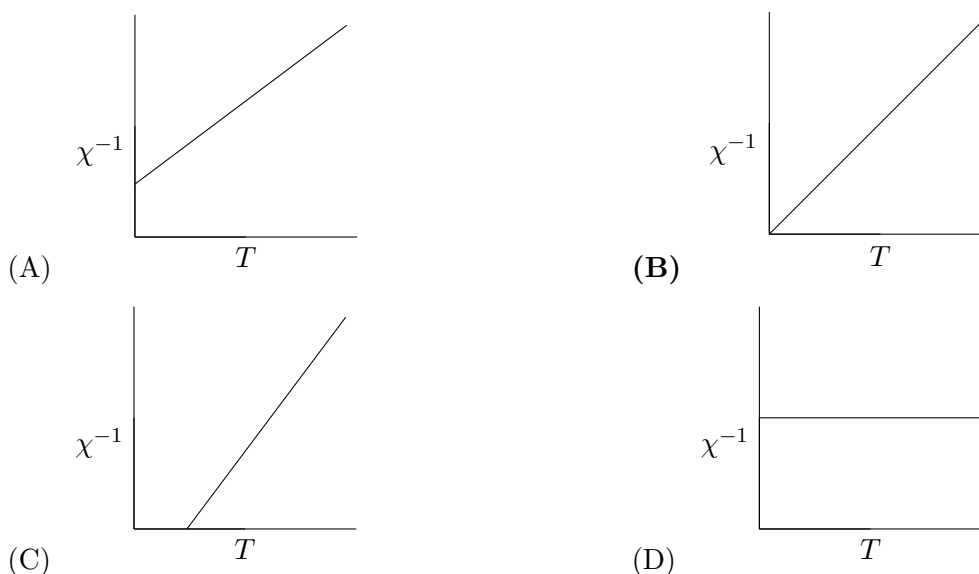


The compounds X, Y, and Z are

- (A) X = AsCl₅; Y = AsCl₃Br₂; Z = AsH₃
- (B) X = AsCl₃; Y = As(CH₃)₃; Z = As
- (C) X = AsCl₅; Y = AsCl₃Br₂; Z = LiAsH₄

(D) $X = \text{AsCl}_3$; $Y = \text{As}(\text{CH}_3)_3$; $Z = \text{AsH}_3$

3.2 The plot of χ^{-1} vs T (χ = molar magnetic susceptibility, T = absolute temperature) for a paramagnetic compound that follows Curie equation is



3.3 Copper crystallizes in a face-centered cubic lattice with lattice parameter 0.420 nm. The radius of copper atom is

- (A) 0.148 nm (B) 0.181 nm (C) 0.210 nm (D) 0.420 nm

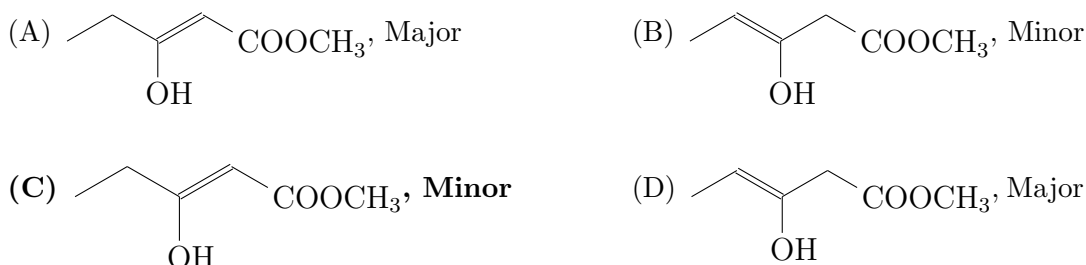
3.4 The polarizing power of the cations is in the order

- (A) $\text{Be}^{2+} > \text{Mg}^{2+} > \text{Ca}^{2+} > \text{Zn}^{2+}$ (B) $\text{Be}^{2+} > \text{Mg}^{2+} > \text{Zn}^{2+} > \text{Ca}^{2+}$
 (C) $\text{Mg}^{2+} > \text{Ca}^{2+} > \text{Zn}^{2+} > \text{Be}^{2+}$ (D) $\text{Zn}^{2+} > \text{Ca}^{2+} > \text{Be}^{2+} > \text{Mg}^{2+}$

3.5 The value of ΔG_f^0 for gaseous mercury at 25°C is 31.85 kJ mol⁻¹. The vapour pressure of mercury at 25°C is (given, K_p is the equilibrium constant in terms of partial pressure)

- (A) 31.85 K_p atm. (B) $\exp\left(\frac{-31.85}{RT}\right)$ atm.
 (C) $\exp\left(\frac{31.85}{RT}\right)$ atm. (D) $\frac{K_p}{31.85}$ atm.

3.6 Select the option showing the structure of methyl 3-hydroxypent-2-enoate and whether it is the major or minor form.



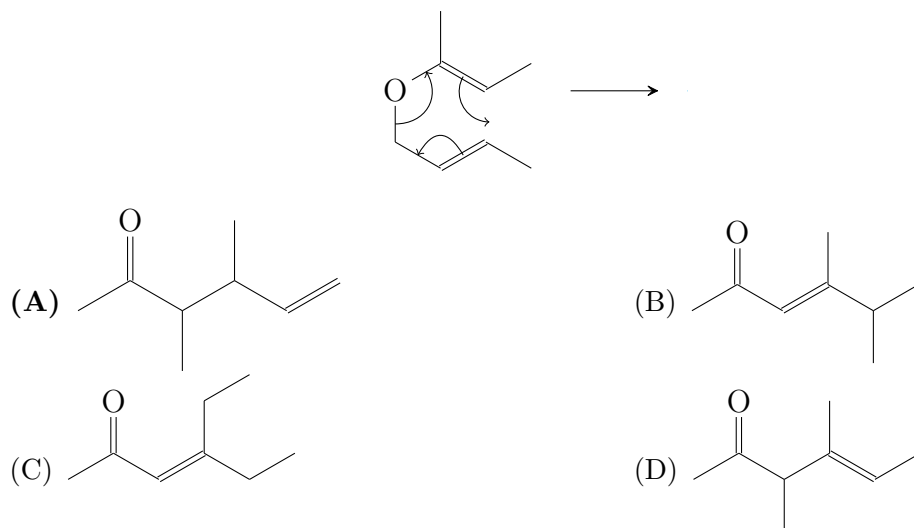
3.7 ^{41}Ar is a radioisotope used to measure the flow of gases from smoke chimneys. A sample of ^{41}Ar decays initially at a rate of 32500 disintegrations/minute. The decay falls to 24500 disintegrations/minute after 75 min. The half-life of the isotope (in minute) is

- (A) $\left(\frac{75}{\ln 75}\right) \times \ln 2$ (B) $\left(\frac{75}{\ln 0.75}\right) \times \ln 2$
 (C) $\left(\frac{-0.75}{\ln 0.75}\right) \times \ln 2$ (D) $\left(\frac{-75}{\ln 0.75}\right) \times \ln 2$

3.8 According to the VSEPR theory, the shape of H_3O^+ ion is

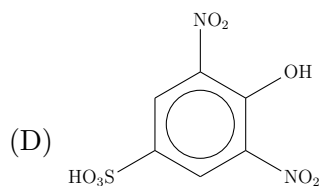
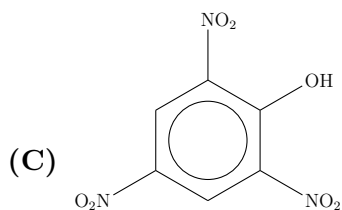
- (A) **trigonal pyramidal.** (B) trigonal planar.
 (C) T shaped. (D) tetrahedral.

3.9 In organic reaction mechanism, curved arrows are used to indicate the movement of electrons. The product formed through the movement of electrons as indicated by the curved arrows in the following reaction is



3.10 Phenol is reacted with conc. H_2SO_4 followed by conc. HNO_3 . The major product is



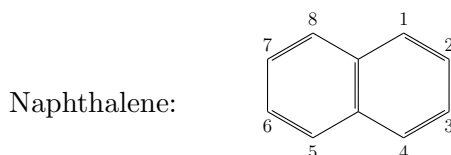


For questions 3.11 to 3.14, **one or more than one** of the 4 options may be correct. Your answer is regarded correct **only if you choose all the correct option(s) and no incorrect option(s)**. A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

3.11 Assume that Pauli's exclusion principle can permit three electrons per orbital rather than two. The correct statement(s) is/are:

- (A) 9 electrons are needed in the outermost shell for a noble gas configuration.
 (B) **3 electrons would be shared in a covalent bond.**
 (C) **The electron dot structure of an element X with $Z = 12$ is $\text{:}\ddot{\text{X}}\text{:}$**
 (D) Assuming MO diagram of N_2 is valid for X_2 ($Z = 12$), the bond order of X_2 is 2.

3.12 Aromatic compounds can be halogenated using molecular halogen in the presence of an acid. Benzene when treated with molecular bromine in the presence of ferric chloride gives bromobenzene. Naphthalene undergoes bromination with bromine in acetic acid. The kinetics of this reaction is interesting. The kinetic expression involves a term that is second order in bromine for the reaction in glacial acetic acid, and first order in bromine for the reaction in 50% aqueous acetic acid.



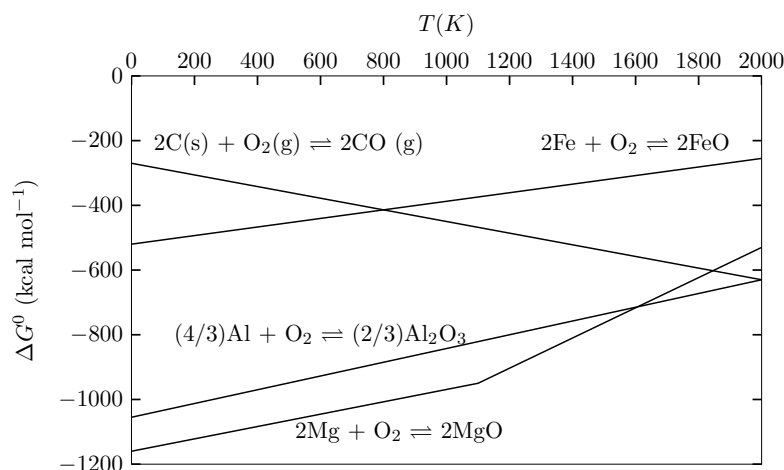
Aluminium chloride is much more active Lewis acid than ferric chloride and the halogenations can be carried out using AlCl_3 . The bond energies of Al-Cl and Al-Br are 502 kJ mol^{-1} and 430 kJ mol^{-1} , respectively. The correct statement(s) is/are

- (A) The reaction of benzene with bromine in the presence of AlCl_3 gives a substantial amount of chlorobenzene.
 (B) **The reaction of naphthalene with bromine in the presence of glacial acetic acid involves reaction of two molecules of bromine with one molecule of naphthalene, and no involvement of acetic acid, in the rate determining step.**
 (C) The reaction of naphthalene with bromine in 50% aqueous acetic acid involves bromine and acetic acid in the rate determining step.
 (D) **When 1-methylnaphthalene is reacted with bromine in the presence of a Lewis acid 4-bromo-1-methylnaphthalene is formed as the major product.**

3.13 Magnetite is an iron oxide ore, which reacts with carbon monoxide to give iron metal and carbon dioxide. When a sample of magnetite is allowed to react with sufficient carbon monoxide, 0.896 L carbon dioxide was produced at NTP. The percentage of iron in magnetite can also be determined by dissolving the ore in acid, then reducing the iron to Fe^{2+} completely and finally titrating it with aqueous acidic KMnO_4 . The atomic weights of Fe, C and O are 58.8, 12.0 and 16.0 a.m.u., respectively. The correct statement(s) is/are

- (A) The mass of iron formed is 1.76 g.
- (B) The number of electrons involved in the redox reaction during titration is 5.
- (C) The paramagnetism of the solution decreases during the reaction of Fe^{2+} with KMnO_4 .
- (D) During the titration, manganese containing species changes its geometry from square planar to tetrahedral.

3.14 In the following figure, Gibb's energy (ΔG^0) is plotted against T for the formation of some oxides.



The correct statement(s) is/are

- (A) The large negative slope of the line representing oxidation of carbon is due to substantial increase in entropy with temperature.
- (B) The reaction $2\text{Al} + 3\text{MgO} \rightarrow \text{Al}_2\text{O}_3 + 3\text{Mg}$ is feasible at 1200°C .
- (C) At 200°C the reducing properties of the metals follow the order $\text{Mg} > \text{Al} > \text{Fe}$.
- (D) At 800°C , the equilibrium constant for the oxidation of magnesium is the highest among the four reactions.

Section 4: Mathematics

Marks for Section 4: 50

This section contains 14 questions.

For questions 4.1 to 4.10, **only one** of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

- 4.1 The set $\left\{ \sum_{n=1}^{\infty} \frac{a_n}{10^n} : a_1 = 5, a_2 = 3, \text{ and } a_n \in \{0, 1, \dots, 9\} \text{ for all } n \geq 3 \right\}$ coincides with the set
- (A) $\{x : 0.53 \leq x < 0.6\}$ (B) $\{x : 0.53 \leq x \leq 0.6\}$
 (C) $\{x : 0.53 \leq x < 0.54\}$ (D) $\{x : 0.53 \leq x \leq 0.54\}$

- 4.2 Let $C_1 = \{(x, y) : |x| + |y| \leq 1\}$, $C_2 = \{(x, y) : x^2 + y^2 \leq 1\}$ and $C_3 = \{(x, y) : \max\{|x|, |y|\} \leq 1\}$ be three regions in xy -plane. If $A(C_j)$ denotes the area of region $C_j, j = 1, 2, 3$, then
- (A) $A(C_3) > A(C_2) > A(C_1)$ (B) $A(C_3) > A(C_1) > A(C_2)$
 (C) $A(C_2) > A(C_3) > A(C_1)$ (D) $A(C_2) > A(C_1) > A(C_3)$

- 4.3 The major axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to the latus rectum of the parabola $y^2 = 4bx$. The eccentricity of the ellipse is
- (A) $\frac{3}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

4.4 Let

$$I = \int_0^{\pi/2} (\cos^2(\cos x) + \sin^2(\sin x)) dx, \quad J = \int_0^{\pi/2} (\cos^2(\sin x) + \sin^2(\cos x)) dx.$$

Then

- (A) $I < J$ (B) $I = J$ (C) $J < I < 2J$ (D) $2J < I$

- 4.5 Let $X = \{z : |z| + |z - 1| = 1\}$ be a subset of complex numbers. Then, in the complex plane X represents
- (A) the triangle with vertices $z_1 = 0, z_2 = 1$ and $z_3 = 1 + i$
 (B) the circle passing through $z_1 = 0$ and $z_2 = 1$ with center $z = \frac{1}{2} + \frac{i}{2}$
 (C) **the line segment joining $z_1 = 0$ and $z_2 = 1$**
 (D) the square with vertices $z_1 = 0, z_2 = 1, z_3 = 1 + i$ and $z_4 = i$

- 4.6 In triangle ABC , let $BC = a, CA = b$ and $AB = c$. Let h_a be the length of the altitude drawn from A on to BC . Define h_b and h_c similarly. If $h_a + h_b, h_b + h_c$ and $h_c + h_a$ are in arithmetic progression then
- (A) c, a, b are in arithmetic progression.
 (B) a, b, c are in arithmetic progression.
 (C) a, b, c are in harmonic progression.
 (D) **c, a, b are in harmonic progression.**

- 4.7 For any real number x , $[x]$ denotes the largest integer less than or equal to x and $\{x\} = x - [x]$. The number of real solutions of $7[x] + 23\{x\} = 191$ is
 (A) 0 (B) 1 (C) 2 **(D) 3**

- 4.8 Let L be the point (t, t) in xy -plane and M be a point on the y -axis such that LM has slope t . Then the locus of the midpoint of LM as t varies over all real numbers is
 (A) $y = 2x^2 - 2x$ (B) $y = -2x^2$ (C) $y = -2x^2 + 2x$ (D) $y = 2x^2 + 2x$

- 4.9 A vertical lamp post is located at one corner of a level ground. The ground is in the shape of a regular hexagon. A man walks from the foot of the post to the next three corners of the ground in clockwise direction and notices that the angles of elevation of the top of the post from each of these corners are α , β and γ respectively, where $\alpha > \beta > \gamma$. Then
 (A) $\cot^2 \gamma = \cot^2 \beta + \cot^2 \alpha$
 (B) $\cot \gamma = \cot \alpha + \cot \beta$
 (C) $\cot \alpha \cot \gamma > \cot^2 \beta$
 (D) $\cot^2 \alpha + \cot^2 \beta + \cot^2 \gamma = \cot \alpha \cot \beta + \cot \beta \cot \gamma + \cot \gamma \cot \alpha$

- 4.10 Suppose \vec{a} , \vec{b} and \vec{c} be three vectors of length 1 , $\frac{1}{2}$ and 1 respectively such that

$$\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = \vec{0}$$

The acute angle between \vec{a} and \vec{c} is

- (A) 15° **(B) 30°** (C) 45° (D) 60°

For questions 4.11 to 4.14, **one or more than one** of the 4 options may be correct. Your answer is regarded correct **only if you choose all the correct option(s) and no incorrect option(s)**. A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

- 4.11 Which of the following functions defined on the set of real numbers is/are periodic ?

- (A) $1 - e^{|\sin x|}$ (B) $e - e^{\sin^2 x}$
 (C) $\cos x - \log(2 + \sin x)$ (D) $5 - x|\sin^3 x|$

- 4.12 Consider a function $f(x) = x^{\frac{1}{x}}$ defined over positive real numbers. Which of the following statements is/are true?

- (A) $f(x)$ has a maximum at $\frac{1}{e}$
(B) The maximum among $f(1), f(2), f(3), f(4)$, and $f(5)$ is $f(3)$
 (C) The minimum among $f(1), f(e), f(e^2), f(e^3)$, and $f(e^4)$ is less than 1
 (D) $e^\pi > \pi^e$

- 4.13 Let $f(x) = \sin x e^{\cos x}$ and $g(x) = \cos x e^{\sin x}$ be two functions. Then

- (A) $f(x)$ is increasing and $g(x)$ is decreasing on $(0, \pi/2)$

- (B) $f(x)$ and $g(x)$ attain their maximum at the same point in $(0, \pi/2)$
- (C) $f(x)$ and $g(x)$ have the same maximum on $(0, \pi/2)$
- (D) $\int_0^{\pi/2} f(x)dx = \int_0^{\pi/2} g(x)dx$

4.14 Each of the three children in a family is equally likely to be a boy or a girl independent of the others. Define the events

E : “all the children in the family are of the same sex”

F : “there is at most one boy in the family”

G : “the family includes a boy and a girl”

Then

(A) E is independent of F

(B) E is independent of G

(C) F is independent of G

(D) E, F, G are independent

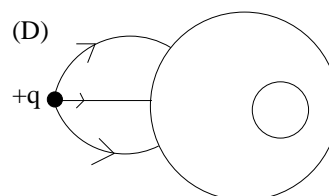
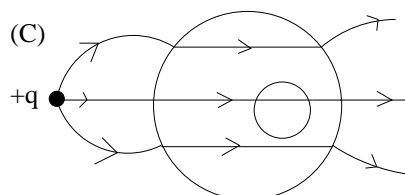
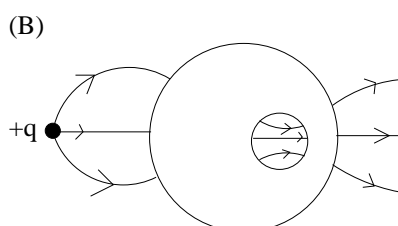
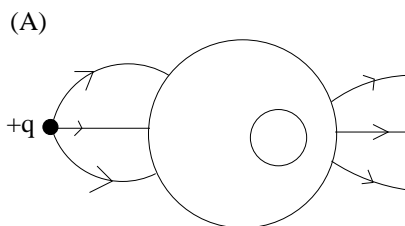
Section 5: Physics

Marks for Section 5: 50

This section contains 14 questions.

For questions 5.1 to 5.10, **only one** of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 marks.

5.1 A positive point charge q is placed near a three dimensional isolated, neutral solid conductor with a cavity as shown. The possible electric lines of force are best depicted in:



5.2 Consider two identical masses m attached to 3 identical springs as shown in the figure. They can be set in motion in two different ways as follows:

(I) $x_1(t) = x_0 \cos \omega t$ and $x_2(t) = x_0 \cos \omega t$

(II) $x_1(t) = x_0 \cos \omega t$ and $x_2(t) = -x_0 \cos \omega t$

Here $x_1(t)$ and $x_2(t)$ denote the displacements from the unstretched positions. Then the potential energy stored in the system is

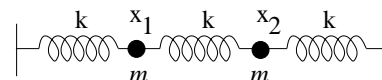
(A) larger for I than II at $t = 0$

(B) larger for I than II at $t = \pi/\omega$

(C) equal for I and II at $t = 0$

(D) smaller for I than II at $t =$

$2\pi/\omega$



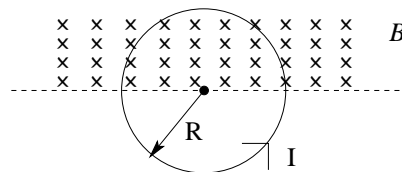
5.3 In a Young's double slit experiment 10 fringes are observed in a given segment of the screen when light of wavelength 500.0 nm is used. When the experiment is performed in a medium of refractive index 1.2, the number of fringes observed in the same segment remains 10. The wavelength of light used in this case is

- (A) 400.0 nm (B) 417.7 nm (C) 500.0 nm **(D) 600.0 nm**

5.4 A body of mass m with specific heat C at temperature T_h is brought into thermal equilibrium by contact with an identical body which was initially at a lower temperature T_l . No heat is lost to the surroundings and the temperatures are in kelvin. The change in entropy of the system is

- (A) $mC \ln \frac{(T_h + T_l)^2}{4T_l T_h}$ (B) $mC \ln \frac{(T_h - T_l)^2}{4T_l T_h}$
 (C) $mC \ln \frac{T_h}{T_l}$ (D) 0

5.5 The figure below depicts a circular loop of radius R carrying a fixed current I . The upper half of the loop is placed in a uniform magnetic field of magnitude B , perpendicular to the plane of the paper, as shown. The magnitude of the force on the loop is (neglect gravity)



- (A) BIR **(B) $2 BIR$** (C) πBIR (D) $2\pi BIR$

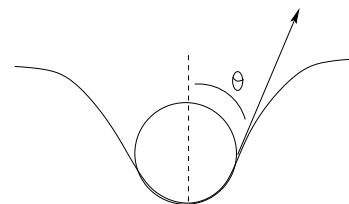
5.6 The dimensions of charge q can be written in terms of length L , energy E , temperature K , and permittivity of free space ϵ_0 as $q = [L^\alpha E^\beta K^\gamma \epsilon_0^\delta]$. Here

- (A) $\alpha = 1, \beta = -2, \gamma = 0$ and $\delta = 1$
 (B) $\alpha = 1/2, \beta = 1/2, \gamma = 1$ and $\delta = -1/2$
(C) $\alpha = 1/2, \beta = 1/2, \gamma = 0$ and $\delta = 1/2$
 (D) $\alpha = 1/2, \beta = -1/2, \gamma = 0$ and $\delta = 1/2$

5.7 A girl of mass 50 kg jumps from the shore onto a stationary boat of equal mass floating on a still pond. She jumps back to the shore almost immediately. Her speed in both cases is 2 m/s with respect to the shore. Consider her entire motion to be horizontal and in one dimension. Ignore friction and drag forces. The speed of the boat with respect to the shore after she jumps back is

- (A) 1 m/s (B) 2 m/s (C) 3 m/s **(D) 4 m/s**

5.8 The surface of a liquid is just able to support the weight of a six-legged insect. The leg ends can be assumed to be spheres each of radius 3.2×10^{-5} m and the weight of the insect is distributed equally over the six legs. The coefficient of surface tension in this case is 0.1 N/m and the angle of the footfall with respect to the vertical is $\theta = \pi/3$ radians (see figure). The mass of the insect is close to

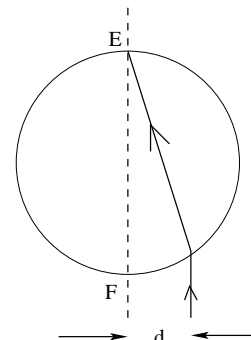


- (A) 5×10^{-5} kg **(B) 5×10^{-6} kg** (C) 1×10^{-6} kg (D) 1×10^{-7} kg

5.9 A free particle with initial kinetic energy 9 eV and de Broglie wavelength 1 nm enters a region of constant potential energy V_0 such that the new de Broglie wavelength is now 1.5 nm. Then V_0 is

- (A) 5 eV (B) 6 eV (C) 13.5 eV (D) 15 eV

5.10 The figure represents the circular cross section of a transparent solid cylinder of radius R and refractive index $\sqrt{3}$ placed in air. A ray of light enters the cylinder parallel to the diameter FE and displaced from it by a distance d as shown. It emerges from the cylinder at the point E.

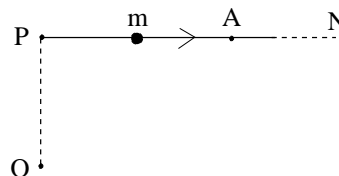


Then d is

- (A) $R/2$ (B) $R/\sqrt{3}$
 (C) $\sqrt{3}R/2$ (D) R

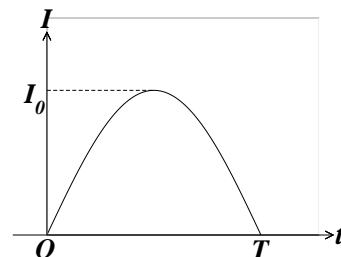
For questions 5.11 to 5.14, one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 marks.

5.11 A particle of mass m is moving on a straight path (PAN) with constant speed as shown on the right. Let ω and α denote the magnitude of the angular velocity and angular acceleration of the particle with respect to the point O which is not on the path. Then



- (A) $\alpha = 0$ and torque on the particle is zero.
 (B) $\alpha \neq 0$ but torque on the particle is zero.
 (C) α behaves non-monotonically as the particle moves from P to N .
 (D) ω behaves monotonically as the particle moves from P to N .

5.12 The figure on the right represents an idealized sinusoidal current flow between a cloud at a height of 1 km and the earth during lightning discharge. Here $I_0 = 157$ kA, $T = 0.2$ ms and the breakdown electric field of air is 300 kV/m. Then

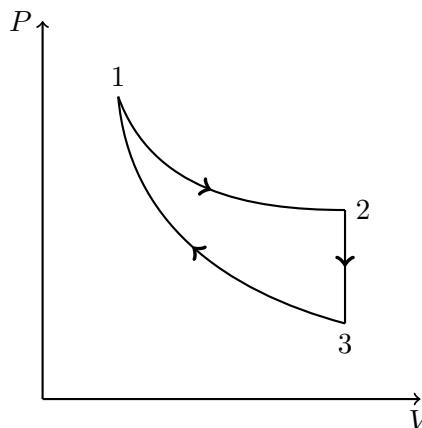


- (A) the total charge released by the lightning is 20 C.
 (B) the electrostatic energy of the cloud - earth system is 10 kJ.
 (C) the average current is 100 kA.
 (D) The capacitance of the cloud - earth system is 6.67×10^{-5} F.

5.13 Consider the sequential radioactive decay process: $A \xrightarrow{K_1} B \xrightarrow{K_2} C$, with decay constants, K_1 and K_2 such that $2K_1 = K_2$. At time $t = 0$ the number $N_A(t)$ of the species A is N_0 and the numbers $N_B(t)$ and $N_C(t)$ of species B and C respectively, are zero. Then

- (A) the sum $N_A(t) + N_B(t) + N_C(t)$ is a constant at any given time t .
- (B) $N_B(t)$ exhibits a maximum.
- (C) $N_C(t)$ approaches N_0 as $t \rightarrow \infty$.
- (D) for t smaller than the half life of species A , $N_B(t)$ is greater than $N_A(t)$.

5.14 The three processes in a thermodynamic cycle shown in the figure are: Process $1 \rightarrow 2$ is isothermal; Process $2 \rightarrow 3$ is isochoric (volume remains constant); Process $3 \rightarrow 1$ is adiabatic. The total work done by the ideal gas in this cycle is 10 J. The internal energy decreases by 20 J in the isochoric process. The work done by the gas in the adiabatic process is -20 J. Then



- (A) the net change in the internal energy in the cycle is 0 J.
- (B) the work done by the gas in the isochoric process is 0 J.
- (C) The change in the internal energy in the adiabatic process is 30 J.
- (D) the heat added to the system in the isothermal process is 30 J.