Very Short Answers Questions

1. Write the following as a single matrix.

Sol. i) [2 1 3] + [0 0 0]

= [2+0 1+0 3+0] = [2 1 3]

ii) $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0-1 \\ 1+1 \\ -1+0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$

iii) $\begin{bmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 3+4 & 9+0 & 0+2 \\ 1+7 & 8+1 & -2+4 \end{bmatrix} = \begin{bmatrix} 7 & 9 & 2 \\ 8 & 9 & 2 \end{bmatrix}$

iv) $\begin{bmatrix} -1 & 2 \\ 1 & -2 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1+0 & 2+1 \\ 1-1 & -2+0 \\ 3-2 & -1+1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 0 & -2 \\ 1 & 0 \end{bmatrix}$

2. If A = $\begin{bmatrix} -1 & 3 \\ 4 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 1 \\ 3 & -5 \end{bmatrix}$ and X = $\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ and A + B = X, then find the values of x_1, x_2, x_3 and x_4.

Sol. A + B = X

$$\begin{bmatrix} -1 & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 7 & -3 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$$

$$\therefore x_1 = 1,\ x_2 = 4,\ x_3 = 7,\ x_4 = -3$$

3. If A = $\begin{bmatrix} -1 & -2 & 3 \\ 1 & 2 & 4 \\ 2 & -1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -2 & 5 \\ 0 & -2 & 2 \\ 1 & 2 & -3 \end{bmatrix}$ and C = $\begin{bmatrix} -2 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$ then find A + B + C.

Sol. A + B + C =

$$\begin{bmatrix} -1 & -2 & 3 \\ 1 & 2 & 4 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 5 \\ 0 & -2 & 2 \\ 1 & 2 & -3 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1-2 & -2-2+1 & 3+5+2 \\ 1+0+1 & 2-2+1 & 4+2+2 \\ 2+1+2 & -1+2+0 & 3-3+1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 & 10 \\ 2 & 1 & 8 \\ 5 & 1 & 1 \end{bmatrix}$$

4. If $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{bmatrix}$ and X = A + B then find the matrix X.

Sol. $X = A + B = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 0 \\ 1 & 3 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{bmatrix}$

$\therefore X = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 3 \\ 5 & 2 & 3 \end{bmatrix}$

5. If $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$, find the values of x, y, z and a.

Sol. Given $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$

$\therefore x - 3 = 5 \Rightarrow x = 3 + 5 = 8$

$2y - 8 = 2 \Rightarrow 2y = 8 + 2 = 10 \Rightarrow y = 5$

$z + 2 = -2 \Rightarrow z = -2 - 2 = -4$

$a - 4 = 6 \Rightarrow a = 4 + 6 = 10$

6. Find the following products wherever possible.

Hint: (1×3) by $(3 \times 1) = 1 \times 1$.

Sol. i) $$\begin{bmatrix} -1 & 4 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1\cdot 5 + 4\cdot 1 + 2\cdot 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5+4+6 \end{bmatrix} = \begin{bmatrix} 5 \end{bmatrix}$$

ii) $$\begin{bmatrix} 2 & 1 & 4 \\ 6 & -2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + 1\cdot 2 + 4\cdot 1 \\ 6\cdot 1 + (-2)\cdot 2 + 3\cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2+4 \\ 6-4+3 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$$

iii) $$\begin{bmatrix} 3 & -2 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 12-4 & -3-10 \\ 4+12 & -1+30 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -13 \\ 16 & 29 \end{bmatrix}$$

iv) $$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} -2 & -3 & 4 \\ 2 & 2 & -3 \\ 1 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -4+4+1 & -6+4+2 & 8-6-2 \\ -2+0+2 & -3+0+4 & 4+0-4 \\ -4+2+2 & -6+2+4 & 8-3-4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

v) $\begin{bmatrix} 3 & 4 & 9 \\ 0 & -1 & 5 \\ 2 & 6 & 12 \end{bmatrix}\begin{bmatrix} 13 & -2 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

First matrix is a 3×3 matrix and second matrix is 2×3 matrix.

Number of columns in first matrix ≠

Number of rows in second matrix.

∴ Matrix product is not possible.

vi) $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 6 & -2 & 3 \end{bmatrix}$

Number of columns in first matrix = 1

Number of rows in second matrix = 2

Number of columns in first matrix ≠

Number of rows in second matrix

Multiplication of matrices is not possible.

vii) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1-1 & 1-1 \\ -1+1 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

viii) $\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$$= \begin{bmatrix} 0+abc-abc & b^2c-b^2c & bc^2-bc^2 \\ -a^2c+a^2c & -abc+abc & -ac^2+ac^2 \\ a^2b-a^2b & ab^2-ab^2 & abc-abc \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

7. If A = $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, do AB and BA exist? If they exist, find them. Do A and B commutative with respect to multiplication of matrices.

Sol. Given A = $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-8+6 & 3-10+3 \\ -8+8+10 & -12+10+5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2-12 & -4+6 & 6+15 \\ 4-20 & -8+10 & 12+25 \\ 2-4 & -4+2 & 6+5 \end{bmatrix} = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix} AB \neq BA$$

$\therefore$ A and B are not commutative with respect to multiplication of matrices.

8. Find A^2 where A = $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

Sol. $A^2 = A.A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 16-2 & 8+2 \\ -4-1 & -2+1 \end{bmatrix} = \begin{bmatrix} 14 & 10 \\ -5 & -1 \end{bmatrix}$$

9. If A = $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, find A^2.

Sol. $A^2 = A, A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

10. If A = $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, B = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and C = $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ then show that

(i) $A^2 = B^2 = C^2 = -I$,

(ii) AB = –BA = –C ($i^2 = -1$ and I is the unit matrix of order 2)

Sol. i) $A^2 = A.A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -1$

$B^2 = B.B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -1$

$C^2 = C.C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -1$

$\therefore A^2 = B^2 = C^2 = -1$

ii) $AB = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = -\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} = -C$

$BA = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} = C$

$\therefore$ AB = –BA = –C.

11. If A = $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 2 & 0 \\ 1 & 0 & 4 \end{bmatrix}$, find AB. Find BA if exists.

Sol. Given A = $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 3 & 2 & 0 \\ 1 & 0 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 2 & 0 \\ 1 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6+1 & 4+0 & 0+4 \\ 3+3 & 2+0 & 0+12 \end{bmatrix} = \begin{bmatrix} 7 & 4 & 4 \\ 6 & 2 & 12 \end{bmatrix}$$

Order of AB is 2×3

BA does not exist since number of columns in B $\neq$ No. of rows in A.

12. If A = $\begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and A^2 = 0, then find the value of k.

Sol. $A^2 = 0 \Rightarrow \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 4-4 & 8+4k \\ -2-k & -4+k^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow 8 + 4k = 0 \Rightarrow 4k = -8 \Rightarrow k = -2$

13. If A = $\begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}$ then find $(AB^T)^T$.

Sol. $B^T = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}^T = \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}$

$$AB^T = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 5 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0+0 & 0+0-2 \\ 1+1+0 & 0+1-10 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 2 & -9 \end{bmatrix}$$

$$(AB^T)^T = \begin{bmatrix} -2 & -2 \\ 2 & -9 \end{bmatrix}^T = \begin{bmatrix} -2 & 2 \\ -2 & -9 \end{bmatrix}$$

14. If $A = \begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$ find $2A + B^T$ and $3B^T - A$.

Sol. $A = \begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix} \Rightarrow 2A = 2\begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 10 & 0 \\ -2 & 8 \end{bmatrix}$

$$B = \begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow B^T = \begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$$

$$2A+B^T=\begin{bmatrix}-4 & 2\\ 10 & 0\\ -2 & 8\end{bmatrix}+\begin{bmatrix}-2 & 4\\ 3 & 0\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-4-2 & 2+4\\ 10+3 & 0+0\\ -2+1 & 8+2\end{bmatrix}=\begin{bmatrix}-6 & 6\\ 13 & 0\\ -1 & 10\end{bmatrix}$$

$$B^T=\begin{bmatrix}-2 & 3 & 1\\ 4 & 0 & 2\end{bmatrix}^T=\begin{bmatrix}-2 & 4\\ 3 & 0\\ 1 & 2\end{bmatrix}$$

$$3B^T-A=3\begin{bmatrix}-2 & 4\\ 3 & 0\\ 1 & 2\end{bmatrix}-\begin{bmatrix}-2 & 1\\ 5 & 0\\ -1 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-6 & 12\\ 9 & 0\\ 3 & 6\end{bmatrix}-\begin{bmatrix}-2 & 1\\ 5 & 0\\ -1 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-6+2 & 12-1\\ 9-5 & 0-0\\ 3+1 & 6-4\end{bmatrix}=\begin{bmatrix}-4 & 11\\ 4 & 0\\ 4 & 2\end{bmatrix}$$

15. If $A=\begin{bmatrix}2 & -4\\ -5 & 3\end{bmatrix}$ then find $A+A^T$ and $A.A^T$.

Sol. $A=\begin{bmatrix}2 & -4\\ -5 & 3\end{bmatrix}$

$$\Rightarrow A^T=\begin{bmatrix}2 & -4\\ -5 & 3\end{bmatrix}^T=\begin{bmatrix}2 & -5\\ -4 & 3\end{bmatrix}$$

$$A+A^T=\begin{bmatrix}2 & -4\\ -5 & 3\end{bmatrix}+\begin{bmatrix}2 & -5\\ -4 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2+2 & -4-5\\ -5-4 & 3+3\end{bmatrix}=\begin{bmatrix}20 & -22\\ -22 & 34\end{bmatrix}$$

16. If A = $\begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ **is a symmetric matrix, then find x.**

Sol. A is a symmetric matrix $\Rightarrow A^T = A$

$$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$$

Equating 2^{nd} row, 3^{rd} column elements we get x = 6.

17. If A = $\begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ **is a skew symmetric matrix, find x.**

Hint: A is a skew symmetric matrix $\Rightarrow A^T = A$

Sol. A is a skew symmetric matrix

$\Rightarrow A^T = A$

$$\begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & x \\ 1 & -2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & 2 \\ 1 & -x & 0 \end{bmatrix}$$

Equating second row third column elements we get x = 2.

18. Is $\begin{bmatrix} 0 & 1 & 4 \\ -1 & 0 & 7 \\ -4 & -7 & 0 \end{bmatrix}$ symmetric or skew symmetric 7.

Sol. Let A = $\begin{bmatrix} 0 & 1 & 4 \\ -1 & 0 & 7 \\ -4 & -7 & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} 0 & 1 & 4 \\ -1 & 0 & 7 \\ -4 & -7 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & -4 \\ 1 & 0 & -7 \\ 4 & 7 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 1 & 4 \\ -1 & 0 & 7 \\ -4 & -7 & 0 \end{bmatrix} = -A$$

$\therefore$ A is a skew symmetric matrix.

19. Find the determinants of the following matrices.

i) $\begin{bmatrix} 2 & 1 \\ 1 & -5 \end{bmatrix}$

Sol. det A = ad – bc = 2(–5) – 1(1) = –10 – 1 = –11

ii) $\begin{bmatrix} 4 & 5 \\ -6 & 2 \end{bmatrix}$

Sol. det A = $4\cdot 2-(-6)\cdot 5 = 8+30 = 38$

iii) $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

Sol. det A = $-i^2 - 0 = 1 - 0 = 1$

iv) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Sol. det A = $0(0 - 1) - 1(0 - 1) + 1(1 - 0)$

$= 1 + 1 = 2$

v) $\begin{bmatrix} 1 & 4 & 2 \\ 2 & -1 & 4 \\ -3 & 7 & 6 \end{bmatrix}$

Sol. det A = $1(-6 - 28) - 4(12 + 12) + 2(14 - 3)$

$= -34 - 96 + 22 = -108$

vi) $\begin{bmatrix} 1 & 4 & 2 \\ 2 & -1 & 4 \\ -3 & 7 & 6 \end{bmatrix}$

Sol. det A = $2(-3 - 2) + 1(4 - 1) + 4(8 + 3)$

$= -10 + 3 + 44 = 37$

vii) $\begin{bmatrix} 1 & 2 & -3 \\ a & -1 & 7 \\ 2 & 4 & -6 \end{bmatrix}$

Sol. det A = 0 since R_1 and R_3 are proportional.

viii) $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

Sol. $\det A = a(bc - f^2) - h(ch - fg) + g(hf - bg)$

$= abc - af^2 - ch^2 + fgh + fgh - bg^2$

$= abc + 2fgh - af^2 - bg^2 - ch^2$

ix) $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

Sol. $\det A = a(bc - a^2) - b(b^2 - ac) + c(ab - c^2)$

$= abc - a^3 - b^3 + abc + abc - c^3$

$= 3abc - a^3 - b^3 - c^3$

x) $\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$

Sol. $\det A = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$

$= 1(225 - 256) - 4(100 - 144) + 9(64 - 81)$

$= -31 + 176 - 153 = -184 + 176 = -8$

20. If A= $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and det A = 45, then find x.

Sol. det A = 45 $\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{vmatrix} = 45$

$3x + 24 = 45 \Rightarrow 3x - 45 + 24 = 0$

$\Rightarrow 3x - 21 = 0 \Rightarrow x = \frac{21}{3} = 7$

21. Find the adjoint and inverses of the following matrices.

i) $\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$ if A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

Sol. Adj A = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix}$

$|A| = 12 - (-12) = 24$

$A^{-1} = \frac{AdjA}{DetA} = \frac{1}{24}\begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix}$

ii) $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Sol. Adj A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, det A = 1

$A^{-1} = \frac{AdjA}{DetA} = \frac{1}{\cos^2\alpha + \sin^2\alpha}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

iii) $\begin{bmatrix} \overset{+}{1} & \overset{-}{0} & \overset{+}{2} \\ \overset{-}{2} & \overset{+}{1} & \overset{-}{0} \\ \overset{+}{3} & \overset{-}{2} & \overset{+}{1} \end{bmatrix}$

Sol. $A_1 = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$

$$B_1 = -\begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix} = -2$$

$$C_1 = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = 1$$

$$A_2 = -\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = -(0-4) = 4$$

$$B_2 = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1 - 6 = 5$$

$$C_2 = -\begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix} = -(2-0) - 2$$

$$A_3 = \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} = 0 - 2 = -2$$

$$B_3 = -\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = -(0-4) = 4$$

$$C_3 = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$\text{Adj}A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\det A = a_1A_1 + b_1B_1 + c_1C_1$$
$$= 1(1) + 0(-2) + 2(1) = 1 + 0 + 2 = 3$$

$$A^{-1} = \frac{\text{Adj}A}{\det A} = \frac{1}{3}\begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$$

iv) $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

Sol. $A_1 = \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = 0 - 2 = -2$

$$B_1 = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -(1-2) = 1$$

$$C_1 = \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} = 2 - 0 = 2$$

$$A_2 = -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -(1-4) = 3$$

$$B_2 = \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} = 2 - 4 = -2$$

$$C_2 = -\begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} = -(4-2) = -2$$

$$A_3 = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$B_3 = -\begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = -(2-2) = 0$$

$$C_3 = \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = -1$$

$$\text{Adj}A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & -2 & 0 \\ 2 & -2 & -1 \end{bmatrix}$$

$$\text{Det}A = a_1A_1 + b_1B_1 + c_1C_1$$
$$= 2(-2) + 1(1) + 2(2) = -4 + 1 + 4 = 1$$

$$A^{-1} = \frac{\text{Adj}A}{\text{det}A} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & -2 & 0 \\ 2 & -2 & -1 \end{bmatrix}$$

22. If A = $\begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$, $a^2 + b^2 + c^2 + d^2 = 1$ then find inverse of A.

Sol. det A = (a + ib)(a – ib) – (c + id)(–c + id)

$= a^2 - i^2 b^2 - (-c^2 + i^2 d^2)$

$= a^2 + b^2 + c^2 + d^2 \; (-i^2 = 1) = 1$

$$\text{Adj}A = \begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}A}{\text{Det}A} = \begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$$

23. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$**, find $(A^T)^{-1}$.**

Sol. $A^T = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$$A_1 = \begin{vmatrix} -1 & 2 \\ 4 & 1 \end{vmatrix} = -1-8 = -9$$

$$B_1 = -\begin{vmatrix} -2 & 2 \\ 3 & 1 \end{vmatrix} = -(-2-6) = 8$$

$$C_1 = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix} = (-8+3) = -5$$

$$A_2 = -\begin{vmatrix} 0 & -2 \\ 4 & 1 \end{vmatrix} = -(0+8) = -8$$

$$B_2 = \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = 1+6 = 7$$

$$C_2 = -\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} = -(4-0) = -4$$

$$A_3 = \begin{vmatrix} 0 & -2 \\ -1 & 2 \end{vmatrix} = 0-2 = -2$$

$$B_3 = -\begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = -(2-4) = 2$$

$$C_3 = \begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix} = -1-0 = -1$$

$$\text{Adj}A^T = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$\det A^T = 1(-9)+0(8)-2(-5) = -9+10 = 1$$

$$(A^T)^{-1} = \frac{\text{Adj}(A^T)}{\det A^T} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

24. If A = $\begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$, then show that Adj A = 3A^T find A^{-1}.

Sol. $A_1 = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = 1 - 4 = -3$

$B_1 = -\begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -(2+4) = -6$

$C_1 = \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = -4 - 2 = -6$

$A_2 = -\begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -(-2-4) = 6$

$B_2 = \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = -1 + 4 = 3$

$C_2 = -\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2+4) = -6$

$A_3 = \begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = 4 + 2 = 6$

$B_3 = -\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2+4) = -6$

$C_3 = \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -1 + 4 = 3$

$$\text{Adj}A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} \ldots(1)$$

$$A^T = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$3A^T = 3\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} \ldots(2)$$

From (1) and (2) we get Adj A = $3A^T$

$$\begin{aligned} \text{Det}A &= a_1A_1 + b_1B_1 + c_1C_1 \\ &= (-1)(-3) + (-2)(-6) + (-2)(-6) \\ &= 3 + 12 + 12 = 27 \end{aligned}$$

$$A^{-1} = \frac{\text{Adj}A}{\det A} = \frac{1}{27}\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

25. If abc ≠ 0, find the inverse of $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$**.**

Sol. $A_1 = \begin{vmatrix} b & 0 \\ 0 & c \end{vmatrix} = bc$

$B_1 = -\begin{vmatrix} 0 & 0 \\ 0 & c \end{vmatrix} = 0 \qquad C_1 = \begin{vmatrix} 0 & b \\ 0 & 0 \end{vmatrix} = 0$

$A_2 = -\begin{vmatrix} 0 & 0 \\ 0 & c \end{vmatrix} = 0 \qquad B_2 = \begin{vmatrix} a & 0 \\ 0 & c \end{vmatrix} = ac$

$C_2 = -\begin{vmatrix} a & 0 \\ 0 & 0 \end{vmatrix} = 0 \qquad A_3 = \begin{vmatrix} 0 & 0 \\ b & 0 \end{vmatrix} = 0$

$B_3 = -\begin{vmatrix} a & 0 \\ 0 & 0 \end{vmatrix} = 0 \qquad C_3 = \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} = ab$

det A = $a_1A_1 + b_1B_1 + c_1C_1$

= 0(–1) + 1(1) + 1(1) = 1 + 1 = 2

$$A^{-1} = \frac{\text{Adj}A}{\det A} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \frac{1}{2}\begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} c-b+b-c & c+a+a-c & a-b+a+b \\ b+c+b-c & c-a+a-c & b-a+a+b \\ b+c+c-b & c-a+c+a & b-a+a-b \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 2a & 2a \\ 2b & 0 & 2b \\ 2c & 2c & 0 \end{bmatrix}$$

$$ABA^{-1} = \frac{1}{2}\begin{bmatrix} 0 & 2a & 2a \\ 2b & 0 & 2b \\ 2c & 2c & 0 \end{bmatrix} \cdot \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 2a+2a & -2a+2a & 2a-2a \\ -2b+2b & 2b+2b & 2b-2b \\ -2c+2c & 2c-2c & 2c+2c \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4a & 0 & 0 \\ 0 & 4b & 0 \\ 0 & 0 & 4c \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \text{Diagonal matrix.}$$

26. If 3A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, then show that $A^{-1} = A^T$.

Sol. $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix} \Rightarrow A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$

Now $A \cdot A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix} \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A.A^T = I$

$\therefore A^{-1} = A^T$

27. If A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then show that $A^{-1} = A^3$.

Sol. $A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$

$A^4 = A^2A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 1$

$\therefore A^4 = I$

$\det A = 3(1) - 3(-2) + 4(-2) = 1$

$\because A \neq 0 \Rightarrow A^{-1}$exists

$\because A^4 = I$

Multiply with A^{-1}

$A^4(A^{-1}) = I(A^{-1})$

$\Rightarrow A^3(AA^{-1}) = A^{-1} \Rightarrow A^3(I) = A^{-1}$

$\therefore A^{-1} = A^3$

28. If AB = I or BA = I, then prove that A is invertible and B = A^{-1}.

Sol. Given $AB = I \Rightarrow |AB| = |I|$

$= |A|\,|B| = 1$

$= |A| \neq 0$

$\therefore$ A is a non-singular matrix.

and $BA = I \Rightarrow |BA| = |I|$

$\Rightarrow |B|\,|A| = 1 \Rightarrow |A| \neq 0$

$\therefore$ A is a non-singular matrix.

AB = I or BA = I, A is invertible.

$\therefore A^{-1}$ exists.

$AB = I \Rightarrow A^{-1}AB = A^{-1}I$

$\Rightarrow IB = A^{-1} \Rightarrow B = A^{-1}$

$\therefore B = A^{-1}$

30. Find the rank of the following matrices.

1. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Sol. det A = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$

and |a| = 1 ≠ 0

$\therefore \rho(A) = 1.$

2. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol. det A = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1 \neq 0$

$\therefore \rho(A) = 2$

3. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

Sol. det A = $\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$

$|1| = 1 \neq 0$

$\therefore \rho(A) = 1$

4. $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Sol. det A = $\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = -1 \neq 0$

$\therefore \rho(A) = 2$

5. $\begin{bmatrix} 1 & 0 & -4 \\ 2 & -1 & 3 \end{bmatrix}$

Sol. $\begin{vmatrix} 1 & -4 \\ 2 & 3 \end{vmatrix} = 3 + 8 = 11 \neq 0$

$\therefore \rho(A) = 2$

6. $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 4 & 3 \end{bmatrix}$

Sol. $\begin{vmatrix} 2 & 6 \\ 4 & 3 \end{vmatrix} = 6 - 24 = -18 \neq 0$

$\therefore \rho(A) = 2$

7. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Sol. det A = $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= 1(1-0) - 0(0-0) + 0(0-0)$
$= 1 - 0 + 0 = 1 \neq 0$

$\therefore \rho(A) = 3$

8. $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

Sol. det A = $\begin{vmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{vmatrix}$

$= 1(6-0) - 2(8+1) + 0(0+3)$
$= 6 - 18 = -12 \neq 0$

$\therefore \rho(A) = 3$

9. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

Sol. det A = $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix}$

$= 1(6-4) - 2(4-3) + 0(8-9)$
$= 2 - 2 + 0 = 0$

$\therefore\ \rho(A) \neq 3,\ \rho(A) < 3$

Take $\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0$

$\therefore\ \rho(A) = 2$

10. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Let A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, det A = 0, $\rho(A) \neq 3$.

Sol. All 2×2 sub matrix det is zero.

$\therefore\ \rho(A) \neq 2$

$|1| = 1 \neq 0,\ \therefore\ \rho(A) = 1$

11. $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$

Sol. Take sub matrix B = $\begin{vmatrix} 1 & 2 & 0 \\ 3 & 4 & 1 \\ -2 & 3 & 2 \end{vmatrix}$

$$= 1(8-3) - 2(6+2)$$
$$= 5 - 16 = -11 \neq 0$$

Rank of the given matrix is 3.

12. $\begin{bmatrix} 0 & 1 & 1 & -2 \\ 4 & 0 & 2 & 5 \\ 2 & 1 & 3 & 1 \end{bmatrix}$

Sol. Take sub matrix A = $\begin{bmatrix} 0 & 1 & 1 \\ 4 & 0 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$$= -1(12-4) + 1(4-0)$$
$$= -8 + 4 = -4 \neq 0$$

$\therefore \rho(A) = 3$

31. Find the trace of A if A = $\begin{bmatrix} 1 & 2 & -1/2 \\ 0 & -1 & 2 \\ -1/2 & 2 & 1 \end{bmatrix}$

Sol. The elements of the principal diagonal of A are 1, –1, 1.

Hence the trace of A is 1 + (–1) + 1 = 1.

32. If A = $\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$ then find –5A.

Sol. $-5A = -5\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -20 & 25 \\ 10 & -15 \end{bmatrix}$

33. If A = $\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 3 \end{bmatrix}$ then find A – B and 4B – 3A.

Sol. $A - B = \begin{bmatrix} 0-1 & 1+2 & 2-0 \\ 2-0 & 3-1 & 4+1 \\ 4+1 & 5-0 & 6-3 \end{bmatrix} = \begin{bmatrix} -1 & 3 & 2 \\ 2 & 2 & 5 \\ 5 & 5 & 3 \end{bmatrix}$

and $4B - 3A = 4\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 3 \end{bmatrix} - 3\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} 4 & -8 & 0 \\ 0 & 4 & -4 \\ -4 & 0 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 3 & 6 \\ 6 & 9 & 12 \\ 12 & 15 & 18 \end{bmatrix}$

$= \begin{bmatrix} 4 & -11 & -6 \\ -6 & -5 & -16 \\ -16 & -15 & -6 \end{bmatrix}$

34. If A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$ then find AB and BA.

Sol. The number of columns of A = 3 = the number of rows of B.

Hence AB is defined and

$$AB = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 5 & -5 \\ 7 & -8 \end{bmatrix}$$

Since the number of columns B = 2 ≠ 3 = the number of rows of A.

∴ BA is not defined.

35. If A = $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ then examine whether A and B commute with respect to multiplication of matrices.

Sol. Both A and B are square matrices of order 3. Hence both AB and BA are define and are matrices of order 3.

$$AB = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -2 \\ 1 & 1 & 10 \\ -1 & 5 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 7 \\ -4 & 5 & 3 \\ 5 & 4 & 1 \end{bmatrix}$$

Which shows that AB ≠ BA

Therefore A and B do not commute with respect to multiplication of matrices.

36. If A = $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ then show that A^2 = –I where i^2 = –1.

Sol. $A^2 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$= (-1)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -1$

37. If A = $\begin{bmatrix} -2 & 1 & 0 \\ 3 & 4 & -5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 \\ 4 & 3 \\ -1 & 5 \end{bmatrix}$ then find A + B^T.

Sol. $A + B^T = \begin{bmatrix} -2 & 1 & 0 \\ 3 & 4 & -5 \end{bmatrix} + \begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} -1 & 5 & -1 \\ 5 & 7 & 0 \end{bmatrix}$

38. Find the minors of –1 and 3 in the matrix $\begin{bmatrix} 2 & -1 & 4 \\ 0 & -2 & 5 \\ -3 & 1 & 3 \end{bmatrix}$.

Sol. Minor of –1 = $\begin{vmatrix} 0 & 5 \\ -3 & 3 \end{vmatrix} = 0 + 15 = 15$

Minor of 3 = $\begin{vmatrix} 2 & -1 \\ 0 & -2 \end{vmatrix} = -4 - 0 = -4$

39. Find the cofactors of the elements 2, –5 in the matrix $\begin{bmatrix} -1 & 0 & 5 \\ 1 & 2 & -2 \\ -4 & -5 & 3 \end{bmatrix}$**.**

Sol. The element 2 is (2, 2)-th element of the given matrix.

Hence cofactor of 2 = $(-1)^{2+2}\begin{vmatrix} -1 & 5 \\ -4 & 3 \end{vmatrix}$

$= (-1)^4(-3+20) = 17$

The element –5 is (3, 2)-th element of the given matrix.

Hence cofactor of –5 $= (-1)^{3+2}\begin{vmatrix} -1 & 5 \\ 1 & -2 \end{vmatrix}$

$= -1(2 - 5) = 3$

40. If A = $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$**, then find AA^T. Do A and A^T commute with respect to multiplication of matrices?**

Sol. $A^T = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

$AA^T = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

$A^TA = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$

Since $AA^T \neq A^TA$, A and A^T do not commute with respect to multiplication of matrices.

41. (a) If A = $\begin{bmatrix} 0 & 4 & -2 \\ -4 & 0 & 8 \\ 2 & -8 & x \end{bmatrix}$ is a skew symmetric matrix, find the value of x.

Sol. A is a skew symmetric matrix and x is an element of the diagonal.

Hence x = 0.

42. Find whether the following system of linear homogeneous equations has a non-trivial solution.

$x - y + z = 0$

$x + 2y - z = 0$

$2x + y + 3z = 0$

Sol. The coefficient matrix is $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$

Its determinant is $9 \neq 0$

Hence the system has the trivial solution

$x = y = z = 0$ only.

43. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$, prove that $(AB)^T = B^T A^T$.

Sol. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$

$= \begin{bmatrix} 3+8 & 1+10 \\ 9+16 & 3+20 \end{bmatrix} = \begin{bmatrix} 11 & 11 \\ 25 & 23 \end{bmatrix}$

$\therefore (AB)^T = \begin{bmatrix} 11 & 25 \\ 11 & 23 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 3+8 & 9+16 \\ 1+10 & 3+20 \end{bmatrix} = \begin{bmatrix} 11 & 25 \\ 11 & 23 \end{bmatrix}$

$\therefore (AB)^T = B^T A^T$

44. If A = $\frac{1}{3}\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & 7 \end{vmatrix}$ and $AA^T = A^T A = I_3$, find x and y.

Sol. $A = \frac{1}{3}\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & 7 \end{vmatrix} \Rightarrow 3A = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & 7 \end{vmatrix}$

$$(3A)^T = 3A^T = \begin{bmatrix} 1 & 2 & x \\ 2 & 1 & 2 \\ 2 & -2 & y \end{bmatrix}$$

$$AA^T = I_3 \Rightarrow (3A)(3A)^T = 9I_3$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}\begin{bmatrix} 1 & 2 & x \\ 2 & 1 & 2 \\ 2 & -2 & y \end{bmatrix} = 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+4+4 & 2+2-4 & x+4+2y \\ 2+2-4 & 4+1+4 & 2x+2-2y \\ x+4+2y & 2x+2-2y & x^2+4+y^2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$