DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

T.B.C. : B–MCZA–N–LZS

Test Booklet Series

Serial No. **105829**

TEST BOOKLET

MATHEMATICS

A

Time Allowed : *Two Hours and Thirty Minutes*

Maximum Marks : 300

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.

2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**

3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. *DO NOT* write *anything else* on the Test Booklet.

4. This Test Booklet contains **120** items (questions). Each item is printed both in **Hindi** and **English**. Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.

5. You have to mark all your responses *ONLY* on the separate Answer Sheet provided. See directions in the Answer Sheet.

6. All items carry equal marks.

7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.

8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet*. You are permitted to take away with you the Test Booklet.

9. Sheets for rough work are appended in the Test Booklet at the end.

10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.
    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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ध्यान दें : अनुदेशों का हिन्दी रूपान्तर इस पुस्तिका के पिछले पृष्ठ पर छपा है।

1. Let X be the set of all citizens of India. Elements x, y in X are said to be related if the difference of their age is 5 years. Which one of the following is correct ?
   (a) The relation is an equivalence relation on X.
   (b) The relation is symmetric but neither reflexive nor transitive.
   (c) The relation is reflexive but neither symmetric nor transitive.
   (d) None of the above

2. Consider the following relations from A to B where A = {u, v, w, x, y, z} and B = {p, q, r, s}.
   1. {(u, p), (v, p), (w, p), (x, q), (y, q), (z, q)}
   2. {(u, p), (v, q), (w, r), (z, s)}
   3. {(u, s), (v, r), (w, q), (u, p), (v, q), (z, q)}
   4. {(u, q), (v, p), (w, s), (x, r), (y, q), (z, s)}
   Which of the above relations are *not* functions ?
   (a) 1 and 2
   (b) 1 and 4
   (c) 2 and 3
   (d) 3 and 4

3. If α and β are the roots of the equation $ax^2 + bx + c = 0$, where $a \neq 0$, then $(a\alpha + b)(a\beta + b)$ is equal to :
   (a) ab
   (b) bc
   (c) ca
   (d) abc

4. Let S denote set of all integers. Define a relation R on S as 'aRb if $ab \geq 0$ where $a, b \in S$'. Then R is :
   (a) Reflexive but neither symmetric nor transitive relation
   (b) Reflexive, symmetric but not transitive relation
   (c) An equivalence relation
   (d) Symmetric but neither reflexive nor transitive relation

5. The roots of the equation $2a^2x^2 - 2abx + b^2 = 0$ when a < 0 and b > 0 are :
   (a) Sometimes complex
   (b) Always irrational
   (c) Always complex
   (d) Always real

6. What is the sum of the two numbers $(11110)_2$ and $(1010)_2$?
   (a) $(101000)_2$
   (b) $(110000)_2$
   (c) $(100100)_2$
   (d) $(101100)_2$

7. Let N denote the set of all non-negative integers and Z denote the set of all integers. The function $f : Z \rightarrow N$ given by $f(x) = |x|$ is :
   (a) One-one but not onto
   (b) Onto but not one-one
   (c) Both one-one and onto
   (d) Neither one-one nor onto

8. If P and Q are two complex numbers, then the modulus of the quotient of P and Q is :
   (a) Greater than the quotient of their moduli
   (b) Less than the quotient of their moduli
   (c) Less than or equal to the quotient of their moduli
   (d) Equal to the quotient of their moduli

9. Let $z = x + iy$ where x, y are real variables and $i = \sqrt{-1}$. If $|2z - 1| = |z - 2|$, then the point z describes :
   (a) A circle
   (b) An ellipse
   (c) A hyperbola
   (d) A parabola

(Contd.)

10. The sum of an infinite GP is x and the common ratio r is such that $|r| < 1$. If the first term of the GP is 2, then which one of the following is correct ?

(a) $-1 < x < 1$

(b) $-\infty < x < 1$

(c) $1 < x < \infty$

(d) None of the above

11. A box contains 3 white and 2 black balls. Two balls are drawn at random one after the other. If the balls are not replaced, what is the probability that both the balls are black ?

(a) 2/5

(b) 1/5

(c) 1/10

(d) None of the above

12. For two variables x and y, the two regression coefficients are $b_{yx} = -3/2$ and $b_{xy} = -1/6$. The correlation coefficient between x and y is :

(a) $-1/4$

(b) $1/4$

(c) $-1/2$

(d) $1/2$

13. The variance of numbers $x_1, x_2, x_3, ... x_n$ is V. Consider the following statements :

1. If every x_i is increased by 2, the variance of the new set of numbers is V.

2. If the numbers x_i is squared, the variance of the new set is V^2.

Which of the following statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

14. What is the mean of the squares of the first 20 natural numbers ?

(a) 151.5

(b) 143.5

(c) 65

(d) 72

15. p, q, r, s, t are five numbers such that the average of p, q and r is 5 and that of s and t is 10. What is the average of all the five numbers ?

(a) 7.75

(b) 7.5

(c) 7

(d) 5

16. The cumulative frequency of the largest observed value must always be :

(a) Less than the total number of observations

(b) Greater than the total number of observations

(c) Equal to total number of observations

(d) Equal to mid point of the last class interval

17. It has been found that if A and B play a game 12 times, A wins 6 times, B wins 4 times and they draw twice. A and B take part in a series of 3 games. The probability that they win alternately, is :

(a) 5/12

(b) 5/36

(c) 19/27

(d) 5/27

18. Out of 7 consonants and 4 vowels, words are to be formed by involving 3 consonants and 2 vowels. The number of such words formed is :

(a) 25200

(b) 22500

(c) 10080

(d) 5040

19. Let X denote the number of scores which exceed 4 in 18 tosses of a symmetrical die. Consider the following statements :

1. The arithmetic mean of X is 6.
2. The standard deviation of X is 2.

Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

20. How many different words can be formed by taking four letters out of the letters of the word 'AGAIN' if each word has to start with A ?

(a) 6
(b) 12
(c) 24
(d) None of the above

21. The sum of the series formed by the sequence $3, \sqrt{3}, 1 \ldots$ upto infinity is :

(a) $\dfrac{3\sqrt{3}(\sqrt{3}+1)}{2}$

(b) $\dfrac{3\sqrt{3}(\sqrt{3}-1)}{2}$

(c) $\dfrac{3(\sqrt{3}+1)}{2}$

(d) $\dfrac{3(\sqrt{3}-1)}{2}$

22. If $|z+\bar{z}|=|z-\bar{z}|$, then the locus of z is :

(a) A pair of straight lines
(b) A line
(c) A set of four straight lines
(d) A circle

23. The number 251 in decimal system is expressed in binary system by :

(a) 11110111
(b) 11111011
(c) 11111101
(d) 11111110

24. What is the argument of the complex number $\dfrac{(1+i)(2+i)}{3-i}$ where $i=\sqrt{-1}$?

(a) 0
(b) $\dfrac{\pi}{4}$
(c) $-\dfrac{\pi}{4}$
(d) $\dfrac{\pi}{2}$

25. Consider the following statements in respect of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$:

1. The matrix A is skew-symmetric.
2. The matrix A is symmetric.
3. The matrix A is invertible.

Which of the above statements is/are correct ?

(a) 1 only
(b) 3 only
(c) 1 and 3
(d) 2 and 3

26. Consider two matrices $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 1 & -4 \end{bmatrix}$. Which one of the following is correct ?

(a) B is the right inverse of A
(b) B is the left inverse of A
(c) B is the both sided inverse of A
(d) None of the above

(Contd.)

27. One of the roots of

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0 \text{ is :}$$

(a) abc

(b) a + b + c

(c) −(a + b + c)

(d) −abc

28. If A is any matrix, then the product AA is defined only when A is a matrix of order m × n where :

(a) m > n

(b) m < n

(c) m = n

(d) m ≤ n

29. The determinant of an odd order skew symmetric matrix is always :

(a) Zero

(b) One

(c) Negative

(d) Depends on the matrix

30. If any two adjacent rows or columns of a determinant are interchanged in position, the value of the determinant :

(a) Becomes zero

(b) Remains the same

(c) Changes its sign

(d) Is doubled

For the next three (03) items that follow :

In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects.

31. The number of students who had taken only Physics is :

(a) 2

(b) 3

(c) 5

(d) 6

32. The number of students who had taken only two subjects is :

(a) 7

(b) 8

(c) 9

(d) 10

33. Consider the following statements :

1. The number of students who had taken only one subject is equal to the number of students who had taken only two subjects.

2. The number of students who had taken at least two subjects is four times the number of students who had taken all the three subjects.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

For the next three (03) items that follow :

In the expansion of $\left(x^3 - \dfrac{1}{x^2}\right)^n$ where n is a positive integer, the sum of the coefficients of x^5 and x^{10} is 0.

34. What is n equal to ?

(a) 5

(b) 10

(c) 15

(d) None of the above

35. What is the value of the independent term ?

(a) 5005

(b) 7200

(c) −5005

(d) −7200

(Contd.)

36. What is the sum of the coefficients of the two middle terms ?

    (a) 0

    (b) 1

    (c) −1

    (d) None of the above

For the next three (03) items that follow :

Given that $C(n, r) : C(n, r + 1) = 1 : 2$ and $C(n, r + 1) : C(n, r + 2) = 2 : 3$.

37. What is n equal to ?

    (a) 11

    (b) 12

    (c) 13

    (d) 14

38. What is r equal to ?

    (a) 2

    (b) 3

    (c) 4

    (d) 5

39. What is $P(n, r) : C(n, r)$ equal to ?

    (a) 6

    (b) 24

    (c) 120

    (d) 720

40. The complete solution of $3 \tan^2 x = 1$ is given by :

    (a) $x = n\pi \pm \dfrac{\pi}{3}$

    (b) $x = n\pi + \dfrac{\pi}{3}$ only

    (c) $x = n\pi \pm \dfrac{\pi}{6}$

    (d) $x = n\pi + \dfrac{\pi}{6}$ only

    where $n \in Z$

41. What is the value of cos 36° ?

    (a) $\dfrac{\sqrt{5} - 1}{4}$

    (b) $\dfrac{\sqrt{5} + 1}{4}$

    (c) $\dfrac{\sqrt{10 + 2\sqrt{5}}}{4}$

    (d) $\dfrac{\sqrt{10 - 2\sqrt{5}}}{4}$

42. Consider the following statements :

    1. Value of $\sin \theta$ oscillates between −1 and 1.

    2. Value of $\cos \theta$ oscillates between 0 and 1.

    Which of the above statements is/are correct ?

    (a) 1 only

    (b) 2 only

    (c) Both 1 and 2

    (d) Neither 1 nor 2

43. If x and y are positive and $xy > 1$, then what is $\tan^{-1} x + \tan^{-1} y$ equal to ?

    (a) $\tan^{-1}\left(\dfrac{x + y}{1 - xy}\right)$

    (b) $\pi + \tan^{-1}\left(\dfrac{x + y}{1 - xy}\right)$

    (c) $\pi - \tan^{-1}\left(\dfrac{x + y}{1 - xy}\right)$

    (d) $\tan^{-1}\left(\dfrac{x - y}{1 + xy}\right)$

(Contd.)

44. Consider the following statements :

1. $n\left(\sin^2 67\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ\right) > 1$ for all positive integers $n \geq 2$.

2. If x is any positive real number, then $nx > 1$ for all positive integers $n \geq 2$.

Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

45. Consider the following statements :

1. If 3θ is an acute angle such that $\sin 3\theta = \cos 2\theta$, then the measurement of θ in radian equals to $\frac{\pi}{10}$.

2. One radian is the angle subtended at the centre of a circle by an arc of the same circle whose length is equal to the diameter of that circle.

Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

46. From an aeroplane above a straight road the angles of depression of two positions at a distance 20 m apart on the road are observed to be 30° and 45°. The height of the aeroplane above the ground is :

(a) $10\sqrt{3}$ m
(b) $10(\sqrt{3}-1)$ m
(c) $10(\sqrt{3}+1)$ m
(d) 20 m

47. Consider the following statements :

1. There exists no triangle ABC for which sin A + sin B = sin C.

2. If the angles of a triangle are in the ratio 1 : 2 : 3, then its sides will be in the ratio $1 : \sqrt{3} : 2$.
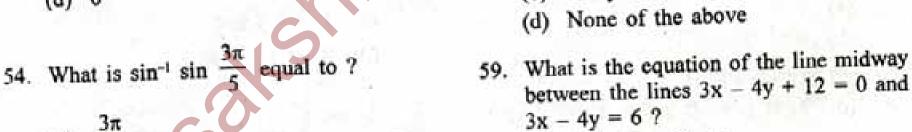
Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

48. Consider the following statements :

1. $\sin|x| + \cos|x|$ is always positive.

2. $\sin(x^2) + \cos(x^2)$ is always positive.

Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

49. What is $\dfrac{1+\sin A}{1-\sin A} - \dfrac{1-\sin A}{1+\sin A}$ equal to ?

(a) sec A – tan A
(b) 2 sec A . tan A
(c) 4 sec A . tan A
(d) 4 cosec A . cot A

50. What is $\dfrac{\cot 224^\circ - \cot 134^\circ}{\cot 226^\circ + \cot 316^\circ}$ equal to ?

(a) –cosec 88°
(b) –cosec 2°
(c) –cosec 44°
(d) –cosec 46°

(Contd.)

51. Consider the following statements :
    1. $\tan^{-1} 1 + \tan^{-1} (0.5) = \pi/2$
    2. $\sin^{-1} (1/3) + \cos^{-1} (1/3) = \pi/2$
    Which of the above statements is/are correct ?
    (a) 1 only
    (b) 2 only
    (c) Both 1 and 2
    (d) Neither 1 nor 2

52. If $A + B + C = \pi$, then what is $\cos(A + B) + \cos C$ equal to ?
    (a) 0
    (b) $2 \cos C$
    (c) $\cos C - \sin C$
    (d) $2 \sin C$

53. What is $\cos 20° + \cos 100° + \cos 140°$ equal to ?
    (a) 2
    (b) 1
    (c) 1/2
    (d) 0

54. What is $\sin^{-1} \sin \dfrac{3\pi}{5}$ equal to ?
    (a) $\dfrac{3\pi}{5}$
    (b) $\dfrac{2\pi}{5}$
    (c) $\dfrac{\pi}{5}$
    (d) None of the above

55. What is $\sin^2(3\pi) + \cos^2(4\pi) + \tan^2(5\pi)$ equal to ?
    (a) 0
    (b) 1
    (c) 2
    (d) 3

56. Consider the following points :
    1. $(0, 5)$
    2. $(2, -1)$
    3. $(3, -4)$
    Which of the above lie on the line $3x + y = 5$ and at a distance $\sqrt{10}$ from $(1, 2)$?
    (a) 1 only
    (b) 2 only
    (c) 1 and 2 only
    (d) 1, 2 and 3

57. What is the equation of the line through $(1, 2)$ so that the segment of the line intercepted between the axes is bisected at this point ?
    (a) $2x - y = 4$
    (b) $2x - y + 4 = 0$
    (c) $2x + y = 4$
    (d) $2x + y + 4 = 0$

58. What is the equation of straight line passing through the point $(4, 3)$ and making equal intercepts on the coordinate axes ?
    (a) $x + y = 7$
    (b) $3x + 4y = 7$
    (c) $x - y = 1$
    (d) None of the above

59. What is the equation of the line midway between the lines $3x - 4y + 12 = 0$ and $3x - 4y = 6$?
    (a) $3x - 4y - 9 = 0$
    (b) $3x - 4y + 9 = 0$
    (c) $3x - 4y - 3 = 0$
    (d) $3x - 4y + 3 = 0$

60. What is the sum of the major and minor axes of the ellipse whose eccentricity is 4/5 and length of latus rectum is 14.4 unit ?
    (a) 32 unit
    (b) 48 unit
    (c) 64 unit
    (d) None of the above

(Contd.)

For the next three (03) items that follow :

A straight line passes through (1, –2, 3) and perpendicular to the plane 2x + 3y – z = 7.

61. What are the direction ratios of normal to plane ?
    (a) < 2, 3, –1 >
    (b) < 2, 3, 1 >
    (c) < –1, 2, 3 >
    (d) None of the above

62. Where does the line meet the plane ?
    (a) (2, 3, –1)
    (b) (1, 2, 3)
    (c) (2, 1, 3)
    (d) (3, 1, 2)

63. What is the image of the point (1, –2, 3) in the plane ?
    (a) (2, –1, 5)
    (b) (–1, 2, –3)
    (c) (5, 4, 1)
    (d) None of the above

For the next two (02) items that follow :

Consider the spheres $x^2 + y^2 + z^2 – 4y + 3 = 0$ and $x^2 + y^2 + z^2 + 2x + 4z – 4 = 0$.

64. What is the distance between the centres of the two spheres ?
    (a) 5 unit
    (b) 4 unit
    (c) 3 unit
    (d) 2 unit

65. Consider the following statements :
    1. The two spheres intersect each other.
    2. The radius of first sphere is less than that of second sphere.
    Which of the above statements is/are correct ?
    (a) 1 only
    (b) 2 only
    (c) Both 1 and 2
    (d) Neither 1 nor 2

For the next three (03) items that follow :

The vertices of a triangle ABC are A(2, 3, 1), B(–2, 2, 0) and C(0, 1, –1).

66. What is the cosine of angle ABC ?
    (a) $\dfrac{1}{\sqrt{3}}$
    (b) $\dfrac{1}{\sqrt{2}}$
    (c) $\dfrac{2}{\sqrt{6}}$
    (d) None of the above

67. What is the area of the triangle ?
    (a) $6\sqrt{2}$ square unit
    (b) $3\sqrt{2}$ square unit
    (c) $10\sqrt{3}$ square unit
    (d) None of the above

68. What is the magnitude of the line joining mid points of the sides AC and BC ?
    (a) $\dfrac{1}{\sqrt{2}}$ unit
    (b) 1 unit
    (c) $\dfrac{3}{\sqrt{2}}$ unit
    (d) 2 unit

For the next two (02) items that follow :

Consider the vectors $\vec{a} = \hat{i} – 2\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} – 4\hat{j} + 7\hat{k}$.

69. What is the scalar projection of $\vec{a}$ on $\vec{b}$?
    (a) 1
    (b) 19/9
    (c) 17/9
    (d) 23/9

(Contd.)

70. What is the vector perpendicular to both the vectors ?

    (a) $-10\hat{i} - 3\hat{j} + 4\hat{k}$

    (b) $-10\hat{i} + 3\hat{j} + 4\hat{k}$

    (c) $10\hat{i} - 3\hat{j} + 4\hat{k}$

    (d) None of the above

For the next two (02) items that follow :

Let a vector $\bar{r}$ make angles 60°, 30° with x and y-axes respectively.

71. What angle does $\bar{r}$ make with z-axis ?

    (a) 30°

    (b) 60°

    (c) 90°

    (d) 120°

72. What are the direction cosines of $\bar{r}$?

    (a) $\left\langle \dfrac{1}{2}, \dfrac{\sqrt{3}}{2}, 0 \right\rangle$

    (b) $\left\langle \dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}, 0 \right\rangle$

    (c) $\left\langle \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}, 0 \right\rangle$

    (d) $\left\langle -\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}, 0 \right\rangle$

For the next two (02) items that follow :

Let $|\bar{a}| = 7$, $|\bar{b}| = 11$, $|\bar{a} + \bar{b}| = 10\sqrt{3}$

73. What is $|\bar{a} - \bar{b}|$ equal to ?

    (a) $2\sqrt{2}$

    (b) $2\sqrt{10}$

    (c) 5

    (d) 10

74. What is the angle between $(\bar{a} + \bar{b})$ and $(\bar{a} - \bar{b})$?

    (a) $\dfrac{\pi}{2}$

    (b) $\dfrac{\pi}{3}$

    (c) $\dfrac{\pi}{6}$

    (d) None of the above

75. A line passes through the points $(6, -7, -1)$ and $(2, -3, 1)$. What are the direction ratios of the line ?

    (a) $< 4, -4, 2 >$

    (b) $< 4, 4, 2 >$

    (c) $< -4, 4, 2 >$

    (d) $< 2, 1, 1 >$

76. What is $\lim\limits_{x \to 0} \dfrac{(1+x)^n - 1}{x}$ equal to ?

    (a) 0

    (b) 1

    (c) n

    (d) n − 1

77. What is $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{1 - \cos x}}$ equal to ?

    (a) $\sqrt{2}$

    (b) $-\sqrt{2}$

    (c) $\dfrac{1}{\sqrt{2}}$

    (d) Limit does not exist

78. What is the derivative of $\sqrt{\dfrac{1+\cos x}{1-\cos x}}$?

    (a) $\dfrac{1}{2}\sec^2\dfrac{x}{2}$

    (b) $-\dfrac{1}{2}\text{cosec}^2\dfrac{x}{2}$

    (c) $-\text{cosec}^2\dfrac{x}{2}$

    (d) None of the above

79. What is $\displaystyle\int_0^1 \dfrac{e^{\tan^{-1}x}\,dx}{1+x^2}$ equal to ?

    (a) $e^{\frac{\pi}{4}}-1$

    (b) $e^{\frac{\pi}{4}}+1$

    (c) $e-1$

    (d) e

80. What is the slope of the tangent to the curve $y = \sin^{-1}(\sin^2 x)$ at $x = 0$?

    (a) 0

    (b) 1

    (c) 2

    (d) None of the above

81. The solution of $\dfrac{dy}{dx}=|x|$ is :

    (a) $y=\dfrac{x|x|}{2}+c$

    (b) $y=\dfrac{|x|}{2}+c$

    (c) $y=\dfrac{x^2}{2}+c$

    (d) $y=\dfrac{x^3}{2}+c$

    where c is an arbitrary constant

82. What is the solution of $\dfrac{dy}{dx}+2y=1$ satisfying $y(0) = 0$?

    (a) $y=\dfrac{1-e^{-2x}}{2}$

    (b) $y=\dfrac{1+e^{-2x}}{2}$

    (c) $y = 1 + e^x$

    (d) $y=\dfrac{1+e^x}{2}$

For the next two (02) items that follow :

Consider the curve $y = e^{2x}$.

83. What is the slope of the tangent to the curve at $(0, 1)$?

    (a) 0

    (b) 1

    (c) 2

    (d) 4

84. Where does the tangent to the curve at $(0, 1)$ meet the x-axis ?

    (a) $(1, 0)$

    (b) $(2, 0)$

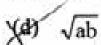   (c) $(-1/2, 0)$

    (d) $(1/2, 0)$

For the next two (02) items that follow :

Consider an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

85. What is the area of the greatest rectangle that can be inscribed in the ellipse ?

    (a) ab

    (b) 2 ab

    (c) ab/2

    (d) $\sqrt{ab}$

(Contd.)

86. What is the area included between the ellipse and the greatest rectangle inscribed in the ellipse ?
    (a) $ab(\pi - 1)$
    (b) $2ab(\pi - 1)$
    (c) $ab(\pi - 2)$
    (d) None of the above

For the next two (02) items that follow :

Consider the integrals

$$I_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}} \quad \text{and} \quad I_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}\, dx}{\sqrt{\sin x} + \sqrt{\cos x}}.$$

87. What is $I_1 - I_2$ equal to ?
    (a) 0
    (b) $2I_1$
    (c) π
    (d) None of the above

88. What is I_1 equal to ?
    (a) $\pi/24$
    (b) $\pi/18$
    (c) $\pi/12$
    (d) $\pi/6$

For the next two (02) items that follow :

Consider the function $f(x) = \dfrac{1 - \sin x}{(\pi - 2x)^2}$

where $x \neq \dfrac{\pi}{2}$ and $f\left(\dfrac{\pi}{2}\right) = \lambda$

89. What is $\displaystyle\lim_{x \to \frac{\pi}{2}} f(x)$ equal to ?
    (a) 1
    (b) 1/2
    (c) 1/4
    (d) 1/8

90. What is the value of λ if the function is continuous at $x = \dfrac{\pi}{2}$?
    (a) 1/8
    (b) 1/4
    (c) 1/2
    (d) 1

91. If $f(9) = 9$ and $f'(9) = 4$ then what is

    $\displaystyle\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ equal to ?

    (a) 36
    (b) 9
    (c) 4
    (d) None of the above

92. What is $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \sin x \, dx$ equal to ?
    (a) 0
    (b) 2
    (c) −2
    (d) π

93. What is the general solution of the differential equation $x \, dy - y \, dx = y^2$?
    (a) $x = cy$
    (b) $y^2 = cx$
    (c) $x + xy - cy = 0$
    (d) None of the above

    where c is an arbitrary constant

94. Consider the following statements :

    1. The function $f(x) = \sqrt[3]{x}$ is continuous at all x except at $x = 0$.
    2. The function $f(x) = [x]$ is continuous at $x = 2.99$ where [.] is the bracket function.

    Which of the above statements is/are correct ?
    (a) 1 only
    (b) 2 only
    (c) Both 1 and 2
    (d) Neither 1 nor 2

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(Contd.)

95. Consider the following statements :
1. The function $f(x) = |x|$ is not differentiable at $x = 1$.
2. The function $f(x) = e^x$ is differentiable at $x = 0$.

Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

96. If $z = f \circ f(x)$ where $f(x) = x^2$, then what is $\dfrac{dz}{dx}$ equal to ?

(a) x^3
(b) $2x^3$
(c) $4x^3$
(d) $4x^2$

For the next two (02) items that follow :

Consider the function $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

97. What is the maximum value of the function ?

(a) 1/2
(b) 1/3
(c) 2
(d) 3

98. What is the minimum value of the function ?

(a) 1/2
(b) 1/3
(c) 2
(d) 3

For the next three (03) items that follow :

Let $f(x)$ be a function defined in $1 \le x < \infty$ by

$$f(x) = \begin{cases} 2 - x & \text{for } 1 \le x \le 2 \\ 3x - x^2 & \text{for } x > 2. \end{cases}$$

99. Consider the following statements :
1. The function is continuous at every point in the interval $[1, \infty)$.
2. The function is differentiable at $x = 1.5$.

Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

100. What is the differentiable coefficient of $f(x)$ at $x = 3$?

(a) 1
(b) 2
(c) −1
(d) −3

101. Consider the following statements :
1. $f'(2 + 0)$ does not exist.
2. $f'(2 - 0)$ does not exist.

Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

102. What is $\displaystyle\int_{0}^{\frac{\pi}{2}} \ln(\tan x)\,dx$ equal to ?

(a) $\ln 2$
(b) $-\ln 2$
(c) 0
(d) None of the above

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(Contd.)

For the next three (03) items that follow :

The general solution of the differential equation $(x^2 + x + 1)dy + (y^2 + y + 1)dx = 0$ is $(x + y + 1) = A(1 + Bx + Cy + Dxy)$ where B, C and D are constants and A is parameter.

103. What is B equal to ?
- (a) −1
- (b) 1
- (c) 2
- (d) None of the above

104. What is C equal to ?
- (a) 1
- (b) −1
- (c) 2
- (d) None of the above

105. What is D equal to ?
- (a) −1
- (b) 1
- (c) −2
- (d) None of the above

106. Consider the following statements :
1. The function $f(x) = \sin x$ decreases on the interval $(0, \pi/2)$.
2. The function $f(x) = \cos x$ increases on the interval $(0, \pi/2)$.

Which of the above statements is/are correct ?
- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

107. What is the number of arbitrary constants in the particular solution of differential equation of third order ?
- (a) 0
- (b) 1
- (c) 2
- (d) 3

108. What is the equation of a curve passing through (0, 1) and whose differential equation is given by $dy = y \tan x \, dx$?
- (a) $y = \cos x$
- (b) $y = \sin x$
- (c) $y = \sec x$
- (d) $y = \csc x$

109. Consider the following statements in respect of the differential equation

$$\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0:$$

1. The degree of the differential equation is not defined.
2. The order of the differential equation is 2.

Which of the above statements is/are correct ?
- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

110. What is the equation of parabola whose vertex is at (0, 0) and focus is at (0, −2) ?
- (a) $y^2 + 8x = 0$
- (b) $y^2 − 8x = 0$
- (c) $x^2 + 8y = 0$
- (d) $x^2 − 8y = 0$

For the next four (04) items that follow :

Number X is randomly selected from the set of odd numbers and Y is randomly selected from the set of even numbers of the set $\{1, 2, 3, 4, 5, 6, 7\}$. Let $Z = (X + Y)$.

111. What is $P(Z = 5)$ equal to ?
- (a) 1/2
- (b) 1/3
- (c) 1/4
- (d) 1/6

112. What is $P(Z = 10)$ equal to ?
- (a) 0
- (b) 1/2
- (c) 1/3
- (d) 1/5

113. What is $P(Z > 11)$ equal to ?
- (a) 0
- (b) 1/4
- (c) 1/6
- (d) 1/12

(Contd.)

114. What is P(Z is the product of two prime numbers) equal to ?

    (a) 0
    (b) 1/2
    (c) 1/4
    (d) None of the above

For the next three (03) items that follow :

The number of telephone calls received in 245 successive one minute intervals at an exchange is given below in the following frequency distribution.

Number of calls	0	1	2	3	4	5	6	7
Frequency	14	21	25	43	51	40	39	12

115. What is the mean of the distribution ?

    (a) 3.76
    (b) 3.84
    (c) 3.96
    (d) 4.05

116. What is the median of the distribution ?

    (a) 3.5
    (b) 4
    (c) 4.5
    (d) 5

117. What is the mode of the distribution ?

    (a) 3
    (b) 4
    (c) 5
    (d) 6

For the next three (03) items that follow :

The mean and standard deviation of 100 items are 50, 5 and that of 150 items are 40, 6 respectively.

118. What is the combined mean of all 250 items ?

    (a) 43
    (b) 44
    (c) 45
    (d) 46

119. What is the combined standard deviation of all 250 items ?

    (a) 7.1
    (b) 7.3
    (c) 7.5
    (d) 7.7

120. What is the variance of all the 250 items ?

    (a) 50.6
    (b) 53.3
    (c) 55.6
    (d) 59.3