II

Total No. of Questions - 24

Total No. of Printed Pages - 4

Regd. No.

Part - III

MATHEMATICS, Paper - I (A)

(Algebra, Vector Algebra and Trigonometry)

(English Version)

Time : 3 Hours ***Max. Marks : 75***

Note : This question paper consists of **three** sections *A*, *B* and *C*.

SECTION A

I. **Very short answer** type questions. **10 × 2 = 20**

i) Answer **all** questions.

ii) Each question carries **two** marks.

1. If the function f is defined by $f(x) = \begin{cases} 3x-2, & x>3 \\ x^2-2, & -2 \le x \le 2 \\ 2x+1, & x<-3 \end{cases}$

then find the values of $f(4)$ and $f(2.5)$.

2. Find the domain of the real valued function $f(x) = \dfrac{1}{(x^2-1)(x+3)}$.

3. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = O$, then find the value of k.

4. If ω is a complex (nonreal) cube root of 1, then show that

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0.$$

5. Find the unit vector in the direction of vector $\overline{a} = 2i + 3j + k$.

6. If the vectors $-3i + 4j + \lambda k$ and $\mu i + 8j + 6k$ are collinear vectors, then find λ and μ.

7. Find the angle between vectors $i + 2j + 3k$ and $3i - j + 2k$.

8. Find the period of the function $Cos\left(\frac{4x+9}{5}\right)$.

9. Find the minimum and maximum values of $3Sin\,x - 4Cos\,x$.

10. If $Sinh\,x = \frac{3}{4}$, find $Cosh(2x)$ and $Sinh(2x)$.

SECTION B

II. Short answer type questions. **5 × 4 = 20**

i) Attempt **any five** questions.

ii) Each question carries **four** marks.

11. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, then show that $A^{-1} = A^T$.

12. If i, j, k are unit vectors along with positive directions of coordinate axes, then show that the four points $4i + 5j + k$, $-j - k$, $3i + 9j + 4k$ and $-4i + 4j + 4k$ are coplanar.

13. Find the area of the triangle whose vertices are $A(1, 2, 3)$, $B(2, 3, 1)$ and $C(3, 1, 2)$.

14. Prove that $\frac{Tan\,\theta + Sec\,\theta - 1}{Tan\,\theta - Sec\,\theta + 1} = \frac{1 + Sin\,\theta}{Cos\,\theta}$.

15. Solve the equation, $Cot^2x-(\sqrt{3}+1)Cot\,x+\sqrt{3}=0;\ 0<x<\frac{\pi}{2}$.

16. Prove that $2Sin^{-1}\left(\frac{3}{5}\right)-Cos^{-1}\left(\frac{5}{13}\right)=Cos^{-1}\left(\frac{323}{325}\right)$.

17. In ΔABC, show that $a^2Cot\,A+b^2Cot\,B+c^2Cot\,C=\frac{abc}{R}$.

SECTION C

III. **Long answer** type questions. **5 × 7 = 35**

i) Attempt **any five** questions.

ii) Each question carries **seven** marks.

18. If $f:A\to B,\ g:B\to C$ are bijective functions, then prove that $(gof)^{-1}=f^{-1}og^{-1}$.

19. Using mathematical induction, prove the statement for all $n\in N$:

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots\ldots\text{ upto } n \text{ terms }=\frac{n}{24}\left[2n^2+9n+13\right].$$

20. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}\neq 0$, then show that $abc=-1$.

21. Solve the following equations by the Gauss-Jordan method. $2x-y+3z=9,\ x+y+z=6,\ x-y+z=2$.

22. If $A=(1,-2,-1),\ B=(4,0,-3),\ C=(1,2,-1)$ and $D=(2,-4,-5)$, then find the distance between AB and CD lines.

23. If A, B, C are the angles in a triangle, then prove that

$$Sin\left(\frac{A}{2}\right)+Sin\left(\frac{B}{2}\right)+Sin\left(\frac{C}{2}\right)=$$

$$1+4\,Sin\left(\frac{\pi-A}{4}\right)Sin\left(\frac{\pi-B}{4}\right)Sin\left(\frac{\pi-C}{4}\right).$$

24. In ΔABC, if $a=13, b=14, c=15$, then show that $R=\frac{65}{8}$, $r=4$,

$r_1=\frac{21}{2}$, $r_2=12$ and $r_3=14$.