## JEE-MAIN Mathematics Model Paper with Solutions

1.If $z$ is a complex number satisfying $z^{4}+z^{3}+2 z^{2}+z+1=0$, then the set of possible values of $|z|$ is
(a) $\{1,2\}$
(b) $\{1\}$
(c) $\{1,2,3\}$
(d) $\{1,2,3,4\}$
2. $\alpha, \beta, \boldsymbol{y}$ are the roots of $x^{3}-3 x^{2}+3 x+7=0$ ( $w$ is cube root of unity) then $\left(\frac{\alpha-1}{\beta-1}+\frac{\beta-1}{\gamma-1}+\frac{\gamma-1}{\alpha-1}\right)$ is
(a) $\frac{3}{\omega}$
(b) $\omega^{2}$
(c) $2 \omega^{2}$
(d) $3 \omega$
3. Let $f:(0, \infty) \rightarrow R$ and $F(x)=\int_{0}^{x} f(t) d t$. If $F\left(x^{2}\right)=x^{2}(1+x)$, then $\mathbf{f}(4)$ equals
a) $\frac{5}{4}$
b) 7
c) 4
d) 2
4.If $x=\sum_{n=0}^{\infty} a^{n}, y=\sum_{n=0}^{\infty} b^{n}, z=\sum_{n=0}^{\infty} c^{n}$ where $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ are in AP and $|a|<1,|b|<1,|c|<1$, then $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ are in (a,b,c are distinct)
a) AP
b) GP
c) HP
d) AGP
5.If number of terms in the expansion of $(x-2 y+3 z)^{n}$ are 45 , then maximum value of ${ }^{n} C_{r}$ is
a) 70
b) 126
C) 35
d) none
6. $\int \mathrm{e}^{x}\left(\tan ^{-1} \mathrm{x}+\frac{2 \mathrm{x}}{\left(1+\mathrm{x}^{2}\right)^{2}}\right) \mathrm{dx}$ is equal to
a) $e^{x}\left(\tan ^{-1} x-\frac{1}{1+x^{2}}\right)+C$
b) $\mathrm{e}^{\mathrm{x}}\left(\tan ^{-1} \mathrm{x}+\frac{1}{1+\mathrm{x}^{2}}\right)+\mathrm{C}$
c) $e^{x}\left(\cot ^{-1} x-\frac{1}{1+x^{2}}\right)+C$
d) $e^{x}\left(\tan ^{-1} x+\frac{2}{1+x^{2}}\right)+C$
7.The number of eight digit numbers (each having distinct digits) that are divisible by 9 is $\qquad$
a) 15 (7!)
b) 20 (7!)
c) 24 (7!)
d) 36 (7!)
8. If the exponents of 5 and 7 in $100_{C_{50}}$ are respectively $x$ and $y$ then
a) $x<y$
b) $x=y$
c) $x>y$
d) $|x-y|=2$
9. 10 identical mangoes are to be distributed among 5 persons. The probability that at least one of them will receive none, is
a) $\frac{35}{143}$
b) $\frac{108}{143}$
C) $\frac{18}{143}$
d) $\frac{125}{143}$
10. If $\left(1+\tan 1^{0}\right)\left(1+\tan 2^{0}\right)\left(1+\tan 3^{\circ}\right) \ldots \ldots . .\left(1+\tan 45^{\circ}\right)=2^{n}$ then $n=$
a) 20
b) 21
c) 22
d) 23
11. Let $a, b$ and $c$ be real numbers such that $a+2 b+c=4$ then the maximum value of $a b+b c+c a$ is
a) 1
b) 2
c) 3
d) 4
12.The solution of $\frac{d v}{d t}+\frac{k}{m} v=-g$ is
a) $v=c e^{-\frac{k}{m} t}-\frac{m g}{k}$
b) $v=c-\frac{m g}{k} e^{-\frac{k}{m} t}$
c) $v e^{-\frac{k}{m} t}=c-\frac{m g}{k}$
d) $v e^{\frac{k}{m_{t}}}=c-\frac{m g}{k} \frac{d v}{d t}+\frac{K}{m} v=-g$
13.If $t_{n}=\frac{1}{4}(n+2)(n+3)$ for $n=1,2,3, \ldots$ then $\frac{1}{t_{1}}+\frac{1}{t_{2}}+\frac{1}{t_{3}}+\ldots+\frac{1}{t_{2003}}=$
a. $\frac{4006}{3006}$
b. $\frac{4003}{3007}$
c. $\frac{4006}{3008}$
d $\frac{4006}{3009}$
14.The lines $\frac{x+4}{3}=\frac{y+6}{5}=\frac{z-1}{-2}$ and $3 x-2 y+z+5=0=2 x+3 y+4 z-k$ are coplanar for $\mathbf{k}=$
a). 4
b). 0
c). 6
d). 5
15. Rolle's theorem holds in [1, 2] for the function $f(x)=x^{3}+b x^{2}+c x$ at the point $\frac{4}{3}$. The values of $b$, $c$ are respectively
a) $-5,-8$
b) $-5,8$
c) $5,-8$
d) $8,-5$
16.If $A=\{1,2,3,4,5\}$, and relation $\mathbf{R}$ on $\mathbf{A}$ is defined by $R=\left\{(x, y) / x<y\right.$ and $\left.\left|x^{2}-y^{2}\right|<9\right\}$

## then $\mathbf{R}$

a. $\{(1,1),(2,2),(3,3)(4,4),(5,5)\}$
b. $\{(2,1),(3,2),(3,2)(4,3),(5,4)\}$
c. $\{(1,2),(1,3),(2,3)(3,4),(4,5)\}$
d. $\{(1,2),(1,3),(2,3)(3,4)\}$
17. $p \wedge(q \wedge r)$ is logically equivalent to
a) $p \vee(q \wedge r)$
b). $(p \wedge q) \wedge r$
c). $(p \wedge q) \vee r$
d). $p \rightarrow(q \wedge r)$
18. $\lim _{n \rightarrow \infty} \sum_{r=1}^{n} \cot ^{-1}\left(r^{2}+r+1\right)=$
a) $\frac{\pi}{4}$
b) $\frac{\pi}{2}$
c) $\pi$
d) 0
19. A function $f$ from the set of integers $\mathbb{Z}$ to $\mathbb{Z}$ is defined as follows:
$f(n)= \begin{cases}n+3 & \text { if } n \text { is odd } \\ \frac{n}{2} & \text { if } n \text { is even }\end{cases}$
Suppose $\mathbf{k}$ is odd and $f(f(f(k)))=27$. Then the sum of the digits of $\mathbf{k}$ is
a) 3
b) 6
c) 9
d) 12
20. If $\mathbf{y}=\mathbf{x}+\mathbf{2}$ is a normal to the parabola $y^{2}=4 a x$, then $\mathbf{a}=$
a) $\frac{1}{3}$
b) $-\frac{1}{2}$
c) $-\frac{1}{3}$
d) 2
21.The curve $x y=c(c>0)$ and the circle $x^{2}+y^{2}=1$ touch at two points, then distance between the points of contacts is
a) 1
b) 2
c) $2 \sqrt{2}$
d) $\sqrt{2}$
22. Statement I: Asymptotes of a hyperbola, $3 x+4 y=2$ and $4 x-3 y=5$ are bisectors of transverse and conjugate axes of hyperbola.

Statement II: Transverse and conjugate axes of hyperbola are bisectors of asymptotes.
a) Both statements are true and statement II is the correct explanation of statement I.
b) Both statements are true but statement II is not correct explanation of statement I.
c) Statement I is true, statement II is false
d) Statement I is false, statement II is true
23. In $\triangle A B C$ orthocentre is $(6,10)$ circumcentre is $(2,3)$ and equation of side $\overleftrightarrow{B C}$ is $2 \mathrm{x}+\mathrm{y}=17$. Then the radius of the circumcircle of $\triangle A B C$ is
a. 4
b. 5
C. 2
d. 3
24. A circle of radius ' $r$ ' passes through origin $O$ and cuts the co-ordinate axes at $A$ and $B$.

The locus of the foot of the perpendicular from $O$ to $A B$ is
a) $\left(x^{2}+y^{2}\right)=r^{2}$
b) $\left(x^{2}+y^{2}\right)^{2} \cdot\left(\frac{1}{x^{2}}+\frac{1}{y^{2}}\right)=2 r^{2}$
C) $\left(x^{2}+y^{2}\right)^{2} \cdot\left(\frac{1}{x^{2}}+\frac{1}{y^{2}}\right)=4 r^{2}$
d) $\frac{1}{x^{2}}+\frac{1}{y^{2}}=\frac{4}{r^{2}}$
25. Let $D_{r}=\left|\begin{array}{ccc}a & 2^{r} & 2^{16}-1 \\ b & 3\left(4^{r}\right) & 2\left(4^{16}-1\right) \\ c & 7\left(8^{r}\right) & 4\left(8^{16}-1\right)\end{array}\right|$ then the value of $\sum_{r=1}^{16} D_{r}$ is
a). 0
b). $a+b+c$
C). $a b+b c+c a$
d). $a b c$
26. The area enclosed between the curve the $y=\log _{e}(x+e)$ and the coordinate axes is
a) 3
b) 4
c) 1
d) 2

27 The population $p(t)$ at a time $t$ of a certain mouse species satisfies the differential equation $\frac{d}{d t} p(t)=0.5 p(t)-450$. If $\mathbf{p}(\mathbf{0})=\mathbf{8 5 0}$, then the time at which the population becomes zero is
a) $2 \ln 18$
b) $\ln 9$
c) $\frac{1}{2} \ln 18$
d) $\ln 18$
28. Let $\overline{\mathrm{a}}=2 \overline{\mathrm{i}}+\overline{\mathrm{j}}-2 \overline{\mathrm{k}}, \overline{\mathrm{b}}=\overline{\mathrm{i}}+\overline{\mathrm{j}}$. If $\overline{\mathrm{c}}$ is a vector such that $\overline{\mathrm{a}} \cdot \overline{\mathrm{c}}=|\overline{\mathrm{c}}|,|\overline{\mathrm{c}}-\overline{\mathrm{a}}|=2 \sqrt{2}$ and the angle between $\overline{\mathrm{a}} \times \overline{\mathrm{b}}$ and $\overline{\mathrm{c}}$ is $30^{\circ}$ then $|(\overline{\mathrm{a}} \times \overline{\mathrm{b}}) \times \overline{\mathrm{c}}|=$
a) $\frac{1}{2}$
b) $\frac{3 \sqrt{3}}{2}$
c) 3
d) $3 / 2$
29. $f(x)=\left\{\begin{array}{cl}x^{2}\left(\frac{e^{1 / x}-e^{-1 / x}}{e^{1 / x}+e^{-1 / x}}\right), & x \neq 0 \\ 0, & x=0\end{array}\right.$. Then
a) $f(x)$ is discontinuous at $x=0$
b) $f(x)$ is continuous but non-differentiable at $x=0$
c) $f(x)$ is differntiable at $x=0$
d) $f^{\prime}(0)=2$
30. $\mathbf{N}$ is the set of all natural numbers and $\mathbf{R}$ is a relation on $N \times N$ defined by $(a, b) R(c, d)$ if and only if $a+d=b+c$. Then $\mathbf{R}$ is
a). only reflexive
b). Only symmetric
c). Only transitive
d). Equivalence relation

## Solutions

1.Sol. b

The given equation is $\left(z^{2}+z+1\right)\left(z^{2}+1\right)=0$.
$z= \pm i, w, w^{2}, w$ being an imaginary cube root of unity. Thus $|z|=1$.
2.Sol: a

We have $x^{3}-3 x^{2}+3 x+7=0$

$$
\begin{aligned}
& \Rightarrow \quad(\mathrm{x}-1)^{3}+8=0 \\
& \Rightarrow \quad\left(\frac{(\mathrm{x}-1)}{-2}\right)^{3}=1 \\
& \Rightarrow \quad\left(\frac{\mathrm{x}-1}{-2}\right)=1, \omega, \omega^{2} \\
& \Rightarrow \quad \mathrm{x}=-1 ; 1-2 \omega ; 1-2 \omega^{2} \\
& \therefore \quad \alpha=-1, \beta=1-2 \omega ; \gamma=1-2 \omega^{2} \\
& \therefore \quad \text { required expression }=3 \omega^{2} .
\end{aligned}
$$

3.Sol: ans-c

Since,

$$
P\left(x^{2}\right)=x^{2}(1+x)
$$

$$
\therefore \quad \int_{0}^{2} f(t) d t=x^{2}(1+x)
$$

$\therefore \quad$ Differentiating both sides, we get

$$
\begin{array}{ll}
\therefore & 2 f\left(x^{2}\right)=2 x+3 x^{2} \\
\therefore & \Rightarrow \\
\therefore & f\left(x^{2}\right)=1+\frac{3 x}{2} \\
& \Rightarrow
\end{array} \quad f\left(2^{2}\right)=1+\frac{3}{2}(2)=4 \Rightarrow f(4)=4
$$

4.Sol;ans(c)

$$
\therefore \quad \text { Here, } \quad x=\frac{1}{1-a}, y=\frac{1}{1-b}, z=\frac{1}{1-c} \Rightarrow 1-a=\frac{1}{x}, 1-b=\frac{1}{y}, 1-c=\frac{1}{z}
$$

$$
\begin{array}{ll}
\therefore & \Rightarrow a=1-\frac{1}{x}, b=1-\frac{1}{y}, c=1-\frac{1}{z} \\
\therefore & \text { Since, } a, b, c \text { are in AP. } \\
\therefore & \Rightarrow 1-\frac{1}{x}, 1-\frac{1}{y}, 1-\frac{1}{z} \text { are in AP. } \\
\therefore & x, y, z \text { are in HP. }
\end{array}
$$

5.Sol; ans.(a)

$$
\begin{array}{llrl}
\therefore & & & \\
& & & \\
& \Rightarrow & (n+3-1 & C_{3-1}={ }^{n+2} C_{2}=45 \\
& & \Rightarrow & \left.n^{2}+3 n-1\right)=90 \\
& & \Rightarrow & \\
& & & n=-11,8 \\
& \therefore & & \max .{ }^{n} C_{r}= \\
& & =\text { max. }{ }^{8} C_{r}=70
\end{array}
$$

6.Sol:.ans-a

$$
\begin{aligned}
& \int \mathrm{e}^{\mathrm{x}}\left(\tan ^{-1} \mathrm{x}+\frac{1}{1+\mathrm{x}^{2}}+\frac{-1}{1+\mathrm{x}^{2}}+\frac{2 \mathrm{x}}{\left(1+\mathrm{x}^{2}\right)^{2}}\right) \mathrm{dx} \\
& =\mathrm{e}^{\mathrm{x}}\left(\tan ^{-1} \mathrm{x}-\frac{1}{1+\mathrm{x}^{2}}\right)+\mathrm{C}
\end{aligned}
$$

7.sol; ans-d $0,1,2,3 \ldots \ldots . .9$

## Sum $=36$

Let have to reject two digits $(0,9),(1,8),(2,7),(3,6)(4,5)$

$$
=8!+4(8!-7!)=36(7!)
$$

8.Sol:.ans-b

$$
\begin{array}{ll}
E_{5}(100!)=20+4=24 \quad E_{7}(100!)=14+2=16 \\
E_{5}(50!)=10+2=12 & E_{7}(50!)=7+1=8 \\
E_{5}(N)=24-2(12)=0 & E_{5}(N)=16-2(8)=0
\end{array}
$$

\ $x=y$
9.Sol: ans d 10 mangoes can be distributed among 5 persons in ${ }^{10+5-1} C_{5-1}={ }^{14} C_{4}$ ways.
$\therefore$ Total number of elementary events $={ }^{14} C_{4}$
Required probability $=1$-probabilty that each person receives at least one mangoe

$$
=1-\frac{{ }^{10-1} C_{5-1}}{{ }^{14} C_{4}}=1-\frac{{ }^{9} C_{4}}{{ }^{14} C_{4}}=1-\frac{18}{143}=\frac{125}{143}
$$

10. sol: ans-d $A+B=45^{\circ}$

$$
\Rightarrow(1+\tan A)(1+\tan B)=2
$$

11.sol: ans-d
$\therefore \quad$ Let $a b+b c+c a=x$
$\therefore \quad \Rightarrow 2 b^{2}+2(c-2) b-4 c+c^{2}+x=0$
$\therefore \quad$ Since $b \in R$,
$\therefore c^{2}-4 c+2 x-4 \leq 0$
$\therefore \quad$ Since $c \in R$
$\therefore x \leq 4$
12.Sol:ans a
$\therefore \quad$ Integrating factor (I.F. $)=e^{\int \frac{k}{m} d t}=e^{\frac{K}{m} t}$

$$
\begin{array}{ll}
\therefore & \therefore V e^{\frac{K}{m} t}=-\int g e^{K . t / m}+c \\
\therefore & V e^{\frac{K}{m} t}=\frac{-g m}{K} e^{\frac{K}{m} t}+c \\
\therefore & V=C \cdot e^{\frac{-K}{m} t}-\frac{m g}{K}
\end{array}
$$

13.Sol: ans-d

$$
\begin{aligned}
& \frac{1}{t_{1}}+\frac{1}{t_{2}}+\ldots+\frac{1}{t_{2003}}=4\left[\frac{1}{3.4}+\frac{1}{4.5}+\ldots+\frac{1}{2005 \times 2006}\right] \\
= & 4\left[\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\ldots+\frac{1}{2005}-\frac{1}{2006}\right]=4\left[\frac{1}{3}-\frac{1}{2006}\right]=\frac{4 \times 2003}{3 \times 2006}=\frac{4006}{3009}
\end{aligned}
$$

14.Sol: ans-a

A point on the first line be A $(-4+3 \lambda,-6+5 \lambda, 1-2 \lambda)$
$\therefore \quad$ If the lines are coplanar A lies on $3 x-2 y+z+5=0$ and $2 x+3 y+4 z-k=0$
$\therefore \quad 3(-4+3 \lambda)-2(-6+5 \lambda)+(1-2 \lambda)+5=0 \Rightarrow-3 \lambda+6=0 \Rightarrow \lambda=2$
$\therefore \quad 2(-4+3 \lambda)+3(-6+5 \lambda)+4(1-2 \lambda)-k=0$
$\therefore \quad \Rightarrow 13 \lambda-22=K \Rightarrow K=26-22=4$
15.sol: ans-b
$\therefore \quad$ Rolle's theorem holds in $[1,2]$ for $f(x)$ at
$\therefore \quad x=\frac{4}{3} \quad f(1)=f(2) \& f^{\prime}\left(\frac{4}{3}\right)=0 \Rightarrow 3 b+c=-7 \quad$ and
$\therefore \quad 8 b+3 c=-16 \Rightarrow b=-5, c=8$
16.Sol: ans- d

$$
\begin{array}{ll} 
& \text { If } A=\{1,2,3,4,5\} \\
\therefore & R=\left\{(x, y) / x<y \text { and }\left|x^{2}-y^{2}\right|<9\right\} \text { then } R=\{(1,2),(1,3)(2,3)(3,4)\}
\end{array}
$$

satisfies the given condition
17.Sol;ans-b

| $q$ | r | $q \wedge r$ | p | $p \wedge(q \wedge r)$ | $p \wedge q$ | $(p \wedge q) \wedge r$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | T | T | T | T | T |
| T | F | F | F | F | F | F |
| F | T | F | T | F | F | F |
| F | F | F | F | F | F | F |

18.Sol: ans-a
$\sum_{r=1}^{n} \operatorname{Tan}^{-1}(r+1)-\operatorname{Tan}^{-1} r=\operatorname{Tan}^{-1}(n+1)-\operatorname{Tan}^{-1} 1 \rightarrow \frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$ as $n \rightarrow \infty$
19.Sol: ans-b.
$f(f(k))=f(k+3)=\frac{k+3}{2}$ and so, $f\left(\frac{k+3}{2}\right)=27$. If $\frac{k+3}{2}$ is odd, then $\frac{k+3}{2}+3=27$ gives $\mathrm{k}=45$
$\therefore \quad$ Clearly $k=45 \Rightarrow \frac{k+3}{2}=24$ is even. So $\frac{k+3}{2}$ is even and $f\left(\frac{k+3}{2}\right)=\frac{k+3}{4}=27$ gives $k=105$
20.Sol:ans-c.
$y=m x-2 a m-a^{3}$

$$
\begin{array}{ll}
\therefore & y=x+2 \\
\therefore \quad m=1 \text { and }-2 a m-\mathrm{am}^{3}=2 \text { р a }=-\frac{1}{3}
\end{array}
$$

21.Sol: ans-b The distance between points of a contact $=$ diameter of circle $=2$.
22.sol: ans-a Statement 2 is true by theory. Since asymptotes are perpendicular statement 1 is also true. But not correct explanation.
23.Sol: ans=b
$\therefore \quad$ Image of orthocenter of $\triangle A B C$ w.r.t. $\stackrel{\rightharpoonup}{\boldsymbol{B C}}$ lies on the circle.
$\therefore$
24. Sol:ans-c .

$$
\begin{equation*}
a^{2}+b^{2}=4 r^{2} \tag{1}
\end{equation*}
$$



AB equations $\frac{x}{a}+\frac{y}{b}=1$
Let $\mathrm{P}(\mathrm{h}, \mathrm{k})$ be then foot of perpendicular from $(0,0)$ to $\overline{A B}$,
$\frac{h-0}{b}=\frac{k-0}{a}=\frac{a b}{a^{2}+b^{2}} \Rightarrow h=\frac{a b^{2}}{a^{2}+b^{2}}, k=\frac{a^{2} b}{a^{2}+b^{2}}$
Consider $\left(h^{2}+k^{2}\right)^{2} \cdot\left(\frac{1}{h^{2}}+\frac{1}{k^{2}}\right)=a^{2}+b^{2}=4 r^{2}$, Eliminate a, b gives
$\therefore$ locus of P is $\left(x^{2}+y^{2}\right)^{2} \cdot\left(\frac{1}{x^{2}}+\frac{1}{y^{2}}\right)=4 r^{2}$.
25.Sol: ans-a

$$
\begin{array}{rlrll}
a & \sum_{r=1}^{16}\left(2^{r}\right) & 2^{16}-1 & & \\
\sum_{r=1}^{16} D_{r}==1 b & 3 \sum\left(3^{r}\right) & 2\left(4^{16}-1\right)|=2| b & 2\left(4^{16}-1\right) & 2\left(4^{16}-1\right) \mid=2(0)=0 \\
c & 4 \sum\left(8^{r}\right) & 4\left(8^{16}-1\right)
\end{array}
$$

26.Sol: ans-c

$$
\begin{aligned}
& \text { Area }=\int_{1-e}^{0} \log (x+e) d x=x \log x+e-\int \frac{x}{x+e} d x \\
& =x \log (x+e)-x+e \log (x+e)]_{1-e}^{0} \\
& =1
\end{aligned}
$$

27.Sol: ans-a.

Given differential equation is linear in $t$

$$
\begin{aligned}
& \therefore I . F=e^{\int-(0.5) d t} e^{-0.5 t} \\
& P(t) \cdot e^{-0.5 t}=\int(-450) e^{-0.5 t} d t \\
& =450 \frac{e^{-0.5 t}}{-0.5}+C \\
& =900 e^{-0.5 t}+C
\end{aligned}
$$

$$
\begin{aligned}
& P(0)=850 \Rightarrow 850=900+C \\
& \Rightarrow C=-50 \\
& \therefore P(t)=900-50 e^{-0.5 t} \\
& \text { If } P(t)=0 \text { then } 50 e^{-0.5 t}=900 \\
& 0.5 t=\log _{e}^{18} \\
& t=2 \log _{e}^{18}
\end{aligned}
$$

28.Sol; ans-d.

$$
\begin{aligned}
& |\overline{\mathrm{c}}-\overline{\mathrm{a}}|^{2}=2 \sqrt{2} \Rightarrow|\overline{\mathrm{c}}|^{2}+|\overline{\mathrm{a}}|^{2}-2 \overline{\mathrm{a}} \cdot \overline{\mathrm{c}}=8 \\
& \Rightarrow|\overline{\mathrm{c}}|^{2}+9-2|\overline{\mathrm{c}}|=8 \Rightarrow(|\overline{\mathrm{c}}|-1)^{2}=0 \quad \therefore|\overline{\mathrm{c}}|=1 \\
& \overline{\mathrm{a}} \times \overline{\mathrm{b}}=2 \overline{\mathrm{i}}-2 \overline{\mathrm{j}}+\overline{\mathrm{k}} \Rightarrow|\overline{\mathrm{a}} \times \overline{\mathrm{b}}|=3 \\
& |(\overline{\mathrm{a}} \times \overline{\mathrm{b}}) \times \overline{\mathrm{c}}|=3 \times 1 \times \frac{1}{2}=3 / 2
\end{aligned}
$$

29.sol: ans-c

At $x=0$,
L.H.L. $=\lim _{x \rightarrow 0-} f(x)=\lim _{x \rightarrow 0+} f(0-h) \quad=\lim _{h \rightarrow 0} h^{2}\left(\frac{e^{-1 / h}-e^{1 / h}}{e^{-1 / h}+e^{1 / h}}\right) 0\left(\frac{0-1}{0+1}\right)=0$
$=$ R.H.L $=\lim _{x \rightarrow 0+} f(x)=\lim _{h \rightarrow 0} f(0+h) \quad=\lim _{h \rightarrow 0} h^{2}\left(\frac{e^{1 / h}-e^{-1 / h}}{e^{1 / h}+e^{-1 / h}}\right)=0\left(\frac{1-0}{1+0}\right)=0$
And $\mathrm{f}(0)=0$
$\Rightarrow$ L.H.L $=$ R.H. $L=f(0)$ Hence, $\mathrm{f}(\mathrm{x})$ is continuous at $\mathrm{x}=0$.
.30. solans-d
$(a, b) R(c, d) \Rightarrow a+d=b+c$
For all $(a, b) \in N \times N(a, b) R(a, b) \quad \because a+b=b+a$
It is reflexive
If $(a, b) R(c, d) \Rightarrow a+d=b+c$ then $(c, d) R(a, b) \Rightarrow c+b=d+a$
It is symmetric
Also $(a, b) R(c, d) \quad(c, d) R(e, f)$
then $(a, b) R(e, f)$
It is equivalence relation

## JEE MAIN Physics Model Paper Key With Solutions

1. A particle moves in the $x-y$ plane with velocity $\vec{v}=a \hat{i}+b \hat{j}$. At the instant $t=\frac{a \sqrt{3}}{b}$ the magnitude of tangential, normal and total accelerations are
a) $\frac{\sqrt{3}}{2} b, \frac{b}{2}$ and b Respectively
b) $\frac{b}{2}, \frac{b}{2}$ and b Respectively
c) $\sqrt{2} b, \sqrt{2} b$ and $b$ Respectively
d) 2b, 2b and 2b Respectively
2. A solid sphere of uniform density and radius $R$ applies a gravitational force of attraction equal to $F_{1}$ on a particle placed at a distance 3R from the center of the sphere. A spherical cavity of radius $R / 2$ is now made in the sphere as shown in the
 figure. The sphere with cavity now applies a gravitational force $F_{2}$ on the same particle. The ratio $F_{2} / F_{1}$ is
a) $\frac{9}{50}$
b) $\frac{41}{50}$
c) $\frac{3}{25}$
d) $\frac{22}{25}$
3. In the figure shown, match the following :

## List-I

## List-II

A) $\mathrm{x}=2 \mathrm{~h}, \mathrm{y}=\mathrm{h}$
B) $\mathrm{x}=\mathrm{h}, \mathrm{y}=3 \mathrm{~h}$
C) $\mathrm{x}=3 \mathrm{~h}, \mathrm{y}=\mathrm{h}$
P) $R=2 \sqrt{3} h$
Q) $R=2 \sqrt{2} h$
R) $R=2 h$
a) A-Q, B-P, C-P
b) A-Q, B-R, C-P
c) A-R, B-Q, C-P
d) A-Q, B-P, C-Q
4. Consider a usual set-up of Young's double slit experiment with slits of equal intensity as shown in figure. Take $O$ as origin and the $Y$ axis as indicated. If average intensity between $y_{1}=-\frac{\lambda D}{4 d}$ and $y_{2}=+\frac{\lambda D}{4 d}$ equal $n$ times the intensity
 of maxima, then n equals (take average over phase difference)
a) $\frac{1}{2}\left(1+\frac{2}{\pi}\right)$
b) $2\left(1+\frac{2}{\pi}\right)$
c) $\left(1+\frac{2}{\pi}\right)$
d) $\frac{1}{2}\left(1-\frac{2}{\pi}\right)$
5. In the potentiometer arrangement shown in figure, null point is obtained at length $l$. Match the following.

## List-I

A) If $E_{1}$ is increased
B) If $R$ is increased
C) If $E_{2}$ is increased

## List-II

P) $l$ should increase
Q) $l$ should decrease
R) $l$ should remain the same to again get the null point
a) A-Q, B-R, C-P
b) A-Q, B-Q, C-P
c) A-R, B-P, C-Q
d) A-Q, B-P, C-P
6. Three concentric conducting spherical shells have radii $\mathrm{r}, 2 \mathrm{r}$ and 3 r and charges $\mathbf{q}_{1}, \mathbf{q}_{2}$ and $\mathbf{q}_{3}$ respectively. Innermost and outermost shells are earthed as shown in figure. select the correct alternative
a) $q_{1}+q_{3}=-q_{2}$
b) $\mathrm{q}_{1}=-\frac{\mathrm{q}_{2}}{4}$
c) $\frac{q_{3}}{q_{1}}=3$
d) All are correct
7. In an experiment refractive index of glass was observed to be $1.45,1.56,1.54,1.44,1.54$ and 1.53 . The mean absolute error in the experiment is
a) $\pm 0.04$
b) 0.02
c) -0.03
d) $\pm 0.01$
8. A smooth square platform ABCD is moving towards right with a uniform speed $\mathbf{v}$. At what angle 0 must a particle be projected from $A$ with speed $u$ so that it strikes the point $B$
a) $\sin ^{-1}\left(\frac{u}{v}\right)$
*b) $\cos ^{-1}\left(\frac{\mathrm{v}}{\mathrm{u}}\right)$
c) $\cos ^{-1}\left(\frac{u}{v}\right)$
d) $\sin ^{-1}\left(\frac{v}{u}\right)$

9. Two blocks of mass 4 kg and 2 kg are connected by a heavy string and placed on rough horizontal plane. The 2 kg block is pulled with a constant force $F$. The coefficient of friction between the blocks and the ground is 0.5 . What is the value of $F$
 so that tension in the string is constant throughout during the motion of the blocks : $\left(\mathrm{g}=\mathbf{1 0} \mathbf{~ m} / \mathrm{s}^{\mathbf{2}}\right)$
a) 40 N
b) 30 N
c) 50 N
d) 60 N
10. The potential energy of a particle of mass $m$ is given by $U=\frac{1}{2} k x^{2}$ for $\mathbf{x}<\mathbf{0}$ and $\mathbf{U}=\mathbf{0}$ for $x \geq 0$. If total mechanical energy of the particle is $E$. Then its speed at $x=\sqrt{\frac{2 E}{k}}$ is
a) Zero
b) $\sqrt{\frac{2 \mathrm{E}}{\mathrm{m}}}$
c) $\sqrt{\frac{E}{m}}$
d) $\sqrt{\frac{E}{2 m}}$
11. A force $F$ is applied at the top of a ring of mass $M$ and radius $R$ placed on a rough horizontal surface as shown in figure. Friction is sufficient to prevent slipping. The friction force acting on the ring is

a) $\frac{\mathrm{F}}{2}$ Towards right
b) $\frac{F}{3}$ towards left
c) $\frac{2 \mathrm{~F}}{3}$ towards right
d) zero
12. A simple pendulum has time period $T_{1}$. The point of suspension is now moved upward according to the relation $y=K t^{2},\left(K=1 \mathrm{~m} / \mathbf{s}^{2}\right)$ where $y$ is the vertical displacement. The time period now becomes $T_{2}$. The ratio of $\frac{T_{1}^{2}}{T_{2}^{2}}$ is $\left(g=10 \mathrm{~m} / \mathrm{s}^{2}\right)$
a) $6 / 5$
b) $5 / 6$
c) 1
d) $4 / 5$
13. Statement 1: Air is more elastic than iron.

Statement 2: Elasticity is directly proportional to compressibility and air is more compressible than iron.
a) If both Assertion and Reason are true and reason is correct explanation of Assertion.
b) If both Assertion and Reason are true but reason is not the correct explanation of Assertion.
c) If Assertion is true but reason is false.
d) If Assertion is false but reason is true
14. An object of specific gravity $\boldsymbol{p}$ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz . The object is immersed in water, so that one half of its volume is submerged. The new fundamental frequency (in Hz ) is
a) $300\left(\frac{2 \rho-1}{2 \rho}\right)^{1 / 2}$
b) $300\left(\frac{2 \rho}{2 \rho-1}\right)^{1 / 2}$
c) $300\left(\frac{2 \rho}{2 \rho-1}\right)$
d) $300\left(\frac{2 \rho-1}{2 \rho}\right)$
15. Regarding speed of sound in gas match the following

## List-I

## List-II

A) Temperature of gas is made 4 times and pressure 2 times
P) speed becomes $2 \sqrt{2}$ times
B) Only pressure is made 4 times without change in
Q) speed becomes 2 times temperature
C) Only temperature is changed to 4 times
R) speed remains unchanged
D) Molecular mass of the gas is made 4 times
S) speed remains half
a) A-Q, B-R, C-P, D-S
b) A-Q, B-R, C-S, D-P
c) A-Q, B-R, C-Q, D-S
d) A-Q, B-P, C-Q, D-S

## Passage: [Following 2 questions]

Suppose $A_{i}$ be the amplitude of incident wave, $A_{r}$ of reflected wave and $A_{t}$ of transmitted wave then


$$
A_{r}=\frac{v_{2}-v_{1}}{v_{1}+v_{2}} A_{i} A_{t}=\frac{2 v_{2}}{v_{1}+v_{2}} A_{i}
$$

Power of a traveling wave is given by $P=\frac{1}{2} \rho \omega^{2} A^{2}$ sv and intensity $I=\frac{1}{2} \rho \omega^{2} A^{2} v$.
16. If $P_{i}, P_{r}$ and $P_{t}$ are powers of incident, reflected and transmitted waves and $I_{i}, I_{r}$ and $I_{t}$ the corresponding intensities, then
a) $P_{i}=P_{r}+P_{t}$
b) $I_{i}=I_{r}+I_{t}$
c) Both (a) and (b) are correct
d) Both (a) and (b) are wrong
17. Under what conditions $75 \%$ of incident energy is transmitted
a) $\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}=\frac{1}{2}$
*b) $\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}=\frac{1}{3}$
c) $\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}=\frac{1}{4}$
d) $\frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}=\frac{2}{3}$
18. One mole of a monoatomic ideal gas undergoes the process $A \rightarrow B$ in the given P-V diagram. The specific heat for this process is
a) $\frac{3 R}{2}$
b) $\frac{13 R}{6}$
c) $\frac{5 R}{2}$
d) $2 R$

19. Consider the two insulating sheets with thermal resistances $R_{1}$ and $R_{2}$ as shown in figure. The temperature $\boldsymbol{\theta}$ is
a) $\frac{\theta_{1} \theta_{2} R_{1} R_{2}}{\left(\theta_{1}+\theta_{2}\right)\left(R_{1}+R_{2}\right)}$
b) $\frac{\theta_{1} R_{1}+\theta_{2} R_{2}}{R_{1}+R_{2}}$
c) $\frac{\left(\theta_{1}+\theta_{2}\right) R_{1} R_{2}}{R_{1}^{2}+R_{2}^{2}}$
d) $\frac{\theta_{1} R_{2}+\theta_{2} R_{1}}{R_{1}+R_{2}}$
20. A horizontal ray of light passes through a prism of $\boldsymbol{\mu}=1.5$ whose apex angle is $\boldsymbol{4}^{\circ}$ and then strikes a vertical mirror $M$ as shown. For the ray after reflection to become horizontal, the mirror must be rotated through an angle of

a) $2^{\circ}$
b) $3^{\circ}$
c) $4^{\circ}$
d) $1^{\circ}$
21. A cell develops the same power across two resistances $R_{1}$ and $\mathbf{R}_{2}$ separately. The internal resistance of the cell is
a) $R_{1}+R_{2}$
b) $\frac{R_{1}+R_{2}}{2}$
c) $\sqrt{\mathrm{R}_{1} \mathrm{R}_{2}}$
d) $\frac{\sqrt{R_{1} R_{2}}}{2}$
22. A conducting wire bent in the form of a parabola $y^{2}=2 x$ carries a current $i=2 \mathrm{~A}$ as shown in figure. This wire is placed in a uniform magnetic field $\vec{B}=-4 \hat{k}$ tesla. The magnetic force on the wire is (in newton)

a) $-16 \hat{\mathrm{i}}$
b) $32 \hat{i}$
c) $-32 \hat{i}$
d) $16 \hat{\mathrm{i}}$
23. Figure shows a square current carrying loop ABCD of side 2 m and current $i=1 / 2 A$. The magnetic moment $\vec{M}$ of the loop is
a) $(\hat{i}-\sqrt{3} \hat{k}) A-m^{2}$
b) $(\hat{j}-\hat{k}) A-m^{2}$
c) $(\sqrt{3} \hat{i}+\hat{k}) A-m^{2}$
d) $(\hat{i}+\hat{k}) A-m^{2}$

24. Some magnetic flux is changed from a coil of resistance $10 \Omega$ As a result an induced current is developed in it, which varies with time shown in figure. the magnitude of change in flux through the coil webers is

a) 2
b) 4
c) 6
d) 8
25. Two concentric and coplanar circuit coils have radii ' $a$ ' and ' $b$ ' ( $\gg$ a) as shown in figure. resistance of the inner coil is $R$. Current in the outer coil is increased from 0 to $i$, then the total charge circulating the inner coil is

a) $\frac{\mu_{0} \mathrm{ia}^{2}}{2 R b}$
b) $\frac{\mu_{0} \mathrm{iab}}{2 R}$
c) $\frac{\mu_{0} i}{2 a} \frac{\pi b^{2}}{R}$
d) $\frac{\mu_{0} \mathrm{ib}}{2 \pi R}$
26. Assertion: Reactance offered by an Inductor increases with the frequency of the AC source. Reason: The current leads the potential in a purely inductive network by $\pi / 2$.
a) If both Assertion and Reason are true and reason is correct explanation of Assertion.
b) If both Assertion and Reason are true but reason is not the correct explanation of Assertion.
*c) If Assertion is true but reason is false.
d) If assertion is false and reason is true.
27. A radioactive substance $X$ decays into another radioactive substance $Y$. Initially only $X$ was present. $\boldsymbol{\lambda}_{\mathrm{x}}$ and $\boldsymbol{\lambda}_{\mathrm{y}}$ are the disintegration constants of $X$ and $Y . N_{x}$ and $N_{y}$ are the number of nuclei of $X$ and $Y$ at any time $t$. Number of nuclei $N_{y}$ will be maximum when
a) $\frac{N_{y}}{N_{x}-N_{y}}=\frac{\lambda_{y}}{\lambda_{x}-\lambda_{y}}$
b) $\frac{N_{x}}{N_{x}-N_{y}}=\frac{\lambda_{x}}{\lambda_{x}-\lambda_{y}}$
c) $\lambda_{y} N_{y}=\lambda_{x} N_{x}$
d) $\lambda_{y} N_{x}=\lambda_{x} N_{y}$
28. The angular momentum of an electron in an orbit is quantized because it is a necessary condition for the compatibility with
a) The wave nature of electron
b) Particle nature of electron
c) Paulli's exclusion behaviour
d) None of the above
29. In $X$-ray tube when the accelerating voltage $V$ is halved, the difference between the wavelengths of $K_{\boldsymbol{\alpha}}$ line and minimum wavelength of continuous $X$-ray spectrum
a) Remains constant
b) Becomes more than two times
c) Becomes half
d) Becomes less than two times
30. The circuit shown in the figure contains two diodes each with a forward resistance of $\mathbf{5 0} \boldsymbol{\Omega}$ and with infinite backward resistance. If the battery voltage is 6 V , the current through the $100 \Omega$ resistance (in amperes) is

a) Zero
b) 0.02
c) 0.03
d) 0.036

## KEY

| 1) a | 2) b | 3) a | 4) a | 5) d |
| :---: | :---: | :---: | :---: | :---: |
| 6) d | 7) a | 8) b | 9) b | 10) b |
| 11) d | 12) a | 13) | 14) a | 15) c |
| 16) a | 17) b |  | 19) d | 20) a |
| 21) c | 22) b | 23) a | 24) a | 25) a |
| 26) c | 27) c | 28) a | 29) b | 30) d |

## SOLUTIONS

1. (a)
$\vec{v}=a \hat{i}+b \hat{j}$
$\therefore \vec{a}=b \hat{j}$
$\therefore$ Total acceleration $=\mathrm{b}$
Speed, $\mathrm{v}=\left(\mathrm{a}^{2}+\mathrm{b}^{2} \mathrm{t}^{2}\right)^{1 / 2}$
$\therefore$ Tangential acceleration $=\frac{\mathrm{dv}}{\mathrm{dt}}=\frac{1}{2}\left(\mathrm{a}^{2}+\mathrm{b}^{2} \mathrm{t}^{2}\right)^{-1 / 2}\left(2 \mathrm{~b}^{2} \mathrm{t}\right)$
At $t=\frac{\sqrt{3} a}{b}$

Tangential acceleration $=t=\frac{\sqrt{3} b}{2}$ and normal acceleration $=\sqrt{b^{2}-\frac{3 b^{2}}{4}}=\frac{b}{2}$
2. (b)

From superposition principle, $\mathrm{F}_{1}=\mathrm{F}_{\mathrm{r}}+\mathrm{F}_{\mathrm{c}}$
Here, $\mathrm{F}_{\mathrm{r}}=$ force due to remaining part $=\mathrm{F}_{2}$
And $\mathrm{F}_{\mathrm{c}}=$ force due to mass on the cavity
Now, $F_{1}=\frac{G M m}{(3 R)^{2}}=\frac{G M m}{9 R^{2}}, F_{c}=\frac{G\left(\frac{M}{8}\right) m}{\left(\frac{5}{2 R}\right)^{2}}=\frac{G M m}{50 R^{2}}$
$\therefore \mathrm{F}_{2}=\mathrm{F}_{1}-\mathrm{F}_{\mathrm{c}}=\frac{41 \mathrm{GMm}}{450 \mathrm{R}^{2}} \Rightarrow \frac{\mathrm{~F}_{2}}{\mathrm{~F}_{1}}=\frac{41}{50}$
3. (a)
$\mathrm{R}=2 \sqrt{\mathrm{~h}(\mathrm{H}-\mathrm{h})}$
Here, $\mathrm{h}=$ height from top and $\mathrm{H}-\mathrm{h}$ from bottom.
4. (a)

Phase difference corresponding to $\mathrm{y}_{1}=-\pi / 2$ and that for $\mathrm{y}_{2}=+\pi / 2$
$\therefore$ Average intensity between $\mathrm{y}_{1}$ and $\hat{y}_{2}=\frac{1}{\pi} \int_{-\pi / 2}^{\pi / 2} \mathrm{I}_{\max } \cos ^{2}(\phi / 2) \mathrm{d} \phi=\mathrm{I}_{\max }(\pi+2) / 2 \pi$
$\therefore$ The required ratio $=\frac{1}{2}\left(1+\frac{2}{\pi}\right)$
5. (d)

With increase in the value of $\mathrm{E}_{1}$, current passing through potentiometer wire will increase. With increase in $R_{1}$, current passing through potentiometer wire will decrease.
6. (d)

Potential of innermost shell is zero.
$\therefore \frac{\mathrm{q}_{1}}{\mathrm{r}}+\frac{\mathrm{q}_{2}}{2 \mathrm{r}}+\frac{\mathrm{q}_{3}}{3 \mathrm{r}}=0 \Rightarrow 6 \mathrm{q}_{1}+3 \mathrm{q}_{2}+2 \mathrm{q}_{3}=0$
Similarly, potential on outermost shell is also zero.
$\therefore \frac{\mathrm{q}_{1}}{3 \mathrm{r}}+\frac{\mathrm{q}_{2}}{3 \mathrm{r}}+\frac{\mathrm{q}_{3}}{3 \mathrm{r}}=0 \Rightarrow \mathrm{q}_{1}+\mathrm{q}_{3}=-\mathrm{q}_{2}$
Solving eqs.(1) and (2), we get
$\mathrm{q}_{1}=-\frac{\mathrm{q}_{2}}{4}, \frac{\mathrm{q}_{3}}{\mathrm{q}_{1}}=3$ and $\frac{\mathrm{q}_{3}}{\mathrm{q}_{2}}=-\frac{3}{4}$.
$\therefore$ Options (a), (b) and (c) are correct.
7. (a)

Mean values of refractive index $\mu=\frac{1.45+1.56+1.44+1.54+1.53+1.54}{6}=1.51$
Absolute errors are $1.51-1.46=0.06$

$$
\begin{aligned}
& 1.51-1.56=-0.05 \\
& 1.51-1.54=-0.03
\end{aligned}
$$

$$
1.51-1.44=0.07
$$

$$
1.51-1.54=-0.03
$$

$$
1.51-1.53=-0.02
$$

Mean absolute error $=\frac{0.06+0.05+0.03+0.07+0.03+0.02}{6}=0.04$
8. (b)

Particle will strike the point B if velocity of particle with respect to platform is along AB or component of its relative velocity along AD is zero. i.e.
$\mathrm{u} \cos \theta=\mathrm{v}$ or $\theta=\cos ^{-1}\left(\frac{\mathrm{v}}{\mathrm{u}}\right)$
9. (b)

For $\mathrm{a}=0$, tension is constant throughout, $\mathrm{T}=4 \mu \mathrm{~g}=20 \mathrm{~N}$
$\mathrm{F}=\mathrm{T}+2 \mu \mathrm{~g}=30 \mathrm{~N}$
10. (b)

Potential energy of particle at $x=\sqrt{\frac{2 \mathrm{E}}{\mathrm{k}}}$ is zero. $(x>0)$
$\therefore \mathrm{KE}=\mathrm{E}$
$\Rightarrow \frac{1}{2} \mathrm{mv}^{2}=\mathrm{E} \Rightarrow \mathrm{v} \sqrt{\frac{2 \mathrm{E}}{\mathrm{m}}}$
11. (d)

Let f be the friction on the ring towards right, ' a ' its linear acceleration and $\alpha$ the angular acceleration about center of mass.

Point of contact $P$ is momentarily at rest, i.e., ring will rotate about $P$.
$\therefore \alpha=\frac{\tau_{\mathrm{P}}}{\mathrm{I}_{\mathrm{P}}}=\frac{\mathrm{F}(2 \mathrm{R})}{2 \mathrm{MR}^{2}}=\frac{\mathrm{F}}{\mathrm{MR}}$


Now $F+\mathrm{f}=\mathrm{Ma}=\mathrm{MR} \alpha=\mathrm{F}$ or $\mathrm{f}=0$.
12. (a)
$\mathrm{y}=\mathrm{Kt}^{2}$
$\frac{d^{2} y}{d t^{2}}=2 K \Rightarrow a_{y}=2 \mathrm{~m} / \mathrm{s}^{2}\left(\right.$ as $\left.K=1 \mathrm{~m} / \mathrm{s}^{2}\right)$
$\mathrm{T}_{1}=2 \pi \sqrt{\frac{1}{\mathrm{~g}}}$ and $\mathrm{T}_{2}=2 \pi \sqrt{\frac{1}{\mathrm{~g}+\mathrm{a}_{\mathrm{y}}}}$
$\therefore \frac{\mathrm{T}_{1}^{2}}{\mathrm{~T}_{2}^{2}}=\frac{\mathrm{g}+\mathrm{a}_{\mathrm{y}}}{\mathrm{g}}=\frac{10+2}{10}=\frac{6}{5}$
13. (d)
14. (a)

The diagrammatic representation of the given problem is shown in figure. The expression of fundamental frequency is $v=\frac{1}{2 l} \sqrt{\frac{T}{\mu}}$


$$
\rho_{\mathrm{w}}=1 \mathrm{~g} / \mathrm{cm}^{3}
$$

In air $T=m g=(V \rho) g$
$\therefore \mathrm{v}=\frac{1}{21} \sqrt{\frac{\mathrm{~A} \rho \mathrm{~g}}{\mu}}$
When the object is half immersed in water
$\mathrm{T}^{\prime}=\mathrm{mg}-$ upthrust $=\mathrm{V}_{\mathrm{\rho}} \mathrm{~g}-\left(\frac{\mathrm{V}}{2}\right) \rho_{\mathrm{w}} \mathrm{g}=\left(\frac{\mathrm{V}}{2}\right) \mathrm{g}\left(2 \rho-\rho_{\mathrm{w}}\right)$
The new fundamental frequency is $v^{\prime}=\frac{1}{2 l} \times \sqrt{\frac{T}{m}}=\frac{1}{21} \sqrt{\frac{\left(\mathrm{~V}_{\mathrm{g}} / 2\right)\left(2 \rho-\rho_{\mathrm{w}}\right)}{\mu}} .$.
$\therefore \frac{\mathrm{v}^{\prime}}{\mathrm{v}}=\sqrt{\frac{2 \rho-\omega_{\mathrm{w}}}{2 \rho}} \Rightarrow \mathrm{v}^{\prime}=\mathrm{v}\left(\frac{2 \rho-\omega_{\mathrm{w}}}{2 \rho}\right)^{1 / 2}=300\left(\frac{2 \rho-1}{2 \rho}\right)^{1 / 2} \mathrm{~Hz}$.
15. (Q, R, Q, S)
$\mathrm{v}=\sqrt{\frac{\gamma \mathrm{RT}}{\mathrm{M}}}=\sqrt{\frac{\gamma \mathrm{P}}{\rho}}$
Speed does not change with change in pressure unless temperature is charged.
16. (a)

Energy per unit time (i.e., P) at junction will remain constant. Not the energy per unit area per unit time (i.e., intensity). Because is transmission area of medium may change.
17. (b)

In reflected and incident wave all other factor are common. So, it is only amplitude which can change the power $75 \%$ is transmitted. Hence, $25 \%$ or $1 / 4^{\text {th }}$ will reflected back.
$\therefore \mathrm{A}_{\mathrm{r}}=\frac{\mathrm{A}_{\mathrm{i}}}{2}\left(\right.$ as $\left.\mathrm{P} \propto \mathrm{A}^{2}\right)$
Or $\frac{A_{i}}{A_{r}}=2 \Rightarrow \frac{v_{1}+v_{2}}{v_{2}-v_{1}}=2 \Rightarrow \frac{v_{1}}{v_{2}}=\frac{1}{3}$
18. (b)

Specific heat $C=\frac{\Delta \mathrm{Q}}{\Delta \mathrm{T}}=\frac{1}{\Delta \mathrm{~T}}(\Delta \mathrm{U}+\mathrm{W})=\mathrm{C}_{\mathrm{v}}+\frac{\mathrm{W}}{\Delta \mathrm{T}}$
For the given process $W=4 V_{0} \frac{9 \mathrm{P}_{0}}{2}=18 \mathrm{P}_{0} \mathrm{~V}_{0}$
Also, $\Delta \mathrm{T}=\mathrm{T}_{2}-\mathrm{T}_{1}=\frac{\left(6 \mathrm{P}_{0}\right)\left(5 \mathrm{~V}_{0}\right)}{\mathrm{R}}-\frac{\left(3 \mathrm{P}_{0}\right) \mathrm{V}_{0}}{\mathrm{R}}=\frac{27 \mathrm{P}_{0} \mathrm{~V}_{0}}{\mathrm{R}}$
and $\mathrm{C}_{\mathrm{v}}=\frac{3}{2} \mathrm{R}$
$\therefore \mathrm{C}=\frac{3 \mathrm{R}}{2}+\frac{2 \mathrm{R}}{3}=\frac{13 \mathrm{R}}{6}$
19. (d)

For the two sheets $\mathrm{H}_{1}=\mathrm{H}_{2}(\mathrm{H}=$ rate of heat transfer $)$
Or $\frac{\theta_{1}-\theta}{\mathrm{R}_{1}}=\frac{\theta-\theta_{2}}{\mathrm{R}_{2}}$
Solving this we get, $\theta=\frac{\theta_{1} R_{2}+\theta_{2} R_{1}}{R_{1}+R_{2}}$
20. (a)
$\delta=(\mu-1) \mathrm{A}=(1.5-1)(4)=2^{\circ}$
$\mathrm{i}=\delta=2^{\circ}$
Let the mirror be rotated by an angle $\theta$
so that $\mathrm{i}^{\prime}=\left(2^{\circ}+\theta\right)$
Then, $\delta_{\text {total }}=180^{\circ}$
Or $\delta+180^{\circ}-2 \mathrm{i}^{\prime}=180^{\circ}$
$\therefore \delta=2 \mathrm{i}^{\prime}$

Or $2^{\circ}=2(2+\theta) \Rightarrow \theta=-2^{\circ}$
Here, negative sign implies that i gets decreased or $\mathrm{i}^{\prime}=0$.
i.e. light should fall normally on mirror.
21. (c)

Let $r$ be the internal resistance of the cell and $E$ its EMF. When connected across the resistance $\mathrm{R}_{1}$ in the circuit, current passing through the resistance is: $i=\frac{E}{R_{1}+r}$
$\therefore \mathrm{P}_{1}=\mathrm{i}^{2} \mathrm{R}_{1}=\left(\frac{\mathrm{E}}{\mathrm{R}_{1}+\mathrm{r}}\right)^{2} \mathrm{R}_{1}$
Similarly $P_{2}=\left(\frac{E}{R_{2}+r}\right)^{2} R_{2}$
Given that $\mathrm{P}_{1}=\mathrm{P}_{2}$
Substituting the values, we get $r=\sqrt{R_{1} R_{2}}$.
22. (b)
$\overrightarrow{\mathrm{F}}_{\mathrm{AOB}}=\overrightarrow{\mathrm{F}}_{\mathrm{AB}}=\mathrm{i}(\overrightarrow{\mathrm{l}} \times \overrightarrow{\mathrm{B}})$
Here, $A B=2 \sqrt{2 \times 2}=4 \mathrm{~m}$
$\therefore \overrightarrow{\mathrm{F}}_{\mathrm{AB}}=2[(-4 \hat{\mathrm{j}}) \times(-4 \hat{\mathrm{k}})]=32 \hat{\mathrm{i}}$
23. (a)
$\overrightarrow{\mathrm{DA}}=-2 \cos 30^{\circ} \hat{\mathrm{i}}-2 \sin 30^{\circ} \hat{\mathrm{k}}=(-\sqrt{3} \hat{\mathrm{i}}-\hat{\mathrm{k}})$
$\overrightarrow{\mathrm{AB}}=2 \hat{\mathrm{j}}$
$\therefore \overrightarrow{\mathrm{M}}=\mathrm{i}(\overrightarrow{\mathrm{DA}} \times \overrightarrow{\mathrm{AB}})=\frac{1}{2}[(-\sqrt{3} \hat{\mathrm{i}}-\hat{\mathrm{k}}) \times(2 \hat{\mathrm{j}})]=-\sqrt{3} \hat{\mathrm{k}}+\hat{\mathrm{i}}=(\mathrm{i}-\sqrt{3} \hat{\mathrm{k}}) \mathrm{A}-\mathrm{m}^{2}$
24. (a)
$|\mathrm{dq}|=\frac{\mathrm{d} \phi}{\mathrm{R}}=\mathrm{idt}=$ Area under i-t graph
$\therefore \mathrm{d} \phi=($ Area under i-t graph $)(\mathrm{R})=\frac{1}{2}(4)(0.1)(10)=2 \mathrm{~Wb}$.
25. (a)
$\mathrm{dq}=\frac{\mathrm{d} \phi}{\mathrm{R}}$
$\phi_{\mathrm{i}}=0$
$\phi_{f}=\left(\frac{\mu_{0}}{2 \pi} \frac{\mathrm{i}}{\mathrm{b}}\right)\left(\pi \mathrm{a}^{2}\right)=\frac{\mu_{0} \mathrm{ia}^{2}}{2 \mathrm{~b}}$
$\therefore \mathrm{d} \phi=\frac{\mu_{0} \mathrm{ia}^{2}}{2 \mathrm{~b}}$
So, $d q=\frac{\mu_{0} \mathrm{ia}^{2}}{2 R b}$
26. (c)
27. (c)

Net rate of formation of Y at any time t is : $\frac{\mathrm{dN}}{\mathrm{y}} \mathrm{dt}=\lambda_{\mathrm{x}} \mathrm{N}_{\mathrm{x}}-\lambda_{\mathrm{y}} \mathrm{N}_{\mathrm{y}}$
$\mathrm{N}_{\mathrm{y}}$ is maximum when $\frac{\mathrm{d} \mathrm{N}_{\mathrm{y}}}{\mathrm{dt}}=0 \Rightarrow \lambda_{\mathrm{x}} \mathrm{N}_{\mathrm{x}}=\lambda_{\mathrm{y}} \mathrm{N}_{\mathrm{y}}$.
28. (a)
$\operatorname{mvr}=\frac{\mathrm{nh}}{2 \pi}$
$\therefore \frac{\mathrm{h}}{\mathrm{mv}}=\frac{2 \pi \mathrm{r}}{\mathrm{n}}$
$\frac{\mathrm{h}}{\mathrm{mv}}=$ de-Broglie wavelength
29. (d)
$\Delta \lambda=\lambda_{\mathrm{K} \alpha}-\lambda_{\text {min }}$
When V is halved $\lambda_{\text {min }}$ becomes two times but $\lambda_{\mathrm{K} \alpha}$ remains the same.
$\therefore \Delta \lambda^{\prime}=\lambda_{\mathrm{K} \alpha}-2 \lambda_{\min }=2(\Delta \lambda)-\lambda_{\mathrm{K} \alpha}$
$\therefore \Delta \lambda^{\prime}<2(\Delta \lambda)$
30. (b)

In the circuit, diode $D_{1}$ is forward biased, while $D_{2}$ is reverse biased. Therefore, current i (through $D_{1}$ and $100 \Omega$ resistance) will be $\mathrm{i}=\frac{6}{50+100+150}=0.02 \mathrm{~A}$

Here, $50 \Omega$ is the resistance of $D_{1}$ in forward biasing.

## JEE MAIN Chemistry Model Paper with Key

1. Calculate the de - Broglie wave length of the electron in the ground state of hydrogen atom given that its kinetic energy is $13.6 \mathrm{ev} \quad\left(1 \mathrm{ev}=1.602 \times 10^{-19} \mathrm{~J}\right)$
1) $3.328 \times 10^{-10} \mathrm{~m}$
2) $2.328 \times 10^{-10} \mathrm{~m}$
3) $3.328 \times 10^{10} \mathrm{~m}$
4) $2.338 \times 10 \mathrm{~m}$
2. Mass of sodium metal that contains same number of atoms that are present in $\mathbf{3 6 g}$ of Al is
1) 23 g
2) 36 g
3) 30.66 g
4) 46 g
3. The order of reactivity of halogens towards halogenation of alkane is
1) $\mathrm{F}_{2}>\mathrm{Br}_{2}>\mathrm{Cl}_{2}$
2) $\mathrm{F}_{2}>\mathrm{Cl}_{2}>\mathrm{Br}_{2}$
3) $\mathrm{Cl}_{2}>\mathrm{F}_{2}>\mathrm{Br}_{2}$
4) $\mathrm{Cl}_{2}>\mathrm{Br}_{2}>\mathrm{F}_{2}$
4. Stability of ter-butyl cation is explained by
1) Electrometric
2)Mesomeric effect
2) Resonance effect
3) Both Inductive and Hyper congugation
5. Which of the following is electrophile
1) ROH
2) $B F_{3}$
3) $\mathrm{NH}_{3}$
4) $\bar{O} R$
6. The maximum amount of $\mathrm{BaSO}_{4}$ that can be obtained on mixing of $0.5 \mathrm{~mol} \mathrm{BaCl}_{2}$ with 1 mol $\mathrm{H}_{2} \mathrm{SO}_{4}$ is
1) 0.5 mol
2) 0.1 mol
3) 0.15 mol
4) 0.2 mol


The product ' $B$ ' is

4) All
this aqueous soluti

1) 759 torr
2) 7.60 torr
3) 76.0 torr
4) 752.40 torr
8. A solid is made of two elements $\mathrm{P} \& \mathrm{Q}$. Atoms P are in ccp arrangement and atoms Q occupy all the octahedral voids and half of the tetrahedral voids present then the simplest formula of the compound is
1) $\mathrm{PQ}_{2}$
2) $\mathbf{P}_{2} \mathbf{Q}$
3) $P Q$
4) $P_{2} Q_{2}$
9. Which shows the highest lattice energy?
1) RbF
2) CsF
3) NaF
4) KF
10. In the reactions given below

$\mathrm{R}-\mathrm{Cl} \xrightarrow{(i) A_{3} C N,(i) \mathrm{LiAlH}_{4}}$ Product B
The compounds $A$ and $B$ are :
1) Chain isomers
2) position isomers
3) functional isomers
4) metamers

11 A gaseous reaction was carried out, first keeping the volume constant and next keeping the Pressure constant. In the second experiment, there was an increase in volume. The heats of reaction were different, because

1) in the first case the energy was spent to keep the volume constant
2) in the second case the energy was spent to expand the gases
3) specific heat of compressed gases is more
4) specific heats of rarefied gases is more

12 Which of the following equations represents that provides the enthalpy of formation of $\mathrm{CH}_{3} \mathrm{Cl}$

1) $\mathrm{C}_{(s)}+\mathrm{HCl}_{(g)}+\mathrm{H}_{2_{(g)}} \rightarrow \mathrm{CH}_{3} \mathrm{Cl}_{(g)}$
2) $\mathrm{C}_{(s)}+3 \mathrm{H}_{(g)}+\mathrm{Cl}_{(g)} \rightarrow \mathrm{CH}_{3} \mathrm{Cl}_{(g)}$
3) $\mathrm{C}_{(s)}+\frac{3}{2} \mathrm{H}_{2_{(g)}}+\frac{1}{2} \mathrm{Cl}_{2_{(g)}} \rightarrow \mathrm{CH}_{3} \mathrm{Cl}_{(g)}$
4) $\mathrm{CH}_{4_{(g)}}+\mathrm{Cl}_{2(\mathrm{~g})} \rightarrow \mathrm{CH}_{3} \mathrm{Cl}_{(\mathrm{g})}+\mathrm{HCl}_{(\mathrm{g})}$
13. Hybridisation of Cu in the complex $\left[\mathrm{Cu}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}\right]^{2+}$ is $\mathrm{dsp}^{2}$, shape of the ion will be
1) Square planar
2) Tetrahedral
3) Distorted rectangle
4) octahedral
14. $\mathrm{N}_{\mathbf{2}}$ and $\mathrm{O}_{\mathbf{2}}$ are converted into mono anions $\mathrm{N}_{2}^{-}$and $\mathrm{O}_{2}^{-}$respectively, which of the following statements is wrong ?
1)The nitrogen - nitrogen bond in $\mathrm{N}_{2}^{-}$is weaker than in $\mathrm{N}_{2}$
2)In $\mathrm{O}_{2}^{-}, \mathrm{O}-\mathrm{O}$ bond order increases
3) $\mathrm{N}_{2}^{-}$is paramagnetic
4) $\mathrm{O}_{2}$ has more magnetic moment than $\mathrm{O}_{2}^{-}$.

## 15 Of the following acids

| I : hypo phosphorous acid | II: hydrofluoric acid |
| :--- | :--- |
| III: oxalic acid | IV: glycine |

1. I, II are monobasic, III dibasic acid and IV amphoteric
2. II monobasic, I, III dibasic acid, IV amphoteric
3. I monobasic, II, III dibasic, IV amphoteric
4. I, II, III dibasic, IV amphoteric
5. The correct stability order for the following species is

I

II

III

IV
1) $\mathrm{I}>$ II $>$ IV $>$ III
2) III $>$ IV $>$ II $>$ I
3) IV $>$ III $>$ II $>I$
4) IV $>$ III $>$ I $>$ II
17. Which of the following acids has the highest dissociation constant
1) 2-Fluoro propanoic acid
2) 3-Fluoro propanoic acid
3) 3-Bromo propanaoic acid
4) 2-Bromo propanoic acid
18. At which of the four conditions, the density of nitrogen will be the largest?
1) STP
2) 273 K and 2 atm
3) 546 K and 1 atm
4) 546 K and 2 atm
19. $\alpha-\mathrm{D}-(+)$ glucose and $\beta-\mathrm{D}-(+)$ glucose are
1) Anomers
2) Enantiomers
3) Conformers
4) Epimers
20. The compound formed in the positive test for nitrogen with the Lassaigne solution of an organic compound is
1) $\mathrm{Fe}_{4}\left[\mathrm{Fe}(\mathrm{CN})_{6}\right]_{3}$
2) $\mathrm{Fe}(\mathrm{CN})_{3}$
3) $\mathrm{Na}_{3}\left[\mathrm{Fe}(\mathrm{CN})_{6}\right]$
4) $\mathrm{Na}_{4}\left[\mathrm{Fe}(\mathrm{CN})_{5} \mathrm{NOS}\right]$
21. A solution containing 2.625 g of $\mathrm{CoCl}_{3} \cdot 6 \mathrm{NH}_{3}$ is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $\mathrm{AgNO}_{3}$ to give 4.78 g of $\mathbf{~ A g C l}$. The formula of the complex is
1) $\left[\mathrm{CoCl}\left(\mathrm{NH}_{3}\right)_{5}\right] \mathrm{Cl}_{2} \cdot \mathrm{NH}_{3}$
2) $\left[\mathrm{Co}\left(\mathrm{NH}_{3}\right)_{6}\right] \mathrm{Cl}_{3}$
3) $\left[\mathrm{CoCl}_{2}\left(\mathrm{NH}_{3}\right)_{4}\right] \mathrm{Cl} .2 \mathrm{NH}$
4) $\left[\mathrm{CoCl}_{3}\left(\mathrm{NH}_{3}\right)_{3}\right] .3 \mathrm{NH}_{3}$
22. Total number of lone pair of electrons on central atom of $\mathrm{XeOF}_{4}$
1) 0
2) 1
3) 2
4) 3
23. The heat of neutralisation of $\mathbf{H C l ~ b y ~} \mathrm{NaOH}$ is $-55.9 \mathrm{~kJ} \mathrm{~mol}^{-1}$. If the heat of neutralisation of HCN by $\mathbf{N a O H}$ is $\mathbf{- 1 2 . 1} \mathbf{~ k J} / \mathbf{m o l}$, the energy of dissociation of HCN is:
1) -43.8 kJ
2) 43.8 kJ
3) 68 kJ
4) -68 kJ
24. Buna-N synthetic rubber is a co-polymer of
1) $\mathrm{CH}_{2}=\mathrm{CH}-\mathrm{CN} \& \mathrm{CH}_{2}=\mathrm{CH}-\underset{\substack{1 \\ \mathrm{CH}_{3}}}{\mathrm{C}}=\mathrm{CH}_{2}$
2) $\mathrm{CH}_{2}=\mathrm{CH}-\underset{\substack{\mid \\ \mathrm{Cl}}}{\mathrm{C}}=\mathrm{CH}_{2} \& \mathrm{CH}_{2}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}_{2}$
3) $\mathrm{CH}_{2}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}_{2} \& \mathrm{CH}_{2}=\mathrm{CH}-\mathrm{C}_{6} \mathrm{H}_{5}$
$\mathrm{CH}_{2}=\mathrm{CH}-\mathrm{CN} \& \mathrm{CH}_{2}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}_{2}$
25. At a certain temperature $\mathbf{2}$ moles of carbonmonoxide and $\mathbf{3}$ moles of chlorine were allowed to reach equilibrium according to the reaction $\mathbf{C O}+\mathbf{C l}_{\mathbf{2}} \longrightarrow \mathbf{C o C l}_{\mathbf{2}}$ in a 5 lit vessel. At equilibrium if one mole of CO is present then equilibrium constant for the reaction is :
1) 2
2) 2.5
3) 3.0
4) 4
26. In a certain reaction $10 \%$ of the reactant decomposes in the first hour, $20 \%$ in second hour, $\mathbf{3 0 \%}$ in third hour and so on. What are the dimensions of rate constant
1) hour $^{-1}$
2) $\mathrm{mol} \mathrm{lit}^{-1}$ hour $^{-1}$
3) lit $\mathrm{mol}^{-1}$ hour $^{-1}$
4) $\mathrm{mol} \mathrm{hour}^{-1}$
27. The slope of the straight line graph between $\log x / m$ and $\log P$ for the adsorption of a gas on solid is
1) $k$
2) $\log \mathrm{k}$
3) $n$
4) $1 / n$
28. The BOD values of four samples of water $A, B, C$ and $D$ are $156 \mathrm{ppm}, 120 \mathrm{ppm}, 20 \mathrm{ppm}$ and 5 ppm respectively. The most polluted and least polluted water samples are
1) $A \& B$
2) $B \& C$
3) $A \& D$
4) $C \& D$
29. Which of the following molecules is/are having zero dipolemoment
I. $\mathrm{NCl}_{3}$
II. $\mathrm{PCl}_{5}$
III. $S F_{6}$
IV. $\mathrm{SCl}_{4}$
1) only II , IV
2) only I , II3) only II , III
3) only III, IV
30. Name of the structure of silicates in which three oxygen atoms of silicate ion are shared
1) Pyrosilicate
2) Sheet silicate
3) Linear chain silicate
4) Three dimensional silicate

## KEY

| $1)$ | 1 | $2)$ | 3 | $3)$ | 4 | $4)$ | 4 | $5)$ | 2 | $6)$ | 1 | $7)$ | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $8)$ | 1 | $9)$ | 3 | $10)$ | 3 | $11)$ | 2 | $12)$ | 3 | $13)$ | 1 | $14)$ | 2 |
| $15)$ | 1 | $16)$ | 1 | $17)$ | 1 | $18)$ | 2 | $19)$ | 1 | $20)$ | 1 | $21)$ | 2 |
| $22)$ | 2 | $23)$ | 4 | $24)$ | 2 | $25)$ | 2 | $26)$ | 4 | $27)$ | 3 | $28)$ | 3 |
| $29)$ | 3 | $30)$ | 1 |  |  |  |  |  |  |  |  |  |  |

