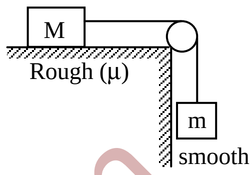
JEE MAIN MODEL TEST -1

PHYSICS

Comprehension: [Following 2 questions]

A block of mass M is placed on a rough horizontal surface. It is connected by means of a light inextensible string passing over a smooth pulley. The other end of string is connected to a block of mass m. there is no friction between vertical surface and block m.



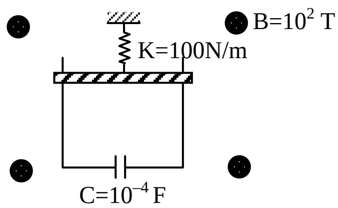
1. The minimum value of coefficient of friction μ, such that system remains at rest, is

a) $\frac{m+M}{m}$ b) $\frac{M}{m}$ *c) $\frac{m}{M}$ d) $\frac{m}{M+m}$

2. If the coefficient of friction μ is less than the above value, then downward acceleration of block m is

*a) $\frac{m-\mu M}{m+M}g$ b) $\frac{m+\mu M}{m+M}g$ c) $\frac{M-\mu m}{m+M}g$ d) μg

3. A conducting rod of length l = 1 m and mass m = 3 kg is attached a spring of spring constant K = 100 N/m and free to slide in two vertical rails. A uniform magnetic field B = 10^2 T exist in horizontal direction and a capacitor C = 10^{-4} F, if attached with the rails. The rod is released in equilibrium position and it performs a S.H.M. in vertical plane. The time period of the rod is given as $T=\frac{\pi x}{10}$, where x is an integer. The value of x is



a) 1 b) 2 c) 3 *d) 4

4. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius 4R is made from an iron plate of thickness t/4. Then the relation between the moment of inertia I_X and I_Y is

a) $I_Y = 32\ I_X$ b) $I_Y = 16\ I_X$ c) $I_Y = I_X$ d) $I_Y = 64\ I_X$

5. Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin(100\pi t + \pi/3)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1, with respect to the velocity of particle 2 is

a) $-\pi/6$ b) $\pi/3$ c) $-\pi/3$ d) $\pi/6$

6. A thin glass (refractive index 1.5) lens has optical power of –5 D in air. Its optical power in a liquid medium with refractive index 1.6 will be

a) 1 D   b) –1 D   c) 25 D   d) –25 D

7. If the sum of two unit vectors is also a unit vector, then magnitude of their difference and angle between the two given unit vector is

a) $\sqrt{2}, 90°$ *b) $\sqrt{3}, 120°$ c) $\sqrt{2}, 120°$ d) $\sqrt{3}, 60°$

8. **Statement-1 :** A food packet is dropped from a rescue plane. Path of the food packet will be straight line for the pilot but parabolic for the person on the ground. And

Statement-2 : Food packet has initial velocity same as that of plane.

a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

b) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation for Statement-1

c) Statement-1 is true, Statement-2 is false

*d) Statement-1 is false, Statement-2 is true

9. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, **where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is D = [U(x = ∞) – $U_{\text{at equilibrium}}$], D is**

a) $\frac{b^2}{2a}$ b) $\frac{b^2}{12a}$ c) $\frac{b^2}{4a}$ d) $\frac{b^2}{6a}$

10. The photoelectric work function of potassium is 3.0 eV. If light having a wavelength of 2475 Å falls on potassium, then the stopping potential in volts is

a) 1   *b) 2   c) 3   d) 4

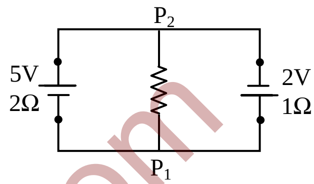
11. A radioactive mixture containing a short-lived specie A and a long-lived specie B, both emitting α-particles. Initially, the mixture emits 10,000 α-particles per minute. 10 minutes later it emits at the rate of 7000 particles per minute. If the half lives of the species are 10 min and 1 year respectively then the ratio of activities of A and B in the initial mixture was

a) 3 : 7   b) 4 : 6   *c) 6 : 4   d) 1 : 3

12. An ideal gas is taken through a process in which the process equation is given as $P = KV^{\alpha}$, where K and α are positive constant. The value of α for which in this process molar heat capacity becomes zero is (γ is adiabatic exponent of given gas)

a) $\alpha = -\frac{\gamma}{K}$ b) $\alpha = +\frac{\gamma}{K}$ c) $\alpha = +\gamma$ *d) $\alpha = -\gamma$

13. A 5 V battery with internal resistance 2 Ω and a 2 V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure. The current in 10 Ω resistor is



a) 0.27 A, P_2 to P_1 b) 0.03 A, P_1 to P_2

c) 0.03 A, P_2 to P_1 d) 0.27 A, P_1 to P_2

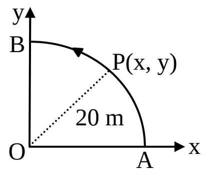
14. The stem of a thermometer has upon it a scale of equal parts. One of the divisions mark is at ice point. Call it 0° X. The steam point is at 90 divisions above the ice point. At what point on this scale, with the mercury stand at 60°C?

a) 60° X *b) 54° X c) 66.6° X d) 50° X

15. Points P, Q and R are in a vertical line such that PQ = QR. A ball of P is allowed to fall freely. The ratio of the times of descent through PQ and QR is

a) $\frac{1}{\sqrt{2}+1}$ b) $\frac{1}{2\sqrt{2}+1}$ c) $\frac{1}{2\sqrt{2}-1}$ *d) $\frac{1}{\sqrt{2}-1}$

16. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in meters and t is in seconds. The radius of the path is 20 m. The acceleration of P when t = 2 s is nearly



a) 13 m/s^2 b) 12 m/s^2

c) 7.2 m/s^2 d) 14 m/s^2

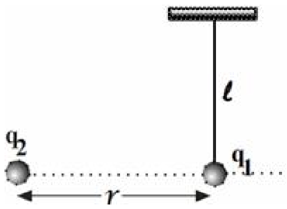
17. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, F_1/F_2 is

a) $\frac{R_2}{R_1}$ b) $\left(\frac{R_1}{R_2}\right)^2$ c) 1 d) $\frac{R_1}{R_2}$

18. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. The ratios of their diameters if each is to have the same tension is [Given : Young's modulus of elasticity for copper and iron are 110×10^9 Nm^{-2} and 190×10^9 Nm^{-2} respectively]

a) 1.2 b) 2.314 *c) 1.314 d) 3.414

19. An isolated particle of mass m and of charge q_1 is suspended freely by a silk thread of length *l*. Another charge q_2 is brought near it by a distance r (r >> *l*) as shown. When q_1 is in equilibrium, the tension in the thread will be



a) Equal to mg *b) Greater than mg 3) Less than mg 4) Zero

20. **Statement (A)**: A wire carrying no current, when placed in a uniform magnetic field experience no force.

Statement(R): An electron in the wire does not experience any force due to the magnetic field at any instant.

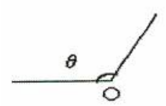
a) Both A and R are correct and R is a correct explanation of A

b) Both A and R are correct and R does not explains A

*c) A is correct but R is wrong

d) A is wrong but R is correct.

21. A thin uniform rod of length 'L' is bent at its mid point as shown in the figure. The distance of the centre of mass from the point 'O' is



a) $\frac{L}{2}\sin\frac{\theta}{2}$ b) $\frac{L}{2}\cos\frac{\theta}{2}$ c) $\frac{L}{4}\sin\frac{\theta}{2}$ d) $\frac{L}{4}\cos\frac{\theta}{2}$

22. There is a body of weight W on a table. It is moved with constant velocity by a force which makes an angle θ with the horizontal. If ϕ is the angle of repose, then the pulling force is

a) $\frac{W\cos\phi}{\cos(\theta-\phi)}$ b) $\frac{W\tan\phi}{\cos(\theta-\phi)}$ *c) $\frac{W\sin\phi}{\cos(\theta-\phi)}$ d) $\frac{W\sin\phi}{\cos(\theta+\phi)}$

23. A mass M is suspended from a light spring. An additional mass m added displaces the spring further by a distance x . Now the combined mass will oscillate on the spring with period

a) $T=2\pi\sqrt{(mg/x(M+m))}$ *b) $T=2\pi\sqrt{(x(M+m)/mg)}$

c) $T=(\pi/2)\sqrt{(mg/x(M+m))}$ d) $T=2\pi\sqrt{((M+m)/mgx)}$

24. Two particles of masses p and q (p > q) are separated by a distance "d". the shift in the centre of mass when the two particles are interchanged is

a) $\frac{d(p+q)}{(p-q)}$ *b) $\frac{d(p-q)}{(p+q)}$ c) $\frac{dp}{(p-q)}$ d) $\frac{dq}{(p-q)}$ $\frac{dq}{(p-q)}$

25. The height at which the acceleration due to gravity becomes g/9 (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is

a) 2R b) $R/\sqrt{3}$ c) R/2 d) $\sqrt{2}R$

26. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom, is

a) 10 b) 20 c) 25.5 d) 5

27. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then

a) $\vec{v}=\vec{E}\times\frac{\vec{B}}{B^2}$ b) $\vec{v}=\vec{B}\times\frac{\vec{E}}{B^2}$ c) $\vec{v}=\vec{E}\times\frac{\vec{B}}{E^2}$ d) $\vec{v}=\vec{B}\times\frac{\vec{E}}{E^2}$

28. An energy source will supply a constant current into the load, if its internal resistance is

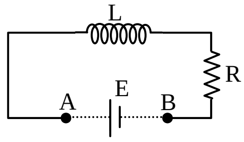
a) Equal to the resistance of the load b) Very large as compared to the load resistance

c) Zero d) Non-zero but less than the resistance of the load

29. In an LCR series AC circuit, the voltage across each of the components. L, C and R is 50 V. The voltage across the LC combination will be

a) 50 V b) $50\sqrt{2}$ V c) 100 V d) zero

30. An inductor (L = 100 mH), a resistor (R = 100 Ω) and a battery (E = 100 V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is



a) 1/e A b) eA c) 0.1 A d) 1 A

SOLUTIONS

1. (c)
2. (a)
3. (d)

At equilibrium, ky = mg …(1)

When rod is at a displacement x from equilibrium

$mg - iBl - k(y+x) = m\frac{dv}{dt}$ …(2)

and from (b) : $i = \frac{dq}{dt} = CBl\frac{dv}{dt}$ …(3)

Now, from (2) and (3),

$-CB^2l^2\frac{dv}{dt} - kx = m\frac{dv}{dt}$

$(m + CB^2l^2)\frac{dv}{dt} + kx = 0$

$\frac{d^2x}{dt^2} + \left[\frac{k}{(m+CB^2l^2)}\right]x = 0 \Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0$

$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m+CB^2l^2}{k}} = 2\pi\sqrt{\frac{3+10^{-4}\times 10^4 \times 1^2}{100}} = 2\pi\sqrt{\frac{4}{100}} = 2\pi\times\frac{2}{10} = \frac{4\pi}{10}$

4. (d)

Sol. Mass of disc (X), $m_X = \pi R^2 t\rho$

where ρ = density of material of disc.

$\therefore I_X = \frac{1}{2}m_X R^2 = \frac{1}{2}\pi R^2 t\rho R^2$

$\Rightarrow I_x = \frac{1}{2}\pi\rho t R^4$ ...(i)

Mass of disc (Y): $m_Y = \pi(4R)^2 \frac{t}{4}\rho = 4\pi R^2 t\rho$

and $I_Y = \frac{1}{2} m_Y (4R)^2 = \frac{1}{2} 4\pi R^2 t\rho \cdot 16R^2$

or $I_Y = 32\pi t\rho R^4$ ...(ii)

$\therefore \frac{I_Y}{I_X} = \frac{32\pi t\rho R^4}{\frac{1}{2}\pi\rho t R^4} = 64 \Rightarrow I_Y = 64 I_X$.

5. (a)

Sol. Given, $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$

$\Rightarrow \frac{dy_1}{dt} = v_1 = 0.1 \times 100\pi \cos(100\pi t + \frac{\pi}{3})$

or $v_1 = 10\pi \sin\left(100\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right)$

or $v_1 = 10\pi \sin\left(100\pi t + \frac{5\pi}{6}\right)$

and $y_2 = 0.1\cos\pi t$

$\frac{dy_2}{dt} = v_2 = -0.1\sin\pi t \Rightarrow v_2 = 0.1\sin(\pi t + \pi)$

Hence, phase difference $\Delta\phi = \phi_1 - \phi_2$

$= \left(100\pi t + \frac{5\pi}{6}\right) - (\pi t + \pi) = \frac{5\pi}{6} - \pi = -\frac{\pi}{6}$ (at $t = 0$)

6. (a)

Sol. $\frac{1}{f_a} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(i)

$= (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

and $\frac{1}{f_m} = \left(\frac{\mu_g - \mu_m}{\mu_m}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(ii)

$$\frac{1}{f_m}=\left(\frac{1.5}{1.6}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Thus, $\frac{f_m}{f_a}=\frac{(1.5-1)}{\left(\frac{1.5}{1.6}-1\right)}=-8$

Or $f_m=-8\times f_a=-8\times\frac{-1}{5}\quad\left(\because f_a=\frac{1}{p}=-\frac{1}{5}m\right)$

$= 1.6$ m

$\therefore\ P_m=\frac{\mu}{f_m}=\frac{1.6}{1.6}=1\,D.$

7. (b)

8. (d)

9. (c)

Sol. $U(x)=\frac{a}{x^{12}}-\frac{b}{x^6}$

$U(x=\infty)=0$

as, $\quad F=-\frac{dU}{dx}=-\left[\frac{12a}{x^{13}}+\frac{6b}{x^7}\right]$

at equilibrium, F = 0

$\therefore\ x^6=\frac{2a}{b}$

$$\therefore\ U_{\text{at equilibrium}}=\frac{a}{\left(\frac{2a}{b}\right)^2}-\frac{b}{\left(\frac{2a}{b}\right)}=-\frac{b^2}{4a}$$

$$\therefore D=\left[U(x=\infty)-U_{\text{at equilibrium}}\right]=\frac{b^2}{4a}$$

10. (b)

$hv = \phi + KE$

$v = 2475$ Å, $\phi = 3.0$ eV

we know, $KE = eV_0$ (where V_0 is stopping potential)

$hv = \phi + eV_0$

$v_0 = 2$ V.

11. (c)

Since $T_{1/2}(A) << T_{1/2}(B)$, the activity of B remains relativity unchanged after 10 min.

Initially, A + B = 10000

After 10 min, $\frac{A}{2} + B = 7000$

This gives A : B = 6 : 4

12. (d)

For polytropic process, pv^n = constant

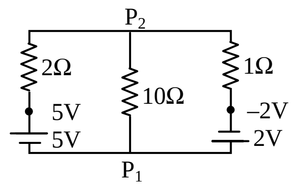
$$C = \frac{R}{\gamma - 1} + \frac{R}{1 - n}$$

$$C = \frac{R}{\gamma - 1} + \frac{R}{1 + \alpha}$$

for C = 0, $\alpha = -\gamma$

13. (c)

Sol. Let potential at P_1 is 0 V and potential at P_2 is V_0. Now apply KCL at P_2



$$\frac{V_0 - 5}{2} + \frac{V_0 - 0}{10} + \frac{V_0 - (-2)}{1} = 0 \Rightarrow V_0 = \frac{5}{16}$$

So, current through 10 Ω resistor is $V_0/10$ from P_2 to P_1.

14. (b)

$$\frac{x - 0}{90} = \frac{60}{100} \Rightarrow x = 54$$

15. (d)

For motion from P to Q : $y = \frac{1}{2} g t_1^2$

For motion from P to R : $2y = \frac{1}{2} g (t_1 + t_2)^2$

Here, t_1 is the time taken from P to Q and t_2 is the time taken from Q to R.

Now, $2t_1^2 = (t_1 + t_2)^2 \Rightarrow \sqrt{2} t_1 = t_1 + t_2$

$$\Rightarrow t_1(\sqrt{2} - 1) = t_2 \Rightarrow \frac{t_1}{t_2} = \frac{1}{\sqrt{2} - 1}$$

16. (d)

Sol. $S = t^3 + 5$

$\therefore$ speed, $v = \frac{ds}{dt} = 3t^2$

and rate of change of speed = $\frac{dv}{dt} = 6t$

$\therefore$ tangential acceleration at

$t = 2s$, $a_t = 6 \times 2 = 12\ ms^{-2}$

at $t = 2s$, $v = 3(2)^2 = 12\ ms^{-1}$

$\therefore$ centripetal acceleration,

$$a_c = \frac{v^2}{R} = \frac{144}{20} ms^{-2}$$

$\therefore$ net acceleration $= \sqrt{a_t^2 + a_i^2} \approx 14\ m/s^2$.

17. (d)

Sol. Since ω is constant, v would also be constant. So, no net force or torque is acting on ring. The force experienced by any particle is only along radial direction, or we can say the centripetal force.

The force experienced by inner part,

$F_1 = m\omega^2 R_1$ and the force experienced by outer part, $F_2 = m\omega^2 R_2$.

$\Rightarrow \frac{F_1}{F_2} = \frac{R_1}{R_2}$.

18. (c)

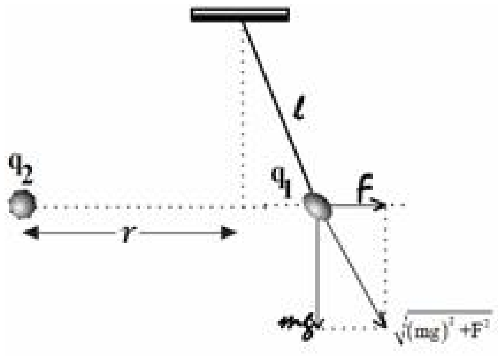
Since each wire is to have same tension therefore, each wire has same extension. Moreover, each wire has the same initial length. so, straight is same for each wire.

Now, $Y = \frac{stress}{strain} = \frac{F/\pi D^2/4}{strain} \Rightarrow Y \propto \frac{1}{D^2} \Rightarrow D \propto \frac{1}{\sqrt{Y}}$

$$\frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}} = \sqrt{\frac{19}{11}} = 1.314$$

19. (b)

At equilibrium, the state will be as shown.



So the tension of the string : $T=\sqrt{F^2+(mg)^2}$

Hence T > mg.

20. Conceptual

21. (d)

$$x_{cm}=\frac{-\frac{m}{2}(L/4)-\frac{m}{2}(\cos\theta)(L/4)}{m}$$

$$y_{cm}=\frac{\frac{m}{2}\sin\theta(L/4)}{m}$$

$$r_{cm}=\sqrt{x_{cm}^2+y_{cm}^2}$$

22. (c)

$$\Rightarrow\frac{\mu W}{\cos\theta+\tan\phi\sin\theta}(\therefore\mu=\tan\phi)$$

$$=\frac{\left(\frac{\sin\phi}{\cos\phi}\right)W}{\frac{\cos(\theta-\phi)}{\cos\phi}}=\frac{W\sin\phi}{\cos(\theta-\phi)}$$

23. (b)

$$K=\frac{mg}{x}$$

$$T=2\pi\sqrt{\frac{(M+m)}{K}}$$

24. $x_{cm} = \frac{dq}{(p+q)},\ x'_{cm} = \frac{dp}{p+q}$

$$\text{shift} = x'_{cm} - x_{cm} = \frac{d(p-q)}{p+q}$$

25. (a)

Sol. $g' = \frac{GM}{(R+h)^2}$, acceleration due to gravity at height h

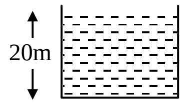
$$\Rightarrow \frac{g}{9} = \frac{GM}{R^2} \cdot \frac{R^2}{(R+h)^2} = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \frac{1}{9} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{3}$$

$\Rightarrow$ 3R = R + h $\Rightarrow$ 2R = h.

26. (b)

Sol. Applying the Bernoulli's theorem just inside and outside the hole. Take reference line for gravitational potential energy at the bottom of the vessel.



Let p_0 is the atmospheric pressure, ρ the density of liquid and v the velocity at which water coming out.

$$p_{inside} + \rho gh + 0 = p_{outside} + \frac{\rho v^2}{2}$$

$$\Rightarrow p_0 + \rho gh = p_0 + \frac{\rho v^2}{2}$$

$$\Rightarrow v = \sqrt{2gh} = \sqrt{2\times 10\times 20} = 20\,ms^{-1}$$

27. (a)

Sol. As $\vec{v}$ of charged particle is remaining constant, it means force acting on charged particle is zero.

So, $q(\vec{v}\times\vec{B}) = q\vec{E}$

$$\Rightarrow \vec{v}\times\vec{B} = \vec{E} \Rightarrow \vec{v} = \frac{\vec{E}\times\vec{B}}{B^2}$$

28. (c)

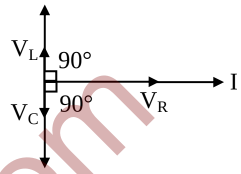
Sol. $I = \frac{E}{R+r}$; $I = \frac{E}{R} = \text{constant}$

where, R = external resistance,

r = internal resistance = 0

29. (d)

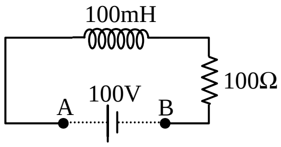
Sol. In an LCR series AC circuit, the voltage across inductor L leads the current by 90° and the voltage across capacitor C lags behind the current by 90°.



Hence, the voltage across LC combination will be zero.

30. (a)

Sol. This is a combined example of growth and decay of current in an LR circuit.



The current through circuit just before shorting the battery, $I_0 = \frac{E}{R} = 1A$.

[as inductor would be shorted in steady state]

After this decay of current starts in the circuit according to the equation

$I = I_0 e^{-t/\tau}$, where τ = L/R.

$I = 1 \times e^{-(1\times10^{-3})/(100\times10^{-3}/100)} = \left(\frac{1}{e}\right) A$

CHEMISTRY

1. Which one of the following sets of ions represents a collection of iso-electronic species

*a) $K^+, Cl^-, Ca^{2+}, Sc^{3+}$ b) $Ba^{2+}, Sr^{2+}, K^+, S^{2-}$ c) $N^{3-}, O^{2-}, F^-, S^{2-}$ d) $Li^+, Na^+, Mg^{2+}, Ca^{2+}$

2. For the following three reactions I,II and III, equilibrium constants are given

I) $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g); K_1$ II) $CH_4(g) + H_2O(g) \rightleftharpoons CO(g) + 3H_2(g); K_2$

III) $CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g); K_3$

Which of the following relations is correct ?

a) $K_1\sqrt{K_2} = K_3$ b) $K_2K_3 = K_1$ *c) $K_3 = K_1K_2$ d) $K_3K_2^3 = K_1^2$

3. The exothermic formation of ClF_3 is represented by the equation

$Cl_2(g) + 3F_2(g) \rightleftharpoons 2ClF_3(g); \Delta H_r = -329\,kJ$. Which of the following will increase the quantity of ClF_3 in an equilibrium mixture of Cl_2, F_2 and ClF_3 ?

*a) adding F_2 b) Increasing the volume of the container

c) Removing Cl_2 d) Increasing the temperature

4. Solubility product of silver bromide 5×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is

a) 6.2×10^{-5} g b) 5.0×10^{-8} g c) 1.2×10^{-10} g *d) 1.2×10^{-9} g

5. 2 mol of an ideal gas expanded isothermally & reversibly from 1 litre to 10 litres at 300 K. What is the enthalpy change?

a) 4.98 KJ b) 11.47 KJ c) –11.47 KJ *d) 0 KJ

6. For the process $H_2O(l)$ (1 bar, 373 K) $\rightarrow$ $H_2O(g)$ (1 bar, 373 K), the correct set of thermodynamic parameters is

*a) $\Delta G = 0, \Delta S = +ve$ b) $\Delta G = 0, \Delta S = -ve$ c) $\Delta G = +ve, \Delta S = 0$ d) $\Delta G = -ve, \Delta S = +ve$

7. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because

a) a and b for Cl_2 < a and b for C_2H_6

b) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6

*c) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

d) a and b for Cl_2 > a and b for C_2H_6

8. What type of crystal defect is indicated in the diagram given below?

$Na^+ \; Cl^- \; Na^+ \; Cl^- \; Na^+ \; Cl^-$

$Cl^- \rightarrow Cl^- \; Na^+ \rightarrow Na^+$

$Na^+ \; Cl^- \rightarrow Cl^-, Na^+ \; Cl^-$

$Cl^- \; Na^+ \; Cl^- \; Na^+ \rightarrow Na^+$

a) Frenkel defect *b) Schottky defect c) Interstitial defect d) Frenkel and Schottky defects

9. $H_2SO_4 + xHI \longrightarrow H_2S + yI_2 + zH_2O$. The values of x, y and z are

*1) x = 8, y = 4 and z = 4 2) x = 4, y = 4 and z = 5

3) x = 8, y = 4 and z = 5 4) x = 2, y = 1 and z = 2

10. Consider the reaction, $2A + B \rightarrow$ products when concentration of B alone was doubled, the half-life did not change. When the concentration of A alone was doubled, the rate increased by two times. The unit of rate constant for this reaction is

*a) $L\ mol^{-1}\ s^{-1}$ b) no unit c) $mol\ L^{-1}\ s^{-1}$ d) s^{-1}

11. In XeF_2, XeF_4 and XeF_6, the number of lone pair of electrons on Xe are respectively

a) 2, 3, 1 b) 1, 2, 3 c) 4, 1, 2 *d) 3, 2, 1

12. The freezing point (in °C) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$ (Mol.wt.329) in 100 g of water ($K_f = 1.86$ K kg mol^{-1}) is

*a) -2.3×10^{-2} b) -5.7×10^{-2} c) -5.7×10^{-3} d) -1.2×10^{-2}

13. The ionic mobility of alkali metal ions in aqueous solution is maximum for

a) K^+ b*) Rb^+ c) Li^+ d) Na^+

14. The weight - volume percentage $\left(\frac{w}{v}\right)$ of 10V-H_2O_2 solution is

1) 6% 2) 2% 3) 11% *4) 3%

15. Which of the complex in its 1 molal solution with 100% ionization will show a boiling point of 102.08°? (K_b for water = 0.52 K $molality^{-1}$)

1) $Fe_4[Fe(CN)_6]$ *2) $K_3[Fe(CN)_6]$ 3) $K_4[Fe(CN)_6]$ 4) none of these

16. Identify the incorrect with respect to ozone

1) Ozone is formed in the upper atmosphere by a photochemical reaction involving dioxygen

2) Ozone is more reactive than dioxygen

3) Ozone is diamagnetic whereas dioxygen is paramagnetic

*4) Ozone protects the earth's inhabitants by absorbing gamma-radiations

17. In Langmuir's model of adsorption of a gas on a solid surface

a) the rate of dissociation of adsorbed molecules from the surface does not depend on the surface covered

b) the adsorption at a single site on the surface may involve multiple molecules at the same time.

*c) the mass of gas striking a given area of surface is proportional to the pressure of the gas

d) the mass of gas striking a given area of surface is independent of the pressure of the gas

18. In a hydrogen-oxygen fuel cell, combustion of hydrogen occurs to

a) generate heat

*b) create potential difference between the two electrodes

c) produce high purity water

d) remove adsorbed oxygen from electrode surfaces

19. The equivalent conductance of two strong electrolytes at infinite dilution in H_2O (where ions move freely through a solution) at 25°c are given below

$\wedge^0_{CH_3COONa} = 91.0\, S\, cm^2 / equiv$

$\wedge^0_{HCl} = 426.2\, S\, cm^2 / equiv$

What additional information/quantity one needs to calculate $\wedge^0$ of an aqueous solution of acetic acid ?

a) $\wedge^0$ of NaCl

b) $\wedge^0$ of CH_3COOK

c) the limiting equivalent conductance of $H^+ \left(\lambda^0_{H^+}\right)$

*d) $\wedge^0$ of chloroacetic acid ($ClCH_2COOH$)

20. $[Co(NH_3)_4(NO_2)_2]Cl$ exhibits

a) Linkage isomerism, geometrical isomerism and optical isomerism

b) Linkage isomerism, ionization isomerism and optical isomerism

*c) Linkage isomerism, ionization isomerism and geometrical isomerism

d) Ionization isomerism, geometrical isomerism and optical isomerism

21. A liquid was mixed with ethanol and a drop of concentrated H_2SO_4 was added. A compound with a fruity smell was formed. The liquid was

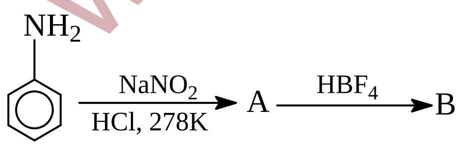
a) CH_3OH

b) HCHO

c) CH_3COCH_3

*d) CH_3COOH

22. In the chemical reactions,



the compounds A and B respectively are

a) nitrobenzene and chlorobenzene

b) nitrobenzene and fluorobenzene

c) phenol and benzene

*d) benzene diazonium chloride and fluorobenzene

23. The polymer containing strong interolecular forces e.g. hydrogen bonding, is

a) teflon *b) nylon 6,6 c) polystyrene d) natural rubber

24. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is :

a) Trichloromethanol b) 2, 2, 2-Trichloropropanol

c) Chloroform *d) 2, 2, 2-Trichloroethanol

25. Which of the following reagents may be used to distinguish between phenol and benzoic acid ?

a) Tollen's reagent b) Molisch reagent *c) Neutral $FeCl_3$ d) Aqueous NaOH

26. The number of stereoisomers obtained by bromination of trans-2-butene is

*a) 1 b) 2 c) 3 d) 4

27. Which of the following compounds exhibits stereoisomerism?

a) 2-methylbutene-1 b) 3-methylbutyne-1

c) 3-methylbutanoic acid *d) 2-methylbutanoic acid

28. Which of the following has the highest nucleophilicity?

a) F^- b) OH^- *c) CH_3^- d) NH_2^-

29. Most stable carbonium ion is

a) $p-NO_2-C_6H_4-CH_2^+$ b) $C_6H_5CH_2^+$

c) $p-Cl-C_6H_4-CH_2^+$ *d) $p-CH_3O-C_6H_4-CH_2^+$

30. In the following groups:

I) –OAc II) –OMe III) $-OSO_2Me$ IV) $-OSO_2CF_3$

The order of leaving group ability is

a) I > II > III > IV *b) IV > III > I > II c) III > II > I > IV d) II > III > IV > I

MATHAMATICS

Instructions : Question No. 1 to 30 FOUR (4) marks each and 1/4th marks will be deducted for indicating incorrect response of each question.

1. Each student in a class of 40, studies at least one of the subjects English, Mathematics and Economics. 16study English, 22 Economics and 26 Mathematics, 5 study English and Economics, 14 Mathematics and Economics and 2 study all the three subjects. The number of students who study English and Mathematics but not Economics is

1. 7 2. 5 3. 10 4. 4

2. The weighted mean of first n natural numbers whose weights are equal to the number of selections out of n natural numbers of corresponding numbers respectively is

1. $\frac{n.2^{n-1}}{2^n-1}$ 2. $\frac{3n(n+1)}{2(2n+1)}$ 3. $\frac{(n+1)(2n+1)}{6}$ 4. $\frac{n(n+1)}{2}$

3. Which of the following statements is a tautology?

1. $(\sim p \vee q) \wedge (p \vee \sim q)$ 2. $(\sim p \vee \sim q) \to p \vee q$

3. $(p \vee \sim q) \wedge (p \vee q)$ 4. $(\sim p \vee \sim q) \vee (p \vee q)$

4.The foot of the perpendicular from (1,0,2) on the line $\frac{x+1}{3}=\frac{y-2}{-2}=\frac{z+1}{-1}$ **is the point**

1. $(1,2,-3)$ 2. $\left(\frac{1}{2},1,-\frac{3}{2}\right)$ 3. $(2,4,-6)$ 4. $(2,3,6)$

5. It is known that $\sum_{r=1}^{\infty}\frac{1}{(2r-1)^2}=\frac{\pi^2}{8}$**. Then** $\sum_{r=1}^{\infty}\frac{1}{r^2}$ **is equal to**

1. $\frac{\pi^2}{24}$ 2. $\frac{\pi^2}{3}$ 3. $\frac{\pi^2}{6}$ 4. None

6. A tangent PT is drawn to the circle $x^2+y^2=4$ **at the point** $P(\sqrt{3},1)$**. A straight line L, perpendicular to PT is a tangent to the circle** $(x-3)^2+y^2=1$**. A possible equation of L is**

1) $x-\sqrt{3}y=1$ 2) $x-\sqrt{3}y=-1$ 3) $x+\sqrt{3}y=1$ 4) $x+\sqrt{3}y=5$

7. The number of tangents to the curve y = cos(x + y), $-2\pi \le x \le 2\pi$, that are perpendicular to the line 2x – y = 3.

1) 1 2) 2 3) 3 4) 4

8. A particle moves in a straight line with a velocity given by $\frac{dx}{dt} = x+1$ (x is the distance traveled). The time taken by a particle to travel a distance of 99 meters is

1) $\log_{10} e$ 2) $2\log_e 10$ 3) $2\log_{10} e$ 4) $1/2\log_{10} e$

9. The distance between the line $\overline{r} = 2i - 2j + 3k + \lambda(i - j + 4k)$ and the plane $r.(i+5j+k) = 5$

1) $\frac{10}{3\sqrt{3}}$ 2) $\frac{10}{9}$ 3) $\frac{10}{3}$ 4) $\frac{3}{10}$

10.. Find the equation of the curve passing through (1, 2) whose differential equation is $y(x+y^3)dx = x(y^3 - x)dy$

1) $xy = 1$ 2) $x^2 - y^2 = 1$

3) $y^3 + 2x = 5x^2 y$ 4) $x^2 - y + 3 = 0$

11. The sum of two positive integers is 200 then chance that their product is greater than $3/4$ times their greatest product then probability is

1) $\frac{51}{99}$ 2) $\frac{99}{199}$ 3) $\frac{1}{2}$ 4) $\frac{1}{3}$

12. If $f(x)$ is function such that $f(x+5) + f(x+6) = 0 \; \forall \; x \in R$ the period of function

1) 1 2) 2 3) 3 4) 2π

13. If the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ meet the ellipse $\frac{x^2}{1} + \frac{y^2}{a^2} = 1$ in four distinct points and $a = b^2 - 10b + 25$, then the value of b does not satisfy

1) $(-\infty, 4]$ 2) $(4,6)$ 3) $[6,\infty)$ 4) none

14. $\int_0^{\pi} [\cot x] dx$, Where [.] denotes the greatest integer function, is equal to

1) 1 2)-1 3) $-\frac{\pi}{2}$ 4) $\frac{\pi}{2}$

15. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ $I_4 = \int_1^2 2^{x^3} dx$ then

1) $I_1 > I_2$ 2) $I_2 > I_1$ 3) $I_3 > I_4$ 4) $I_3 = I_4$ 17.

16. If $x^2+4y^2-8x+12=0$ **is satisfied by real values of** x **and** y **then** $'y'\in$

1) $[2,6]$ 2) $[2,5]$ 3) $[-1,1]$ 4) $[-2,-1]$

17. If α **is non-real and** $\alpha=\sqrt[5]{1}$, **then the value of** $2^{|1+\alpha+\alpha^2+\alpha^{-2}-\alpha^{-1}|}$ **is equal to**

1) -1 2) -2 3) 1 4) none of these

18. Statement 1: If $27a+9b+3c+d=0$, **then the equation** $f(x)=4ax^3+3bx^2+2cx+d=0$

Has at least one real root lying between (0, 3).

Statements 2: If f(x) is continuous in $[a,b]$, **derivable in (a, b) such that f(a)=f(b), then at least one point** $c\in(a,b)$ **such that** $f'(c)=0$.

1) If both the statements are TRUE and Statement 2 is the correct explanation of Statement – 1.

2) If both the statements are TRUE and Statement 2 is NOT the correct explanation of Statement – 1

3) If Statement – 1 is TRUE and Statement 2 is FALSE.

4) If Statement – 1 is FALSE and Statement 2 is TRUE.

19. The coefficient of x^{20} **in the expansion of** $(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$ **is**

1) ${}^{30}C_{10}$ 2) ${}^{30}C_{25}$ 3) 1 4) 13

20. If one of the lines of $my^2+(1-m^2)xy-mx^2=0$ **is a bisector of the angle between the lines xy = 0, then m =**

1. -2 **2.** ± 1 **3.** 0 4 6

21. If M (x_o,y_o) **is the point on the curve** $3x^2-4y^2=72$ **which is nearest to the line** $3x+2y+1=0$, **then the value of** (x_o+y_o) **is equal to**

1) 3 2) -3 3) 9 4) -9

22. Let $f(x)=\lim_{h\to 0}\frac{\sin(x+h)^{\ln(x+h)}-(\sin x)^{\ln x}}{h}$ *then* $f\left(\frac{\pi}{2}\right)$ **is**

1) Equal to 0 2) Equal to 1 3) $\ln\frac{\pi}{2}$ 4) Non Existent

23. If $|z+2i|\le 1$ **and** $z_1=6-3i$ **then the maximum value of** $|iz+z_1-4|$ **is equal to**

1) 2 2) 6 3) 3 4) $\frac{1}{2}$

24. **Let** $f(x) = |x-1| + |x+1|$. **Then f is differentiable in**

1) $(-\infty, \infty)$ 2) $(-\infty, 0)$ 3) $(-1, 1)$ 4) $[-1, 1]$

25. **If** $\hat{a}, \hat{b}$ and $\hat{c}$ **are unit vectors satisfying** $\hat{a} - \sqrt{3}\hat{b} + \hat{c} = 0$**, then the angle between** $\hat{a}$ **and** $\hat{c}$ **is**

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

26. **The number of real roots of the equation** $x^3 - 3x + 1 = 0$ **is**

1) 2 2) 3 3) 0 4) 1

27. **Statement-1: Number of ways in which 10 identical toys can be distributed among three students if each receives atleast two toys is** ${}^9C_2$

Statement-2: Number of positive integral solutions of $x + y + z + w = 7$ **is** ${}^6C_3$.

1) Statement-1 is True, Statement-2 is False
2) Statement-1 is False, Statement-2 is True
3) Statement-1 is True, Statement-2 is True; Statement-2 is a correct Explanation For Statement-1
4) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

28. **Let P & Q be 3x3 matrices with** $P \neq Q$. **If** $P^3 = Q^3$ **&** $P^2Q = PQ^2$ **then** $|P^2 + Q^2|$

1) 1 2) 0 3) -1 4) 2

29. **If a and b are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement. Then probability that** $\underset{x \to 0}{Lt}\left(\frac{a^x + b^x}{2}\right)^{2/x} = 6$ **is**

1) $\frac{1}{3}$ 2) $\frac{1}{4}$ 3) $\frac{1}{9}$ 4) $\frac{2}{9}$

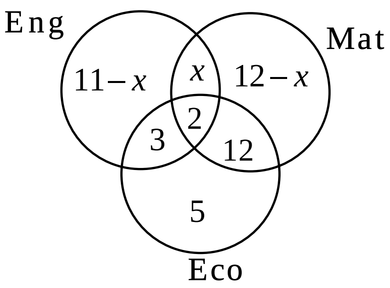
30. A Triangle is inscribed in a circle. The vertices of the triangle divide the circle into three arcs of length 3,4 and 5 units, then area of the triangle is

1. $\frac{9\sqrt{3}(1+\sqrt{3})}{\pi^2}$ **2.** $\frac{9\sqrt{3}(\sqrt{3}-1)}{\pi^2}$ **3.** $\frac{9\sqrt{3}(1+\sqrt{3})}{2\pi^2}$ **4.** $\frac{9\sqrt{3}(\sqrt{3}-1)}{2\pi^2}$

Solutions

HINTS: 1. **sol:**option-2

28-2x+22=40



$2x=10$

$x=5$

2:**sol:** option-1 The required mean $\overline{X}=\dfrac{1.{}^nC_1+2.{}^nC_2+3.{}^nC_3+...+n.{}^nC_n}{{}^nC_1+{}^nC_2+....+{}^nC_n}$

$$=\frac{\sum_{r=0}^{n} r.{}^nC_r}{\sum_{r=1}^{n} {}^nC_r}=\frac{\sum_{r=1}^{n} r.\frac{n}{r}\,{}^{n-1}C_{r-1}}{\sum_{r=1}^{n} {}^nC_r}=\frac{n\sum_{r=1}^{n} {}^{n-1}C_{r-1}}{\sum_{r=1}^{n} {}^nC_r}=\frac{n\left(2^{n-1}\right)}{\left(2^n-1\right)}$$

3. Sol: option-4

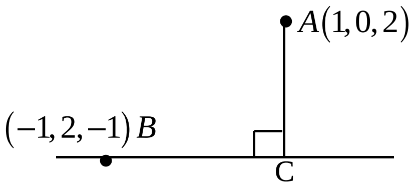
In truth table last column is all true

It is tautology

p	q	~p	~q	$p\vee q$	$\sim p\vee\sim q$	$(\sim p\vee\sim q)\vee(p\vee q)$
T	T	F	F	T	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	T	T

4. Sol:. Option-2

$C=(-1+3\lambda,\ 2-2\lambda,\ -1-\lambda)$



D.r's of $\overline{AC}=(-2+3\lambda,2-2\lambda,-3-\lambda)$

D.r's of the line $\overline{BC} = (3,-2,-1)$

$\overline{AC} \perp \overline{BC} \Rightarrow (-2+3\lambda).3+(2-2\lambda)(-2)+(-3-\lambda)(-1)=0$

$\Rightarrow -6+9\lambda-4+4\lambda+3+\lambda=0 \Rightarrow 14\lambda-7=0 \Rightarrow \lambda=\frac{1}{2}$

$C=\left(\frac{1}{2},1,-\frac{3}{2}\right)$

5.Sol: Option-3

Here , $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.......\infty=\frac{\pi^2}{8}$

Let $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.......\infty=x$

Then, $x=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.......\infty$

$=\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.......\infty\right)+\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+.......\infty\right)$

$=\frac{\pi^2}{8}+\frac{1}{4}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.......\infty\right)=\frac{\pi^2}{8}+\frac{1}{4}x$

$\Rightarrow \frac{3x}{4}=\frac{\pi^2}{8} \Rightarrow x=\frac{\pi^2}{6}$

6. Sol;- option-1

Equation of Tangent at $P(\sqrt{3},1)$ is $x.\sqrt{3}+y.1-4=0$ --------------- (1)

Let required tangent to $(x-3)^2+y^2=1$ perpendicular to (1) is $x-\sqrt{3}y+k=0$

Use $r=d$.

7. *Ans-* ***option-2***

$\Rightarrow sin(x+y)=1 \Rightarrow cos(x+y)=0 \Rightarrow y=0 \Rightarrow sin\, x=1$ $\Rightarrow$ *No. of points= 2*

---

8 sol : option-2 $\int \frac{dx}{x+1}=\int dt$

$\log(x+1)=t+c$

$\Rightarrow (0,0) \Rightarrow c=0$

$x=99 \Rightarrow t=\log 100=2\log_e 10$

9.Sol: option-2distance = $\frac{\bar{a}.\bar{n}-\bar{d}}{|n|}$

$= \frac{(2i+2j+3k).(i+5j+k)-5}{\sqrt{1^2+5^2+1^2}} = \frac{10}{3\sqrt{3}}$

10. Sol.:option-3 $-x^2y^2.\frac{xdy-ydx}{x^2}+x(ydx+xdy)=0$

$\frac{-y}{x}d\left(\frac{y}{x}\right)+\frac{dxy}{x^2y^2}=0$

On integration $-\frac{\left(\frac{y}{x}\right)^2}{2}-\frac{1}{xy}=c$ passes through (1, 2) $\Rightarrow c=\frac{-5}{3}$

$\Rightarrow y^3+2x-5x^2y=0$

11.

Sol. option-3

$\Rightarrow P(E)=\frac{50}{100}=\frac{1}{2}$

12. Sol:.option-2

Replace $'x'$ with $x+1 \Rightarrow f(x+6)+f(x+7)=0 \rightarrow$ (2)

(1) – (2) $\rightarrow$ $f(x+5)=f(x+7)$ of $X+5=x$

$\Rightarrow f(x)=f(x+2)$

$\Rightarrow$ Period = 2

13.Sol: option-2

now according to condition $a>1$

$b^2-10b+25=a>1$

$\Rightarrow b^2-10b+24>0$

$\Rightarrow (b-4)(b-6)>0$ *i.e.* $b<4$ & $b>6$

14.Sol: option-3

$I = \int_0^{\pi} [\cot x] dx = \int_0^{\pi} [\cot(\pi - x)] dx = \int_0^{\pi} [-\cot x] dx$

Adding we have , $2I = \int_0^{\pi} \{[\cot x] + [-\cot x]\} dx$

$= \int_0^{\pi} -1 dx = [-x]_0^{\pi} = -\pi$

$\therefore I = -\frac{\pi}{2}$

15.Sol; option-1

$O < x < 1$
$\Rightarrow x^2 > x^3$
$\Rightarrow 2^{x^2} > 2^{x^3}$
$\Rightarrow \int_0^1 2^{x^2} dx > \int_0^1 2^{x^3} dx$
$\Rightarrow I_1 > I_2$

16
Sol:option-3
$x^2 - 8x + (4y^2 + 12) = 0$ is a quadratic in $'x'$, $'x'$ is real then discriminate ≥ 0

17.
Sol:. option-2
$\because \alpha^5 = 1$

$\therefore |1 + \alpha + \alpha^2 + \alpha^{-2} - \alpha^{-1}| = |1 + \alpha + \alpha^2 + \alpha^3 - \alpha^4|$

$= |1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 - 2\alpha^4|$

$= \left|\frac{1 - \alpha^5}{1 - \alpha} - 2\alpha^4\right| = |2\alpha^4| = 2|\alpha|^4 = 2 \times 1 = 2$

18.
Sol: option-1

Consider
$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$
$\Rightarrow f(x) = ax^4 + bx^3 + cx^2 + dx + e$
$f(0) = e$

And
$f(3) = 81a + 27b + 9c + 3d + e$
$= 3(27a + 9b + 3c + d) + e = 0$

Hence,Rolle's theorem is applicable for f(x),
$\Rightarrow$ there exists at least one c in (a,b) such that $f'(c) = 0$.

19.

Sol: option-2

Expression $= \left(1 + x^2\right)^{40} . \left(x + \frac{1}{x}\right)^{-10} = \left(1 + x^2\right)^{30} . x^{10}$

The coefficient of x^{20} in $x^{10}\left(1 + x^2\right)^{30}$

= the coefficient of x^{10} in $\left(1 + x^2\right)^{30}$

$= {}^{30}C_5 = {}^{30}C_{25}$

20.

Sol;option-2

$my^2 + (1-m^2)xy - mx^2 = 0 \Rightarrow y^2 - x^2 + (\frac{1}{m} - m)\text{xy=0}$

$(y - mx)(y + \frac{x}{m}) = 0$

The bisectors of xy=0 (coordinate axes) are y=x and y = - x

(i) $\Rightarrow m = 1, -1$

21.Sol:optton-2

Slope of the give line = $-\frac{3}{2}$

The points on the curve at which the tangent is parallel to the given line. So, differentiating both sides with respect to x of $3x^2 - 4y^2 = 72$ we get

$\frac{dy}{dx} = \frac{3x}{4y} = \frac{-3}{2}(given)$

$\Rightarrow \frac{x}{y} = -2$

Now $3\left(\frac{x}{y}\right)^2 - 4 = \frac{72}{y^2} \Rightarrow y^2 = 9 \Rightarrow y = -3, 3$

So, points are $(-6,3)$ *and* $(6,-3)$

Now, distance of $(-6,3)$ from the given line = $\left|\frac{-18+6+1}{\sqrt{13}}\right| = \frac{11}{\sqrt{13}}$

And distance of $(6,-3)$ from the given line = $\left|\frac{18-6+1}{\sqrt{13}}\right| = \frac{13}{\sqrt{13}}$

Clearly , the required point is on $(-6,3) = (x_o, y_o)$ (given)

So, $x_o = -6, y_o = 3$

Hence $\left(x_0 + y_o\right) = -6 + 3 = -3$

22.

Sol: optton-1

Let $g(x) = (\sin x)^{\ln x} = e^{\ln x . \ln(\sin x)}$

$$f(x) = g'(x) = (\sin x)^{\ln x}\left[\cot x(\ln x) + \frac{\ln(\sin x)}{x}\right]$$

Hence $f\left(\frac{\pi}{2}\right) = g'\left(\frac{\pi}{2}\right) = 1(0+0) = 0$

23

Sol; optton-2.

$|iz + z_1 - 4| = |z + 2i - 3 - 4i|$

$\leq |z + 2i| + |3 + 4i|$

$\leq 1 + 5$

≤ 6

24.

Sol: optton-3

.

$$f(x) = \begin{cases} -2x \ for \ \ x \leq -1 \\ 2 \ for \ -1 \leq x \leq 1 \\ 2x \ for \ x \geq 1 \end{cases}$$

F is diff in $(-1,1)$

25.

Sol: optton-2 , squaring on both sides , $3 = 1 + 1 + 2\hat{a}.\hat{c}$

$\cos\theta = \frac{1}{2}$

26..

Sol: optton-2

. Let $f(x) = x^3 - 3x + 1$

Then $f^1(x) = 3\left(x^2 - 1\right)$

Let $f^1(x) = 0 \Rightarrow x = \pm 1$ and f(1) f(–1) < 0

27.

sol: optton-2

$10 - 3 \times 2 = 4$

$s_1 + s_2 + s_3 = 4$

No. of non negative integral solutions of $s_1 + s_2 + s_3 = 4$ is $(4+3-1)c_{3-1} = 6c_2$

28..

Sol: optton-2 $P^3 = Q^3$ ------ (1)

$P^2Q = PQ^2$ -------(2)

(1) – (2) $P^2(P-Q)+Q^2(P-Q)=0$

$(P-Q)(P^2+Q^2)=0$

$|P^2+Q^2|=0$

29.

Sol optton-3;

$\underset{x\to 0}{Lt}\left(\frac{a^x+b^x}{2}\right)^{2/x}=6$

$= e^{\underset{x\to 0}{Lt}\left(\frac{a^x-1}{x}\right)+\left(\frac{b^x-1}{x}\right)}=6$

$= e^{\log a+\log b}=6$

ab = 6

$(a,b)=(1,6),(6,1),(2,3),(3,2)$

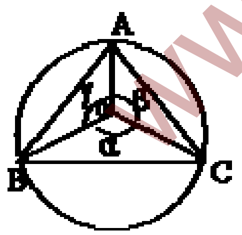
Required probability = $\frac{4}{6\times 6}=\frac{1}{9}$

30.Sol; optton-1

Let ‘r’ be radius of circle.

$3+4+5=r(\alpha+\beta+\gamma)=r.2\pi$

$\Rightarrow r=\frac{6}{\pi}$



$\therefore \Delta ABC=\Delta OBC+\Delta OCA+\Delta OAB$

$=\frac{1}{2}r^2[\sin(\frac{3}{r})+\sin(\frac{4}{r})+\sin(\frac{5}{r})]$

$$= \frac{1}{2} \times \frac{36}{\pi^2} \times [1 + \frac{\sqrt{3}}{2} + \frac{1}{2}]$$

$$= \frac{9\sqrt{3}(1+\sqrt{3})}{\pi^2}$$