MATHEMATICS PAPER IIA

TIME : 3hrs Max. Marks.75

Note: This question paper consists of three sections A,B and C.

SECTION - A

Very Short Answer Type Questions.

10X2 =20

1. If $x^2 + bx + c = 0$, $x^2 + cx + b = 0$ $(b \neq c)$ have a common root, then show that $b + c + 1 = 0$.

2. If $-1, 2$ and α are the roots of $2x^3 + x^2 - 7x - 6 = 0$, then find α

3. Simplify $-2i(3 + i)(2 + 4i)(1 + i)$ and obtain the modulus of that complex number.

4. If $z = 2 - 3i$, then show that $z^2 - 4z + 13 = 0$.

5. Find all values of $(-i)^{\frac{1}{6}}$

6. Find the number of different ways of preparing a garland using 7 distinct red roses and 4 distinct yellow roses such that no two yellow roses come together.

7. Find the number of positive division of 1080.

8. If ${}^{22}C_r$ is the largest binomial coefficient in the expansion of $(1 + x)^{22}$, find the value of ${}^{13}C_r$.

9.

X=x	–2	–1	0	1	2	3
P(X=x)	0.1	k	0.2	k	0.3	k

is the probability distribution of a random variable x. find the value of K.

10. Find the mean deviation about the median for the following data.

13, 17, 16, 11, 13, 10, 16, 11, 18, 12, 17

SECTION- B

Short Answer Type Questions.

Answer Any Five of The Following 5 X 4 = 20

11. Prove that $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4 if $x \in R$.

12. The points P, Q denote the complex numbers z_1, z_2 in the Argand diagram. O is origin. If $z_1\bar{z}_2+\bar{z}_1 z_2=0$ then show that $\angle POQ = 90°$.

13. Prove that $\frac{{}^{4n}C_{2n}}{{}^{2n}C_n}=\frac{1.3.5.....(4n-1)}{\left\{1.3.5......(2n-1)^2\right\}}$

14. If the letters of the word EAMCET are permuted in all possible ways and if the words thus formed are arranged in the dictionary order. Find the rank of the word EAMCET.

15. Find the coefficient of x^4 in the expansion of $\frac{3x}{(x-2)(x+1)}$.

16. State and prove Addition Theorem on Probability.

17. A speaks the truth in 75% of the cases, B is 80% cases. What is the probability that their statements about an incident do not match?

SECTION- C

Long Answer Type Questions.

Answer Any Five of the Following 5 X 7 = 35

18. Solve $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$. Given that it has two pairs of equal roots

19. Solve $(x - 1)^n = x^n$, n is a positive integer.

20. If x = $\frac{1}{5} + \frac{1 \cdot 3}{5 \cdot 10} + \frac{1 \cdot 3 \cdot 5}{5 \cdot 10 \cdot 15} + \ldots \infty$, find $3x^2 + 6x$.

21. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1 + x)^{th}$ are in A.P. then show that $n^2 - (4r + 1)n + 4r^2 - 2 = 0$.

22. Two dice are rolled at random. Find the probability distribution of the sum of the numbers on them. Find the mean of the random variable.

23. Three boxes B_1, B_2 and B_3 contain balls detailed below.

	White	Black	Red
B_1	2	1	2
B_2	3	2	4
B_3	4	3	2

A die is thrown, B_1 is chosen if either 1 or 2 turns up, B_2 is chosen if 3 or 4 turns up and B_3 is chosen if 5 or 6 turns up. Having chosen a box in this way, a ball is chosen at random from this box. If the ball drawn is of red colour, what is the probability that it comes from box B_2?

24. The following table gives the daily wages of workers in a factor. Compute the standard deviation and the coefficient of variation of the wages of the workers.

Wages (Rs.)	125-175	175-225	225-275	275-325	325-375	375-425	425-475	475-525	525-575
Number of workers	2	22	19	14	3	4	6	1	1

MATHS IIA PAPER 1 - SOLUTIONS

1. If $x^2 + bx + c = 0$, $x^2 + cx + b = 0$ $(b \neq c)$ have a common root, then show that $b + c + 1 = 0$.

Sol: $x^2 + bx + c = 0$

$x^2 + cx + b = 0$

α is common root.

$\alpha^2 + b\alpha + c = 0$

$\alpha^2 + c\alpha + b = 0$

$\alpha(b - c) + c - b = 0$

$\alpha(b - c) = b - c$

$\alpha = 1$

$\therefore 1 + b + c = 0$

2. If $-1, 2$ and α are the roots of $2x^3 + x^2 - 7x - 6 = 0$, then find α

Sol: $-1, 2, \alpha$ are roots of $2x^3 + x^2 - 7x - 6 = 0$

Sum $= -1 + 2 + \alpha = -\frac{1}{2}$

$\alpha = -\frac{1}{2} - 1 = -\frac{3}{2}$

3. Simplify $-2i(3 + i)(2 + 4i)(1 + i)$ and obtain the modulus of that complex number.

Sol: $z = -2i(3 + i)(2 + 4i)(1 + i)$

$= -2i(2 + 14i)(1 + i)$

$= -2i(2 + 2i + 14i - 14)$

$= -2i(-12 + 16i)$

$= 24i + 32$

$= 8(4 + 3i)$

$|z|^2 = 64 \cdot 25$

$|z| = 8 \times 4 = 40$

4. If z = 2 – 3i, then show that $z^2 - 4z + 13 = 0$.

Sol: $z = 2 - 3i \Rightarrow z - 2 = -3i \Rightarrow (z-2)^2 = (-3i)^2$

$\Rightarrow z^2 + 4 - 4z = -9$

$\Rightarrow z^2 - 4z + 13 = 0.$

5. Find all values of $(-i)^{\frac{1}{6}}$

Solution :

$$(-i)^{\frac{1}{6}} = \left\{\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)\right\}^{\frac{1}{6}}$$

$$= cis\left(\frac{2k\pi - \pi/2}{6}\right) k = 0, 1, 2, 3, 4, 5$$

$$\therefore (-1)^{\frac{1}{6}} = cis(4k-1)\frac{\pi}{12} \quad k = 0, 1, 2, 3$$

6. Find the number of different ways of preparing a garland using 7 distinct red roses and 4 distinct yellow roses such that no two yellow roses come together.

Sol: First arrange 7 red roses in a circular form (garland form) in (7 - 1)! = 6! ways. Now, there are 7 gaps and 4 yellow roses can be arranged in these 7 gaps in 4 $^7P_4$ ways.

Thus, the total number of circular permutations is 6! x$^7P_4$.

But, in the case of garlands, clock-wise and anti clock-wise arrangements look alike. Hence, the number of required ways is. $\frac{1}{2}(6! \times {}^7P_4))$

7. Find the number of positive division of 1080.

Sol: $1080 = 2^3 \times 3^3 \times 5^1$

$\therefore$ The number of positive divisions of 1080

$= (3+1)(3+1)(1+1)$

$= 4 \times 4 \times 2 = 32$

8. If $^{22}C_r$ is the largest binomial coefficient in the expansion of $(1 + x)^{22}$, find the value of $^{13}C_r$.

Sol. Here n = 22 is an even integer. There is only one largest binomial coefficient and it is

$$^{n}C_{(n/2)} = {}^{22}C_{11} = {}^{22}C_r \Rightarrow r = 11$$

$$\therefore {}^{13}C_r = {}^{13}C_{11} = {}^{13}C_2 = \frac{13\times 12}{1\times 2} = 78$$

9.

X=x	**–2**	**–1**	**0**	**1**	**2**	**3**
P(X=x)	**0.1**	**k**	**0.2**	**k**	**0.3**	**k**

is the probability distribution of a random variable x. find the value of K.

Sol. We know that $\sum_{i=1}^{n} p(x_i) = 1$

0.1 + k + 0.2 + k + 0.3 + k = 1

$\Rightarrow$ 3k + 0.6 = 1

3k = 1– 0.6 = 0.4$\Rightarrow$ k = $\frac{0.4}{4} = 0.1$

10. Find the mean deviation about the median for the following data.

i) 13, 17, 16, 11, 13, 10, 16, 11, 18, 12, 17

ii) 4, 6, 9, 3, 10, 13, 2

Sol. Expressing the given data in the ascending order.

We get 10, 11, 11, 12, 13, 13, 16, 16, 17, 17, 18

Mean (M) of these 11 observations is 13.

The absolute values of deviations are $|x_i - M| = 3, 2, 2, 1, 0, 0, 3, 3, 4, 4, 5$

$$\therefore \text{Mean deviation about Median} = \frac{\sum_{i=1}^{11} |x_i - M|}{n} = \frac{3+2+2+1+0+0+3+3+4+4+5}{11}$$

$$= \frac{27}{11} = 2.45$$

11. Prove that $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$ **does not lie between 1 and 4 if** $x \in R$**.**

Sol: $y=\frac{x+1+3x+1-1}{(3x+1)(x+1)}$

$$y=\frac{4x+1}{3x^2+4x+1}$$

$$3yx^2+x(4y-4)+y-1=0$$

Discriminant ≥ 0

$$4(y-1)^2-4\cdot 3y(y-1)\geq 0$$

$$16(y-1)^2-12y(y-1)\geq 0$$

$$4(y-1)[4(y-1)-3y)]\geq 0$$

$$4(y-1)(y-4)\geq 0$$

$$(y-1)(y-4)\geq 0$$

$\Rightarrow y\geq 4$ or $y\leq 1.$

12. The points P, Q denote the complex numbers z_1, z_2 in the Argand diagram. O is origin. If $z_1\bar{z}_2+\bar{z}_1z_2=0$ **then show that** $\angle POQ = 90°$**.**

Sol: $z_1\bar{z}_2+\bar{z}_1z_2=0$

$$\frac{z_1\bar{z}_2+\bar{z}_1z_2}{z_2\bar{z}_2}=0 \Rightarrow \text{Real of } \frac{z_1}{z_2}=0$$

$$\left(\frac{z_1}{z_2}+\frac{\bar{z}_1}{z_2}\right)=0$$

Imaginary part of $\left(\frac{z_1}{z_2}\right)$ is k.

$$\left(\frac{z_1}{z_2}\right)+\left(\frac{\bar{z}_1}{z_2}\right)=0$$

Or $\frac{z_1}{z_2}$ is purely imaginary, $\frac{z_1}{z_2}=ki.$

$$\Rightarrow \text{Arg}\left(\frac{z_1}{z_2}\right)=\frac{\pi}{2}.$$

13. Prove that $\frac{{}^{4n}C_{2n}}{{}^{2n}C_n}=\frac{1.3.5.....(4n-1)}{\left\{1.3.5......(2n-1)^2\right\}}$

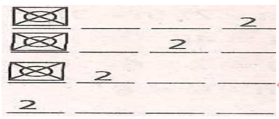
Sol: $\frac{{}^{4n}C_{2n}}{{}^{2n}C_n}=\frac{\frac{(4n)!}{(2n)!(2n)!}}{\frac{(2n)!}{n!n!}}$

$$=\frac{(4n)!}{(2n)!^2}\times\frac{n!n!}{(2n)!}$$

$$=\frac{(4n)(4n-1)(4n-2)(4n-3)(4n-4).......5.4.3.2.1}{\left[(2n)(2n-1)(2n-2)(2n-3)(2n-4).......5.4.3.2.1\right]^2}\times\frac{n!n!}{(2n)!}$$

$$=\frac{\left[(4n-1)(4n-3).....5.3.1\right]\left[(2n)(2n-1)(2n-2).....2.1.2^{2n}\right]}{\left[\{(2n-1)(2n-3)......5.3.1\}\{n(n-1)(n-2)....2.1.2^n\}\right]^2}\times\frac{n!n!}{(2n)!}$$

$$=\frac{\left[(4n-1)(4n-3).....5.3.1\right](2n)!\,2^{2n}}{\left[(2n-1)(2n-3).....5.3.1\right]^2[n!]^2(2n)^2}\frac{(n!)^2}{(2n)!}$$



$$\therefore\frac{{}^{4n}C_{2n}}{{}^{2n}C_n}=\frac{1.3.5......(4n-1)}{\left[1.3.5....(2n-1)^2\right]}$$

14. If the letters of the word EAMCET are permuted in all possible ways and if the words thus formed are arranged in the dictionary order. Find the rank of the word EAMCET.

Sol: The dictionary order of the letters of the word EAMCET IS

A C EE M T

In the dictionary order first gives the words which begins with the letters A.

If we fill the first place with A, remaining 5 letters can be arranged in $\frac{5!}{2!}$ ways (since there are 2 E's) on proceeding like this, we get

A ------------->$\frac{5!}{2!}$ words

C ------------->$\frac{5!}{2!}$ words

E A C -------------> 3! words

E A E -------------> 3! words

E A M C E T -------------> 1 word

Hence the rank of the word EAMCET is

$=2\times\frac{5!}{2!}+2\times 3!+1$

$=120+12+1=133$

15. Find the coefficient of x^4 in the expansion of $\frac{3x}{(x-2)(x+1)}$.

Sol. $\frac{3x}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}$

Multiplying with $(x-2)(x+1)$

$3x = A(x+1) + B(x-2)$

Put $x = -1$, $-3 = B(-3) \Rightarrow B = 1$

Put $x = 2$, $6 = A(3) \Rightarrow A = 2$

$\therefore \frac{3x}{(x-2)(x+1)}=\frac{2}{x-2}+\frac{1}{x+1}$

$=\frac{2}{-2\left(1-\frac{x}{2}\right)}+\frac{1}{1+x}=-\left(1-\frac{x}{2}\right)^{-1}+(1+x)^{-1}$

$=-\left[1+\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{8}+\frac{x^4}{16}+...\right]$

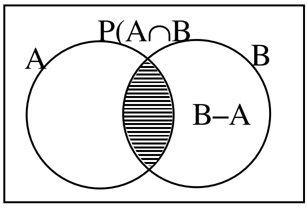
$+\left(1-x+x^2-x^3+x^4...\right)$

$\therefore$ Coefficient of $x^4 = -\frac{1}{16}+1=\frac{15}{16}$

16. State and prove Addition Theorem on Probability.

If A, B are two events in a sample space S, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Sol. From the figure (venn diagram) it can be observed that $(B - A) \cup (A \cap B) = B$, $(B - A) \cap (A \cap B) = \phi$.



$$\therefore P(B) = P[(B-A) \cup (A \cap B)]$$
$$= P(B-A) + P(A \cap B)$$
$$\Rightarrow P(B-A) = P(B) - P(A \cap B) \quad ...(1)$$

Again from the figure, it can be observed that

$$A \cup (B-A) = A \cup B, A \cap (B-A) = \phi$$
$$\therefore P(A \cup B) = P[A \cup (B-A)]$$
$$= P(A) + P(B-A)$$
$$= P(A) + P(B) - P(A \cap B) \text{ since from (1)}$$
$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

17. A speaks the truth in 75% of the cases, B is 80% cases. What is the probability that their statements about an incident do not match?

Sol. Let E_1, E_2 be the events that A and B respectively speak truth about an incident.

Then $P(E_1) = \frac{75}{100} = \frac{3}{4}$, $P(E_2) = \frac{80}{100} = \frac{4}{5}$

So that $P\left(E_1^C\right) = \frac{1}{4}$, $P\left(E_2^C\right) = \frac{1}{5}$

Let E be the event that their statements do not match about the incident. Then this happens in two mutually exclusive ways.

i) A speaks truth, B tells lie

ii) A tells lie, B speaks truth. These two events are represented by $E_1 \cap E_2^C$, $E_1^C \cap E_2$.

$\therefore P(E) = P(E_1 \cap E_2^C) + P(E_1^C \cap E_2)$

$= P(E_1)P(E_2^C) + P(E_1^C)P(E_2)$

($\because E_1, E_2$ are independent)

$= \frac{3}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{4}{5} = \frac{7}{20}$

18. Solve $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$. Given that it has two pairs of equal roots

Sol: Given equation is

$x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$

Let the roots be $\alpha, \alpha, \beta, \beta$

Sum of the roots , $2(\alpha + \beta)$

$= -4 \Rightarrow \alpha + \beta = -2$

Let $\alpha\beta = p$

The equation having root α, β is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

I.e., $x^2 + 2x + p = 0$

$\therefore x^4 + 4x^3 - 2x^2 - 12x + 9$

$= \left[x^2 - (\alpha + \beta)x + \alpha\beta\right]^2$

$= x^4 + 4x^3 + (2p + 4)x^2 + 4px + p^2$

Comparing coefficients of x on both sides

$4p = -12 \Rightarrow p = -3$

$x^2 + 2x + p = 0$

$\therefore x^4 + 4x^3 + (2p + 4)x^2 + 4px + p^2$

Comparing coefficients of x on both sides

$4p = -12 \Rightarrow p = -3$

$x^2 + 2x + p = 0 \Rightarrow x^2 + 2x - 3 = 0$

$\Rightarrow (x+3)(x-1) = 0$

$\Rightarrow x = -3, 1$

$\therefore$ The root s of the given equation are $-3, -3, 1, 1$

19. Solve $(x – 1)^n = x^n$, n is a positive integer.

Sol: $\left(\frac{x-1}{x}\right)^n = 1$

$$\frac{x-1}{x} = (1)^{1/n}$$

$$\frac{x-1}{x} = [\cos 2m\pi + i\sin 2m\pi]^{1/n}$$

$$\frac{x-1}{x} = \cos\frac{2m\pi}{n} + i\sin\frac{2m\pi}{n}$$

$$1 - \frac{1}{x} = e^{i\frac{2m}{n}\pi}$$

$$1 - \cos\frac{2m\pi}{n} - i\sin\frac{2m\pi}{n} = \frac{1}{x}$$

$$2\sin^2\frac{m\pi}{n} - 2i\sin\frac{m\pi}{n}\cos\frac{m\pi}{n} = \frac{1}{x}$$

$$-2\sin^2\frac{m\pi}{n}\left[\cos\frac{m\pi}{n} - i\sin\frac{m\pi}{n}\right] = \frac{1}{x}$$

$$x = \frac{1}{-2i\sin\frac{m\pi}{n}\left[\cos\frac{m\pi}{n} - i\sin\frac{m\pi}{n}\right]}$$

$$x = \frac{\cos\frac{m\pi}{n} + i\sin\frac{m\pi}{n}}{-2i\sin\frac{m\pi}{n}}$$

$\frac{1}{2}\left[1 + i\cot\frac{m\pi}{n}\right]$; m = 1, 2, 3, …(n – 1)

20. If $x = \frac{1}{5} + \frac{1\cdot 3}{5\cdot 10} + \frac{1\cdot 3\cdot 5}{5\cdot 10\cdot 15} + \ldots\infty$, find $3x^2 + 6x$.

Sol. Given that

$$x = \frac{1}{5} + \frac{1\cdot 3}{5\cdot 10} + \frac{1\cdot 3\cdot 5}{5\cdot 10\cdot 15} + \ldots\ldots$$

$$= \frac{1}{5} + \frac{1\cdot 3}{1\cdot 2}\left(\frac{1}{5}\right)^2 + \frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\left(\frac{1}{5}\right)^3 + \ldots\ldots$$

$$\Rightarrow 1 + x = 1 + 1\cdot\frac{1}{5} + \frac{1\cdot 3}{1\cdot 2}\left(\frac{1}{5}\right)^2 + \frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\left(\frac{1}{5}\right)^3 + \ldots$$

$$= 1 + \frac{p}{1!}\frac{1}{5} + \frac{p(p+q)}{2!}\left(\frac{1}{5}\right)^2 + \frac{p(p+q)(p+2q)}{3!}\left(\frac{1}{5}\right)^3 = (1-x)^{-p/q}$$

Here $p = 1, q = 2, \frac{x}{q} = \frac{1}{5} \Rightarrow x = \frac{2}{5}$

$$= \left(1 - \frac{2}{5}\right)^{-1/2} = \left(\frac{3}{5}\right)^{-1/2} = \sqrt{\frac{5}{3}}$$

$$\Rightarrow 1 + x = \sqrt{\frac{5}{3}} \Rightarrow 3(1+x)^2 = 5$$

$$\Rightarrow 3x^2 + 6x + 3 = 5 \Rightarrow 3x^2 + 6x = 2$$

21. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1 + x)^{th}$ are in A.P. then show that $n^2 - (4r + 1)n + 4r^2 - 2 = 0$.

Sol. Coefficient of $T_r = {}^nC_{r-1}$

Coefficient of $T_{r+1} = {}^nC_r$

Coefficient of $T_{r+2} = {}^nC_{r+1}$

Given ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$ are in A.P.

$\Rightarrow 2\,{}^nC_r = {}^nC_{r-1} + {}^nC_{r+1}$

$$\Rightarrow 2\frac{n!}{(n-r)!r!} = \frac{n!}{(n-r+1)!(r-1)!} + \frac{n!}{(n-r-1)!(r+1)!}$$

$$\Rightarrow \frac{2}{(n-r)r} = \frac{1}{(n-r+1)(n-r)} + \frac{1}{(r+1)r}$$

$$\Rightarrow \frac{1}{n-r}\left[\frac{2}{r} - \frac{1}{n-r+1}\right] = \frac{1}{(r+1)r}$$

$$\Rightarrow \frac{1}{n-r}\left[\frac{2n-2r+2-r}{r(n-r+1)}\right] = \frac{1}{r(r+1)}$$

$$\Rightarrow (2n-3r+2)(r+1) = (n-r)(n-r+1)$$

$$\Rightarrow 2nr + 2n - 3r^2 - 3r + 2r + 2 = n^2 - 2nr + r^2 + n - r$$

$$\Rightarrow n^2 - 4nr + 4r^2 - n - 2 = 0$$

$$\therefore n^2 - (4r+1)n + 4r^2 - 2 = 0$$

22. Two dice are rolled at random. Find the probability distribution of the sum of the numbers on them. Find the mean of the random variable.

Sol. When two dice are rolled, the sample space S contains $6 \times 6 = 36$ sample points.

S = {(1, 1), (1, 2)…(1, 6), (2, 1), (2, 2)…(6,6)}

Let x denote the sum of the numbers on the two dice

Then the range x = {2, 3, 4, …12}

Probability Distribution of x is given by the following table.

$X=x_i$	2	3	4	5	6	7	8	9	10	11	12
$P(X=x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
$X_i.p(x_i)$	2/36	6/36	12/36	20/36	30/36	42/36	40/36	36/36	30/36	22/36	12/36

Mean of x = $\mu = \sum_{1-2}^{12} x_1 p(X = x_1)$

$$= 2.\frac{1}{36} + 3.\frac{2}{36} + 4.\frac{3}{36} + 5.\frac{4}{36} + 6.\frac{5}{36} + 7.\frac{6}{36} + 8.\frac{5}{36} + 9.\frac{4}{36} + 10.\frac{3}{36} + 11.\frac{2}{36} + 12.\frac{1}{36}$$

$$= \frac{1}{36}(2+6+12+20+30+42+40+36+30+22+12)$$

$= \frac{252}{36} = 7$

23. Three boxes B_1, B_2 and B_3 contain balls detailed below.

	White	Black	Red
B_1	**2**	**1**	**2**
B_2	**3**	**2**	**4**
B_3	**4**	**3**	**2**

A die is thrown, B_1 is chosen if either 1 or 2 turns up, B_2 is chosen if 3 or 4 turns up and B_3 is chosen if 5 or 6 turns up. Having chosen a box in this way, a ball is chosen at random from this box. If the ball drawn is of red colour, what is the probability that it comes from box B_2?

Sol. Let $P(E_i)$ be the probability of choosing the box B_i (i = 1, 2, 3).

Then $P(E_i) = \frac{2}{6} = \frac{1}{3}$; for i = 1, 2, 3

Having chosen the box B_i, the probability of drawing a red ball, say, $P(R/E_i)$ is given by

$P\left(\frac{R}{E_1}\right) = \frac{2}{5}$, $P\left(\frac{R}{E_2}\right) = \frac{4}{9}$ and $P\left(\frac{R}{E_3}\right) = \frac{2}{9}$

We have to find the probability $P(E_2/R)$

By Bayer's theorem, we get

$$P\left(\frac{E_2}{R}\right) = \frac{P(E_2)P(R/E_2)}{P(E_1)P\left(\frac{R}{E_1}\right) + P(E_2)P\left(\frac{R}{E_2}\right) + P(E_3)P\left(\frac{R}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{4}{9}}{\frac{1}{3}\left(\frac{2}{5} + \frac{4}{9} + \frac{2}{9}\right)} = \frac{\frac{4}{18}}{\frac{18+20+10}{5\times9\times3}} = \frac{5}{12}$$

24. The following table gives the daily wages of workers in a factor. Compute the standard deviation and the coefficient of variation of the wages of the workers.

Wages (Rs.)	**125-175**	**175-225**	**225-275**	**275-325**	**325-375**	**375-425**	**425-475**	**475-525**	**525-575**
Number of workers	**2**	**22**	**19**	**14**	**3**	**4**	**6**	**1**	**1**

Sol. We shall solve this problem using the step deviation method, since the mid points of the class intervals are numerically large.

Here h = 50. Take a = 300. Then $y_i = \dfrac{x_i - 300}{50}$

Mid point x_i	**frequency f_i**	**y_i**	**$f_i y_i$**	**$f_i y_i^2$**
150	2	–3	–6	18
200	22	–2	–44	88
250	19	–1	–19	19
300	14	0	0	0
350	3	1	3	3
400	4	2	8	16
450	6	3	18	54
500	1	4	4	16
550	1	5	5	25
	N = 72		$\Sigma f_i y_i = -31$	$\Sigma f_i y_i^2 = 239$

Mean $\bar{x} = A + \left(\dfrac{\Sigma f_i y_i}{M}\right) \times h = 300 + \left(\dfrac{-31}{72}\right) 50 = 300 - \dfrac{1550}{72} = 278.47$

Variance $(\sigma_x^2) = \dfrac{h^2}{N^2}\left[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2\right]$

$$= \frac{2500}{72\times72}\left[72(239)-(31\times31)\right]$$

$$\sigma_x = \sqrt{2500\left(\frac{239}{72}-\frac{961}{72\times72}\right)} = 88.52$$

Coefficient of variation = $\frac{88.52}{278.47}\times100 = 31.79$.