# MATHEMATICS PAPER IIB

COORDINATE GEOMETRY AND CALCULUS.

Time: 3hrs Max. Marks.75

**Note:** This question paper consists of three sections A,B and C.

## SECTION -A

**Very Short Answer Type Questions.** 10 X 2 =20

1. Show that the equation of a circle having the line segment joining A($x_1$, $y_1$) and B($x_2$, $y_2$) as diameter is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.

2. Show that the points (-6, 1), (2, 3) are Conjugate points with respect to the circle $x^2 + y^2 - 2x + 2y + 1 = 0$

3. Find equation of the tangent and normal to the parabola $y^2 = 6x$ at the positive end of the latus rectum.

4. Find the equation of the common chord of the following pair of circles.
$x^2 + y^2 - 4x - 4y + 3 = 0$, $x^2 + y^2 - 5x - 6y + 4 = 0$

5. If the lines $3x - 4y = 12$ and $3x + 4y = 12$ meets on a hyperbola $S = 0$ then find the eccentricity of the hyperbola $S = 0$.

6. $\int \frac{ax^{n-1}}{bx^n + C} dx$, where $n \in N$, a, b, c are real numbers, $b \neq 0$ and
$x \in I \subset \left\{x \in R : x^n \neq -\frac{c}{b}\right\}$

7. $\int \frac{\sec x}{(\sec x + \tan x)^2} dx$ on $I \subset R - \left\{(2n+1)\frac{\pi}{2}, n \in Z\right\}$.

8. Find the area bounded by the curve $y = \log x$, the X-axis and the straight line $x = e$.

9. Obtain the differential equation which corresponds to each of the following family ellipses with centres at the origin and having coordinate axes as axes.

10. Evaluate $\int_0^1 x \cdot e^{-x^2} dx$

## SECTION -B

**Short Answer Type Questions.**

Answer Any Five of the Following 5 X 4 = 20

11. Find the equation of tangents the circle $x^2 + y^2 - 10 = 0$ at the points whose abscissa are 1

12. Find the equation of the common tangent of the circles $x^2 + y^2 + 10x - 2y + 22 = 0$, $x^2 + y^2 + 2x - 8y + 8 = 0$at their point of contact

13. A man running on a race course notices that the sum of the distances of the two flag posts from him is always 10 m. and the distance between the flag posts is 8 m. Find the equation of the race course traced by the man.

14. Show that the tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at points whose eccentric angles differ by $\pi/2$ intersect on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$.

15. Find the equations of the tangents to the hyperbola $x^2 - 4y^2 = 4$ which are (i) Parallel (ii) Perpendicular to the line $x + 2y = 0$.

16. Find the area enclosed between $y = x^2 - 5x$ and $y = 4-2x$.

17. Solve $2\frac{dy}{dx}=\frac{y}{x}+\frac{y^2}{x^2}$

**SECTION- C**

**Long Answer Type Questions.**

Answer Any Five of the Following 5 X 7= 35

18. Find the equation of the circle which intersects each of the following circles orthogonally

$x^2 + y^2 + 2x + 4y + 1 = 0$; $x^2 + y^2 - 2x + 6y - 3 = 0$; $2x^2 + 2y^2 + 6x + 8y - 3 = 0$

19. Show that the circles $x^2+y^2 - 6x - 2y +1 = 0$; $x^2+y^2 + 2x - 8y + 13 = 0$ Touch each other. Find the point of contact and the equation of the common tangent at their point of contact.

20. Show that the common tangent to the circle $2x^2 + 2y^2 = a^2$ and the parabola $y^2 = 4ax$ intersect at the focus of the parabola $y^2 = -4ax$.

21. $\int \frac{2\cos x + 3\sin x}{4\cos x + 5\sin x} dx$

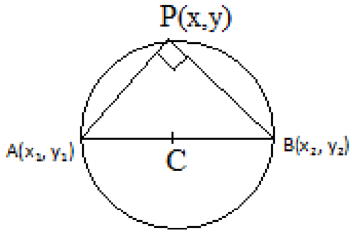
22. Obtain the reduction formula for $I_n = \int \csc^n x \, dx$, n being a positive integer, $n \geq 2$ and deduce the value of $\int \operatorname{cosec}^5 x \, dx$.

23. Evaluate $\int_0^{\pi/2} \frac{\sin^2 x}{\cos x + \sin x} dx$

24. Solve $\frac{dy}{dx}(x^2y^3 + xy) = 1$

# Maths 2B Paper 2 – Solutions

1. Show that the equation of a circle having the line segment joining A($x_1$, $y_1$) and B($x_2$, $y_2$) as diameter is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.




Let P(x,y) be any point on the circle. Given points A($x_1$, $y_1$) and B($x_2$, $y_2$).

Now $\underline{|APB} = \frac{\pi}{2}$. (Angle in a semi circle.)

Slope of AP. Slope of BP =-1

$$\Rightarrow \frac{y-y_1}{x-x_1}\frac{y-y_2}{x-x_2} = -1$$
$$\Rightarrow (y-y_2)(y-y_1) = -(x-x_2)(x-x_1)$$
$$\Rightarrow (x-x_2)(x-x_1)+(y-y_2)(y-y_1)=0$$

**2) Show that the points (-6, 1), (2, 3) areConjugate points with respect to the circle $x^2 + y^2 - 2x + 2y + 1 = 0$**

**Sol**

S= $x^2 + y^2 - 2x + 2y + 1 = 0$

Points are **(-6, 1), (2, 3)**

**Now $S_{12}$ = -6.2+1.3-(-6+2)+(1+3)+1**

**= -12+3+4+4+1 = 0.**

**Therefore given points are conjugate points.**

**3. Find equation of the tangent and normal to the parabola $y^2$ = 6x at the positive end of the latus rectum.**

**Sol.** Equation of parabola $y^2 = 6x$

$4a = 6 \Rightarrow a = 3/2$

Positive end of the Latus rectum is(a, 2a) = $\left(\frac{3}{2}, 3\right)$

Equation of tangent $yy_1 = 2a(x + x_1)$

$yy_1 = 3(x + x_1)$

$3y = 3\left(x + \frac{3}{2}\right)$

$2y – 2x – 3 = 0$ is the equation of tangent

Slope of tangent is 1

Slope of normal is –1

Equation of normal is $y - 3 = -1\left(x - \frac{3}{2}\right)$

$2x + 2y – 9 = 0$

**4. Find the equation of the common chord of the following pair of circles.**

**$x^2 + y^2 – 4x – 4y + 3 = 0,\quad x^2 + y^2 – 5x – 6y + 4 = 0$**

**Sol.** $S = x^2 + y^2 – 4x – 4y + 3 = 0$

$S^1 = x^2 + y^2 – 5x – 6y + 4 = 0$

Common chord is $S – S' = 0$

$(x^2 + y^2 – 4x – 4y + 3)–(x^2 + y^2 – 5x–6y+4) = 0$

$x + 2y – 1 = 0$

**5. If the lines 3x – 4y = 12 and 3x + 4y = 12 meets on a hyperbola S = 0 then find the eccentricity of the hyperbola S = 0.**

**Sol.** Given lines $3x – 4y = 12$, $3x + 4y = 12$

The combined equation of the lines is

(3x – 4y)(3x + 4y) = 144

$9x^2 - 16y^2 = 144$

$$\frac{x^2}{\frac{144}{9}} - \frac{y^2}{\frac{144}{16}} = 1 \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$a^2 = 16, b^2 = 9$

Eccentricity e = $\sqrt{\frac{a^2+b^2}{a^2}}$

$$= \sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

**6.** $\int \frac{ax^{n-1}}{bx^n + C}dx$, **where n** $\in$**N, a, b, c are real numbers, b** $\neq$ **0 and**

$x \in I \subset \left\{x \in R : x^n \neq -\frac{c}{b}\right\}$

**Sol.** $\int \frac{ax^{n-1}}{bx^n + C}dx$

let $bx^n$ + C=t $\Rightarrow$ $nbx^{n-1}dx=dt$, $x^{n-1}dx = \frac{1}{nb}dt$

$$\int \frac{ax^{n-1}}{bx^n + C}dx = \frac{a}{nb}\int \frac{dt}{t} = \frac{a}{nb}\log|t| + dt$$

$$= \frac{a}{nb}\log|bx^n + c| + k$$

7. $\int \frac{\sec x}{(\sec x + \tan x)^2}dx$ on $I \subset R - \left\{(2n+1)\frac{\pi}{2}, n \in Z\right\}$.

**Sol:** $\int \frac{\sec x}{(\sec x + \tan x)^2}$

$$= \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)^3}dx$$

Let sec x + tan x = t

Then (sec x tan x + $\sec^2$ x) dx = dt

$\Rightarrow \sec x(\sec x+\tan x)dx = dt$

$\therefore \int \frac{\sec x}{(\sec x+\tan x)^2}dx$

$= \int \frac{dt}{t^3} = \int t^{-3}dt = \frac{t^{-2}}{-2}$

$= -\frac{1}{2t^2} = -\frac{1}{2(\sec x+\tan x)^2}+c$

**8. Find the area bounded by the curve y = log x, the X-axis and the straight line x = e.**

**Sol:** Area bounded by the curve y = $\log_c$x,

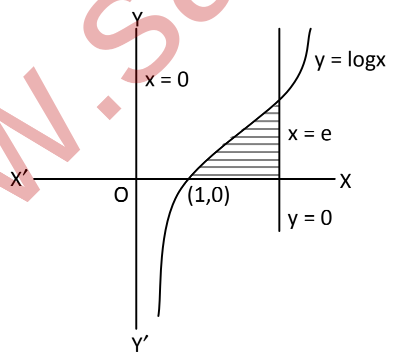
X-axis and the straight line x = e is

$= \int_1^e \log_e x\,dx$

$= [x\log x]_1^e - \int_1^e dx$

($\because$ When x = e, y = $\log_e$e = 1)

= (e – 0) – (e – 1) = 1 sq.units.



**9. Obtain the differential equation which corresponds to each of the following family**

**ellipses with centres at the origin and having coordinate axes as axes.**

**Sol.** Equation of ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Diff. w.r.t.x,

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0 \Rightarrow y.y_1=-\frac{b^2}{a^2}x$$

Diff. w.r.t.x,

$$y.y_2+y_1.y_1=-\frac{b^2}{a^2} \Rightarrow y.y_2+2y_1=\frac{y.y_1}{x}$$

$$\Rightarrow x\left(y.y_2+2y_1\right)=y.y_1$$

**10.** $\int_0^1 x\cdot e^{-x^2}dx$

**Sol.** $\int_0^1 x\cdot e^{-x^2}dx=\frac{1}{2}\int_0^1 2xe^{-x^2}dx,\quad \text{put} -x^2=t$

$$\Rightarrow -2xdx=dt \Rightarrow 2xdx=-dt$$
$$x=1 \Rightarrow t=1,\ x=0 \Rightarrow t=0$$

$$I=\frac{1}{2}\int_0^{-1}-e^t dt=\frac{1}{2}\left[-e^t\right]_0^{-1}$$

$$=\frac{1}{2}\left[e^0-e^{-1}\right]=\frac{1}{2}\left(1-\frac{1}{e}\right)$$

**11. Find the equation of tangents the circle $x^2 + y^2 - 10 = 0$ at the points whose abscissa are 1**

**Sol.** Equation of the circle is $S = x^2 + y^2 = 10$

Let the point be (1, y)

$1 + y^2 = 10 \Rightarrow y^2 = 9$

$Y = \pm 3.$

Co – ordinates of P are (1,3) and (1, -3)
Equation of the tangent at P (1, 3) is $S_1=0$.
$\Rightarrow x.\,1 + y.\,3 = 10$
$\Rightarrow x + 3y - 10 = 0$
Equation of the tangent of P(1, -3) is $S_2=0$
$\Rightarrow x.1 + y(-3) = 10 \Rightarrow x - 3y - 10 = 0$

**12. Find the equation of the common tangent of the following circles at their point of contact.**

**$x^2 + y^2 + 10x - 2y + 22 = 0,\; x^2 + y^2 + 2x - 8y + 8 = 0.$**

**Sol.** $S=x^2 + y^2 + 10x - 2y + 22 = 0$

Centre A = (-5, 1), radius $r_1 = 2$

$S' = x^2 + y^2 + 2x - 8y + 8 = 0.$

Centre B = (-1, 4) radius $r_2 = 3$

$AB = \sqrt{16+9} = 5$

Therefore AB =5 = 3+2 = $r_1+r_2$.

Given circles touch each other externally.

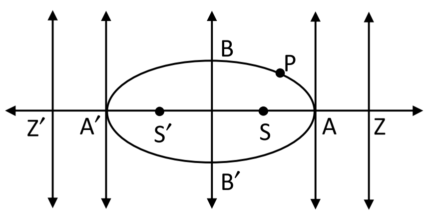
When circles touch each other, their common tangent is $S - S' = 0$

$\therefore (x^2 + y^2 + 10x - 2y + 22) - (x^2 + y^2 + 2x - 8y + 8) = 0$

$8x + 6y + 14 = 0$ (or) $4x + 3y + 7 = 0$

**13. A man running on a race course notices that the sum of the distances of the two flag posts from him is always 10 m. and the distance between the flag posts is 8 m. Find the equation of the race course traced by the man.**

**Sol:**




Given AA′ = 2a = 10 ⇒ a = 5

(Taking flag posts located at A and A′)

Also given the distance between two fixed points S and S′ = 8 m.

$\therefore$ 2ae = 8 $\Rightarrow$ ae = 4

$\therefore$ $b^2 = a^2(1 - e^2)$

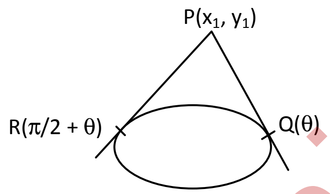
$\Rightarrow$ $a^2 - a^2e^2 = 25 - 16 = 9$

$\therefore$ $b^2 = 9$

Hence the equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

14. **Show that the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at points whose eccentric angles differ by π/2 intersect on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.**

**Sol.**




Equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Equation of the tangent at Q(θ) is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

Equation of the tangent at $R\left(\frac{\pi}{2} + \theta\right)$ is

$$\frac{x}{a}\cos\left(\frac{\pi}{2} + \theta\right) + \frac{y}{b}\sin\left(\frac{\pi}{2} + \theta\right) = 1$$

$$-\frac{x}{a}\sin\theta + \frac{y}{b}\cos\theta = 1$$

Suppose $P(x_1, y_1)$ is the point of intersection of the tangents at Q and R

$$\therefore \quad \frac{x_1}{a}\cos\theta + \frac{y_1}{b}\sin\theta = 1 \qquad ..... (1)$$

$$\frac{-x_1}{a}\sin\theta + \frac{y_1}{b}\cos\theta = 1 \qquad ..... (2)$$

Squaring and adding (1) and (2)

$$\left(\frac{x_1}{a}\cos\theta + \frac{y_1}{b}\sin\theta\right)^2 + \left(\frac{-x_1}{a}\sin\theta + \frac{y_1}{b}\cos\theta\right)^2 = 1+1$$

$$\frac{x_1^2}{a^2}\cos^2\theta + \frac{y_1^2}{b^2}\sin^2\theta + \frac{2x_1y_1}{ab}\cdot\cos\theta\sin\theta$$

$$+\frac{x_1^2}{a^2}\sin^2\theta + \frac{y_1^2}{b^2}\cos^2\theta - \frac{2x_1y_1}{ab}\cos\theta\sin\theta = 2$$

$$\frac{x_1^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y_1^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$$

$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 2$$

Locus of $P(x_1, y_1)$ is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

**15. Find the equations of the tangents to the hyperbola $x^2 - 4y^2 = 4$ which are (i) Parallel (ii) Perpendicular to the line x + 2y = 0.**

**Sol.** Equation of the hyperbola is $x^2 - 4y^2 = 4$

$$\frac{x^2}{4} - \frac{y^2}{1} = 1 \Rightarrow a^2 = 4, b^2 = 1$$

**i) Given line is x + 2y = 0**

Since tangent is parallel to x + 2y = 0, slope of the tangent is $m = -\frac{1}{2}$

$$c^2 = a^2m^2 - b^2 = 4\cdot\frac{1}{4} - 1 = 1 - 1 = 0$$

c = 0

Equation of the parallel tangent is:

$y = mx + c = -\frac{1}{2}x$

$\Rightarrow 2y = -x \Rightarrow x + 2y = 0$

**ii) The tangent is perpendicular to x + 2y = 0**

Slope of the tangent $m = \frac{-1}{(-1/2)} = 2$

$c^2 = a^2m^2 - b^2 = 4\cdot4 - 1 = 15$

$c = \pm\sqrt{15}$

Equation of the perpendicular tangent is

$y = 2x \pm \sqrt{15}$.

**16. Find the area enclosed between $y = x^2 - 5x$ and y = 4-2x.**

Sol: Equations of the curves are

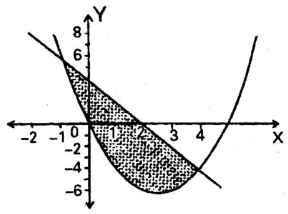
$y = x^2 - 5x$ ……….(1)

$y = 4-2x$ ……….(2)

$x^2 - 5x = 4 - 2x$ , $x^2 - 5x = 4 - 2x$

$x^2 - 3x - 4 = 0$

$(x+1)(x-4) = 0$ x =-1,4



Required area $\int_{-1}^{4}\left[(4-2x)-(x^2-5x)\right]dx$

$= \int_{-1}^{4}(4+3x-x^2)dx = \left(4x+\frac{3}{2}x^2-\frac{x^3}{3}\right)_{-1}^{4}$

$= \left(16+\frac{3}{2}16-\frac{64}{3}\right)-\left(-4+\frac{3}{2}+\frac{1}{3}\right)$

$= 16+24-\frac{64}{3}+4-\frac{3}{2}-\frac{1}{3}$

$= 44-\frac{64}{3}-\frac{3}{2}-\frac{1}{3}$

$= \frac{264-128-9-2}{6}=\frac{125}{6}$

**17. Solve** $2\frac{dy}{dx}=\frac{y}{x}+\frac{y^2}{x^2}$

**Sol.** Put y = vx $\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$2v+2x\frac{dv}{dx}=v+v^2 \Rightarrow 2x\frac{dv}{dx}=v^2-v$

$\frac{dv}{v(v-1)}=2\frac{dx}{x} \Rightarrow \int\left(\frac{1}{v-1}-\frac{1}{v}\right)dv=2\int\frac{dx}{x}$

log(v – 1) – log v = 2 logx + log c

$$\log\frac{v-1}{v}=\log cx^2 \Rightarrow \frac{v-1}{v}=cx^2$$

$$\frac{\frac{y}{x}-1}{\frac{y}{x}}=cx^2 \Rightarrow \frac{y-x}{y}=cx^2$$

Solution is : $(y - x) = cx^2y$

**18) Find the equation of the circle which intersects each of the following circles orthogonally**

**$x^2 + y^2 + 2x + 4y + 1 = 0$; $x^2 + y^2 - 2x + 6y - 3 = 0$; $2x^2 + 2y^2 + 6x + 8y - 3 = 0$**

**Sol.** Let equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

This circle is orthogonal to

$x^2 + y^2 + 2x+ 4y + 1 = 0$; $x^2 + y^2 - 2x +6y - 3 = 0$; $x^2+ y^2 + 3x + 4y – 3/2 = 0$

$2g (1) + 2f(2) = c + 1$ -(i)

$2g\left(\frac{3}{2}\right) + 2f(2) = c - \frac{3}{2}$ - (ii)

$2g (-1) +2f(3) = c – 3$ - (iii)

(iii) – (i)

$-5g + 2f = \frac{-3}{2}$ (or) $– 10g + 4f = -3$ –(iv)

(iii) – (i)

$- 4g +2f = -4$

$F – 2g = -2$

Solving (iv) and (v) we get

$F = -7, g = - 5/2, c = -34$

∴ Equation of circle be

$x^2 + y^2 – 5x –14y - 34 = 0$

**19) Show that the circles $x^2+y^2 – 6x - 2y +1 = 0$; $x^2+y^2 + 2x - 8y + 13 = 0$ Touch each other. Find the point of contact and the equation of the common tangent at their point of contact.**

**Sol.** Equations of the circles are

$S \equiv x^2+y^2 - 6x - 2y + 1 = 0$

Centers A (3, 1), radius $r_1 = \sqrt{9+1-1} = 3$

$S' \equiv x^2+y^2 + 2x - 8y + 13 = 0$

Centers B(-1,4) , radius $r_2 = \sqrt{1+16-13} = 2$

$AB = \sqrt{(3+1)^2+(1-4)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$AB = 5 = 3+2 = r_1 + r_2$

∴ The circles touch each other externally. The point of contact P divides AB internally in the ratio $r_1 : r_2 = 3:2$

Co – ordinates of P are

$\left(\frac{3(-1)+2.3}{5}, \frac{3.4+2.1}{5}\right)$ i.e., $p\left(\frac{3}{5}, \frac{14}{5}\right)$

Equation of the common tangent is

$S_1 = 0$

$\Rightarrow -8x + 6y - 12 = 0 \Rightarrow 4x - 3y + 6 = 0$

**20. Show that the common tangent to the circle $2x^2 + 2y^2 = a^2$ and the parabola $y^2 = 4ax$ intersect at the focus of the parabola $y^2 = -4ax$.**

**Sol.**

Given parabola is $y^2 = 4ax$

Let $y = mx + \frac{a}{m}$ be the tangent. But this is also the tangent to $2x^2 + 2y^2 = a^2$

$\Rightarrow$ Perpendicular distance from centre (0, 0) = radius

$$\Rightarrow \left|\frac{a/m}{\sqrt{m^2+1}}\right| = \frac{a}{\sqrt{2}} \Rightarrow \frac{a^2/m^2}{m^2+1} = \frac{a^2}{2}$$

$$\Rightarrow \frac{2a^2}{m^2} = a^2(m^2+1)$$

$$\Rightarrow 2 = m^4 + m^2 \Rightarrow m^4 + m^2 - 2 = 0$$

$\Rightarrow (m^2-1)(m^2+2)=0 \;\; (\because m^2+2 \neq 0)$

$m^2-1=0 \Rightarrow m=\pm 1$

Therefore, equations of the tangents are

y = -x –a  and  y =  x + a  .

The point of intersection of these two tangents is ( - a, 0) which is the focus of the parabola $y^2 = -4ax$.

**21.** $\int \frac{2\cos x + 3\sin x}{4\cos x + 5\sin x} dx$

**Sol.** Let 2 cos x + 3 sin x = A(4 cosx + 5 sin x) + B(–4 sin x + 5 cos x)

Equating the coefficient of sinx and cosx, we get 4A + 5B = 2, 5A – 4B = 3.

A   B   1

+5  –2  4  +5
–4  –3  5  –4

$$\frac{A}{-15-8} = \frac{B}{-10+12} = \frac{1}{-16-25}$$

$$A = \frac{23}{41}, B = -\frac{2}{41}$$

$$\int \frac{2\cos x + 3\sin x}{4\cos x + 5\sin x} dx =$$

$$= \frac{23}{41}\int dx - \frac{2}{41}\int \frac{-4\sin x + 5\cos x}{4\cos x + 5\sin x} dx$$

$$= \frac{23}{41}x - \frac{2}{41}\log|4\cos x + 5\sin x| + C$$

**22. Obtain the reduction formula for $I_n = \int \csc^n x\, dx$, n being a positive integer, n ≥ 2 and deduce the value of $\int \operatorname{cosec}^5 x\, dx$.**

**Sol.** $I_n = \int \csc^n x\, dx = \int \csc^{n-2} x \cdot \csc^2 x\, dx$

$$= \csc^{n-2} x(-\cot x) + \int \cot x (n-2)\csc^{n-3} x(\cot x)dx$$

$$= -\csc^{n-2} x \cot x + (n-2)\int \csc^{n-2} x(\csc^2 x - 1)dx$$

$$= -\csc^{n-2} x \cot x + (n-2)I_{n-2} - (n-2)I_n$$

$$I_n(1+n-2) = -\csc^{n-2} x \cdot \cot x + (n-2)I_{n-2}$$

$$I_n = \frac{-\csc^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} I_{n-2}$$

$$n = 5 \Rightarrow I_5 = -\frac{\csc^3 x \cdot \cot x}{4} + \frac{3}{4} I_3$$

$$I_3 = -\frac{\csc x \cdot \cot x}{2} + \frac{1}{2} I_1$$

$$I_1 = \int \csc x \, dx = \log\left|\tan\frac{x}{2}\right|$$

$$I_3 = -\frac{\csc x \cdot \cot x}{2} + \frac{1}{2}\log\left|\tan\frac{x}{2}\right|$$

$$I_5 = -\frac{\csc^3 x \cdot \cot x}{4} - \frac{3}{8}\csc x \cot x + \frac{3}{8}\log\left|\tan\frac{x}{2}\right| + C$$

**23. Evaluate** $\int_0^{\pi/2} \frac{\sin^2 x}{\cos x + \sin x} dx$

**Sol,.** $I = \int_0^{\pi/2} \frac{\sin^2 x}{\cos x + \sin x} dx$ ----1.

$$= \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx$$

$$= \int_0^{\pi/2} \frac{\cos^2 x dx}{\sin x + \cos x}$$ ----2.

Adding 1.and 2.

$$2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx$$

$$\Rightarrow I == \frac{1}{2} \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx$$

Consider $\int_0^{\pi/2} \frac{dx}{\sin x + \cos x}$

Put tan(x/2) = t

$$dx = \frac{2dt}{1+t^2}, \ \cos x = \frac{1-t^2}{1+t^2}, \sin x = \frac{2t}{1+t^2}$$

$$\int_0^{\pi/2} \frac{dx}{\sin x + \cos x} = \int_0^1 \frac{2tdt}{2t + (1-t^2)}$$

$$= 2\int_0^1 \frac{dt}{(\sqrt{2})^2 - (t-1)^2} = 2 \cdot \frac{1}{2\sqrt{2}} \left[ \log \frac{\sqrt{2}+t-1}{\sqrt{2}-t+1} \right]_0^1$$

$$= \frac{1}{\sqrt{2}} \left( \log 1 - \log \frac{\sqrt{2}-1}{\sqrt{2}+1} \right)$$

$$= \frac{1}{\sqrt{2}} \log \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{1}{\sqrt{2}} \log(\sqrt{2}+1)^2 = \frac{2}{\sqrt{2}} \log(\sqrt{2}+1)$$

$$I = \frac{1}{\sqrt{2}} \log(\sqrt{2}+1)$$

**24. Solve** $\frac{dy}{dx}(x^2y^3 + xy) = 1$

**Sol.** $\frac{dy}{dx}(x^2y^3 + xy) = 1$

$$\frac{dx}{dy} = xy + x^2y^3$$

$$\Rightarrow \frac{dx}{dy} - xy = x^2y^3 \text{---- (1)}$$

Which is Bernoulli's equation

Dividing with $x^2$,

$$\frac{1}{x^2}\frac{dx}{dy} - \frac{1}{x}y = y^3$$

Put $z = -\frac{1}{x}$ so that $\frac{dz}{dy} = \frac{1}{x^2}\frac{dx}{dy}$

$$\Rightarrow \frac{dz}{dy} + z \cdot y = y^3 \text{----2)}$$

Which is linear d.eq.in z

$$\text{I.F.} = e^{\int y dy} = e^{y^2/2}$$

Sol is $z.\text{I.F} = \int Q.\ \text{I.F. } dy$

$$z \cdot e^{y^2/2} = \int y^3 e^{y^2/2} \cdot dy$$

put $\frac{y^2}{2} = t \Rightarrow y\,dy = dt$

$$= \int t \cdot dt \cdot e^t = e^t(t-1) = e^{y^2/2}\left(\frac{y^2}{2} - 1\right)$$

$$z \cdot e^{y^2/2} = e^{y^2/2}\left(\frac{y^2}{2} - 1\right) + c$$

$$z = \frac{y^2}{2} - 1 + c \cdot e^{-y^2/2} \Rightarrow -\frac{1}{x} = \frac{y^2}{2} - 1 + c \cdot e^{-y^2/2}$$

$$-1 = x\left(\frac{y^2}{2} - 1 + c \cdot e^{-y^2/2}\right)$$

Hence solution is $1+x\left(\frac{y^2}{2}-1+c\cdot e^{-y^2/2}\right)=0$