డీమాయర్ సిద్ధాంతం

Very Short Answer Questions

1. **n** పూర్ణాంకం అయితే $(1+i)^{2n}+(1-i)^{2n}=2^{n+1}\cos\frac{n\pi}{2}$ అని చూపండి.

Solution : -

$1+i=r\{\cos\theta+i\sin\theta\}$ అనుకొనుము.

$(r\cos\theta)^2+(r\sin\theta)^2=2\Rightarrow r=\sqrt{2}$

$\cos\theta=\frac{1}{\sqrt{2}}\ \ \sin\theta=\frac{1}{\sqrt{2}}\ P.V\ of\ \theta=\pi/4$

$\therefore 1+i=\sqrt{2}\left\{\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right\}$ ఇదేవిధంగా $(1-i)=\sqrt{2}\left\{\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right\}$

$$(1+i)^{2n}+(1-i)^{2n}=\left(\sqrt{2}\right)^{2n}\left\{\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right\}^{2n}+\left(\sqrt{2}\right)^{2n}\left\{\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right\}^{2n}$$

$=2^n\left\{\cos\frac{2n\pi}{4}+i\,\cancel{\sin}\,\frac{2n\pi}{4}\ \cos\frac{2n\pi}{4}-i\,\cancel{\sin}\,\frac{2n\pi}{4}\right\}$

$=\ 2^{n+1}\cos\frac{n\pi}{2}$

2. ఈ క్రింది వాని విలువలను కనుగొనుము

(i) $\left(1+i\sqrt{3}\right)^3$

Solution : -

$1+i\sqrt{3}=2\left\{\cos\frac{\pi}{3}+i\sin\pi/3\right\}$

$\left(1+1\sqrt{3}\right)^3=8\left\{\cos\frac{\pi}{3}+i\sin\pi/3\right\}^3$

$=8\{\cos\pi+i\sin\pi\}\quad\left\{\because(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta\right\}$

$=8\{-1+0\}=-8$

(ii) $(1-i)^8$

Solution $(1-i)^8 = \left(\sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)\right)^8 = \left(\sqrt{2}\left\{\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right\}\right)^8 = 2^4\{\cos 2\pi - i\sin 2\pi\}$

(iii) $(1+i)^{16}$

Solution $(1+i)^{16} = \left\{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right\}^{16} = 2\{\cos 2\pi + i\sin 2\pi\}$

$= 256$

(iv) $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 - \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^3$

Solution . $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 - \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$

$\left\{\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right\}^5 - \left\{\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right\}^5$

$\cancel{\cos\frac{5\pi}{6}} + i\sin\frac{5\pi}{6} - \cancel{\cos\frac{5\pi}{6}} + 1\sin\pi/6$

$2i\sin\frac{5\pi}{6} = (\cancel{2}i)\frac{1}{\cancel{2}} = i$

3. $(1-i\sqrt{3})^{\frac{1}{3}}$ యొక్క అన్ని విలువలను కనుగొనుము

$(1-i\sqrt{3})^{\frac{1}{3}} = \left\{2\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\right\}^{\frac{1}{3}}$

$= \left\{2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)\right\}^{\frac{1}{3}}$

$= 2^{\frac{1}{3}}\left\{\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)\right\}^{\frac{1}{3}}$

$= 2^{\frac{1}{3}}\left\{\cos\frac{\left(2k\pi - \frac{\pi}{3}\right)}{3} + isn\left(\frac{2k\pi - \frac{\pi}{3}}{3}\right)\right\} \quad k = 0, 1, 2$

$= 3\sqrt{2}\, cis(6k-1)\frac{\pi}{9} \quad k = 0, 1, 2$

4. $(-i)^{\frac{1}{6}}$ యొక్క అన్ని విలువలను కనుగొనుము

Solution : -

$$(-i)^{\frac{1}{6}}=\left\{\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right)\right\}^{\frac{1}{6}}$$

$$=cis\left(\frac{2k\pi-\pi/2}{6}\right)k=0,1,2,3,4,5$$

$$\therefore(-1)^{\frac{1}{6}}=cis(4k-1)\frac{\pi}{12}\quad k=0,1,2,3$$

5. $(1+i)^{2/3}$ యొక్క అన్ని విలువలను కనుగొనుము

$$(1+i)^{2/3}=\left[\left\{\sqrt{2}\left(\cos\frac{\pi}{4}+1\sin\frac{\pi}{4}\right)\right\}^{2}\right]^{\frac{1}{3}}$$

$$=\left\{2\left(\cos\frac{\pi}{2}+i\sin\pi/2\right)\right\}^{\frac{1}{3}}$$

$$=2^{\frac{1}{3}}cis\left(\frac{2k\pi+\frac{\pi}{2}}{3}\right)k=0,1,2$$

$$=2^{\frac{1}{3}}cis(4k+1)\frac{\pi}{6}k=0,1,2$$

6. $(-16)^{\frac{1}{4}}$ యొక్క అన్ని విలువలను కనుగొనుము

$$(-16)^{\frac{1}{4}}=\left(2^{4}\right)^{\frac{1}{4}}(-1)^{\frac{1}{4}}$$

$$=2(cis\pi)^{\frac{1}{4}}=2cis\left(\frac{2k\pi+\pi}{4}\right)k=0,1,2,3$$

$$=2cis(2k+1)\frac{\pi}{4}\quad k=0,1,2,3$$

7. $(-32)^{\frac{1}{5}}$ యొక్క అన్ని విలువలను కనుగొనుము

$$(-32)^{\frac{1}{5}} \quad (2^5)^{\frac{1}{5}}(-1)^{\frac{1}{5}} = 2\{\cos\pi + i\sin\pi\}^{\frac{1}{5}}$$

8. $1, \omega, \omega^2$ లు**1** యొక్క సంకీర్ణ ఘన మూలాలు అయితే $\frac{1}{2+\omega}+\frac{1}{1+2\omega}=\frac{1}{1+\omega}$ అని చూపండి.

Solution : -

$$\text{L.H.S } \frac{1}{2+\omega}+\frac{1}{1+2\omega}$$

$$\frac{1+2\omega+2+\omega}{(2+\omega)(1+2\omega)} = \frac{3(1+\omega)}{2+4\omega+\omega+2\omega^2}$$

$$= \frac{3(1+\omega)}{2(1+\omega^2)+5\omega}$$

$$= \frac{3(-\omega^2)}{-2\omega+5\omega} \quad \because 1+\omega = -\omega^2$$

$$1+\omega^2 = \omega$$

$$= \frac{-3\omega^2}{3\omega} = -\omega$$

$$= -\frac{1}{\omega^2} = \frac{1}{1+\omega}$$

9. $1, \omega, \omega^2$ లు**1** యొక్క సంకీర్ణ ఘన మూలాలు అయితే $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11}) = 49$ అని చూపండి.

Solution : - $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11}) =$

$$\{(2-\omega)(2-\omega^2)\}\{(2-\omega)(2-\omega^2)\} \quad \{\because \omega^{10}=\omega \quad \omega^{11}=\omega^2\}$$

$$=\{4-2(\omega+\omega^2+\omega^3)\}\{4-2(\omega+\omega^2)+\omega^3\}$$

$$=(4+2+1)(4+2+1)=49$$

10. $1, \omega, \omega^2$ లు**1** యొక్క సంకీర్ణ ఘన మూలాలు అయితే

$(x+y+z)(x+y\omega+z\omega)\left(x+y\omega^2+2\omega\right)=x^3+y^3+z^3-3xyz$ అని చూపండి.

Solution: -

$$(x+y+z)\left\{x+y\omega+z\omega^2\right\}\left\{x+y\omega^2+z\omega\right\}$$

$$(x+y+z)\left\{x^2+xy\omega^2+xz\omega+xy\omega+xy\omega+y^2\omega^3+yz\omega^2+xz\omega^2+yz\omega+z^2\omega^3\right\}$$

$$(x+y+z)\left\{x^2+y^2+z^2+xy\left(\omega^2+\omega\right)+yz\left(\omega+\omega^2\right)+zx\left(\omega+\omega^2\right)\right\}$$

$$(x+y+z)\left\{x^2+y^2+z^2-xy-yz-zx\right\}$$

$$x^3+y^3+z^3-3xyz$$

11. i) **x = cisθ** అయితే $\left(x^6+\frac{1}{x^6}\right)$ విలువ కనుగొనుము

Sol: i) $x=e^{i\theta}$

$$x^6=e^{i6\theta}$$

$$\frac{1}{x^6}=e^{-6i\theta}$$

$$x^6+\frac{1}{x^6}=e^{i6\theta}+e^{-6\theta i}$$

$$=\cos 6\theta+i\sin 6\theta+\cos 6\theta-i\sin 6\theta$$

$$=2\cos 6\theta.$$

ii) **x = cisθ** అయితే **8**యొక్క ఘన మూలం కనుగొనుము

$$x=(8)^{1/3}$$

$$x^3-8=0$$

$$(x-2)(x^2+2x+4)=0$$

$$x=2, x=\frac{-2\pm\sqrt{4-16}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{3}\,i}{2}$$

$$x = -1 \pm \sqrt{3}\,i$$

Roots are 2, 2ω, 2ω².

12 ఏకకపు ఘన మూలాలుω, ω^2 అయిన $z^2 - z + 1 = 0$ మూలాలు$-\omega$, $-\omega^2$ లు అవుతాయని చూపుము.

Sol: $z^2 - z + 1 = 0$

$$z = \frac{1 \pm \sqrt{1-4}}{2}$$

$$z = \frac{1 \pm \sqrt{3}i}{2}$$

$$z = \frac{-[-1 \pm \sqrt{3}i]}{2}$$

$$z = -\omega, -\omega^2$$

13. $1, \omega, \omega^2$ లు**1** యొక్క సంకీర్ణ ఘన మూలాలు అయితే ఈ క్రింది వాని విలువలను కనుగొనుము

i) $(a+b)^3 + (a\omega + b\omega^2)^2 + (a\omega^2 + b\omega)^3$

Sol: **i)** $(a+b)^3 + (a\omega + b\omega^2)^2 + (a\omega^2 + b\omega)^3$

$$= a^3 + b^3 + 3a^2b + 3ab^2 + a^3\omega^3 + b^3\omega^6 + 3a^2\omega^2 \cdot b\omega^2 + 3a\omega \cdot b^2\omega^4 + a^3\omega^6 + b^3\omega^3 + 3a^2b\omega^4 \cdot \omega + 3b^2\omega^2 \cdot a\omega^2$$

$$= a^3 + b^3 + 3a^2b(1 + \omega + \omega^2) + a^3 + b^3 + 3b^2a(\omega^2 + \omega + 1) + a^3 + b^3$$

$$= 3(a^3 + b^3)$$

ii) $(a+2b)^2 + (a\omega^2 + 2b\omega)^2 + (a\omega + 2b\omega^2)^2$

sol. $(a+2b)^2 + (a\omega^2 + 2b\omega)^2 + (a\omega + 2b\omega^2)^2$

$$= a^2 + 4b^2 + 4ab + a^2\omega^4 + 4b^2\omega^2 + 4ab\omega^3 + a^2\omega^2 + 4b^2\omega^4 + 4ab\omega^3$$

$$= a^2(1 + \omega + \omega^2) + 4b^2(1 + \omega^2 + \omega) + 4ab(1 + \omega^3 + \omega^2)$$

$$= 12ab.$$

iii) $(1-\omega+\omega^2)^3$

sol. $(1-\omega+\omega^2)^3$

Now $1+\omega+\omega^2=0$

$1+\omega^2=-\omega$

$$=(-\omega-\omega)^3$$
$$=(-2)^3\omega^3$$
$$=-8$$

iv) $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$

sol.

$$=(1-\omega-\omega^2+\omega^3)(1-\omega)(1-\omega^2)$$
$$=(1-\omega-\omega^2+\omega^3)(1-\omega-\omega^2+\omega^3)$$
$$=(1+1+1)(1+1+1)$$
$$=9$$

v) $\left(\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right)+\left(\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}\right)$

sol. $\left(\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right)+\left(\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}\right)$

$$=\frac{\omega^2(a+b\omega+c\omega^2)}{c\omega^2+a\omega^3+b\omega^4}+\frac{a\omega^2+b\omega^3+c\omega^4}{\omega^2(b+c\omega+a\omega^2)}$$
$$=\omega^2+\frac{1}{\omega^2}$$
$$=\omega^2+\frac{\omega}{\omega^3}$$
$$\Rightarrow \omega^2+\omega=-1$$

vi) $(1-\omega)^3+(1+\omega^2)^3$

sol. $(1-\omega)^3+(1+\omega^2)^3$

$=(-\omega^2)^3+(-\omega)^3$

$=-1+(-1)$

$=-2.$

vii) $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$ $\left(\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right)+\left(\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}\right)$

$(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$

$1+\omega^2=-\omega$

$=(-2\omega)^5+(-2\omega^2)^5$

$=(-2)^5(\omega^2+\omega)$

$=(-2)^5(-1)=32$.

Short Answer Questions

1. α, β లు $x^2 - 2x + 4 = 0$ సమీకరణం యొక్క మూలాలు అయి $n \in N$ అయితే $\alpha^n + \beta^n = 2^{n+1}\cos\frac{n\pi}{3}$ అని చూపండి.

Solution: -

$$x^2 - 2x + 4 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4-16}}{2} = \frac{2 \pm 2i\sqrt{3}}{2}$$

$$\alpha = 2\left\{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right\} \beta = 2\left\{\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right\}$$

$$\alpha^n + \beta^n = \left\{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right\}^n + \left\{2\left(\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\right)\right\}^n$$

$$= 2^n\left\{\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3} + \cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right\}$$

$$= 2^n\left\{2\cos\frac{n\pi}{3}\right\} = 2^{n+1}\cos\frac{n\pi}{3}$$

2. $\cos\alpha + \cos\beta + \cos\vartheta = 0 = \sin\alpha + \sin\beta + \sin\vartheta = 0$ అయితే ఈ క్రింది వానిని నిరూపించండి.

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\vartheta = 3\cos(\alpha + \beta + \vartheta)$

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\vartheta = 3\sin(\alpha + \beta + \vartheta)$

(iii) $\cos(2\alpha - \beta - \vartheta) + \cos\{2\beta - \vartheta - \alpha\} + \sin(2\vartheta - \alpha - \beta) = 3$

(iv) $\sin(2\alpha - \beta - \vartheta) + \sin(2\beta - \vartheta - \alpha) + \sin(2\vartheta - \alpha - \beta) = 0$

(v) $\cos 2\alpha + \cos 2\beta + \cos 2\vartheta = 0$

(vi) $\sin 2\alpha + \sin 2\beta + \sin 2\vartheta = 0$

(vii) $\cos^2\alpha + \cos^2\beta + \cos^2\vartheta = 0$

(viii) $\sin^2\alpha + \sin^2\beta + \sin^2\vartheta = 3/2$

(ix) $\cos(\alpha + \beta) + \cos(\beta + \vartheta) + \cos(\vartheta + \alpha) = 0$

(x) $\sin(\alpha + \beta) + \sin(\beta + \vartheta) + \sin(\vartheta + \alpha) = 0$

Solution : -

Let $x=\cos\alpha+i\sin\alpha \quad y=\cos\beta+i\sin\beta : z=\cos\vartheta+i\sin\vartheta$

$x+y+z=(\cos\alpha+\cos\beta+\cos\vartheta)+i(\sin\alpha+\sin\beta+\sin\vartheta)$

$x+y+z=0 \Rightarrow x^3+y^3+z^3=3xyz$

Proof of (i) & (ii)

$(\cos\alpha+i\sin\alpha)^3+(\cos\beta+i\sin\beta)^3+(\cos\vartheta+i\sin\vartheta)^3=3cis\alpha\, cis\beta\, cis\vartheta$

$cis3\alpha+cis3\beta+cis3\vartheta=3cis(\alpha+\beta+\vartheta)$

$(\cos3\alpha+i\sin3\alpha)+(\cos3\beta+i\sin3\beta)+(\cos3\vartheta+i\sin3\vartheta)=3\cos(\alpha+\beta+\vartheta)+3i\sin(\alpha+\beta+\vartheta)$

By comparing real and imaginary parts on both sides

$\cos3\alpha+\cos3\beta+\cos3\vartheta \; +3\cos(\alpha+\beta+\vartheta)$

$\sin3\alpha+\sin3\beta+\sin3\vartheta=3\sin(\alpha+\beta+\vartheta)$

Proof of (iii) & (iv)

We know that $\sin3\alpha+\sin3\beta+\sin3\vartheta=3\sin(\alpha+\beta+\vartheta)$

$\frac{x^3+y^3+z^3}{xyz}=3 \Rightarrow \frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=3$

$\frac{cis2\alpha}{cis\beta cis\vartheta}+\frac{cis2\beta}{cis\vartheta.\, cis\alpha}+\frac{cis2\vartheta}{cis\alpha\, cis\beta}=3$

$cis(2\alpha-\beta-\vartheta)+cis(2\beta-\vartheta-\alpha)+\cos(2\vartheta-\alpha-\beta)=3$

$\{\cos(2\alpha-\beta-\vartheta)+i\sin(2\alpha-\beta-\vartheta)\}+\cos(2\beta-\vartheta-\alpha)+1\sin(2\beta-\vartheta-\alpha)$

$+\cos(2\vartheta-\alpha-\beta)+1\sin(2\vartheta-\alpha-\beta)=3$

Comparing real and imaginary parts on both sides

$\cos(2\alpha-\beta-\vartheta)+\cos(2\beta-\vartheta-\alpha)+\cos(2\vartheta-\alpha-\beta)=3$

$\sin(2\alpha-\beta-\vartheta)+\sin(2\beta-\vartheta-\alpha)+\sin(2\vartheta-\alpha-\beta)=0$

Proof of V & VI

We know that $x+y+z=0$

$$\therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{\cos\alpha+1\sin\alpha}+\frac{1}{\cos\beta+i\sin\beta}+\frac{1}{\cos\vartheta+i\sin\vartheta}$$

$$=\cos\alpha-i\sin\alpha+\cos\beta-i\sin\beta+\cos\vartheta-i\sin\vartheta$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$$

$$x+y+z=0\Rightarrow(x+y+z)^2=0\Rightarrow x^2+y^2+z^2+2xy+2y+2zx=0$$

$$x^2+y^2+z^2+2xyz\left\{\frac{1}{z}+\frac{1}{x}+\frac{1}{y}\right\}=0$$

$$(cis\alpha)^2+(cis\beta)^2+(cis\vartheta)^2+2(cis\alpha\, cis\beta\, cis\vartheta)(0)$$

$$\left\{\because \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0\right\}$$

$$cis2\alpha+cis2\beta+cis2\vartheta=0\Rightarrow(\cos2\alpha+\cos2\beta+\cos2\vartheta)+i(\sin2\alpha+\sin2\beta+\sin2\vartheta)=0$$

వాస్తవ మరియు సంకీర్ణ భాగాలను పోల్చగ

$$\cos2\alpha+\cos2\beta+\cos2\vartheta=0$$

$$\sin2\alpha+\sin2\beta+\sin2\vartheta=0$$

Proof of (vii)

$$\cos2\alpha+\cos2\beta+\cos2\vartheta=0$$

$$2\cos^2\alpha+2\cos^2\beta-1+2\cos^2\vartheta-1=0$$

$$2\{\cos^2\alpha+\cos^2\beta+\cos^2\vartheta\}=3$$

$$\therefore\cos^2\alpha+\cos^2\beta+\cos^2\vartheta=\frac{3}{2}$$

Proof viii

$$\cos2\alpha+\cos2\beta+\cos2\vartheta=0$$

$$1-2\sin^2\alpha+1-2\sin^2\beta+1-2\sin^2\vartheta=0$$

$$3=2\{\sin^2\alpha+\sin^2\beta+\sin^2\vartheta\}$$

$$\Rightarrow\sin^2\alpha+\sin^2\beta+\sin^2\vartheta=\frac{3}{2}$$

Proof of (ix) and (x)

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$$

$$\therefore yz+zx+xy=0$$

$$\therefore cis\alpha\, cis\beta + cis\beta\, cis\vartheta + cis\vartheta\, cis\alpha = 0$$

$$= cis(\alpha+\beta)+\cos(\beta+\vartheta)+cis(\vartheta+\alpha)=0$$

$$\{\cos(\alpha+\beta)+i\sin(\alpha+\beta)\}+\{\cos(\beta+\vartheta)\}+\{\cos(\vartheta+\alpha)+i\sin(\vartheta+\alpha)\}=0$$

వాస్తవ మరియు సంకీర్ణ భాగాలను పోల్చగ

$$\cos(\alpha+\beta)+\cos(\beta+\vartheta)+\cos(\vartheta+\alpha)=0$$

$$\sin(\alpha+\beta)+\sin(\beta+\vartheta)+\sin(\vartheta+\alpha)=0$$

3. **n పూర్ణాంకం అయి $z = cis\theta$ అయితే $\frac{z^{2n}-1}{z^{2n}+1} = i\tan n\theta$ అని చూపండి.**

Solution : -

$$\frac{z^{2n}-1}{z^{2n}+1}=\frac{(\cos\theta+i\sin\theta)^{2n}-1}{(\cos\theta+i\sin\theta)^{2n}+1}$$

$$=\frac{\cos 2n\theta + i\sin 2n\theta - 1}{\cos 2n\theta + i\sin 2n\theta + 1}$$

$$=\frac{-(1-\cos 2n\theta)+i\sin 2n\theta}{(1+\cos 2n\theta)+i\sin 2n\theta}$$

$$=\frac{i^2(2\sin^2 n\theta)+2i\sin n\theta\cos n\theta}{2\cos^2 n\theta + 2i\sin n\theta\cos n\theta}\left\{\because -1=i^2\right\}$$

$$=\frac{2i\sin n\theta\{\cos n\theta + i\sin n\theta\}}{2\cos n\theta\{\cos n\theta + i\sin n\theta\}}=i\tan n\theta$$

4. $(1+x)^n = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ **అయితే**

i) $a_0 - a_2 + a_4 - a_6 + ... = 2^{n/2}\cos\frac{n\pi}{4}$

ii) $a_1 - a_3 + a_5 - a_7 + ... = 2^{n/2}\sin\frac{n\pi}{4}$**. అని చూపండి.**

Sol: $(1+x)^n = a_0 + a_1x + a_2x^2 + ... + a_nx^n$

Put x = i

$(1+i)^n = a_0 + a_1 i + a_2 i^2 + ... + a_n i^n$

$$\left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^n = (a_0 - a_2 + a_4 ...) + i(a_1 - a_3 + a_5 ...)$$

$$2^{n/2}\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right) = (a_0 - a_2 + a_4 ...) + i(a_1 - a_3 + a_5 ...)$$

వాస్తవ భాగాలను పోల్చగ

$$a_0 - a_2 + a_4 ... = 2^{n/2}\cos\frac{n\pi}{4}$$

సంకీర్ణ భాగాలను పోల్చగ

$$a_1 - a_3 + a_5 ... = 2^{n/2}\sin\frac{n\pi}{4}.$$

5. **(i)** $x^4 - 1 = 0$ **సమీకరణాన్ని సాధించండి.**

Solution : -

(i) $x^4 - 1 = 0 \Rightarrow x^4 = 1$

$$\therefore x = (1)^{\frac{1}{4}} = (\cos 0 + i\sin 0)^{\frac{1}{4}}$$

$$= \left\{\cos\frac{2k\pi}{4} + \frac{i\sin 2k\pi}{4}\right\} k = 0, 1, 2, 3$$

$$= \cos 0 + 1\sin 0 \quad cis\frac{\pi}{2} \quad cis\pi \; cis\frac{3\pi}{2}$$

= 1, I, -1, -i

$= \pm 1, \pm i$

(ii) $x^5+1=0$ **సమీకరణాన్ని సాధించండి.**

Solution : -

$$x^5+1=0 \Rightarrow (\cos\pi + i\sin\pi)^{\frac{1}{5}}$$

$$x=\cos\left(\frac{2k\pi+\pi}{5}\right) k=0,1,2,3,4$$

$$\therefore x = cis\frac{\pi}{5}, cis\frac{3\pi}{5} cis\pi, cis\frac{7\pi}{5}, \cos\frac{9\pi}{5}$$

(iii) $x^9-x^5+x^4-1=0$ **సమీకరణాన్ని సాధించండి.**

Solution : -

$$x^9-x^5+x^4-1=0$$

$$x^5(x^4-1)+1(x^4-1)=0$$

$$(x^4-1)=0 : (x^5+1)=0$$

Do (i) , (ii) to get the solution of (iii)

(iv) $x^4+1=0$ **సమీకరణాన్ని సాధించండి.**

Solution : -

$$x^4+1=0 \Rightarrow x=(-1)^{\frac{1}{4}}=(cis\pi)^{\frac{1}{4}}$$

$$x=cis\left(\frac{2k\pi+\pi}{4}\right) k=0,1,2,3$$

$$x=cis\frac{\pi}{4}, cis\frac{3\pi}{4} cis\frac{5\pi}{4} cis\frac{7\pi}{4}$$

6. **n పూర్ణాంకం అయితే** $(p+iq)^{\frac{1}{n}}+(p-iq)^{\frac{1}{n}}=2\left(p^2+q^2\right)^{\frac{1}{2n}}\cos\left\{\frac{1}{n}arc.\tan\frac{q}{p}\right\}$ **అని చూపండి.**

Solution : -

Let $p+iq=r\{\cos\theta+i\sin\theta\}$

$r\cos\theta=p \quad r\sin\theta=q \quad \Rightarrow r^2=p^2+q^2$

$\therefore r=\sqrt{p^2+q^2}$

$\cos\theta=\frac{p}{\sqrt{p^2+q^2}}\ \sin\theta=\frac{q}{\sqrt{p^2+q^2}}$

$\tan\theta=\frac{q}{p}\ \Rightarrow\theta=\tan^{-1}\left(\frac{p}{q}\right)$

$(p+iq)^{\frac{1}{n}}+(p-q)^{\frac{1}{n}}=\{r(\cos\theta+i\sin\theta)\}^{\frac{1}{n}}+\{r(\cos\theta-i\sin\theta)\}^{\frac{1}{n}}$

$=r^{\frac{1}{n}}\left\{\cos\frac{\theta}{n}+i\sin\frac{\theta}{n}+\cos\frac{\theta}{n}-i\sin\frac{\theta}{n}\right\}$

$=\left(\sqrt{p^2+q}^{\,2}\right)^{\frac{1}{n}}\left\{2\cos\frac{1}{n}\theta\right\}$

$=2\left(p^2+q^2\right)^{\frac{1}{2n}}\cos\left(\frac{1}{n}\tan^{-1}\frac{q}{p}\right)$

7. $\left\{\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right\}^{8/3}=-1$ **అని చూపండి.**

Solution : -

$$\text{LHS }=\left\{\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right\}^{8/3}$$

$$\left\{\frac{1+\cos\left(\frac{\pi}{2}-\pi/8\right)+i\sin\left(\frac{\pi}{2}-\pi/8\right)}{1+\cos\left(\frac{\pi}{2}-\pi/8\right)-1\sin\left(\frac{\pi}{2}-\pi/3\right)}\right\}^{8/3}$$

$$\left\{\frac{1+\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}}{1+\cos\frac{3\pi}{8}-i\sin\frac{3\pi}{8}}\right\}^{8/3}=\left\{\frac{2\cos^2\frac{3\pi}{16}+2i\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}}{2\cos^2\frac{3\pi}{16}-2i\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}}\right\}^{8/3}$$

$$\left[\frac{2\cos\frac{3\pi}{16}\left\{\cos\frac{3\pi}{16}+1\sin\frac{3\pi}{16}\right\}}{2\cos\frac{3\pi}{16}\left(\cos\frac{3\pi}{16}-i\sin\frac{3\pi}{16}\right)}\right]^{\frac{8}{3}}$$

$$\left[\frac{\left(\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)\left(\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)}{\left(\cos\frac{3\pi}{16}-i\sin\frac{3\pi}{16}\right)\left(\cos\left(\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)\right)}\right]^{8/3}$$

$$\left[\frac{\left(\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)^2}{\cos^2\frac{3\pi}{16}+\sin^2\frac{3\pi}{16}}\right]^{8/3}$$

$$\left(\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}\right)^{8/3}$$

$$\cos\pi+i\sin\pi=-1$$

8. **$x^{12}-1=0$, $x^4+x^2+1=0$** ల ఉమ్మడి మూలములను కనుక్కోండి

Sol: $x^{12}-1=0$

$x=(1)^{1/12}$

$x=[\cos(2n\pi)+i\sin(2n\pi)]^{1/12}$

$n=0,1,2,...11 \quad ...(1)$

$x^4+x^2+1=0$

$(x^2-1)(x^4+x^2+1)=0$

$x^6-1=0$

$x=(1)^{1/6}$

$x=[\cos(2n\pi)+i\sin(2n\pi)]^{1/6}$

$=\cos\frac{2n\pi}{6}+i\sin\frac{2n\pi}{6},\ n=0,1,2,3,4,5...(2)$

Common roots to (1) and (2)

$\text{cis}\frac{\pi}{3}, \text{cis}\frac{2\pi}{3}, \text{cis}\frac{4\pi}{3}, \text{cis}\frac{5\pi}{3}$.

9. ఏక కప్తు **15** వ మూలాలు, ఏక కప్తు **25** వ మూలాలలో ఉమ్మడి మూలాల సంఖ్యను కనుక్కోండి

Sol: $x = (1)^{1/15}$

$x = [\cos 2n\pi + i\sin 2n\pi]^{1/15}$

$x = \cos\frac{2n\pi}{15} + i\sin\frac{2n\pi}{15}$

$n = 0, 1, 2, 3, \ldots 14$

$n = 3, m = 5$

$x = \cos\frac{2\pi}{25} + i\sin\frac{2\pi}{25}$

$n = 9, m = 15$

$\cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}$

$x = (1)^{1/25}$

$x = [\cos 2m\pi + i\sin 2m\pi]^{1/25}$

$x = \cos\frac{2m\pi}{25} + i\sin\frac{2m\pi}{25}$

$m = 0, 1, 2, 3, \ldots 24$

$n = 6, m = 10$

$x = \cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}$

$n = 12, m = 20$

$\cos\frac{8\pi}{5} + i\sin\frac{8\pi}{5}$

$n = 0, m = 0$

5 roots common.

6. $1, \omega, \omega^2$ లు**1** యొక్క సంకీర్ణ ఘన మూలాలు అయితే $(x-1)^3 + 8 = 0$. యొక్క మూలాలు కనుగొనుము.

Sol: $(x-1)^3 = -8$

$$(x-1) = (-8)^3$$
$$x-1 = -2 \Rightarrow x = -1$$
$$x-1 = -2\omega \Rightarrow x = -2\omega + 1$$
$$x-1 = -2\omega^2 \Rightarrow x = -2\omega^2 + 1$$

5. $(1 + i)^{4/5}$. యొక్క అన్ని మూలాల లబ్దాన్ని కనుగొనుము

Sol: $(1 + i)^{4/5}$

$$= (\sqrt{2})^{4/5}\left[\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right]^{4/5}$$

$$= (2)^{2/5}\left[\cos\left(2n\pi + \frac{\pi}{4}\right) + i\sin\left(2n\pi + \frac{\pi}{4}\right)\right]^{4/5}$$

$n = 0, 1, 2, 3, 4$

$$= (2)^{2/5}\left[\cos\frac{\pi}{4}\cdot\frac{4}{5} + i\sin\frac{\pi}{4}\cdot\frac{4}{5}\right] \quad ...(1)$$

$$= (2)^{2/5}\left[\cos\frac{9\pi}{4}\cdot\frac{4}{5} + i\sin\frac{9\pi}{4}\cdot\frac{4}{5}\right] \quad ...(2)$$

$$\text{Product} = (2^{2/5})^5 e^{i\left[\frac{\pi}{5}(1+9+17+25+33)\right]}$$

$$= 2^2 e^{i\frac{\pi}{5}(85)}$$

$$= 2^2[\cos 17\pi + i\sin 17\pi]$$

$$= -4$$

10 **z సంకీర్ణ సంఖ్య అయి $z^2 + z + 1 = 0$ అయితే**

$$\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\left(z^4+\frac{1}{z^4}\right)^2+\left(z^5+\frac{1}{z^5}\right)^2+\left(z^6+\frac{1}{z^6}\right)^2=12$$ **అని చూపండి.**

Sol: **let** $z = \omega$

L.H.S. =

$$\left(\omega+\frac{1}{\omega}\right)^2+\left(\omega^2+\frac{1}{\omega^2}\right)^2+\left(\omega^3+\frac{1}{\omega^3}\right)^2+$$

$$\left(\omega^4+\frac{1}{\omega^4}\right)^2+\left(\omega^5+\frac{1}{\omega^5}\right)^2+\left(\omega^6+\frac{1}{\omega^6}\right)^2$$

$$=(-1)^2+(-1)^2+(2)^2+(-1)^2+(-1)^2+(2)^2$$

$$=12$$

Long Answer Questions

1. ఏక కప్పు **n** వ మూలాలు $1, \alpha, \alpha^2, \alpha^3, \ldots \alpha^{n-1}$ అయితే

$$1^p + \alpha^p + (\alpha^2)^p + (\alpha^3)^p + \ldots + (\alpha^{n-1})^p = \begin{cases} 0 \text{ if } p \neq kn \\ n \text{ if } p = kn \end{cases}$$ అని చూపండి **(p, k ∈ N.)**

Sol: $x^n - 1 = 0 \Rightarrow x = (1)^{1/n}$

$x = [\cos 2n\pi + i\sin 2n\pi]^{1/n}$

$x = \cos\frac{2m\pi}{n} + i\sin\frac{2m\pi}{n}$

$\alpha = \cos\frac{2m\pi}{n} + i\sin\frac{2m\pi}{n}$

$\alpha^p = \cos\frac{2mp\pi}{n} + i\sin\frac{2mp\pi}{n}$

Now p = kn

1 + 1 + 1 + … n terms = n

If p ≠kn value

$1^p + \alpha^p + (\alpha^2)^p + (\alpha^3)^p + \ldots + (\alpha^{n-1})^p = 0$.

2. **$x^7 - 1 = 0$** మూలాల యొక్క **99**వఘాతాల మొత్తం శూన్యం అనిచూపండి. దీను=ఇ నుంచి **deduce the roots of** $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$. యొక్క మూలాలను రాబట్టండి.

Sol: $x^7 - 1 = 0 \Rightarrow x = (1)^{1/7}$

$x = (\cos 2k\pi + i\sin 2k\pi)^{1/7}$

$x = \cos\frac{2k\pi}{7} + i\sin\frac{2k\pi}{7}$

$k = 0, 1, 2, 3, 4, 5, 6$

$x_1 = 1, x_2 = e^{\frac{2\pi}{7}i}, x_3 = e^{\frac{4\pi}{7}i}, \ldots, x_6 = e^{\frac{12\pi}{7}i}$

$x_1^{99} + x_2^{99} + x_3^{99} + \ldots$

$$1^{99}+e^{\frac{2\pi}{7}\cdot 99i}+e^{\frac{4\pi}{7}\cdot 99i}+\ldots e^{\frac{12\pi}{7}\cdot 99i}=0$$

$x^6+x^5+x^4+x^3+x^2+x+1=0$ యొక్క మూలాలు $\cos\frac{2k\pi}{7}+i\sin\frac{2k\pi}{7}$; k = 1, 2, 3, 4, 5, 6.

$\because (x^7)-1=$

$$(x-1)(x^6+x^5+x^4+x^3+x^2+x+1)=0$$

x = 1 ఒక మూలం

$\Rightarrow \text{cis}\frac{2k\pi}{7}$; k = 1, 2, 3, 4, 5, 6 లు

$x^6+x^5+x^4+x^3+x^2+x+1=0$.యొక్క మూలాలు.

5. n ధన పూర్ణాంకం అయితే$(x-1)^n=x^n$ సాధించండి

Sol: $\left(\frac{x-1}{x}\right)^n=1$

$$\frac{x-1}{x}=(1)^{1/n}$$

$$\frac{x-1}{x}=[\cos 2m\pi+i\sin 2m\pi]^{1/n}$$

$$\frac{x-1}{x}=\cos\frac{2m\pi}{n}+i\sin\frac{2m\pi}{n}$$

$$1-\frac{1}{x}=e^{i\frac{2m}{n}\pi}$$

$$1-\cos\frac{2m\pi}{n}-i\sin\frac{2m\pi}{n}=\frac{1}{x}$$

$$2\sin^2\frac{m\pi}{n}-2i\sin\frac{m\pi}{n}\cos\frac{m\pi}{n}=\frac{1}{x}$$

$$2\sin\frac{m\pi}{n}\left[\sin\frac{m\pi}{n}-i\cos\frac{m\pi}{n}\right]=\frac{1}{x}$$

$$x = \frac{1}{2\sin\frac{m\pi}{n}\left[\sin\frac{m\pi}{n} - i\cos\frac{m\pi}{n}\right]}$$

$$= \frac{1}{2\sin\frac{m\pi}{n}\left[\sin\frac{m\pi}{n} - i\cos\frac{m\pi}{n}\right]} X \frac{\left[\sin\frac{m\pi}{n} + i\cos\frac{m\pi}{n}\right]}{\left[\sin\frac{m\pi}{n} + i\cos\frac{m\pi}{n}\right]}$$

$$= \frac{\left[\sin\frac{m\pi}{n} + i\cos\frac{m\pi}{n}\right]}{2\sin\frac{m\pi}{n}\left[\sin^2\frac{m\pi}{n} + \cos^2\frac{m\pi}{n}\right]}$$

$$= \frac{\left[\sin\frac{m\pi}{n} + i\cos\frac{m\pi}{n}\right]}{2\sin\frac{m\pi}{n}}$$

$$\frac{1}{2}\left[1 + i\cot\frac{m\pi}{n}\right]; m = 1, 2, 3, \ldots(n-1)$$

2. **m, n లు పూర్ణాంకాలు మరియు $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$ అయితే**

$$x^m y^n + \frac{1}{x^m y^n} = \cos(m\alpha + n\beta) \textbf{ and } x^m y^n - \frac{1}{x^m y^n} = 2i\sin(m\alpha + n\beta)$$ **అని చూపండి.**

Sol. $x^m = (\cos\alpha + i\sin\alpha)^m$

$= \cos m\alpha + i\sin m\alpha$

$y^n = (\cos\beta + i\sin\beta)^n$

$= \cos n\beta + i\sin n\beta$

$\therefore x^m y^n = (\cos m\alpha + i\sin m\alpha)(\cos n\beta + i\sin n\beta)$

$= \cos(m\alpha + n\beta) + i\sin(m\alpha + n\beta)$ ... (1)

$$\frac{1}{x^m y^n} = \frac{1}{\cos(m\alpha + n\beta) + i\sin(m\alpha + n\beta)}$$

$= \cos(m\alpha + n\beta) - i\sin(m\alpha + n\beta)$ ...(2)

By adding (1) and (2), we get

$$x^m y^n + \frac{1}{x^m y^n} = \cos(m\alpha + n\beta)$$

By subtracting (2) from (1), we get

$$x^m y^n - \frac{1}{x^m y^n} = 2i\sin(m\alpha + n\beta)$$

3. n ధన పూర్ణాంకం అయితే (1 + i)ⁿ + (1 – i)ⁿ = $2^{\frac{n+2}{2}}\cos\left(\frac{n\pi}{4}\right)$ **అని చూపండి.**

Sol.

$$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$(1-i) = \sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$$

$$(1+i)^n = \left(\sqrt{2}\right)^n\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^n$$

$$= 2^{n/2}\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right) \quad ...(1)$$

$$(1-i)^n = \left(\sqrt{2}\right)^n\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^n$$

$$= 2^{n/2}\left(\cos\frac{n\pi}{4} - i\sin\frac{n\pi}{4}\right) \quad ...(2)$$

By adding (1) and (2), we get

$$(1+i)^n + (1-i)^n = 2^{n/2}\left(2\cos\frac{n\pi}{4}\right) = 2^{\frac{n+2}{2}}\cos\left(\frac{n\pi}{4}\right)$$

4. n ధన పూర్ణాంకం అయితే $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$ **అని చూపండి.**

Sol.L.H.S. =

$$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n =$$

$$=\left(2\cos^2\frac{\theta}{2}+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n+\left(2\cos^2\frac{\theta}{2}-2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n$$

$$=2^n\cos^n\frac{\theta}{2}\left[\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^n+\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)^n\right]$$

$$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}+\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$$

$$=2^n\cos^n\frac{\theta}{2}\left(2\cos\frac{n\theta}{2}\right)$$

$$=2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}=\text{R.H.S.}$$

5. $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ **n అయితే**

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{3}{2}=\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ **అని చూపండి.**

Sol. $(\cos\alpha+i\sin\alpha)+(\cos\beta+i\sin\beta)+(\cos\gamma+i\sin\gamma)$

$$=(\cos\alpha+\cos\beta+\cos\gamma)+i(\sin\alpha+\sin\beta+\sin\gamma)=0+i0$$

$$(\cos\alpha+i\sin\alpha)+(\cos\beta+i\sin\beta)+(\cos\gamma+i\sin\gamma)=0 \quad ...(1)$$

Let $x=\text{cis}\alpha, y=\text{cis}\beta, z=\text{cis}\gamma$ then

$x + y + z = 0$ by (1), then

$x^2+y^2+z^2=-2(xy+yz+zx)$

$$=-2xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

$$=-2xyz[\cos\alpha-i\sin\alpha+\cos\beta-i\sin\beta+\cos\gamma-i\sin\gamma]$$

$$=-2xyz[(\cos\alpha+\cos\beta+\cos\gamma)-i(\sin\alpha+\sin\beta+\sin\gamma)]$$

$= -2xyz(0 - i0) = 0$

$\therefore x^2 + y^2 + z^2 = 0$

$\Rightarrow (\cos\alpha + i\sin\alpha)^2 + (\cos\beta + i\sin\beta)^2 + (\cos\gamma + i\sin\gamma)^2 = 0$

$\Rightarrow \cos 2\alpha + i\sin 2\alpha + \cos 2\beta + i\sin 2\beta + \cos 2\gamma + i\sin 2\gamma = 0$

$\Rightarrow (\cos 2\alpha + \cos 2\beta + \cos 2\gamma) + i(\sin 2\alpha + \sin 2\beta + \sin 2\gamma) = 0$

$\therefore \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$

$2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1 = 0$

$2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 3$

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{2}$

$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = \frac{3}{2}$

$\therefore \sin^2\alpha + \sin^2\beta + \sin^2\gamma = \frac{3}{2}$.

6. $(\sqrt{3}+i)^{1/4}$ యొక్క అన్ని విలువలను కనుగొనుము

Sol. $\sqrt{3}+i$ యొక్క మాప ఆయామ రూపం

$\sqrt{3}+i = 2\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = 2(\cos 30° + i\sin 30°)$

$(\sqrt{3}+i)^{1/4} = \left(2\text{cis}\frac{\pi}{6}\right)^{1/4}$

$= 2^{1/4}\left(\text{cis}\frac{2k\pi + \frac{\pi}{6}}{4}\right)$, k = 0, 1, 2, 3

$= 2^{1/4}\text{cis}\left(\frac{12k\pi + \pi}{24}\right)$, k = 0, 1, 2, 3

$= 2^{1/4}\text{cis}(12k+1)\frac{\pi}{24}$, k = 0, 1, 2, 3

$\therefore$ $\sqrt{3}+i$ యొక్క అన్ని విలువలు

$2^{1/4}\text{cis}\frac{\pi}{24}, 2^{1/4}\text{cis}\frac{13\pi}{24}, 2^{1/4}\text{cis}\frac{25\pi}{24}, 2^{1/4}\text{cis}\frac{37\pi}{24}$

7. $1, \omega, \omega^2$ **లు1 యొక్క సంకీర్ణ ఘన మూలాలు అయితే క్రిందివానిని నిరూపించండి.**

i) $(1-\omega+\omega^2)^6+(1-\omega^2+\omega)^6=128$ $\quad = (1-\omega+\omega^2)^7+(1+\omega-\omega^2)^7$

Sol: $1+\omega+\omega^2 = 1+\dfrac{(-1+i\sqrt{3})}{2}+\dfrac{(-1-i\sqrt{3})}{2}=0$ **and** $\omega^3=\left(\text{cis}\dfrac{2\pi}{3}\right)^3=\text{cis}2\pi=1$

i) $(1-\omega+\omega^2)^6+(1-\omega^2+\omega)^6$

$=(-\omega-\omega)^6+(-\omega^2-\omega^2)^6$

$=2^6(\omega^6+\omega^{12})=2^6(2)=128$

$(1-\omega+\omega^2)^7+(1+\omega-\omega^2)^7$

$=(-\omega-\omega)^7+(-\omega^2-\omega^2)^7$

$=(-2)^7(\omega^7+\omega^{14})$

$=(-2)^7(\theta+\theta^2)$

$=(-128)(-1)=128.$

ii) $(a+b)(a\omega+b\omega^2)(a\omega^2+b\omega)=a^3+b^3$**.**

$(a+b)(a\omega+b\omega^2)(a\omega^2+b\omega)$

$=(a+b)(a^2\omega^3+ab\omega^4+ab\omega^2+b^2\omega^3)$

$=(a+b)\left(a^2+ab(\omega+\omega^2)+b^2\right)$

$=(a+b)(a^2-ab+b^2)$

$=a^3+b^3$

iii) $\mathbf{x^2+4x+7=0\ ,\ x=\omega-\omega^2-2.}$

$x=\omega-\omega^2-2$

$(x+2)=\omega-\omega^2$

$\Rightarrow (x+2)^2=\omega^2+\omega^4-2\omega^3$

$\Rightarrow x^2+4x+4=\omega^2+\omega-2=-1-2=-3$

$\Rightarrow x^2+4x+7=0.$