DIFFERENTIAL EQUATIONS

Very Short Answer Questions

1. మూల బిందువు కేంద్రం గాగల వృత్త కుటుంబానకి అవకలన సమీకరణ పరిమాణాన్ని మరియు తరగతి ని కనుగొనుము

వృత్తాల సమీకరణం $x^2+y^2=a^2---(1)$, **a** పరామితి.

అవకలనం చేయగా

$2x+2y.y_1=0$.

$\Rightarrow x+y.y_1=0$.

పరిమాణం= **1**

2. $\frac{d^2y}{dx^2}=-p^2y$. అవకలన సమీకరణ పరిమాణాన్ని మరియు తరగతి ని కనుగొనుము

Ans. తరగతి = **1**, పరిమాణం= **2**

3. $\left(\frac{d^3y}{dx^3}\right)^2-3\left(\frac{dy}{dx}\right)^2-e^x=4$. అవకలన సమీకరణ పరిమాణాన్ని మరియు తరగతి ని కనుగొనుము

పరిమాణం= **3**, తరగతి = **2**

4. $x^{1/2}\left(\frac{d^2y}{dx^2}\right)^{1/3}+x\cdot\frac{dy}{dx}+y=0$ పరిమాణం**2**, తరగతి **1**

5. $\left(\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3\right)^{6/5}=6y$. అవకలన సమీకరణ పరిమాణాన్ని మరియు తరగతి ని కనుగొనుము

Sol. $\left(\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3\right)^{6/5}=6y$

i.e. $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=(6y)^{5/6}$

పరిమాణం= **2**, తరగతి = **1**

క్రింది అవకలన సమీకరణాల ను సాధించండి

6. $\sqrt{1-x^2}dy+\sqrt{1-y^2}dx=0$.

Sol. $\sqrt{1-x^2}dy+\sqrt{1-y^2}dx=0$

$\sqrt{1-x^2}dy=-\sqrt{1-y^2}dx$

ఇరువైపులా సమాకలనం చేయగా

$$\int\frac{dy}{\sqrt{1-y^2}}=-\int\frac{dx}{\sqrt{1-x^2}}$$

$\sin^{-1}y = -\sin^{-1}x + c$

ఇచ్చిన సమీకరణం యొక్క సాధన.

$\sin^{-1}x + \sin^{-1}y = c$ ఇక్కడ c స్థిర రాశి.

7. $\frac{dy}{dx}=\frac{2y}{x}$.

Sol. $\frac{dy}{dx}=\frac{2y}{x}\Rightarrow\int\frac{dy}{y}=2\frac{dx}{x}$

ఇరువైపులా సమాకలనం చేయగా

$\log c+\log y=2\log x$

$\log cy=\log x^2$

ఇచ్చిన సమీకరణం యొక్క సాధన.

$cy = x^2$ ఇక్కడ c స్థిర రాశి.

8. $xdy-ydx=\sqrt{x^2+y^2}dx$ ను $F\left(\frac{y}{x}\right)=\frac{dy}{dx}$ రూపంలోకి మార్చుము .

Sol. $x\cdot dy-ydx=\sqrt{x^2+y^2}dx$

$$x\frac{dy}{dx}-y=\sqrt{x^2+y^2}\Rightarrow x\frac{dy}{dx}=y+\sqrt{x^2+y^2}$$

$$\frac{dy}{dx}=\frac{y}{x}+\sqrt{\frac{x^2+y^2}{x^2}}=\frac{y}{x}+\sqrt{1+\left(\frac{y}{x}\right)^2}$$

$$F\left(\frac{y}{x}\right)=\frac{dy}{dx}$$

9. $\left(x-y\text{Tan}^{-1}\frac{y}{x}\right)dx+x\tan^{-1}\frac{y}{x}dy=0$ ను $F\left(\frac{y}{x}\right)=\frac{dy}{dx}$ రూపంలోకి మార్చుము .

Sol. $\left(x-y\text{Tan}^{-1}\frac{y}{x}\right)dx+x\tan^{-1}\frac{y}{x}dy=0$

$$x\tan^{-1}\left(\frac{y}{x}\right)dy=-\left(x-y\tan^{-1}\frac{y}{x}\right)dx$$

$$\tan^{-1}\left(\frac{y}{x}\right)\frac{dy}{dx}=-\left(1-\frac{y}{x}\tan^{-1}\frac{y}{x}\right)$$

$$=\frac{y}{x}\tan^{-1}\left(\frac{y}{x}\right)-1$$

$$\frac{dy}{dx}=\frac{\frac{y}{x}\cdot\tan^{-1}\left(\frac{y}{x}\right)-1}{\tan^{-1}\left(\frac{y}{x}\right)}=F\left(\frac{y}{x}\right)$$

10. $x\cdot\frac{dy}{dx}=y(\log y-\log x+1)$ ను $F\left(\frac{y}{x}\right)=\frac{dy}{dx}$ రూపంలోకి మార్చుము .

Sol. $x\cdot\frac{dy}{dx}=y(\log y-\log x+1)$

$$\frac{dy}{dx}=\frac{y}{x}\left(\log\frac{y}{x}+1\right)$$

11. క్రింది అవకలన సమీకరణాల ను సాధించండి

1. $\frac{dy}{dx}=-\frac{12x+5y-9}{5x+2y-4}$

Sol. ఇచ్చిన సమీకరణం నుండి

b = –5, a = 5 ⇒ b = –a

(5x + 2y – 4)dy = –(12x + 5y – 9)dx

(5x + 2y – 4)dy + (12x + 5y – 9)dx = 0

5(x dy+y dx) + 2y dy–4 dy+12x dx – 9 dx = 0

ఇరువైపులా సమాకలనం చేయగా

$5xy + y^2 - 4y + 6x^2 - 9x = c$.

2. $\frac{dy}{dx} = \frac{-3x-2y+5}{2x+3y+5}$

Sol. ఇచ్చిన సమీకరణం నుండి **b = –2, a = 2 ⇒ b = –a**

(2x + 3y + 5)dy = (–3x – 2y + 5)dx

2x dy + 3y dy + 5dy = –3x dx –2y dx + 5 dx

2x dy + 3y dy + 5dy + 3x dx –2y dx + 5 dx = 0

ఇరువైపులా సమాకలనం చేయగా

$$2xy + \frac{3}{2}y^2 + \frac{3}{2}x^2 + 5y - 5x = c$$

$$4xy + 3y^2 + 3x^2 - 10x + 10y = 2c = c'$$

ఇచ్చిన సమీకరణం యొక్క సాధన **$4xy + 3(x^2 + y^2) - 10(x - y) = c$.**

3. $\frac{dy}{dx} = \frac{-3x-2y+5}{2x+3y-5}$

Sol. $\frac{dy}{dx} = \frac{-3x-2y+5}{2x+3y-5}$

Here b = –2, a′ = 2, b = –a′

(2x + 3y – 5)dy = (–3x – 2y + 5)dx

⇒ 2(x dy+y dx) + (3y – 5)dy + (3x – 5)dx = 0

⇒ 2d xy + (3y – 5)dy + (3x – 5)dx = 0

ఇరువైపులా సమాకలనం చేయగా

$$2\int d(xy) + \int (3y-5)dy + \int (3x-5)dx = 0$$

$$\Rightarrow 2xy+3\frac{y^2}{2}-5y+3\frac{x^2}{2}-5x=\frac{c}{2}$$

$$(\text{or})3x^2+4xy+3y^2-10x-10y=c$$

ఇచ్చిన సమీకరణం యొక్క సాధన

4. $2(x-3y+1)\frac{dy}{dx}=4x-2y+1$

Sol. (2x – 6y + 2)dy = (4x – 2y + 1)dx

(2x – 6y + 2)dy – (4x – 2y + 1)dx = 0

2(x dy + y dx) – 6y dy + 2 dy – 4x dx – dx = 0

ఇరువైపులా సమాకలనం చేయగా

$2xy - 3y^2 - 2x^2 + 2y - x = c$.

5. $\frac{dy}{dx}=\frac{x-y+2}{x+y-1}$

Sol. b = –1, a′ = a ⇒ b = –a′

(x + y – 1)dy = (x – y + 2)dx

(x + y – 1)dy – (x – y + 2)dx = 0

(x dy + y dx) + y dy – dy – x dx – 2 dx = 0

ఇరువైపులా సమాకలనం చేయగా

$$xy+\frac{y^2}{2}-\frac{x^2}{2}-y-2x=c$$

$$2xy+y^2-x^2-2y-4x=2c=c'$$

6. $\frac{dy}{dx}=\frac{2x-y+1}{x+2y-3}$

Sol. b = –1, a′ = 1 ⇒ b = –a′

(x + 2y – 3)dy = (2x – y + 1)dx

(x + 2y – 3)dy – (2x – y + 1)dx = 0

(x dy + y dx) + 2y dy – 3 dy – 2x dx – dx = 0

ఇరువైపులా సమాకలనం చేయగా

$xy + y^2 - x^2 - 3x - x = c$

12. క్రింది అవకలన సమీకరణాలకి **I.F.** కనుగొనుము

1. $x\frac{dy}{dx} - y = 2x^2 \sec^2 2x$

Sol. $x\frac{dy}{dx} - y = 2x^2 \sec^2 2x$

$\frac{dx}{dy} - \frac{1}{x}y = 2x\sec^2 2x$ **which is linear in y .**

$$\text{I.F.} = e^{\int p dx} = \int_e -\frac{1}{x}\log x = e^{-\log x} = e^{\log(1/x)} = \frac{1}{x}$$

2. $y\frac{dx}{dy} - x = 2y^3$

Sol. $y\frac{dx}{dy} - x = 2y^3 \Rightarrow \frac{dx}{dy} - \frac{1}{y}x = 2y^2$ ఇది **x** లో ఏకఘాతీయ సమీకరము

$$\text{I.F.} = e^{\int p dy} = e^{\int -\frac{1}{y}dy} = e^{-\log y} = e^{\log(1/y)} = \frac{1}{y}$$

3. $x\frac{dy}{dx} - y = 2x^2 \sec^2 2x$

Sol. $x\frac{dy}{dx} - y = 2x^2 \sec^2 2x$

$\frac{dx}{dy} - \frac{1}{x}y = 2x\sec^2 2x$ ఇది **y** లో ఏకఘాతీయ సమీకరము

$$\text{I.F.} = e^{\int p dx} = \int_e -\frac{1}{x}\log x = e^{-\log x} = e^{\log(1/x)} = \frac{1}{x}$$

4. $y\frac{dx}{dy} - x = 2y^3$

Sol. $y\frac{dx}{dy} - x = 2y^3 \Rightarrow \frac{dx}{dy} - \frac{1}{y}x = 2y^2$ ఇది **x** లో ఏకఘాతీయ సమీకరము

$$\text{I.F.} = e^{\int p dy} = e^{\int -\frac{1}{y}dy} = e^{-\log y} = e^{\log(1/y)} = \frac{1}{y}$$

Short Answer Questions

1. బ్రాకెట్లలో చూపిన స్థిర సంఖ్యలను తొలగించి ఇచ్చిన వక్రాల కుటుంబ సమీకరణాలకు అనుగుణంగా వచ్చే అవకలన సమీకరణాన్ని రాబట్టండి

i). $y=c(x-c)^2;(c)$

$y=c(x-c)^2 ------(1)$

Diff. w.r.t x,

$y_1=c.2(x-c)----(2)$

$\frac{(1)}{(2)}\Rightarrow\frac{y}{y_1}=\frac{x-c}{2}$

$\Rightarrow x-c=\frac{2y}{y_1}$ and $c=x-\frac{2y}{y_1}$

(1) నుండి

$y=\left(x-\frac{2y}{y_1}\right)\left(\frac{2y}{y_1}\right)^2\Rightarrow y_1^3=4y(xy_1-2y)$

ii) $xy=ae^x+be^{-x};(a,b)$

$xy=ae^x+be^{-x}---(1)$

x దృష్ట్యా అవకలనము చేయగా

$y+x.y_1=ae^x-be^{-x}---(2)$

$y_1+y_1+xy_2=ae^x+be^{-x}=xy$

$\therefore 2y_1+xy_2=xy$

ఇదేకావల్సిన అవకలన సమీకరణం

iii) $y=(a+bx)e^{kx};(a,b)$

$y=(a+bx)e^{kx}---(1)$

Diff.w.r.t x,

$\Rightarrow y_1 = k(a+bx)e^{kx} + be^{kx}$

$\Rightarrow y_1 = ky + be^{kx} ---(2)$

Diff.w.r.t. x,

$\Rightarrow y_2 = ky_1 + kbe^{kx}$

$\Rightarrow y_2 = ky_1 + k(y_1 - ky)$

$\Rightarrow y_2 = 2ky_1 - k^2 y$ ఇదేకావల్సిన అవకలన సమీకరణం

iv) $y = a\cos(nx+b);(a,b)$

Ans. $y_2 + n^2 y = 0$

2. $(x - a^2) + (y - b)^2 = r^2$ వృత్తా కుటుంబానికి a , b స్థిర సంఖ్యలు అనుగుణంగా వచ్చే అవకలన సమీకరణాన్ని రాబట్టండి

Sol. $(x - a^2) + (y - b)^2 = r^2$ **...(1)**

x దృష్ట్యా అవకలనము చేయగా

$$2(x-a) + 2(y-b)\frac{dy}{dx} = 0 \qquad \text{... (2)}$$

x దృష్ట్యా మరల అవకలనము చేయగా

$$1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0 \qquad \text{... (3)}$$

(2) నుండి $(x-a) = -(y-b)\frac{dy}{dx}$

(1) నుండి

$$(y-b)^2\left(\frac{dy}{dx}\right)^2 + (y-b)^2 = r^2$$

$$(y-b)^2\left(\left(\frac{dy}{dx}\right)^2 + 1\right) = r^2 \qquad \text{... (4)}$$

(3) నుండి

$$(y-h)\frac{d^2y}{dx^2}=-\left(1+\left(\frac{dy}{dx}\right)^2\right)$$

$$(y-h)=-\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)}{\left(\frac{d^2y}{dx^2}\right)}$$

(4) లో ప్రతిక్షేపించగా

$$\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^3}{\left(\frac{d^2y}{dx^2}\right)^2}=r^2$$

i.e. $r^2\left(\frac{d^2y}{dx^2}\right)^2=\left(1+\left(\frac{dy}{dx}\right)^2\right)^3$

3. క్రింది వక్రాల కుటుంబాలకు అవకలన సమీకరణాల ను రాబట్టండి

4i) నిరూపకాక్షాలు అనంత స్పర్శ రేఖలుగా వున్న అతిపరావలయాలు

Sol. అతిపరావలయ సమీకరణం **$xy=c^2$**

x దృష్ట్యా అవకలనము చేయగా $x\frac{dy}{dx}+y=0$

ii) మూలబిందువు వద్ద కేంద్రం వుండి నిరూపకాక్షాలు అక్షాలు గా వున్న దీర్ఘవృత్తాలు.

Sol. దీర్ఘవృత్త సమీకరణం

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

x దృష్ట్యా అవకలనము చేయగా

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0\Rightarrow y.y_1=-\frac{b^2}{a^2}x$$

x దృష్ట్యా అవకలనము చేయగా

$$y.y_2+y_1.y_1=-\frac{b^2}{a^2}\Rightarrow y.y_2+2y_1=\frac{y.y_1}{x}$$

$$\Rightarrow x\left(y.y_2+2y_1\right)=y.y_1$$

4. క్రింది వక్రాల కుటుంబాలకు అవకలన సమీకరణాల ను రాబట్టండి

i) మూలబిందువు వద్ద y అక్షం స్పృశించే వృత్తాలు.

Sol. వృత్త సమీకరణాలు $x^2 + y^2 + 2gx = 0$

$x^2 + y^2 = -2gx$

x దృష్ట్యా అవకలనము చేయగా

$2x + 2yy_1 = -2g$ ...(ii)

(i) నుండి

$x^2 + y^2 = x(2x + 2yy_1)$ by (ii)

$= 2x^2 + 2xyy_1$

$yy^2 - 2xyy_1 - 2x^2 = 0$

$y^2 - x^2 = 2xy\frac{dy}{dx}$.

ii) అక్షం x అక్షానికి సమాంతరంగా , నాభి లంబం 4a ఉన్న గల పరావలయాల కుటుంబం

Sol.

పరావలయాల కుటుంబం $(y - k)^2 = 4a(x - h)$-----(i)

x దృష్ట్యా అవకలనము చేయగా

$2(y - k)y_1 = 4a$

$(y - k)y_1 = 2a$ ...(2)

x దృష్ట్యా అవకలనము చేయగా

$(y - k)y_2 + y_1^2 = 0$ ...(3)

(2) నుండి $y - k = \frac{2a}{y_1}$

(3) లో ప్రతిక్షేపించగా

$$\frac{2a}{y_1}\cdot y_2 + y_1^2 = 0 \Rightarrow 2ay_2 + y_1^3 = 0$$

iii) మూలబిందువు వద్ద నాభి x అక్షం గుండా అక్షం గల పరావలయాల కుటుంబం

Sol.

పరావలయాల కుటుంబం $y^2 = 4a(x + a)$-----(i)

$$2y\frac{dy}{dx} = 4a \Rightarrow \frac{1}{2}yy' = a \text{ -----(2)}$$

(i) , (2) ల నుండి

$$y^2 = 4\frac{1}{2}yy'\left(x + \frac{1}{2}yy'\right)$$

$$y^2 = 2y'x + 4\cdot\frac{1}{4}y^2y'^2 \Rightarrow y^2 = 2yy'x + y^2y'^2$$

$$y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) = y$$

5. క్రింది అవకలన సమీకరణాల ను సాధించండి

1. $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Sol. $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

ఇరువైపులా సమాకలనం చేయగా

$$\Rightarrow \int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\tan^{-1} y = \tan^{-1} x + \tan^{-1} c$$

2. $\frac{dy}{dx} = e^{y-x}$

Sol. $\frac{dy}{dx} = \frac{e^y}{e^x} \Rightarrow \frac{dy}{e^y} = \frac{dx}{e^x}$

ఇరువైపులా సమాకలనం చేయగా

$$\int e^{-x}dx = \int e^{-y}dy \Rightarrow -e^{-x} = -e^{-y} + c$$

$e^{-y} = e^{-x} + c$ **, c** స్థిర రాశి

3. $(e^x + 1)y\,dy + (y + 1)dx = 0$

Sol. $(e^x + 1)y\,dy = -(y + 1)dx$

$$\frac{ydy}{y+1} = -\frac{dx}{e^x+1}$$

ఇరువైపులా సమాకలనం చేయగా

$$\int\left(1-\frac{1}{y+1}\right)dy=\int-\frac{e^{-x}dx}{e^{-x}+1}$$

$$y-\log(y+1)=\log(e^{-x}+1)+\log c$$
$$\Rightarrow y-\log(y+1)=\log c(e^{-x}+1)$$
$$\Rightarrow y=\log(y+1)+\log c(e^{-x}+1)$$
$$y=\log c(y+1)(e^{-x}+1)$$

ఇచ్చిన సమీకరణం యొక్క సాధన $e^{y}=c(y+1)(e^{-x}+1)$.

4. $\frac{dy}{dx}=e^{x-y}+x^{2}e^{-y}$

Sol. $\frac{dy}{dx}=e^{x-y}+x^{2}e^{-y}=\frac{e^{x}}{e^{y}}+\frac{x^{2}}{e^{y}}$

ఇరువైపులా సమాకలనం చేయగా

$$\int e^{y}\cdot dy=\int(e^{x}+x^{2})dx$$

ఇచ్చిన సమీకరణం యొక్క సాధన $e^{y}=e^{x}+\frac{x^{3}}{3}+c$

5. tany dx + tanx dy = 0

Sol. tan y dx = –tan x dy

$$\frac{dx}{\tan x}=\frac{-dy}{\tan y}\Rightarrow\frac{\cos x}{\sin x}dx=-\frac{\cos y}{\sin y}dy$$

ఇరువైపులా సమాకలనం చేయగా

$$\int\frac{\cos x}{\sin x}dx=-\int\frac{\cos y}{\sin y}dy$$

$$\log\sin x=-\log\sin y+\log c$$
$$\log\sin x+\log\sin y=\log c$$
$$\log(\sin x\cdot\sin y)=\log c\Rightarrow\sin x\cdot\sin y=c$$

6. $\sqrt{1+x^2}dx+\sqrt{1+y^2}dy=0$

Sol. $\sqrt{1+x^2}dx=-\sqrt{1+y^2}dy$

ఇరువైపులా సమాకలనం చేయగా

$$\int\sqrt{1+x^2}dx=-\int\sqrt{1+y^2}dy$$

$$\frac{x}{2}\times\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}x=$$

$$y\frac{\sqrt{1+y^2}}{2}=\frac{1}{2}\sinh^{-1}x+c$$

$$x\sqrt{1+x^2}+y\sqrt{1+y^2}+\log\left[(x+\sqrt{1+x^2})(y+\sqrt{1+y^2})\right]=c$$

7. $y-x\frac{dy}{dx}=5\left(y^2+\frac{dy}{dx}\right)$

Sol. $y-5y^2=(x+5)\frac{dy}{dx}\Rightarrow\frac{dx}{x+5}=\frac{dy}{y(1-5y)}$

ఇరువైపులా సమాకలనం చేయగా

$$\int\frac{dx}{x+5}=\int\frac{dy}{y(1-5y)}=\int\left(\frac{1}{y}+\frac{5}{1-5y}\right)dy$$

$$\ln|x+5|=\ln y-\ln|1-5y|+\ln c$$

$$\ln|x+5|=\ln\left|\frac{cy}{1-5y}\right|\Rightarrow x+5=\left(\frac{cy}{1-5y}\right)$$

8. $\frac{dy}{dx}=\frac{xy+y}{xy+x}$

Sol. $\frac{dy}{dx}=\frac{y(x+1)}{x(y+1)}\Rightarrow\frac{y+1}{y}dy=\frac{x+1}{x}dx$

$$\int\left(1+\frac{1}{y}\right)dy=\int\left(1+\frac{1}{x}\right)dx$$

$$y + \log y = x + \log x + \log c$$

$$y - x = \log\left|\frac{cx}{y}\right|$$

6. క్రింది అవకలన సమీకరణాల ను సాధించండి

1. $(2x + 2y + 3)\dfrac{dy}{dx} = x + y + 1$

Sol. $\dfrac{dy}{dx} = \dfrac{x+y+1}{2x+2y+3} = \dfrac{(x+y)+1}{2(x+y)+3}$

v = x + y అనుకొనుము. అప్పుడు

$$\frac{dv}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dv}{dx} = 1 + \frac{v+1}{2v+3} = \frac{2v+3+v+1}{2v+3} = \frac{3v+4}{2v+3}$$

$$\frac{2v+3}{3v+4}dv = dx$$

$$\frac{2}{3}\int dv + \frac{1}{9}\int \frac{3\cdot dv}{3v+4} = \int dx$$

$$\frac{2}{3}v + \frac{1}{9}\log(3v+4) = x + c$$

6v + log(3v + 4) = 9x + 9c

6(x + y) + log[3(x + y) + 4] = 9x + c

i.e. log(3x + 3y + 4) = 3x – 6y + c

2. $\dfrac{dy}{dx} = \dfrac{4x+6y+5}{2x+3y+4}$

Sol. $\dfrac{dy}{dx} = \dfrac{4x+6y+5}{2x+3y+4} = \dfrac{2(2x+3y)+5}{2x+3y+4}$

v = 2x + 3y అనుకొనుము. అప్పుడు

$$\frac{dv}{dx} = 2 + 3\frac{dy}{dx} \Rightarrow \frac{dv}{dx} = 2 + \frac{3(2v+5)}{v+4}$$

$$\Rightarrow \quad = \frac{2v+8+6v+15}{v+4} = \frac{8v+23}{v+4}$$

$$\frac{v+4}{8v+23}dv = dx$$

$$\frac{1}{8}\int dv + \frac{9}{8}\int \frac{dv}{8v+23} = \int dx$$

$$\frac{1}{8}v + \frac{9}{64}\log(8v+23) = x + c$$

$\Rightarrow$ **8v + 9 log(8v + 23) = 64x + 64c**

$\Rightarrow$ **8(2x + 3y) – 64x + 9log(16x + 24y + 23) = c′**

$$\Rightarrow \quad 2x+3y-8x+\frac{9}{8}\log(16x+24y+23) = c''$$

$$\Rightarrow \quad 3x-6x+\frac{9}{8}\log(16x+24y+23) = c''$$

$$\Rightarrow \quad y-2x+\frac{3}{8}\log(16x+24y+23) = k$$

3. (2x + y + 1)dx + (4x + 2y – 1)dy = 0

Sol. $\frac{dy}{dx} = -\frac{2x+y+1}{4x+2y-1}$

$$\Rightarrow a_1 = 2, b_1 = 1, a_2 = 4, b_2 = 2$$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2} = \frac{b_1}{b_2}$$

2x + y=v అనుకోండి అప్పుడు $\frac{dv}{dx} = 2 + \frac{dy}{dx}$

$$\frac{dv}{dx} = 2 - \frac{v+1}{2v-1} = \frac{4v-2-v-1}{2v-1} = \frac{3(v-1)}{2v-1}$$

$$\frac{2v-1}{3(v-1)}dv = dx \Rightarrow \frac{2v-1}{v-1}dv = 3dx$$

$$\int\left(2+\frac{1}{v-1}\right)dv = 3\int dx$$

$$2v+\log(v-1)=3x+c$$
$$2v-3x+\log(v-1)=c$$
$$2(2x+y)-3x+\log(2x+y-1)=c$$
$$4x+2y-3x+\log(2x+y-1)=c$$

$\Rightarrow$ **x + 2y + log(2x + y – 1) = c**

4. $\frac{dy}{dx}=\frac{2y+x+1}{2x+4y+3}$

Sol. $\frac{dy}{dx}=\frac{2y+x+1}{2x+4y+3}$

v = x + 2y అయితే అప్పుడు

$$\frac{dv}{dx}=1+\frac{2dy}{dx}$$

$$\Rightarrow \frac{dv}{dx}=1+\frac{2(v+1)}{2v+3}=\frac{2v+3+2v+2}{2v+3}=\frac{4v+5}{2v+3}$$

$$\Rightarrow \frac{2v+3}{4v+5}dv=dx \Rightarrow \int\left(\frac{1}{2}+\frac{1}{2(4v+5)}\right)dv=\int dx$$

$$\Rightarrow \frac{1}{2}v+\frac{1}{2}\cdot\frac{1}{4}\log(4v+5)=x+c \Rightarrow$$

$$4v+\log(4v+5)=8x+8c$$
$$4(x+2y)-8x+\log[4(x+2y)+5]=c'$$

ఇచ్చిన సమీకరణం యొక్క సాధన.4x + 8y – 8x + log(4x + 8y + 5) = c′

ఇచ్చిన సమీకరణం యొక్క సాధన.8y – 4x + log(4x + 8y + 5) = c′

5. (x + y – 1)dy = (x + y + 1)dx

Sol. $\frac{dy}{dx}=\frac{x+y+1}{x+y-1}$

$$v=x+y \Rightarrow \frac{dv}{dx}=1+\frac{dy}{dx}$$

$$\frac{dv}{dx}=1+\frac{v+1}{v-1}=\frac{v-1+v+1}{v-1}=\frac{2v}{v-1}$$

$$\int \frac{v-1}{v} dv = 2\int dx \Rightarrow \int \left(1-\frac{1}{v}\right) dv = 2x + c$$

$\Rightarrow$ $v - \log v = 2x + c$

$x + y - \log(x+y) = 2x - c$

$\Rightarrow$ **(x – y) + log (x + y) = c**

క్రింది అవకలన సమీకరణాల ను సాధించండి

1. $\frac{dy}{dx} + y\tan x = \cos^3 x$

Sol. $\frac{dy}{dx} + y\tan x = \cos^3 x$ ఇది **y** లో ఏకఘాతీయ సమీకరము

$$\text{I.F.} = e^{\int p\,dx} = e^{\int \tan x\,dx} = e^{\log(\sec x)} = \sec x$$

సమీకరణానికి సాధన

y.I.F. = $\text{y.I.F} = \int Q.\ \text{I.F.}\ dx$

$\Rightarrow y\sec x = \int \sec x \cos^3 x\ dx = \int \cos^2 x\ dx$

$= \frac{1}{2}\int (1+\cos 2x)dx = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c$

$\frac{2y}{\cos x} = x + \sin x \cdot \cos x + c$

2y = x cos x + sin x · cos² x + c · cos x

2y = x cos x + sin x · cos² x + c · cos x

2. $\frac{dy}{dx} + y\sec x = \tan x$

Sol. $\frac{dy}{dx} + y\sec x = \tan x$ **which is l.d.e. in y**

$$\text{I.F.} = e^{\int \sec x dx} = e^{\log(\sec x + \tan x)} = \sec x + \tan x$$

ఇచ్చిన సమీకంణానికి సాధన

y.I.F. = $\text{y.I.F} = \int Q.\ \text{I.F.}\ dx$

$$y(\sec x+\tan x)=\int \tan x(\sec x+\tan x)dx$$

$$=\int(\sec x\cdot\tan x+\tan^2 x)dx$$

$$=\int(\sec x\cdot\tan x+\sec^2 x-1)dx$$

$$\Rightarrow y(\sec x+\tan x)=\sec x+\tan x-x+c$$

3. $\frac{dy}{dx}-y\tan x=e^x\sec x$ ఇది **y** లో ఏకఘాతీయ సమీకరము

Sol. $\text{I.F.}=e^{-\int\tan x\,dx}=e^{\log\cos x}=\cos x$

ఇచ్చిన సమీకంణానికి సాధన

y.I.F. = $y.\text{I.F}=\int Q.\ \text{I.F.}\ dx$

$$y\cos x=\int e^x\sec x\cos x dx=\int e^x dx=e^x+c$$

4. $x\frac{dy}{dx}+2y=\log x$

Sol. $\text{I.F.}=e^{\int\frac{2}{x}dx}=e^{2\log x}=e^{\log x^2}=x^2$

ఇచ్చిన సమీకంణానికి సాధన

$$\therefore y\cdot x^2=\int x^2\frac{\log x}{x}dx=\int x\log x dx$$

$$=\log x\left(\frac{x^2}{2}\right)-\frac{1}{2}\int x^2\frac{1}{x}dx=\frac{x^2}{2}\log x-\frac{1}{2}\int x\,dx=\frac{x^2}{2}\log x-\frac{x^2}{4}+c$$

5. $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$

Sol. $\frac{dy}{dx}+\frac{1}{1+x^2}\cdot y=\frac{e^{\tan^{-1}x}}{1+x^2}$ ఇది **y** లో ఏకఘాతీయ సమీకరము

$$\text{I.F.}=e^{\int pdx}=e^{\int\frac{dx}{1+x^2}}=e^{\tan^{-1}x}$$

ఇచ్చిన సమీకంణానికి సాధన

y.I.F. = $y.\text{I.F}=\int Q.\ \text{I.F.}\ dx$

$$y \cdot e^{\tan^{-1}x} = \int \frac{(e^{\tan^{-1}x})^2}{1+x^2} dx \qquad \text{... (1)}$$

$$\int \frac{(e^{\tan^{-1}x})^2}{1+x^2} dx \qquad \text{put } \tan^{-1}x = t \Rightarrow \frac{dx}{1+x^2} = dt$$

$$= \int (e^t)^2 dt = \int e^{2t} dt = \frac{e^{2t}}{2} = \frac{e^{2\tan^{-1}x}}{2}$$

ఇచ్చిన సమీకంణానికి సాధన

$$y \cdot e^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} + \frac{c}{2}$$

$$2y \cdot e^{\tan^{-1}x} = e^{2\tan^{-1}x} + c$$

6. $\frac{dy}{dx} + \frac{2y}{x} = 2x^2$

Sol. $\frac{dy}{dx} + \frac{2y}{x} = 2x^2$ ఇది **y** లో ఏకఘాతీయ సమీకరము

$$\text{I.F.} = e^{\int \frac{2}{x} dx} = e^{2\log x} = e^{\log x^2} = x^2$$

ఇచ్చిన సమీకంణానికి సాధన

y.I.F. $= y.\text{I.F} = \int Q.\ \text{I.F. } dx$

$$y \cdot x^2 = \int 2x^4 dx = \frac{2x^5}{5} + c$$

7. $\frac{dy}{dx} + \frac{4x}{1+x^2} y = \frac{1}{(1+x^2)^2}$

Sol. $\text{I.F.} = e^{\int \frac{4x}{1+x^2} dx} = e^{2\log(1+x^2)}$

$$= e^{\log(1+x^2)^2} = (1+x^2)^2$$

∴ ఇచ్చిన సమీకంణానికి సాధన

$$y(1+x^2)^2 = \int dx = x + c$$

8. $x\frac{dy}{dx}+y=(1+x)e^{x}$

Sol. $\frac{dy}{dx}+\frac{1}{x}\cdot y=\frac{(1+x)e^{x}}{x}$

$\text{I.F.}=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

$y\cdot x=\int(1+x)e^{x}dx=x\cdot e^{x}+c$

9. $\frac{dy}{dx}+\frac{3x^{2}}{1+x^{3}}y=\frac{1+x^{2}}{1+x^{3}}$

Sol. $\frac{dy}{dx}+\frac{3x^{2}}{1+x^{3}}y=\frac{1+x^{2}}{1+x^{3}}$ ఇది **y** లో ఏకఘాతీయ సమీకరము

$\text{I.F.}=e^{\int\frac{3x^{2}}{1+x^{3}}dx}=e^{\log(1+x^{3})}=1+x^{3}$

ఇచ్చిన సమీకంణానికి సాధన

y.I.F. = $\text{y.I.F}=\int \text{Q. I.F. dx}$

$y(1+x^{3})=\int(1+x^{2})dx=x+\frac{x^{3}}{3}+c$

10. $\frac{dy}{dx}-y=-2e^{-x}$

Sol. $\text{I.F.}=e^{\int -dx}=e^{-x}$

$y\cdot e^{-x}=-2\int e^{-2x}dx=e^{-2x}+c$

$y=e^{-x}+ce^{x}$

11. $(1+x^{2})\frac{dy}{dx}+y=\tan^{-1}x$.

Sol. $\frac{dy}{dx}+\frac{1}{1+x^{2}}\cdot y=\frac{\tan^{-1}x}{1+x^{2}}$ ఇది **y** లో ఏకఘాతీయ సమీకరము

I.F. $e^{\int\frac{dx}{1+x^{2}}}=e^{\tan^{-1}x}$

Sol is y.I.F. = $\text{y.I.F}=\int \text{Q. I.F. dx}$

$$y \cdot e^{\tan^{-1} x} = \int \tan^{-1} x \frac{e^{\tan^{-1} x}}{1+x^2} dx$$

Put t = $\tan^{-1}$x so that $dt = \frac{dx}{1+x^2}$

R.H.S. = $\int t \cdot e^t dt = t \cdot e^t - \int e^t dt = t \cdot e^t - e^t$

ఇచ్చిన సమీకరణానికి సాధన

$$y.e^{\tan^{-1} x} = e^{\tan^{-1} x}(\tan^{-1} x - 1) + c$$

$$y = \tan^{-1} x - 1 + c \cdot e^{-\tan^{-1} x}$$

12. $\frac{dy}{dx} + y \tan x = \sin x$ **.**

Sol. $\text{I.F.} = e^{\int \tan x \, dx} = e^{\log \sec x} = \sec x$

$$y \sec x = \int \sin x \cdot \sec x \, dx$$

$$= \int \tan x \, dx = \log \sec x + c$$

Long Answer Questions

1. బ్రాకెట్లలో చూపిన స్థిర సంఖ్యలను తొలగించి ఇచ్చిన వక్రాల కుటుంబ సమీకరణాలకు అనుగుణంగా వచ్చే అవకలన సమీకరణాన్ని రాబట్టండి

i) $y = ae^{3x} + be^{4x}; (a, b)$

Sol. $y = ae^{3x} + be^{4x} ----(1)$

x దృష్ట్యా అవకలనము చేయగా

$y_1 = 3ae^{3x} + 4be^{4x} -----(2)$

x దృష్ట్యా అవకలనము చేయగా

$y_2 = 9ae^{3x} + 16be^{4x} -----(3)$

a,b లను తొలగించగా

$$\begin{vmatrix} y & e^{3x} & e^{4x} \\ y_1 & 3e^{3x} & 4e^{4x} \\ y_2 & 9e^{3x} & 16e^{4x} \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} y & 1 & 1 \\ y_1 & 3 & 4 \\ y_2 & 9 & 16 \end{vmatrix} = 0$$

$\Rightarrow y_2 - 7y_1 + 12y = 0$

ii) **$y = ax^2 + bx$; (a, b)**

Sol.

$y = ax^2 + bx$---- (1)

$\Rightarrow y_1 = y_2x + b \Rightarrow y_1x = y_2x^2 + bx ---(2)$

$\Rightarrow y_2 = 2a$ **---- (3)**

(2) , **(3)** ల నుండి

$y_1 = y_2x + b \Rightarrow y_1x = y_2x^2 + bx ---(4)$

$$x^2\frac{d^2y}{dx^2} = 2ax^2 \qquad \text{... (i)}$$

$$-2x\frac{dy}{dx} = -4ax^2 - 2bx \qquad \text{... (ii)}$$

$2y = 2ax^2 + 2bx$ ... (iii)

(1)+(2)+(3) $\Rightarrow$

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

iii) $ax^2 + by^2 = 1$; (a, b)

Sol.

$ax^2 + by^2 = 1$------ (1)

x దృష్ట్యా అవకలనము చేయగా

$$\Rightarrow 2ax + 2byy_1 = 0$$
$$\Rightarrow ax + byy_1 = 0 ------(2)$$

x దృష్ట్యా అవకలనము చేయగా

$$\Rightarrow a + b(yy_2 + y_1y_1) = 0 \Rightarrow a + b(yy_2 + y_1^2) = 0$$

$$\Rightarrow ax + bx(yy_2 + y_1^2) = 0 \text{---- (3)}$$

$$(3)-(2) \Rightarrow bx(yy_2 + y_1^2) - byy_1 = 0$$
$$\Rightarrow x(yy_2 + y_1^2) - yy_1 = 0$$

iv) $xy = ax^2 + \frac{b}{x}$; (a, b)

Sol. $xy = ax^2 + \frac{b}{x}$

$x^2y = ax^2 + b$

x దృష్ట్యా అవకలనము చేయగా

$x^2y_1 + 2xy = 3ax^2$

$xy_1 + 2y = 3ax$ ... (i)

x దృష్ట్యా అవకలనము చేయగా

$xy_2 + y_1 + 2y_1 = 3a$

$xy_2 + 3y_1 = 3a$ … (ii)

(i) / (ii) $\Rightarrow$

$$\frac{xy_1 + 2y}{xy_2 + 3y_1} = \frac{3ax}{3a} = x$$

$$xy_1 + 2y = x^2y_2 + 3xy$$

$$\Rightarrow \quad x^2y_2 + 2xy_1 - 2y = 0$$

$$x^2\left(\frac{d^2y}{dx^2}\right) + 2x\left(\frac{dy}{dx}\right) - 2y = 0$$

2. క్రింది అవకలన సమీకరణాల ను సాధించండి

1. $\frac{dy}{dx} = \frac{1+y^2}{(1+x^2)xy}$

Sol. $\frac{dy}{dx} = \frac{1+y^2}{(1+x^2)xy}$

$$\Rightarrow \frac{ydy}{1+y^2} = \frac{dx}{x(1+x^2)}$$

$$\frac{2ydy}{1+y^2} = \frac{2xdx}{x^2(1+x^2)}$$

ఇరువైపులా సమాకలనం చేయగా

$$\int \frac{2ydy}{1+y^2} = \int\left(\frac{1}{x^2} - \frac{1}{1+x^2}\right)2x\ dx$$

$$\log(1+y^2) = \log x^2 - \log(1+x^2) + \log c$$

$$\log(1+x^2) + \log(1+y^2) = \log x^2 + \log c$$

ఇచ్చిన సమీకరణం యొక్క సాధన.

$(1 + x^2)(1 + y^2) = cx^2$

2. $\frac{dy}{dx}+x^2=x^2\cdot e^{3y}$

Sol. $\frac{dy}{dx}+x^2=x^2\cdot e^{3y}$

$\Rightarrow \frac{dy}{dx}=x^2\cdot e^{3y}-x^2=x^2(e^{3y}-1)$

ఇరువైపులా సమాకలనం చేయగా

$\int\frac{dy}{e^{3y}-1}=\int x^2dx \Rightarrow \int\frac{e^{-3y}}{1-e^{-3y}}=\int x^2dx$

$\log\frac{(1-e^{-3y})}{3}=\frac{x^3}{3}+c$

$\log(1-e^{-3y})=x^3+c'\ (c'=3c)$

$\Rightarrow\ 1-e^{-3y}=e^{x^3}\cdot k\ \ (k=e^{c'})$

3. $(xy^2 + x)dx + (yx^2 + y)dy = 0$

Sol. $(xy^2 + x)dx + (yx^2 + y)dy = 0$

$x(y^2 + 1)dx + y(x^2 + 1)dy = 0$

$\Rightarrow (1 + x^2)(1 + y^2)$

$\frac{xdx}{1+x^2}+\frac{ydy}{1+y^2}=0$

ఇరువైపులా సమాకలనం చేయగా

$\int\frac{xdx}{1+x^2}+\int\frac{ydy}{1+y^2}=0$

$\frac{1}{2}\left[(\log(1+x^2)+\log(1+y^2)\right]=\log c$

$\log(1+x^2)(1+y^2)=2\log c=\log c^2$

$(1 + x^2)(1 + y^2) = k$ **when** $k = c^2$.

4. $\frac{dy}{dx} = 2y\tanh x$

Sol. $\frac{dy}{dx} = 2y\tanh x \Rightarrow \frac{dy}{y} = 2\tanh x dx$

ఇరువైపులా సమాకలనం చేయగా

$$\int \frac{dy}{y} = 2\int \tanh x \ dx$$

$$\log y = 2\log|\cosh x| + \log c$$

$$\ln y = 2\ln\cosh x + \ln c \Rightarrow y = c\cos^2 hx$$

5. $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

$$\frac{dy}{dx} = \sin(x+y) \Rightarrow x + y = t$$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - 1 = \sin t \Rightarrow \frac{dt}{dx} = 1 + \sin t$$

$$\frac{dt}{1+\sin t} = dx$$

ఇరువైపులా సమాకలనం చేయగా

$$\int \frac{dt}{1+\sin t} = \int dx$$

$$\int \frac{1-\sin t}{\cos^2 t} dt = x + c$$

$$\int \sec^2 t dt - \int \tan t \cdot \sec t \ dt = x + c$$

$$\tan t - \sec t = x + c$$

$$\Rightarrow \tan(x+y) - \sec(x+y) = x + c$$

6. $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$

$$\frac{-dy}{y^2+y+1}=\frac{dx}{x^2+x+1}$$

ఇరువైపులా సమాకలనం చేయగా

$$-\int\frac{dy}{y^2+y+1}=\int\frac{dx}{x^2+x+1}$$

$$-\int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\frac{3}{4}}=\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$-\frac{2}{\sqrt{3}}\tan^{-1}\frac{(y+1/2)}{\sqrt{3/2}}=\frac{2}{\sqrt{3}}\tan^{-1}\frac{(x+1/2)}{\sqrt{3/2}}+c$$

$$\tan^{-1}\frac{2x+1}{\sqrt{3}}+\tan^{-1}\frac{2y+1}{\sqrt{3}}=c$$

7. $\frac{dy}{dx}=\tan^2(x+y)$

Sol. $\frac{dy}{dx}=\tan^2(x+y)$ **put v = x + y**

$$\frac{dv}{dx}=1+\frac{dy}{dx}=1+\tan^2 v=\sec^2 v$$

$$\int\frac{dv}{\sec^2 v}=\int dx=\int\cos^2 v\cdot dv=x+c$$

$$\int\frac{(1+\cos 2v)}{2}dv=x+c$$

$$\Rightarrow\int(1+\cos 2v)dv=2x+2c$$

$$v+\frac{\sin 2v}{2}=2x+2c$$

$$2v+\sin 2v=4x+c'$$

$$2(x+y)+\sin 2(x+y)=4x+c'$$

$$x-y-\frac{1}{2}\sin[2(x+y)]=c$$

3. క్రింది అవకలన సమీకరణాల ను సాధించండి

1. $\frac{dy}{dx}=\frac{x-y}{x+y}$

Sol. $\frac{dy}{dx}=\frac{x-y}{x+y}$ **----- (1)**

(1) సమఘాతీయఅవకలన సమీకరణం

Put y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{x-vx}{x+vx}=\frac{x(1-v)}{x(1+v)}$$

$$x\cdot\frac{dv}{dx}=\frac{1-v}{1+v}-v=\frac{1-v-v-v^2}{1+v}=\frac{1-2v-v^2}{1+v}$$

$$\int\frac{(1+v)dv}{1-2v-v^2}=\int\frac{dx}{x}$$

$$-\frac{1}{2}\log(1-2v-v^2)=\log x+\log c$$

$$-\frac{1}{2}\log\left(1-2\cdot\frac{y}{x}-\frac{y^2}{x^2}\right)=\log cx$$

$$\log\frac{(x^2-2xy-y^2)}{x^2}=-2\log cx=\log(cx)^{-2}$$

$$\frac{x^2-2xy-y^2}{x^2}=(cx)^{-2}=\frac{1}{c^2x^2}$$

$$(x^2-2xy-y^2)=\frac{1}{c^2}=k$$

2. $(x^2+y^2)dy = 2xy\,dx$

Sol. $\frac{dy}{dx}=\frac{2xy}{x^2+y^2}$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx

$$\frac{dy}{dx}=v+x\cdot\frac{dv}{dx}$$

$$v + x\cdot\frac{dv}{dx} = \frac{2x(vx)}{x^2+v^2x^2} = \frac{2v}{1+v^2}$$

$$x\cdot\frac{dv}{dx} = \frac{2v}{1+v^2} - v = \frac{2v-v-v^3}{1+v^2} = \frac{v-v^3}{1+v^2}$$

$$\int\frac{1+v^2}{v(1-v^2)}dv = \int\frac{dx}{x}$$

Let $\frac{1+v^2}{v(1-v^2)} = \frac{A}{v} + \frac{B}{1+v} + \frac{C}{1-v}$

$$1+v^2 = A(1-v^2) + BV(1-v) + CV(1+v)$$
$$v = 0 \Rightarrow 1 = A$$
$$v = 1 \Rightarrow 1+1 = C(2) \Rightarrow c = 1$$
$$v = -1 \Rightarrow 1+1 = B(-1)(2) \Rightarrow 2 = -2B \Rightarrow B = -1$$

$$\int\frac{1+v^2}{v(1-v^2)}dv = \int\frac{dv}{v} - \int\frac{dv}{1+v} + \int\frac{dv}{1-v}$$

$$= \log v - \log(1+v) - \log(1-v) = \log\frac{v}{1-v^2}$$

$$\therefore \log\frac{v}{1-v^2} = \log x + \log c = \log cx$$

$$\frac{v}{1-v^2} = cx \Rightarrow v = cx(1-v^2)$$
$$\frac{y}{x} = cx\left(1-\frac{y^2}{x^2}\right) \Rightarrow \frac{y}{x} = cx\frac{(x^2-y^2)}{x^2}$$

సమీకరణ సాధన $y = c(x^2 - y^2)$

3. $\frac{dy}{dx} = \frac{-(x^2+3y^2)}{(3x^2+y^2)}$

Sol. $\frac{dy}{dx} = \frac{-(x^2+3y^2)}{(3x^2+y^2)}$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx

$$\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$$
$$v + x\cdot\frac{dv}{dx} = \frac{-(x^2+3v^2x^2)}{3x^2+v^2x^2} = \frac{-x^2(1+3v^2)}{x^2(3+v^2)}$$

$$x\cdot\frac{dv}{dx}=-v-\frac{1+3v^2}{3+v^2}$$

$$=\frac{-3v-v^3-1-3v^2}{3+v^2}=-\frac{(v+1)^3}{3+v^2}$$

$$\frac{3+v^2}{(v+1)^3}=\frac{-dx}{x}$$

$\frac{3+v^2}{(v+1)^3}=\frac{A}{v+1}+\frac{B}{(v+1)^2}+\frac{C}{(v+1)^3}$ అనుకొనుము. అప్పుడు

$$\frac{v^2+3}{(v+1)^3}=\frac{1}{v+1}-\frac{2}{(v+1)^2}+\frac{4}{(v+1)^3}$$

$$\int\frac{v^2+3}{(v+1)^3}=-\int\frac{dx}{x}$$

$$\int\left(\frac{1}{v+1}-\frac{2}{(v+1)^2}+\frac{4}{(v+1)^3}\right)dv=-\log x+\log c\ \log(v+1)+\frac{2}{v+1}-\frac{4}{2(v+1)^2}=\log\frac{c}{x}$$

సమీకరణ సాధన $\log\left(\frac{y}{x}+1\right)+\frac{2}{\frac{y}{x}+1}-\frac{2}{\left(\frac{y}{x}+1\right)^2}=\log\frac{c}{x}$

$$\frac{2x}{x+y}-\frac{2x^2}{(x+y)^2}=\log\frac{c}{x}-\log\frac{(x+y)}{x}$$

$$\frac{2x^2+2xy-2x^2}{(x+y)^2}=\log\frac{c}{x+y}$$

$$\frac{2xy}{(x+y)^2}=\log\frac{c}{x+y}$$

$$\log\left(\frac{x+y}{c}\right)c=-\log\left(\frac{c}{x+y}\right)=-\frac{2xy}{(x+y)^2}$$

\

4. $y^2\,dx + (x^2 - xy)dy = 0$

Sol. $y^2\,dx + (x^2 - xy)dy = (xy - x^2)dy$

$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ సమఘాతీయఅవకలన సమీకరణం

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{v^2x^2}{x^2(v - v^2)}$$

$$x \cdot \frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v^2 - v^2 + v}{v-1}$$

$$\frac{v-1}{v}dv = \frac{dx}{x} \Rightarrow \int\left(1 - \frac{1}{v}\right)dv = \int\frac{dx}{x}$$

$$v - \log v = \log x + \log k$$

$$v = \log v + \log x + \log k = \log k(vx)$$

$$\frac{y}{x} = \log ky \Rightarrow ky = e^{y/x}$$

5. $\frac{dy}{dx} = \frac{(x+y)^2}{2x^2}$

Sol,. $\frac{dy}{dx} = \frac{(x+y)^2}{2x^2}$ సమఘాతీయఅవకలన సమీకరణం

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{(x+vx)^2}{2x^2} = x^2\frac{(1+v)^2}{2x^2}$$

$$x\frac{dv}{dx} = \frac{(1+v^2)}{2} - v = \frac{1 + v^2 + 2v - 2v}{2}$$

$$2\int\frac{dv}{1+v^2} = \int\frac{dx}{x} \Rightarrow 2\tan^{-1}v = \log x + \log c$$

$$2\tan^{-1}\left(\frac{y}{x}\right) = \log cx$$

6. $(x^2 - y^2)dx - xy\,dy = 0$

Sol. $(x^2 - y^2)dx - xy\,dy = 0$

$(x^2 - y^2)dx = xy\,dy$

$\frac{dy}{dx} = \frac{x^2 - y^2}{xy}$ సమఘాతీయఅవకలన సమీకరణం

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{vx^2} = \frac{x^2(1-v^2)}{vx^2}$$

$$x\frac{dv}{dx} = \frac{1-v^2}{v} - v = \frac{1-v^2-v^2}{v} = \frac{1-2v^2}{v}$$

$$\int \frac{vdv}{1-2v^2} = \int \frac{dx}{x}$$

$$-\frac{1}{4}\log(1-2v^2) = \log x + \log c$$

$$-\frac{1}{4}\log\left(1 - \frac{2y^2}{x^2}\right) = \log x + \log c$$

$$-\frac{1}{4}\log\left(\frac{x^2 - 2y^2}{x^2}\right) = \log x + \log c$$

$$-\frac{1}{4}\left[\log(x^2 - 2y^2) - \log x^2\right] = \log x + \log c$$

$$-\frac{1}{4}\log(x^2 - 2y^2) + \frac{1}{4}\cdot 2\log x = \log x + \log c$$

$$-\frac{1}{4}\log(x^2 - 2y^2) = \frac{1}{2}\log x + \log c$$

$$-\log(x^2 - 2y^2) = -2\log x - 4\log c$$

$\log(x^2 - 2y^2) = -2\log x + \log k$ **where**

$$k = \frac{1}{c^4} = \log\frac{k}{x^2} \Rightarrow x^2 - 2y^2 = \frac{k}{x^2}$$

సమీకరణ సాధన : $x^2(x^2 - 2y^2) = k$

7. $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$

Sol. $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$

$\frac{dy}{dx} = \frac{x^2y - 2xy^2}{x^3 - 3x^2y}$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{x^3v - 2v^2x^3}{x^3 - 3vx^3}$$

$$= \frac{(v - 2v^2)x^3}{(1-3v)x^3} = \frac{v - 2v^2}{1-3v}$$

$$x\frac{dv}{dx} = \frac{v - 2v^2}{1-3v} - v$$

$$= \frac{v - 2v^2 - v(1-3v)}{1-3v} = \frac{-2v^2 + 3v^2}{1-3v}$$

$$x\frac{dv}{dx} = \frac{v^2}{1-3v} \Rightarrow \frac{1-3v}{v^2}dv = \frac{dx}{x}$$

$$\int\left(\frac{1}{v^2} - \frac{3}{v}\right)dv = \int\frac{dx}{x}$$

$$\frac{-1}{v} - 3\log v = \log x + \log c$$

$$\frac{-x}{y} = 3\log\left(\frac{y}{x}\right) = \log x + \log c$$

$$\frac{-x}{y} - \log\left(\frac{y}{x}\right)^3 = \log xc$$

$$\frac{-x}{y} = \log xc + \log\frac{y^3}{x^3}$$

$$\frac{-x}{y} = \log\left(cx \cdot \frac{y^3}{x^3}\right) = \log\left(\frac{cy^3}{x^2}\right)$$

$$\frac{cy^3}{x^2} = e^{-x/y} \Rightarrow cy^3 = \frac{x^2}{e^{x/y}}$$

$$cy^3 \cdot e^{x/y} = x^2$$

8. $y^2\,dx + (x^2 - xy + y^2)dy = 0$

Sol. $y^2\,dx = -(x^2 - xy + y^2)dy$ సమఘాతీయఅవకలన సమీకరణం

$$\frac{dy}{dx} = \frac{-y^2}{x^2 - xy + y^2}$$

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{-v^2x^2}{x^2 - vx^2 + v^2x^2} = \frac{-v^2x^2}{x^2(1 - v + v^2)}$$

$$x\frac{dv}{dx} = \frac{-v^2}{1 - v + v^2} - v$$

$$= \frac{-v^2 - v + v^2 - v^3}{1 - v + v^2} = -\frac{v(1 + v^2)}{1 - v + v^2}$$

$$\frac{1 - v + v^2}{v(1 + v^2)}dv = -\frac{dx}{x} \qquad \text{... (1)}$$

$\frac{1 - v + v^2}{v(1 + v^2)} = \frac{A}{v} + \frac{Bv + C}{1 + v^2}$ అనుకొనుము. అప్పుడు

$$\frac{1 - v + v^2}{v(1 + v^2)} = \frac{1}{v} - \frac{1}{1 + v^2}$$

$$\int \frac{1 - v + v^2}{v(1 + v^2)}dv = \int \frac{dv}{v} - \int \frac{dv}{1 + v^2} = \log v - \tan^{-1} v$$

$$\log v - \tan^{-1} v = -\log x + \log c$$

$$\tan^{-1} v = \log v + \log x - \log c = \log \frac{vx}{c} = \log \frac{y}{c}$$

$$\frac{y}{c} = e^{\tan^{-1} v} = e^{\tan^{-1}(y/x)}$$

సమీకరణ సాధన : $y = c \cdot e^{\tan^{-1}(y/x)}$

9. $(y^2 - 2xy)dx + (2xy - x^2)dy = 0$

Sol. $(y^2 - 2xy)dx + (2xy - x^2)dy = 0$

$(2xy - x^2)dy = -(y^2 - 2xy)dx$

$\frac{dy}{dx} = \frac{2xy - y^2}{2xy - x^2}$ సమఘాతీయఅవకలన సమీకరణం

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{2vx^2 - v^2x^2}{2vx^2 - x^2} = \frac{x^2(2v - v^2)}{x^2(2v - 1)}$$

$$x\frac{dv}{dx} = \frac{2v - v^2}{2v - 1} - v$$

$$= \frac{2v - v^2 - 2v^2 + v}{2v - 1} = \frac{3v(1 - v)}{2v - 1}$$

$$\int \frac{2v - 1}{v(1 - v)}dv = 3\int \frac{dx}{x} \qquad \text{... (1)}$$

$$\text{Let } \frac{2v - 1}{v(1 - v)} = \frac{A}{v} + \frac{B}{1 - v}$$

$$2v - 1 = A(1 - v) + Bv$$

$$v = 0 \Rightarrow -1 = A \Rightarrow A = -1$$

$$v = 1 \Rightarrow 1 = B \Rightarrow B = 1$$

$$\int \left(-\frac{1}{v} + \frac{1}{1 + v}\right)dv = 3\int \frac{dx}{x}$$

$$-\log v - \log(1 - v) = 3\log x + \log c$$

$$\log \frac{1}{v(1 - v)} = \log cx^3$$

$$\frac{1}{v(1 - v)} = cx^3 \Rightarrow v(1 - v) = \frac{1}{cx^3}$$

$$\frac{y}{x}\left(1 - \frac{y}{x}\right) = \frac{1}{cx^3} \Rightarrow \frac{y}{x}\left(\frac{x - y}{x}\right) = \frac{1}{cx^3}$$

$$xy(x - y) = \frac{1}{c} = k \Rightarrow xy(y - x) = -\frac{1}{c} = k$$

10. $\frac{dy}{dx}+\frac{y}{x}=\frac{y^2}{x^2}$

Sol. $\frac{dy}{dx}+\frac{y}{x}=\frac{y^2}{x^2}$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx $\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$v+x\frac{dv}{dx}+v=\frac{v^2x^2}{x^2}\Rightarrow x\frac{dv}{dx}=v^2-2v$$

$$\frac{dv}{v^2-2v}=\frac{dy}{x}$$

Let $\frac{1}{v^2-2v}=\frac{A}{v}+\frac{B}{v-2}$

$$1=A(v-2)+Bv$$

$$v=0\Rightarrow 1=A(-2)\Rightarrow -\frac{1}{2}$$

$$v=2\Rightarrow 1=2B\Rightarrow B=\frac{1}{2}$$

$$-\frac{1}{2}\int\left(\frac{1}{v}-\frac{1}{v-2}\right)dv=\int\frac{dx}{x}$$

$$-\frac{1}{2}[\log v-\log(v-2)]=\log x+\log c$$

$$-\frac{1}{2}\left[\log\frac{v}{v-2}\right]=\log cx$$

$$\log\frac{v}{v-2}=-\log cx=\log(cx)^{-2}$$

$$\frac{v}{v-2}=(cx)^{-2}\Rightarrow\frac{(y/x)}{(y/x)-2}=\frac{1}{c^2x^2}$$

$$\frac{y}{y-2x}=\frac{1}{c^2x^2}\Rightarrow x^2y=\frac{1}{c^2}(y-2x)$$

సమీకరణ సాధన $y-2x=c^2x^2y=kx^2y$ **where k = c²**

11. $xdy - ydx = \sqrt{x^2 + y^2}dx$

Sol. $xdy - ydx = \sqrt{x^2 + y^2}dx$

$$x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$

$$\frac{dy}{dx} - \frac{y}{x} = \frac{\sqrt{x^2 + y^2}}{x}$$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore x\frac{dv}{dx} = \frac{\sqrt{x^2 + y^2}}{x} = \frac{x\sqrt{1+v^2}}{x}$$

$$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x} \Rightarrow \sinh^{-1} v = \log x + \log c$$

$$\log\left[v + \sqrt{1+v^2}\right] = \log cx \Rightarrow v + \sqrt{1+v^2} = cx$$

$$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = cx \Rightarrow y + \sqrt{x^2 + y^2} = cx^2$$

12. (2x – y)dy = (2y – x)dx

Sol. $\frac{dy}{dx} = \frac{2y - x}{2x - y}$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{x(2v-1)}{x(2-v)}$$

$$x\frac{dv}{dx} = \frac{2v-1}{2-v} - v = \frac{2v - 1 - 2v + v^2}{2-v}$$

$$\frac{2-v}{v^2-1}dv = \frac{dx}{x} \Rightarrow 2\int \frac{dv}{v^2-1} - \int \frac{vdv}{v^2-1} = \int \frac{dx}{x}$$

$$2 \cdot \frac{1}{2}\log\frac{v-1}{v+1} - \frac{1}{2}\log(v^2 - 1) = \log x + \log c$$

$$\frac{1}{2}\left(2\log\frac{v-1}{v+1} - \log(v^2-1)\right) = \log cx$$

$$\frac{1}{2}\log\frac{(v-1)^2}{(v+1)^2}\cdot\frac{1}{(v-1)(v+1)}=\log cx$$

$$\log\frac{v-1}{(v+1)^2}=2\log cx=\log c^2x^2$$

$$\therefore\frac{v-1}{(v+1)^3}=c^2x^2\Rightarrow\frac{\frac{y}{x}-1}{\left(\frac{y}{x}+1\right)^3}=c^2x^2$$

$$\frac{\frac{y-x}{x}}{\frac{(y-x)^3}{x^3}}=c^2x^2\Rightarrow\frac{x^2(y-x)}{(x+y)^3}=c^2x^2$$

$(y-x)=c^2(x+y)^3$.

13. $(x^2-y^2)\frac{dy}{dx}=xy$

Sol. $\frac{dy}{dx}=\frac{xy}{(x^2-y^2)}$ సమఘాతీయఅవకలన సమీకరణం

Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$$v+x\frac{dv}{dx}=\frac{x(vx)}{x^2-v^2x^2}=\frac{v}{1-v^2}$$

$$x\frac{dv}{dx}=\frac{v}{1-v^2}-v=\frac{v-v+v^3}{1-v^2}=\frac{v^3}{1-v^2}$$

$$\frac{1-v^2}{v^3}dv=\frac{dx}{x}\Rightarrow\int\frac{dv}{v^3}-\int\frac{dv}{v}=\int\frac{dx}{x}$$

$$-\frac{1}{2v^2}-\log v=\log x+c$$

$$-\frac{1}{2}\frac{x^2}{y^2}=\log vx+c=\log y+c$$

$$\frac{-x^2}{2y^2}=(\log y+c)\Rightarrow -x^2=2y^2(c+\log y)$$

$x^2+2y^2(c+\log y)=0$.

14. **Solve** $2\frac{dy}{dx}=\frac{y}{x}+\frac{y^2}{x^2}$ సమఘాతీయఅవకలన సమీకరణం

Sol. $2\frac{dy}{dx}=\frac{y}{x}+\frac{y^2}{x^2}$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx $\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$2v+2x\frac{dv}{dx}=v+v^2 \Rightarrow 2x\frac{dv}{dx}=v^2-v$$

$$\frac{dv}{v(v-1)}=2\frac{dx}{x} \Rightarrow \int\left(\frac{1}{v-1}-\frac{1}{v}\right)dv=2\int\frac{dx}{x}$$

log(v – 1) – log v = 2 logx + log c

$$\log\frac{v-1}{v}=\log cx^2 \Rightarrow \frac{v-1}{v}=cx^2$$

$$\frac{\frac{y}{x}-1}{\frac{y}{x}}=cx^2 \Rightarrow \frac{y-x}{y}=cx^2$$

సమీకరణ సాధన **(y – x) = cx^2y**

క్రింది అవకలన సమీకరణాల ను సాధించండి

1. $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$.

Sol. $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$

$\Rightarrow \frac{dx}{dy}=-\frac{e^{x/y}\left(1-\frac{x}{y}\right)}{\left(1+e^{x/y}\right)}$ సమఘాతీయఅవకలన సమీకరణం

Put x = vy $\Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$

$$(1+e^v)\frac{dx}{dy}+e^v(1-v)=0$$

$$(1+e^v)\left(v+y\frac{dv}{dy}\right)+e^v(1-v)=0$$

$$v+ve^v+y(1+e^v)\frac{dv}{dy}+e^v-ve^v=0$$

$$y(1+e^v)dv=-(v+e^v)dy$$

$$\int\frac{1+e^v}{v+e^v}dv=-\int\frac{dy}{y}$$

$$\log(v+e^v)=-\log y+\log c\Rightarrow v+e^v=\frac{c}{y}$$

$$\frac{x}{y}+e^{x/y}=\frac{c}{y}\Rightarrow x+y\cdot e^{x/y}=c$$

2. $x\sin\frac{y}{x}\cdot\frac{dy}{dx}=y\sin\frac{y}{x}-x$

Sol. $x\sin\frac{y}{x}\cdot\frac{dy}{dx}=y\sin\frac{y}{x}-x$

$$\Rightarrow\frac{dy}{dx}=\frac{\frac{y}{x}\left(\sin\left(\frac{y}{x}\right)-\frac{x}{y}\right)}{\sin\left(\frac{y}{x}\right)}$$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx $\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$$v+x\frac{dv}{dx}=\frac{v\left(\sin v-\frac{1}{v}\right)}{\sin v}$$

$$x\frac{dv}{dx}=\frac{v\sin v-1-v\sin v}{\sin v}$$

$$-\sin v dv=\frac{1}{x}dx$$

$$\Rightarrow\int-\sin v\cdot dv=+\int\frac{dx}{x}$$

$$\Rightarrow\cos v=\log x+\log c=\log cx$$

$$\Rightarrow cx=e^{\cos v}=e^{\cos(y/x)}.$$

3. $xdy = \left(y + x\cos^2\frac{y}{x}\right)dx$

Sol. $x\frac{dy}{dx} = y + x\cdot\cos^2\frac{y}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x} + \cos^2\frac{y}{x}$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx,

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \Rightarrow v + x\frac{dv}{dx} = v + \cos^2 v$$

$$\frac{dv}{\cos^2 v} = \frac{dx}{x} \Rightarrow \int \sec^2 v\cdot dv = \int\frac{dx}{x}$$

tan v = logx + c

i.e. $\tan\left(\frac{y}{x}\right) = \log x + c$ **.**

4. (x – y log y + y log x)dx + x(log y – log x)dy = 0.

Sol.

$$1 - \frac{y}{x}\log y + \frac{y}{x}\log x + \log\left(\frac{y}{x}\right)\frac{dy}{dx} = 0$$

$1 - \frac{y}{x}\left(\log y - \log\frac{y}{x}\right) + \log\left(\frac{y}{x}\right)\frac{dy}{dx} = 0$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$1 - v[\log v] + \log v\left(v + x\frac{dv}{dx}\right) = 0$$

$$1 - v\log v + v\log v + x\log v\frac{dv}{dx} = 0$$

$$x\log v\frac{dv}{dx} = -1 \Rightarrow \int\log v\,dv = -\int\frac{dx}{x}$$

$$v\log v - \int dv = c - \log x$$

$$v\log v - v = c - \log x$$

$$v + c = v\log v + \log x \Rightarrow v + c = v\log v + \log x$$

$$\frac{y}{x} + c = \frac{y}{x}\log\left(\frac{y}{x}\right) + \log x$$

$$y + cx = y\log\left(\frac{y}{x}\right) + x\log x$$

$$= y\log y - y\log x + x\log x$$
$$= (x - y)\log x + y\log y$$

5. $(ydx + xdy)x\cos\frac{y}{x} = (xdy - ydx)y\sin\frac{y}{x}$

Sol. $(ydx + xdy)x\cos\frac{y}{x} = (xdy - ydx)y\sin\frac{y}{x}$

$$\left(xy\cdot\cos\frac{y}{x} + y^2\sin\frac{y}{x}\right) - \left(xy\cdot\sin\frac{y}{x} - x^2\cos\frac{y}{x}\right)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{xy\cdot\cos\left(\frac{y}{x}\right) + y^2\sin\left(\frac{y}{x}\right)}{xy\sin\left(\frac{y}{x}\right) - x^2\cos\left(\frac{y}{x}\right)}$$

$$= \frac{\left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2\sin\left(\frac{y}{x}\right)}{\left(\frac{y}{x}\right)\sin\left(\frac{y}{x}\right) - \cos\left(\frac{y}{x}\right)} = F\left(\frac{y}{x}\right)$$

సమఘాతీయఅవకలన సమీకరణం

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v\cos v + v^2\sin v}{v\sin v - \cos v}$$

$$x\frac{dv}{dx} = \frac{v\cos v + v^2\sin v}{v\sin v - \cos v} - v$$

$$= \frac{v\cos v + v^2\sin v - v^2\sin v + v\cos v}{v\sin v - \cos v}$$

$$= \frac{2v\cos v}{v\sin v - \cos v}$$

$$\frac{v\sin v - \cos v}{v\cos v} = \frac{2}{x}dv$$

$$\int\left(\tan v - \frac{1}{v}\right)dv = \int\frac{2}{x}dv$$

$$\log\sec v - \log v = 2\log|x|$$

$$\therefore \log x^2 = \log\left|\frac{c}{v\cos v}\right| \Rightarrow x^2 = \frac{c}{v\cos v}$$

But $\frac{y}{x} = v$

సమీకరణ సాధన

$$x^2 = \frac{c}{\frac{y}{x}\cdot\cos\left(\frac{y}{x}\right)} \Rightarrow xy\cos\left(\frac{y}{x}\right) = c$$

6. వాలు $\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x}$, **x > 0, y > 0** అవుతూ **(1, π/4)** బిందువు గుండా పోయే వక్రం సమీకరణం కనుగొనుము.

Sol. $\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x}$ సమఘాతీయఅవకలన సమీకరణం

y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = v - \cos^2 v \Rightarrow \int\frac{dv}{\cos^2 v} = -\int\frac{dx}{x}$$

$$\int\sec^2 v = -\int\frac{dx}{x} \Rightarrow \tan v = -\log|x| + c$$

(1, π/4) బిందువు గుండా పోతోంది, కావున

$$\tan\left(\frac{\pi}{4}\right) = c - \log 1 \Rightarrow c = 1$$

$$\tan v = 1 - \log|x| \Rightarrow \tan\left(\frac{y}{x}\right) 1 - \log|x|$$

7. $(1+2e^{x/y})dx + 2e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$.

Sol. $(1+2e^{x/y})dx = -2e^{x/y}\left(1-\frac{x}{y}\right)dy$

$$= 2e^{x/y}\left(\frac{x}{y}-1\right)dy$$

$$\frac{dy}{dx}=\frac{2e^{x/y}\left(\frac{x}{y}-1\right)}{1+2e^{x/y}}$$ సమఘాతీయఅవకలన సమీకరణం

Put x = vy

$$\frac{dx}{dy}=v+y\frac{dv}{dy}\Rightarrow v+y\frac{dv}{dy}=\frac{2e^{v}(v-1)}{1+2e^{v}}$$

$$y\frac{dv}{dy}=\frac{2e^{v}(v-1)}{1+2e^{v}}-v$$

$$=\frac{2ve^{v}-2e^{v}-v-2v}{1+2e^{v}}e^{v}=\frac{-(2e^{v}+v)}{1+2e^{v}}$$

$$\int\frac{1+2e^{v}}{v+2e^{v}}dv=-\int\frac{dy}{y}$$

$$\log(v+2e^{v})=-\log y+\log c=\log\frac{c}{y}$$

$$v+2e^{v}=\frac{c}{y}$$

Solution is: $\frac{x}{y}+2e^{x/y}=\frac{c}{y}\Rightarrow x+2y\cdot e^{x/y}=c$

8. $x\sec\left(\frac{y}{x}\right)(ydx+xdy)=y\csc\left(\frac{y}{x}\right)(xdy-ydx)$.

Sol. $x\sec\left(\frac{y}{x}\right)(ydx+xdy)=y\csc\left(\frac{y}{x}\right)(xdy-ydx)\Rightarrow x\sec\left(\frac{y}{x}\right)\left(y+x\frac{dy}{dx}\right)=y\csc\left(\frac{y}{x}\right)\left(x\frac{dy}{dx}-y\right)$

$$x\frac{dy}{dx}\left(x\cdot\sec\left(\frac{y}{x}\right)-y\cdot\csc\left(\frac{y}{x}\right)\right)$$

$$=-y\left[y\csc\left(\frac{y}{x}\right)+x\sec\left(\frac{y}{x}\right)\right]$$

$$\frac{dy}{dx}=\frac{-y\left(y\csc\left(\frac{y}{x}\right)+x\sec\left(\frac{y}{x}\right)\right)}{x\left(x\sec\left(\frac{y}{x}\right)-y\csc\left(\frac{y}{x}\right)\right)}$$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = v\left(\frac{v\csc v + \sec v}{v\csc v - \sec v}\right)$$

$$= \frac{v\left(\frac{v}{\sin v} + \frac{1}{\cos v}\right)}{\left(v\frac{1}{\sin v} - \frac{1}{\cos v}\right)} = \frac{v(v\cos v + \sin v)}{v\cos v - \sin v}$$

$$x\frac{dv}{dx} = \frac{v(v\cos v + \sin v)}{v\cos v - \sin v} - v$$

$$= \frac{v(v\cos v + \sin v - v\cos v + \sin v)}{v\cos v - \sin v}$$

$$= \frac{2v\sin v}{v\cos v - \sin v}$$

$$\int \frac{v\cos v - \sin v}{v\sin v}dv = 2\int \frac{dx}{x}$$

$$\int \frac{\cos v}{\sin v}dv - \int \frac{1}{v}dv = 2\int \frac{dx}{x}$$

$$\log\sin v - \log v = 2\log x + \log c$$

$$\log\left(\frac{\sin v}{v}\right) = \log cx^2 \Rightarrow \frac{\sin v}{v} = cx^2$$

$$\frac{x}{y}\sin\left(\frac{y}{x}\right) = cx^2 \Rightarrow \sin\left(\frac{y}{x}\right) = cxy$$

4. క్రింది అవకలన సమీకరణాల ను సాధించండి

1. $\frac{dy}{dx} = \frac{3y - 7x + 7}{3x - 7y - 3}$

Sol.

$$\Rightarrow a_1 = -7, b_1 = 3, a_2 = 3, b_2 = -7$$

$$\frac{a_1}{a_2} = \frac{-7}{3}, \frac{b_1}{b_2} = \frac{3}{-7} \qquad \therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

x = x + h, y = y + k అనుకొనుము. అప్పుడు

$$3k - 7h + 7 = 0 \quad \text{and} 3h - 7k - 3 = 0 \textbf{ and } \frac{dy}{dx} = \frac{dY}{dX}$$

సమీకరణాల ను సాధించగా

h = 0 , k = 1

$\frac{dy}{dx}=\frac{3y-7x}{3x-7y}$ సమఘాతీయఅవకలన సమీకరణం

Put y = vx $\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$v+x\frac{dv}{dx}=\frac{x(3v-7)}{x(3-7v)}$$

$$x\frac{dv}{dx}=\frac{3v-7}{3-7v}-v=\frac{3v-7-3v+7v^2}{3-7v}$$

$$=\frac{7v^2-7}{3-7v}=\frac{7v^2-7}{3-7v}$$

$$\frac{3-7v}{7v^2-7}=\frac{dx}{x}$$

$$\int\frac{3}{7v^2-7}dv-\int\frac{7vdv}{7v^2-7}=\int\frac{dx}{x}$$

$\ln\ x=\frac{3}{14}\ln\left|\frac{v-1}{v+1}\right|-\frac{1}{2}\ln|v^2-1|$ **14 log x–log c**

$$x=3\log\left|\frac{v-1}{v+1}\right|-7\log|v^2-1|\Rightarrow 14\ln x-\ln c$$

$$=3\ln(v-1)-3\ln(v+1)-7\ln(v+1)-7\ln(v-1)$$

$$14\ln x-\ln c=-10\ln(v+1)-4\ln(v-1)$$

$$\ln(v+1)^5+\ln(v-1)^2+\ln x^7=\ln c$$

$$(v+1)^5\cdot(v-1)^2\cdot x^7=c$$

$$\left(\frac{y}{x}+1\right)^5\left(\frac{y}{x}-1\right)^2x^7=c$$

$$(y-x)^2(y+x)^5=c$$

$$[y-(x-1)]^2(y+x-1)^5=c$$

Solution is $[y-x+1)]^2(y+x-1)^5=c$

2. $\frac{dy}{dx}=\frac{6x+5y-7}{2x+18y-14}$

Sol. $\frac{dy}{dx}=\frac{6x+5y-7}{2x+18y-14}$

x = X + h, y = Y + k

$$\frac{dY}{dX}=\frac{dy}{dx}=\frac{6(X+h)+5(Y+k)-7}{2(X+h)+18(Y+k)-14}$$

$$=\frac{(6X+5Y)+(6h+5k-7)}{(2X+18Y)+(2h+18k-14)}$$

$$\begin{array}{ccccc} h & & k & & 1 \\ +5 & -7 & +6 & 5 \\ 18 & -14 & +2 & 18 \end{array}$$

$$\frac{h}{-70+126}=\frac{k}{-14+84}=\frac{1}{108-10}$$

$$h=\frac{56}{98}=\frac{4}{7},k=\frac{70}{98}=\frac{5}{7}$$

$$\frac{dY}{dX}=\frac{6X+5Y}{2X+18Y}$$

$$Y=VX\Rightarrow\frac{dY}{dX}=V+X\frac{dV}{dX}$$

$$V+X\frac{dV}{dX}=\frac{6X+5VX}{2X+18VX}=\frac{X(6+5V)}{X(2+18V)}$$

$$X\frac{dV}{dX}=\frac{6+5V}{2+18V}-V=\frac{6+5V-2V-18V^2}{2+18V}$$

$$=\frac{6+3V-18V^2}{2+18V}=\frac{3(2+V-6V^2)}{2+18V}$$

$$\int\frac{2+18V}{6V^2-V-2}dV=-3\int\frac{dX}{X}$$

Let $\frac{2+18V}{6V^2-V-2}=\frac{A}{3V-2}+\frac{B}{2V+1}$

$\Rightarrow$ **2 + 18V = A(2V + 1) + B(3V – 2)**

$$V=\frac{2}{3}\Rightarrow 2+12=A\left(\frac{4}{3}+1\right)$$

$$14=A-\frac{7}{3}\Rightarrow A=6$$

$$V=-\frac{1}{2}\Rightarrow 2-9=B\left(-\frac{3}{2}-2\right)$$

$$-7=-\frac{7}{2}B\Rightarrow B=2$$

$$\int\left(\frac{6}{3V-2}+\frac{2}{2V+1}\right)dV=-3\int\frac{dX}{X}$$

2 log(3V – 2) + log(2V + 1) = –3 log X + log c

log $(3V-2)^2$ (2V + 1) + log X^3 = log c

log X^3 $(3V-2)^2$ (2V + 1) = log c

X^3 $(3V-2)^2$ (2V + 1) = c

$$X^3\left(\frac{3Y}{X}-2\right)^2\left(\frac{2Y}{X}+1\right)=c$$

$$X^3\frac{(3Y-2X)^2}{X^2}\frac{(2Y+X)}{X}=c$$

$$\left(3\left(y-\frac{5}{7}\right)-2\left(x-\frac{4}{7}\right)\right)^2$$
$$\left(2\left(y-\frac{5}{7}\right)+\left(x-\frac{4}{7}\right)\right)=c$$

$$\frac{(3y-2x-1)^2}{7^2}\frac{(2y+x-2)}{7}=c$$

Solution is:

$(3y-2x-1)^2$ (x + 2y – 2) = 343c = c″

3. $\frac{dy}{dx}+\frac{10x+8y-12}{7x+5y-9}=0$

Sol. $\frac{dy}{dx}+\frac{10x+8y-12}{7x+5y-9}=0$

$x=X+h,\ y=Y+k$

$\Rightarrow \frac{dY}{dX}=\frac{dy}{dx}$

$$\frac{dY}{dX}+\frac{10(X+h)+8(Y+k)-12}{7(X+h)+5(Y+k)-9}=0$$

$$\frac{dY}{dX}+\frac{(10X+8Y)+(10h+8k-12)}{(7X+5Y)+(7h+5k-9)}$$

Choose h and k so that :

10h + 8k – 12 = 0, 7h + 5k – 9 = 0

$$\begin{array}{ccccccc} h & & k & & I & \\ +8 & -12 & 10 & 8 \\ +5 & -9 & +7 & 5 \end{array}$$

$$\frac{h}{-72+60}=\frac{k}{-84+90}=\frac{1}{50-56}$$

$$h=\frac{-12}{-6}=2, k=\frac{6}{-6}=-1$$

$$\frac{dY}{dX}=-\frac{10X+8Y}{7X+5Y}$$

$$Y=VX\Rightarrow \frac{dY}{dX}=V+X\frac{dV}{dX}$$

$$V+X\frac{dV}{dX}=-\frac{10X+8VX}{7X+5VX}=-\frac{X(10+8V)}{X(7+5V)}$$

$$X\frac{dV}{dX}=-\frac{10+8V}{7+5V}-V=\frac{-10-8V-7V-5V^2}{7+5V}$$

$$X\frac{dV}{dX}=-\frac{5(V^2+3V+2)}{7+5V}$$

$$\int\frac{5V+7}{(V+1)(V+2)}dV=-5\int\frac{dX}{X}$$

$$\frac{5V+7}{(V+1)(V+2)}=\frac{A}{V+1}+\frac{B}{V+2}$$

$$5V+7=A(V+2)+B(V+1)$$
$$V=-1\Rightarrow 2=A(-1+2)=A\Rightarrow A=2V$$
$$V=-2\Rightarrow -3=B(-2+1)=-B, B=3$$

$$\int\left(\frac{2}{V+1}+\frac{3}{V+2}\right)dV=-5\int\frac{dX}{X}$$

2 log(V + 1) + 3 log (V + 2) = –5 log X + c

c = 2 log(V + 1) + 3 log(V + 2) + 5 log X = log(V + 1)^2 (V + 2)^3 x^5

$$\log\left(\frac{Y}{X}+1\right)^2\left(\frac{Y}{X}+2\right)^3X^5=$$
$$\log\frac{(Y+X)^2}{X^2}\frac{(Y+2X)^3}{X^3}X^5$$

$$\Rightarrow (Y+X)^2(Y+2X)^3=e^c=c'$$
$$(Y+1-X-2)^2(Y+1-2x-4)^3=c$$

సమీకరణ సాధన **: (x + y – 1)^2 (2x + y – 3)^3 = c**

4. (x – y – 2)dx + (x – 2y – 3)dy = 0

Sol. $\frac{dy}{dx}=\frac{-x+y+2}{x-2y-3}$

x = X + h, y = Y + k అనుకొనుము. అప్పుడు

$$\frac{dy}{dx}=\frac{-(X+h)+(Y+k)+2}{(X+h)-2(Y+k)-3}$$
$$=\frac{-X+y+(k-h+2)}{(X-2Y)+(h-2k-3)}$$

–h + k + 2 = 0, h – 2k – 3 = 0సమీకరణాల ను సాధించగా

$$\begin{array}{ccccc} h & & k & & I \\ -1 & -2 & 1 & -1 \\ -2 & -3 & 1 & -2 \end{array}$$

$$\frac{h}{+3-4}=\frac{k}{-2+3}=\frac{1}{-2+1}$$

h = 1, k = –1

$\frac{dY}{dX}=\frac{-X+Y}{X-2Y}$ సమఘాతీయఅవకలన సమీకరణం

Put Y = VX so that $\frac{dY}{dX}=V+X\frac{dV}{dX}$

$$V+X\frac{dV}{dX}=\frac{X(-1+V)}{X(1-2V)}=\frac{-1+V}{1-2V}$$

$$X\frac{dV}{dX}=\frac{-1+V}{1-2V}-V=\frac{-1+V-V+2V^2}{1-2V}=\frac{2V^2-1}{1-2V}$$

$$\int\frac{(1-2V)dV}{2V^2-1}=\int\frac{dX}{X}$$

$$\int\frac{dX}{X}=\int\frac{-\frac{1}{2}(-4V)-1}{1-2V^2}dV$$

$$=\frac{1}{2}\int\frac{(-4VdV)}{1-2V^2}-\int\frac{dV}{1-2V^2}$$

$$\log|x|=-\frac{1}{2}\log|1-2V^2|-\frac{1}{2}\int\frac{dV}{\left(\frac{1}{\sqrt{2}}\right)^2-V^2}$$

$$=-\frac{1}{2}\log|1-2V^2|-\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\log\left|\frac{\frac{1}{\sqrt{2}}+V}{\frac{1}{\sqrt{2}}-V}\right|+\log c\ \ 2\log|x|=-\log|1-2V^2|$$

$$-\frac{1}{2\left(\frac{1}{\sqrt{2}}\right)}\log\left|\frac{\frac{1}{\sqrt{2}}+V}{\frac{1}{\sqrt{2}}-V}\right|+\log C$$

$$2\log|x|+\log|1-2V^2|=-\frac{1}{\sqrt{2}}\log\left|\frac{1+V\sqrt{2}}{1-V\sqrt{2}}\right|+\log c$$

$$\log X^2(1-2V^2)=-\frac{1}{\sqrt{2}}\log\left|\frac{X+Y\sqrt{2}}{X-Y\sqrt{2}}\right|+\log c$$

$$\log|X^2-2Y^2|=\log c\left(\frac{X-Y\sqrt{2}}{X+Y\sqrt{2}}\right)^{1/\sqrt{2}}$$

$$\therefore X^2-2Y^2=c\left(\frac{X-Y\sqrt{2}}{X+Y\sqrt{2}}\right)^{1/\sqrt{2}}$$

X = x–h = x – 1 , Y = y–k = y + 1

$$(x-1)^2-2(y+1)^2=c\left(\frac{x-1-(y+1)\sqrt{2}}{x-1+(y+1)\sqrt{2}}\right)^{1/\sqrt{2}}$$

$$(x^2-2y^2-2x-4y-1)=c\left(\frac{x-y\sqrt{2}-1-\sqrt{2}}{x+y\sqrt{2}-1+\sqrt{2}}\right)^{1/\sqrt{2}}$$

5. (x – y)dy = (x + y + 1) dx

Sol. $\frac{dy}{dx}=\frac{x+y+1}{x-y}$

x = X + h, y = Y + k

$$\frac{dy}{dx}=\frac{X+h+Y+k+1}{(X+h)-(Y+k)}=\frac{(X+Y)+(h+k+1)}{(X-Y)+(h-k)}$$

h + k + 1 = 0, h – k = 0సమీకరణాల ను సాధించగా

$$\mathbf{h}=-\frac{1}{2},k=-\frac{1}{2}$$

$\therefore \frac{dY}{dX}=\frac{X+Y}{X-Y}$ సమఘాతీయఅవకలన సమీకరణం

Put Y = VX $\Rightarrow \frac{dY}{dX}=V+X\frac{dV}{dX}$

$$V+X\frac{dV}{dX}=\frac{X(1+V)}{X(1-V)}$$

$$X\frac{dV}{dX}=\frac{1+V}{1-V}-V=\frac{1+V-V+V^2}{1-V}=\frac{1+V^2}{1-V}$$

$$\therefore \frac{(1-V)dV}{1+V^2}=\frac{dX}{X}$$

$$\int\frac{dV}{1+V^2}-\frac{1}{2}\int\frac{2VdV}{1+V^2}=\int\frac{dX}{X}$$

$$\tan^{-1}V-\frac{1}{2}\log(1+V^2)=\log x+\log c$$

$$2\tan^{-1}V=\log(1+V)^2+2\log x+2\log c$$

$$=\log c^2x^2(1+V^2)$$

$$2\tan^{-1}\left(\frac{Y}{X}\right)=\log c^2x^2\left(1+\frac{y^2}{x^2}\right)$$

$$2\tan^{-1}\left(\frac{y+\frac{1}{2}}{x+\frac{1}{2}}\right)=\log c^2(Y^2+X^2)$$

$$2\tan^{-1}\left(\frac{2y+1}{2x+1}\right)=\log c^2\left[\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2\right]=\log c^2\left(x^2+y^2+x+y+\frac{1}{2}\right)$$

6. (2x + 3y – 8)dx = (x + y – 3)dy

Sol. $\frac{dy}{dx}=\frac{2x+3y-8}{x+y-3}$

x = X + h, y = Y + k $\Rightarrow \frac{dY}{dX}=\frac{dy}{dx}$

$$\frac{dY}{dX}=\frac{2(X+h)+3(Y+k)-8}{(X+h)+(Y+k)-3}$$

$$=\frac{(2X+3Y)+(2h+3k-8)}{(X+Y)+(h+k-3)}$$

h , k లకొరకు క్రింది సమీకరణాల ను సాధించగా

:

2h + 3k – 8 = 0, h + k – 3 = 0

$$\begin{array}{ccccc} h & & k & & I \\ 3 & -8 & 2 & 3 & \\ 1 & -3 & 1 & 1 & \end{array}$$

$$\frac{h}{-9+8}=\frac{k}{-8+6}=\frac{1}{2-3}$$

$$h=1, k=2$$

$$\therefore \frac{dY}{dX}=\frac{2X+3Y}{X+Y}$$

Put Y = VX so that $\frac{dY}{dX} = V + X\frac{dV}{dX}$

$$V + X\frac{dV}{dX} = \frac{X(2+3V)}{X(1+V)} \Rightarrow X\frac{dV}{dX} = \frac{2+3V}{1+V} - V$$

$$= \frac{2+3V-V-V^2}{1+V} = \frac{2+2V-V^2}{1+V}$$

$$\int \frac{(1+V)dV}{2+2V-V^2} = \int \frac{dX}{X}$$

$$\int \frac{(1+V)dV}{2+2V-V^2}$$

$$\int \frac{(1+V)dV}{2+2V-V^2} = -\frac{1}{2}\int \frac{(2-2V)dV}{2+2V-V^2} + 2\int \frac{dV}{2+2V-V^2}$$

$$= -\frac{1}{2}\log(2+2V-V^2) + 2\int \frac{dV}{(\sqrt{3})^2-(V-1)^2}$$

$$= -\frac{1}{2}\log(2+2V-V^2) + 2\frac{1}{2\sqrt{3}}\log\frac{\sqrt{3}+V-1}{\sqrt{3}-V+1}$$

$$= -\frac{1}{2}\log(2+2V-V^2) + \frac{1}{\sqrt{3}}\log\frac{V+(\sqrt{3}-1)}{-V+(\sqrt{3}+1)}$$

$$= -\frac{1}{2}\log\left(2+\frac{2Y}{X}-\frac{Y^2}{X^2}\right) + \frac{1}{\sqrt{3}}\log\frac{\frac{Y}{X}+(\sqrt{3}-1)}{-\frac{Y}{X}+\sqrt{3}+1}$$

$$= -\frac{1}{2}\log(2X^2+2XY-Y^2) + \frac{1}{\sqrt{3}}\log\frac{Y+(\sqrt{3}-1)X}{Y-(\sqrt{3}+1)Y}$$

$$\therefore \log X + c = -\frac{1}{2}\log(2X^2+2XY-Y^2) + \frac{1}{\sqrt{3}}\log\frac{[Y+\sqrt{3}-1)X]}{Y-(\sqrt{3}+1)X}$$

7. $\frac{dy}{dx}=\frac{x+2y+3}{2x+3y+4}$

Sol. Let x = X + h, y = Y + k $\Rightarrow \frac{dY}{dX}=\frac{dy}{dx}$

$$\frac{dY}{dX}=\frac{(X+h)+2(Y+k)+3}{2(X+h)+3(Y+k)+4}$$

$$=\frac{(X+2Y)+(h+2k+3)}{(2X+3Y)+(2h+3k+4)}$$

h , k లకొరకు క్రింది సమీకరణాల ను సాధించగా

h + 2k + 3 = 0, 2h + 3k + 4 = 0

$$\begin{array}{ccccc} h & & k & & I \\ 2 & 3 & 1 & 2 & \\ 3 & 4 & 2 & 3 & \end{array}$$

$$\frac{h}{8-9}=\frac{k}{6-4}=\frac{1}{3-4}$$

$$h=\frac{-1}{-1}=1, k=\frac{2}{-1}=-2$$

$\frac{dY}{dX}=\frac{X+2Y}{2X+3Y}$ సమఘాతీయఅవకలన సమీకరణం

$$Y=VX\Rightarrow\frac{dY}{dX}=V+X\frac{dV}{dX}$$

$$V+X\frac{dV}{dX}=\frac{X(1+2V)}{X(2+3V)}$$

$$X\frac{dV}{dX}=\frac{1+2V}{2+3V}-V=\frac{1+2V-2V-3V^2}{2+3V}$$

$$\frac{(2-3V)dV}{1-3V^2}=\frac{dX}{X}$$

$$2\int\frac{dV}{1-3V^2}-\frac{1}{2}\int\frac{-6VdV}{1-3V^2}=\int\frac{dX}{X}$$

$$\frac{2}{3}\int\frac{dV}{\left(\frac{1}{\sqrt{3}}\right)^2-V^2}-\frac{1}{2}\log|1-3V|^2=\log X+\log c\ \frac{2}{3}\frac{1}{2\cdot\frac{1}{\sqrt{3}}}\log\left|\frac{\frac{1}{\sqrt{3}}+V}{\frac{1}{\sqrt{3}}-V}\right|-\frac{1}{2}\log|1-3V^2|=\log cx$$

$$\frac{1}{3\sqrt{3}}\log\left|\frac{1+\sqrt{3}V}{1-\sqrt{3}V}\right|\log|1-3V^2|=\log cy$$

$$\frac{1}{3\sqrt{3}}\log\left|\frac{1+\frac{\sqrt{3}Y}{X}}{1-\frac{\sqrt{3}Y}{X}}\right|-\frac{1}{2}\log\left|1-\frac{3y^2}{x^2}\right|=\log cy$$

$$\frac{1}{\sqrt{3}}\log\left|\frac{X+\sqrt{3}Y}{X-\sqrt{3}Y}\right|-\frac{1}{2}\log\left|\frac{X^2-3Y^2}{X^2}\right|=\log cx$$

Where X = x – 1, Y = y + 2

$$\frac{1}{\sqrt{3}}\left[\log(X+\sqrt{3}Y)-\log(X-\sqrt{3})\right]$$

$$-\frac{1}{2}\left(\log(X+\sqrt{3}Y)+\log(X-\sqrt{3}Y)-2\log X\right)$$

$$=\log CX$$

$$\log(X+\sqrt{3}Y)\left(\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{2}}\right)-\log(X-\sqrt{3}Y)$$

$$\left(\frac{1}{2}+\frac{1}{\sqrt{3}}\right)+\log X=\log c+\log X$$

$$\frac{2-\sqrt{3}}{2\sqrt{3}}\log(X+\sqrt{3}Y)\frac{2+\sqrt{3}}{2\sqrt{3}}\log(X-\sqrt{3}Y)$$

$$=\log c$$

$$\text{i.e.}\frac{(2-\sqrt{3})}{2}\log(X+\sqrt{3}Y)\frac{(2+\sqrt{3})}{2}$$

$$\log(X-\sqrt{3}Y)=\sqrt{3}c' \text{ where } c'=\log c$$

8. $\frac{dy}{dx}=\frac{2x+9y-20}{6x+2y-10}$

Sol. $\frac{dy}{dx}=\frac{2x+9y-20}{6x+2y-10}$

x = X + h, y = Y + k అనుకొనుము. అప్పుడు

$$\frac{dY}{dX}=\frac{dy}{dx}$$

$$\frac{dY}{dX}=\frac{2(X+h)+9(Y+k)-20}{6(X+h)+2(Y+k)-10}$$
$$=\frac{(2X+9Y)+(2h+9k-20)}{(6X+2Y)+(6h+2k-10)}$$

h , k లకొరకు క్రింది సమీకరణాల ను సాధించగా

2h + 9k – 20 = 0, 6h + 2k – 10 = 0

$$\begin{array}{ccccc} h & & k & & I \\ 9 & -20 & 2 & 9 \\ 2 & -10 & 6 & 2 \end{array}$$

$$\frac{h}{-90+40}=\frac{k}{-120+20}=\frac{1}{4-54}$$

$$h=\frac{-50}{-50}=1, k=\frac{-100}{-50}=2$$
$$\therefore \frac{dY}{dX}=\frac{2X+9Y}{6X+2Y}$$

సమఘాతీయఅవకలన సమీకరణం

$$y=VX\Rightarrow\frac{dY}{dX}=V+X\frac{dV}{dX}$$
$$V+X\frac{dV}{dX}=\frac{x(2+9V)}{x(6+2V)}$$
$$X\frac{dV}{dX}=\frac{2+9V}{6+2V}-V=\frac{2+9V-6V-2V^2}{6+2V}$$
$$\frac{6+2V\cdot dY}{2+3V-2V^2}=\frac{dX}{X}$$
$$\int\frac{(6+2V)dV}{(1+2V)(2-V)}=\int\frac{dx}{x} \qquad \textbf{... (1)}$$

Let $\frac{6+2V}{(1+2V)(2-V)}=\frac{A}{1+2V}+\frac{B}{2-V}$

6 + 2V = A(2 – V) + B(1 + 2V)

V = 2 ⇒ 10 = B(5) ⇒ B = 2

$$V=-\frac{1}{2}\Rightarrow 5=A\left(\frac{5}{2}\right)\Rightarrow A=2$$
$$\frac{6+2V}{(1+2V)(2-V)}=\frac{2}{1+2V}+\frac{2}{2-V}$$

$$\int \frac{(6+2V)dV}{(1+2V)(2-V)} = \int \frac{2dV}{1+2V} - 2\int \frac{dV}{V-2}$$
$$= \log(1+2V) - 2\log(V-2)$$

(1) నుండి

$$\log(1+2V) - \log(V-2)^2 = \log X - \log c$$
$$\log \frac{1+2V}{(V-2)^2} = \log \frac{X}{c} \Rightarrow \frac{1+2V}{(V-2)^2} = \frac{X}{c}$$
$$1+2V = \frac{X}{c}(V-2)^2 \Rightarrow 1+\frac{2V}{X} = \frac{x}{c}\left(\frac{Y}{X}-2\right)^2$$
$$\frac{X+2Y}{X} = \frac{x}{c}\frac{(Y-2X)^2}{X^2}$$

సమీకరణ సాధన $(X + 2Y) = (Y - 2X)^2$

ఇక్కడ $X = x - 1, Y = y - 2$

$c(x - 1 + 2y - 4) = (y - 2 - 2x + 2)^2$

$c(x + 2y - 5) = (y - 2x)^2 = (2x - y)^2$

5. క్రింది అవకలన సమీకరణాల ను సాధించండి

1. $\cos x \frac{dy}{dx} + y \sin x = \sec^2 x$

Sol. $\frac{dy}{dx} + \tan x \cdot y = \sec^3 x$ **ఇది y లో ఏకఘాతీయ సమీకరము**

$$\text{I.F.} = e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x$$

Sol is y.I.F. $= \text{y.I.F} = \int \text{Q. I.F. } dx$

$$y \cdot \sec x = \int \sec^4 x dx = \int (1+\tan^2 x)\sec^2 x dx$$
$$= \tan x + \frac{\tan^3 x}{3} + c$$

2. $\sec x \cdot dy = (y + \sin x)dx$

Sol. $\frac{dy}{dx} = \frac{y + \sin x}{\sec x} = y\cos x + \sin x \cdot \cos x$

$\frac{dy}{dx} - y\cos x = \sin x \cdot \cos x$ ఇది **y** లో ఏకఘాతీయ సమీకరము

$I.F. = e^{-\int \cos x dx} = e^{-\sin x}$

Sol is y.I.F. $= y.I.F = \int Q.\ I.F.\ dx$

$y \cdot e^{-\sin x} = \int e^{-\sin x} \cdot \sin x \cdot \cos x \cdot dx$

Consider $\int e^{-\sin x} \cdot \sin x \cdot \cos x \cdot dx$

t = – sin x ⇒ dt = –cos x dx

$\int e^{-\sin x} \cdot \sin x \cdot \cos x\, dx = +\int e^t \cdot t\, dt$

$= t \cdot e^t - e^t + c = e^{-\sin x}(-\sin x - 1) + c$

$y \cdot e^{-\sin x} = -e^{-\sin x}(\sin x + 1) + c$

or $y = -(\sin x + 1) + c \cdot e^{\sin x}$

3. $x \log x \cdot \frac{dy}{dx} + y = 2\log x$

Sol. $\frac{dy}{dx} + \frac{1}{x\log x} y = \frac{2}{x}$

$I.F. = e^{\int \frac{dx}{x\log x}} = e^{\log(\log x)} = \log x$

$y\log x = 2\int \frac{\log x}{x} dx = (\log x)^2 + c$

4. $(x + y + 1)\frac{dy}{dx} = 1$

Sol. $(x + y + 1)\frac{dy}{dx} = 1$

$\frac{dx}{dy} = x + y + 1 \Rightarrow \frac{dx}{dy} - x = y + 1$ ఇది **x** లో ఏకఘాతీయ సమీకరము

$I.F. = e^{\int p dy} = e^{\int -dy} = e^{-y}$

ఇచ్చిన సమీకంణానికి సాధన

$x.I.F.=\int Q.IFdy$

$$x\cdot e^{-y}=\int e^{-y}(y+1)dy=-(y+1)e^{-y}+\int e^{-y}dy$$

$$=-(y+1)e^{-y}-e^{-y}=-(y+2)e^{-y}+c$$

$$x=-(y+2)+c\cdot e^{y}$$

5. Solve $x(x-1)\frac{dy}{dx}-y=x^3(x-1)^3$.

Sol. $\frac{dy}{dx}-\frac{1}{x(x-1)}y=x^2(x-1)^2$ ఇది **y** లో ఏకఘాతీయ సమీకరము

$$I.F.=e^{\int pdx}=e^{\int -\frac{dx}{x(x-1)}}=e^{\int\left(\frac{1}{x}-\frac{1}{x-1}\right)dx}$$

$$=e^{\log x-\log(x-1)}=e^{\log\frac{x}{x-1}}=\frac{x}{x-1}$$

Sol is y.I.F. $=y.I.F=\int Q.\ I.F.\ dx$

$$y\cdot\frac{x}{x-1}=\int x^2(x-1)^2\frac{x}{(x-1)}dx=\int x^3(x-1)dx$$

Hence solution is $\frac{xy}{x-1}=\frac{x^5}{5}-\frac{x^4}{4}+c$

6. $(x+2y^3)\frac{dy}{dx}=y$

Sol. $\frac{dy}{dx}=\frac{x+2y^3}{y}=\frac{x}{y}+2y^2$

$$\frac{dx}{dy}-\frac{1}{y}x=2y^2$$

$$I.F.=e^{\int -\frac{1}{y}dy}=e^{-\log y}=e^{\log 1/y}=\frac{1}{y}$$

$$x\cdot\frac{1}{y}=\int 2ydy=y^2+c$$

Solution is : $x=y(y^2+c)$

7. Solve $(1-x^2)\frac{dy}{dx}+2xy=x\sqrt{1-x^2}$ **.**

Sol. $\frac{dy}{dx}+\frac{2x}{1-x^2}y=\frac{x}{\sqrt{1-x^2}}$

$$\text{I.F.}=e^{\int\frac{2x}{1-x^2}dx}=e^{-\log(1-x^2)}$$

$$=e^{\log(1-x^2)^{-1}}=\frac{1}{1-x^2}$$

$$\frac{y}{1-x^2}=\int\frac{xdx}{(1-x^2)^{3/2}}=(1-x^2)^{-1/2}+c$$

$$(\text{or})y=\sqrt{1-x^2}+c(1-x^2)$$

8. $x(x-1)\frac{dy}{dx}-(x-2)y=x^3(2x-1)$

Sol. $\frac{dy}{dx}-\frac{x-2}{x(x-1)}y=\frac{x^3(2x-1)}{x(x-1)}$

$$\text{I.F.}=e^{\int\frac{2-x}{x(x-1)}dx}\Rightarrow\frac{2-x}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}$$

$$2-x=A(x-1)+Bx$$

$$x=0\Rightarrow 2=-A\Rightarrow A=-2$$

$$x=1\Rightarrow 1=B\Rightarrow B=1$$

$$\frac{2-x}{x(x-1)}=\frac{-2}{x}+\frac{1}{x+1}$$

$$\int\frac{2-x}{x(x-1)}dx=-2\int\frac{dx}{x}+\int\frac{dx}{x-1}$$

$$=-\log x+\log(x-1)=\log\frac{x-1}{x^2}$$

$$\text{I.F.}=e^{\log\frac{x-1}{x^2}}=\frac{x-1}{x^2}$$

$$y\frac{x-1}{x^2}=\int\frac{x^3(2x-1)}{x(x-1)}\cdot\frac{x-1}{x^2}dx$$

$$=\int(2x-1)dx=x^2-x+c$$

సమీకరణ సాధన $\mathbf{y(x-1)=x^2(x^2-x+c)}$

9. $\frac{dy}{dx}(x^2y^3+xy)=1$

Sol. $\frac{dy}{dx}(x^2y^3+xy)=1$

$\frac{dx}{dy}=xy+x^2y^3$

$\Rightarrow \frac{dx}{dy}-xy=x^2y^3$ **----(1)**

ఇది బెర్నూలి సమీకరణం

x^2 భాగించగా

$\frac{1}{x^2}\frac{dx}{dy}-\frac{1}{x}y=y^3$

$z=-\frac{1}{x}\Rightarrow\frac{dz}{dy}=\frac{1}{x^2}\frac{dx}{dy}$

$\Rightarrow \frac{dz}{dy}+z\cdot y=y^3$ **----2)**

ఇది z లో ఏకఘాతీయ సమీకరము

$I.F.=e^{\int ydy}=e^{y^2/2}$

సమీకరణ సాధన $z.I.F=\int Q.\ I.F.\ dy$

ఇచ్చిన సమీకంణానికి సాధన

$z\cdot e^{y^2/2}=\int y^3e^{y^2/2}\cdot dy$

$$\text{put } \frac{y^2}{2}=t\Rightarrow y\,dy=dt$$

$=\int t\cdot dt\cdot e^t=e^t(t-1)=e^{y^2/2}\left(\frac{y^2}{2}-1\right)$

$z\cdot e^{y^2/2}=e^{y^2/2}\left(\frac{y^2}{2}-1\right)+c$

$z=\frac{y^2}{2}-1+c\cdot e^{-y^2/2}\Rightarrow -\frac{1}{x}=\frac{y^2}{2}-1+c\cdot e^{-y^2/2}$

$-1=x\left(\frac{y^2}{2}-1+c\cdot e^{-y^2/2}\right)$

$1+x\left(\frac{y^2}{2}-1+c\cdot e^{-y^2/2}\right)=0$

10. $\frac{dy}{dx}+x\cdot\sin 2y=x^3\cos^2 y$

Sol. $\frac{1}{\cos^2 y}\frac{dy}{dx}+\frac{2\sin y\cos y}{\cos^2 y}x=x^3$

$\sec^2 y\frac{dy}{dx}+2\tan y.x=x^3$

$z=\tan y\Rightarrow\frac{dz}{dx}=\sec^2 y\frac{dy}{dx}$

$\frac{dz}{dx}+2zx=x^3$

$\text{I.F.}=e^{\int 2xdx}=e^{x^2}$

$z\cdot e^{x^2}=\int x^3\cdot e^{x^2}dx \qquad ...(1)$

$\int x^3\cdot e^{x^2}dx$

$t=x^2\Rightarrow dt=2x\cdot dx$

$\int x^3\cdot e^{x^2}dx=\int x\cdot x^2\cdot e^{x^2}dx$

$=\frac{1}{2}\int t\cdot e^t dt=\frac{1}{2}e^t(t-1)$

$=z\cdot e^{x^2}=\frac{1}{2}e^{x^2}(x^2-1)+c$

$z=\frac{x^2-1}{2}+c\cdot e^{-x^2}$

$\tan y=\frac{x^2-1}{2}+c\cdot e^{-x^2}$

11. $y^2+\left(x-\frac{1}{y}\right)\frac{dy}{dx}=0$

Sol.

$y^2+\left(x-\frac{1}{y}\right)\frac{dy}{dx}=0$

$\left(x-\frac{1}{y}\right)\frac{dy}{dx}=-y^2$

$$\frac{dx}{dy}=\frac{x-1/y}{-y^2}=-\frac{x}{y^2}+\frac{1}{y^3}$$

$\frac{dx}{dy}+\frac{1}{y^2}\cdot x=\frac{1}{y^3}$ ఇది **x** లో ఏకఘాతీయ సమీకరము

$$\text{I.F.}=e^{\int\frac{1}{y^2}dy}=e^{-1/y}$$

Sol is $x.\text{I.F}=\int Q.\ \text{I.F.}\ dy$

$$x\cdot e^{-1/y}=\int\frac{e^{-1/y}}{y^3}dy \qquad \textbf{... (1)}$$

$$\text{put}-\frac{1}{y}=z\Rightarrow\frac{1}{y^2}dy=dz$$

$$=\int z\cdot e^z dz=e^z(z-1)$$

$$x\cdot e^{-1/y}=-e^{-1/y}\left(-\frac{1}{y}-1\right)+c$$

$$\frac{x}{e^{1/y}}=\frac{1+y}{y\cdot e^{1/y}}+c$$

సమీకరణ సాధన **$xy = 1 + y + cy\,e^{1/y}$.**