

పాక్షిక భిన్నాలు

Very Short Answer Questions

I. ఈ క్రింది వానిని పాక్షికభిన్నాలు గా విడగొట్టండి.

1. $\frac{2x+3}{(x+1)(x-3)}$

Sol. $\frac{2x+3}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$

$$2x+3 = A(x-3) + B(x+1)$$

$$x = -1 \Rightarrow 1 = A(-4) \Rightarrow A = -\frac{1}{4}$$

$$x = 3 \Rightarrow 9 = B(4) \Rightarrow B = \frac{9}{4}$$

$$\frac{2x+3}{(x+1)(x-3)} = \frac{-1}{4(x+1)} + \frac{9}{4(x-3)}$$

2. $\frac{5x+6}{(2+x)(1-x)}$

Sol. Let $\frac{5x+6}{(2+x)(1-x)} = \frac{A}{2+x} + \frac{B}{1-x}$

$$(2+x)(1-x)$$

$$5x+6 = A(1-x) + B(2+x)$$

$$x = -2, -10+6 = A(1+2) \Rightarrow A = -\frac{4}{3}$$

$$x = 1, 5+6 = B(2+1) \Rightarrow B = \frac{11}{3}$$

$$\therefore \frac{5x+6}{(2+x)(1-x)} = -\frac{4}{3(2+x)} + \frac{11}{3(1-x)}$$

Short Answer Questions

1. $\frac{3x+7}{x^2-3x+2}$

Sol:

$$\frac{3x+7}{x^2-3x+2} = \frac{3x+7}{(x-2)(x-1)}$$

Let $\frac{3x+7}{x^2-3x+2} = \frac{A}{(x-2)} + \frac{B}{(x-1)}$

$$\Rightarrow A(x-1)+B(x-2)=3x+7 \quad \dots (1)$$

x = 2 ప్రతిక్షేపించగా

$$A = 13$$

x = 1 ప్రతిక్షేపించగా

$$-B = 10 \text{ i.e., } B = -10$$

$$\therefore \frac{3x+7}{x^2-3x+2} = \frac{13}{x-2} - \frac{10}{x-1}$$

2. $\frac{x+4}{(x^2-4)(x+1)}$

Sol. $\frac{x+4}{(x^2-4)(x+1)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-2}$

$$x+4 = A(x^2-4) + B(x+1)(x-2) + C(x+1)(x+2)$$

$$x = -1 \Rightarrow 3 = A(1-4) = -3A \Rightarrow A = -1$$

$$x = -2 \Rightarrow 2 = B(-2+1)(-2-2) = 4B$$

$$\Rightarrow B = \frac{2}{4} = \frac{1}{2}$$

$$x = 2 \Rightarrow 6 = C(2+1)(2+2) = 12C \Rightarrow C = \frac{1}{2}$$

$$\therefore \frac{x+4}{(x^2-4)(x+1)} = -\frac{1}{x+1} + \frac{1}{2(x+2)} + \frac{1}{2(x-2)}$$

3. $\frac{2x^2 + 2x + 1}{x^3 + x^2}$

Sol. Let $\frac{2x^2 + 2x + 1}{x^3 + x^2} = \frac{2x^2 + 2x + 1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$x^2(x+1)$ తో గుణించగ

$$2x^2 + 2x + 1 = A x(x + 1) + B(x + 1) + Cx^2$$

Put $x = 0, 1 = B$

Put $x = -1, 2 - 2 + 1 = C(1) \Rightarrow C = 1$

x^2 పద గుణకాలను పొల్పగ

$$2 = A + C \Rightarrow A = 2 - C = 2 - 1 = 1$$

$$\therefore \frac{2x^2 + 2x + 1}{x^3 + x^2} = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x+1}$$

4. $\frac{2x+3}{(x-1)^3}$

Sol. $x - 1 = y$ అనుకోండి అప్పుడు

$$x = y + 1$$

$$\Rightarrow \frac{2x+3}{(x-1)^3} = \frac{2(y+1)+3}{y^3} = \frac{2y+5}{y^3}$$

$$= \frac{2}{y^2} + \frac{5}{y^3} = \frac{2}{(x-1)^2} + \frac{5}{(x-1)^3}$$

$$\therefore \frac{2x+3}{(x-1)^3} = \frac{2}{(x-1)^2} + \frac{5}{(x-1)^3}$$

5. $\frac{x^2 - 2x + 6}{(x-2)^3}$

Sol: $x - 2 = y$ అనుకోండి అప్పుడు

$$\therefore \frac{x^2 - 2x + 6}{(x-2)^3} = \frac{(y+2)^2 - 2(y+2) + 6}{y^3}$$

$$= \frac{y^2 + 4y + 4 - 2y - 4 + 6}{y^3}$$

$$= \frac{y^2 + 2y + 6}{y^3}$$

$$= \frac{1}{y} + \frac{2}{y^2} + \frac{6}{y^3}$$

$$= \frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{6}{(x-2)^3}$$

$$\therefore \frac{x^2 - 2x + 6}{(x-2)^3} = \frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{6}{(x-2)^3}$$

6. $\frac{2x^2 + 3x + 4}{(x-1)(x^2 + 2)}$

Sol. $\frac{2x^2 + 3x + 4}{(x-1)(x^2 + 2)} = \frac{A}{x-1} + \frac{Bx + C}{x^2 + 2}$

$$2x^2 + 3x + 4 = A(x^2 + 2) + (Bx + C)(x - 1)$$

$$x = 1 \Rightarrow 2 + 3 + 4 = A(1 + 2)$$

$$9 = 3A \Rightarrow A = 3$$

x^2 పద గుణకాలను పోల్చగా

$$2 = A + B \Rightarrow B = 2 - A = 2 - 3 = -1$$

స్థిరాంకములను పోల్చగా

$$4 = 2A - C \Rightarrow C = 2A - 4 = 6 - 4 = 2$$

$$\frac{2x^2 + 3x + 4}{(x-1)(x^2 + 2)} = \frac{3}{x-1} + \frac{-x+2}{x^2 + 2}$$

7. $\frac{3x-1}{(1-x+x^2)(x+2)}$

Sol.Let $\frac{3x-1}{(1-x+x^2)(x+2)} = \frac{A}{2+x} + \frac{Bx+C}{1-x+x^2}$

$(2+x)(1-x+x^2)$ తో గుణించగా

$$3x - 1 = A(1 - x + x^2)(Bx + C)(2 + x)$$

$$x = -2 \Rightarrow -7 = A(1+2+4) = 7A \Rightarrow A = -1$$

x^2 పద గుణకాలను పోల్చగా

$$0 = A + B \Rightarrow B = -A = 1$$

స్థిరాంకములను పోల్చగా $-1 = A + 2C$

$$2C = -1 - A = -1 + 1 = 0 \Rightarrow C = 0$$

$$\frac{3x-1}{(1-x+x^2)(x+2)} = -\frac{1}{2+x} + \frac{x}{1-x+x^2}$$

8. $\frac{x^2-3}{(x+2)(x^2+1)}$

Sol.Let $\frac{x^2-3}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

$(x+2)(x^2+1)$ తో గుణించగా

$$x^2 - 3 = A(x^2 + 1) + (Bx + C)(x + 2)$$

$$x = -2 \Rightarrow 4 - 3 = A(4 + 1)$$

$$1 = 5A \Rightarrow A = \frac{1}{5}$$

x^2 పద గుణకాలను పొల్చగ

$$1 = A + B \Rightarrow B = 1 - A = 1 - \frac{1}{5} = \frac{4}{5}$$

స్థిరాంకములను పొల్చగా $-3 = A + 2C$

$$2C = -3 - A = -3 - \frac{1}{5} = -\frac{16}{5} \Rightarrow C = -\frac{8}{5}$$

$$\frac{x^2 - 3}{(x + 2)(x^2 + 1)} = \frac{1}{5(x + 2)} + \frac{4x - 8}{5(x^2 + 1)}$$

9. $\frac{x^2 + 1}{(x^2 + x + 1)^2}$

Sol. Let $\frac{x^2 + 1}{(x^2 + x + 1)^2} = \frac{Ax + B}{x^2 + x + 1} + \frac{Cx + D}{(x^2 + x + 1)^2}$

$$x^2 + 1 = (Ax + B)(x^2 + x + 1) + (Cx + D)$$

x^3 పద గుణకాలను పొల్చగ

$$A = 0$$

x^2 పద గుణకాలను పొల్చగ

$$A + B = 1 \Rightarrow B = 1$$

x పద గుణకాలను పొల్చగ

$$A + B + C = 0$$

$$\Rightarrow 1 + C = 0 \Rightarrow C = -1$$

స్థిరాంకములను పోల్చగా

$$B + D = 1$$

$$\Rightarrow D = 1 - B = 1 - 1 = 0$$

$$\therefore Ax + B = 1, Cx + D = -x$$

$$\therefore \frac{x^2 + 1}{(x^2 + x + 1)^2} = \frac{1}{x^2 + x + 1} - \frac{x}{(x^2 + x + 1)^2}$$

10. $\frac{x^3 + x^2 + 1}{(x-1)(x^3 - 1)}$

Sol. Let $\frac{x^3 + x^2 + 1}{(x-1)(x^3 - 1)} = \frac{x^3 + x^2 + 1}{(x-1)(x-1)(x^2 + x + 1)}$

Let $\frac{x^3 + x^2 + 1}{(x-1)(x-1)(x^2 + x + 1)} =$

$$\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+x+1} \dots(1)$$

$$= \frac{A(x-1)(x^2+x+1) + B(x^2+x+1) + (Cx+D)(x-1)^2}{(x-1)^2(x^2+x+1)}$$

$$\therefore x^3 + x^2 + 1 = A(x-1)(x^2+x+1) + B(x^2+x+1) + (Cx+D)(x-1)^2 \dots(2)$$

$$x = 1$$

$$1+1+1 = A(0) + B(1+1+1) + (C(1)+D)(0)$$

$$\Rightarrow 3B = 3 \Rightarrow B = 1$$

x^3 పద గుణకాలను పోల్చగా

$$1 = A + C \dots(3)$$

x^2 పద గుణకాలను పోల్చగా

$$1 = A(1-1) + B(1) + C(-2) + D(1)$$

$$\Rightarrow 1 = B - 2C + D$$

$$\because B=1 \Rightarrow 1=1-2C+D \Rightarrow 2C=D \quad \dots(4)$$

$x=0$ అయితే

$$1 = A(-1)(1) + B(1) + D(-1)^2$$

$$\Rightarrow -A + B + D = 1 \Rightarrow -A + 1 + D = 1$$

$$\Rightarrow A = D \quad \dots(5)$$

(3), (4), (5) ల నుండి

$$1 = D + \frac{D}{2} \Rightarrow \frac{3D}{2} = 1 \Rightarrow D = \frac{2}{3}$$

$$(5) \text{ నుండి } A = \frac{2}{3},$$

$$(4) \text{ నుండి } C = \frac{D}{2} = \frac{(2/3)}{2} = \frac{1}{3}$$

$$\begin{aligned} \therefore \frac{x^3 + x^2 + 1}{(x-1)(x^3-1)} &= \frac{(2/3)}{x-1} + \frac{1}{(x-1)^2} + \frac{\left(\frac{1}{3}x + \frac{2}{3}\right)}{x^2 + x + 1} \\ \Rightarrow \frac{x^3 + x^2 + 1}{(x-1)(x^3-1)} &= \frac{2}{3(x-1)} + \frac{1}{(x-1)^2} + \frac{x+2}{3(x^2+x+1)} \end{aligned}$$

11. $\frac{x^2}{(x-1)(x-2)}$

Sol. Let $\frac{x^2}{(x-1)(x-2)} = 1 + \frac{A}{x-1} + \frac{B}{x-2}$

$(x-1)(x-2)$ తో గుణించగా

$$x^2 = (x-1)(x-2) + A(x-2) + B(x-1)$$

$$x = 1, 1 = A(-1) \Rightarrow A = -1$$

$$x = 2, 4 = B(1) \Rightarrow B = 4$$

$$\therefore \frac{x^2}{(x-1)(x-2)} = 1 - \frac{1}{x-1} + \frac{4}{x-2}$$

14. $\frac{x^3}{(x-1)(x+2)}$

Sol. $\frac{x^3}{(x-1)(x+2)} = \frac{x^3}{x^2+x-2}$

$$= \frac{x(x^2+x-2) - 1(x^2+x-2) + 3x-2}{x^2+x-2}$$

$$= x-1 + \frac{3x-2}{x^2+x-2} = x-1 + \frac{3x-2}{(x-1)(x+2)}$$

Let $\frac{3x-2}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

$(x-1)(x+2)$ నుండి

$$3x-2 = A(x+2) + B(x-1)$$

$$x = 1, 1 = A(3) \Rightarrow A = \frac{1}{3}$$

$$x = -2, -8 = B(-3) \Rightarrow B = \frac{8}{3}$$

$$\therefore \frac{x^3}{(x-1)(x+2)} = x-1 + \frac{1}{3(x-1)} + \frac{8}{3(x+2)}$$

15. $\frac{x^3}{(2x-1)(x-1)^2}$

Sol.

$$\frac{x^3}{(2x-1)(x-1)^2} = \frac{1}{2} + \frac{A}{2x-1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$2(2x-1)(x-1)^2$ తో గుణించగా

$$2x^3 = (2x-1)(x-1)^2 + 2A(x-1)^2 + 2B(2x-1)(x-1) + 2C(2x-1)$$

Put $x = \frac{1}{2}$, $2\left(\frac{1}{8}\right) = 2A\left(\frac{1}{4}\right) \Rightarrow A = \frac{1}{2}$

Put $x = 1$, $2(1) = 2C(1) \Rightarrow C = 1$

Put $x = 0$,

$$0 = (-1)(1) + 2A(1) + 2B(-1)(-1) + 2C(-1)$$

$$\Rightarrow 2A + 2B - 2C = 1$$

$$\Rightarrow 2B = 1 + 2C - 2A = 1 + 2 - 1 = 2 \Rightarrow B = 1$$

$$\therefore \frac{x^3}{(2x-1)(x-1)^2} = \frac{1}{2} + \frac{1}{2(2x-1)} + \frac{1}{x-1} + \frac{1}{(x-1)^2}$$

16. $\frac{x^3}{(x-a)(x-b)(x-c)}$

Sol. Let $\frac{x^3}{(x-a)(x-b)(x-c)} = 1 + \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

$(x-a)(x-b)(x-c)$ తో గుణించగా

$$x^3 = (x-a)(x-b)(x-c) + A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$$

$$x = a, a^3 = A(a - b)(a - c)$$

$$\Rightarrow A = \frac{a^3}{(a - b)(a - c)}$$

$$x = b, b^3 = B(b - a)(b - c)$$

$$\Rightarrow B = \frac{b^3}{(b - a)(b - c)}$$

$$x = c, c^3 = C(c - a)(c - b)$$

$$\Rightarrow C = \frac{c^3}{(c - a)(c - b)}$$

$$\frac{x^3}{(x - a)(x - b)(x - c)} = 1 + \frac{a^3}{(a - b)(a - c)(x - a)} + \frac{b^3}{(b - a)(b - c)(x - b)} + \frac{c^3}{(c - a)(c - b)(x - c)}$$

17. $\frac{3x - 18}{x^3(x + 3)}$ ను పాక్షిక భిన్నాలుగా విడగొట్టుము.

Sol. Let $\frac{3x - 18}{x^3(x + 3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x + 3}$

$$\therefore \frac{3x - 18}{x^3(x + 3)} = \frac{Ax^2(x + 3) + Bx(x + 3) + C(x + 3) + Dx^3}{x^3(x + 3)}$$

$$\Rightarrow 3x - 18 = Ax^2(x + 3) + Bx(x + 3) + C(x + 3) + Dx^3 \dots(1)$$

$$x = -3$$

$$3(-3) - 18 = A(0) + B(0) + C(0) + D(-3)^3$$

$$\Rightarrow -27D = -27 \Rightarrow D = 1$$

$$x = 0$$

$$3(0) - 18 = A(0) + B(0) + C(0 + 3) + D(0)$$

$$\Rightarrow 3C = -18 \Rightarrow C = -6$$

x^3 పద గుణకాలను పొల్పగా

$$0 = A + D \Rightarrow A = -D = -1 \Rightarrow A = -1$$

x^2 పద గుణకాలను పొల్పగా

$$0 = 3A + B \Rightarrow B = -3A = -3(-1) = 3 \Rightarrow B = 3$$

$$\frac{3x-18}{x^3(x+3)} = \frac{-1}{x} + \frac{3}{x^2} - \frac{6}{x^3} + \frac{1}{x+3}$$

18. $\frac{2x^2+1}{x^3-1}$ ను పాక్షిక భిన్నాలుగా విడగొట్టుము.

Sol. $\frac{2x^2+1}{x^3-1} = \frac{2x^2+1}{(x-1)(x^2+x+1)}$... (1)

$$\frac{2x^2+1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$
$$\Rightarrow \frac{2x^2+1}{(x-1)(x^2+x+1)} = \frac{A(x^2+x+1) + (Bx+C)(x-1)}{(x-1)(x^2+x+1)}$$

$$\therefore 2x^2+1 = A(x^2+x+1) + (Bx+C)(x-1) \dots (1)$$

$$x = 1$$

$$2(1) + 1 = A(1 + 1 + 1) + (B + C)(0)$$

$$\Rightarrow 3A = 3 \Rightarrow A = 1$$

$x = 0$ అయితే

$$0 + 1 = A(1) + (0 + C)(0 - 1)$$

$$\Rightarrow 1 = A - C \Rightarrow C = 0$$

x^2 పద గుణకాలను పొల్పగా

$$2 = A + B \Rightarrow 2 = 1 + B \Rightarrow B = 1$$

$$\therefore \frac{2x^2+1}{x^3-1} = \frac{1}{x-1} + \frac{(1)(x)+0}{x^2+x+1} = \frac{1}{x-1} + \frac{x}{x^2+x+1}$$

ఈ క్రింది వానిని పాక్షికభిన్నాలు గా విడగొట్టండి

1. $\frac{x^3+x^2+1}{(x^2+2)(x^2+3)}$

Sol. Let $\frac{x^3+x^2+1}{(x^2+2)(x^2+3)} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{x^2+3}$

$$= \frac{(Ax+B)(x^2+3) + (Cx+D)(x^2+2)}{(x^2+2)(x^2+3)}$$

$$\therefore x^3+x^2+1 = (Ax+B)(x^2+3) + (Cx+D)(x^2+2) \dots(1)$$

$$\Rightarrow x^3+x^2+1 = (A+C)x^3 + (B+D)x^2 + (3A+2C)x + (3B+2D)$$

x^3, x^2, x పద గుణకాలను పోల్చగా

$$A + C = 1, B + D = 1,$$

$$3A + 2C = 0, 3B + 2D = 1$$

$$\Rightarrow A = -2, C = 3, B = -1, D = 2$$

$$\therefore \frac{x^3+x^2+1}{(x^2+2)(x^2+3)} = \frac{-2x-1}{x^2+2} + \frac{3x+2}{x^2+3}$$
$$= \frac{3x+2}{x^2+3} - \frac{2x+1}{x^2+2}$$

2. $\frac{3x^3 - 2x^2 - 1}{x^4 + x^2 + 1}$.

Sol. $x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2$

$$= (x^2 + 1)^2 - x^2 = (x^2 + x + 1)(x^2 - x + 1)$$

$$\frac{3x^3 - 2x^2 - 1}{x^4 + x^2 + 1} = \frac{3x^3 - 2x^2 - 1}{(x^2 + x + 1)(x^2 - x + 1)}$$
$$= \frac{Ax + B}{x^2 + x + 1} + \frac{Cx + D}{x^2 - x + 1}$$

$x^4 + x^2 + 1$ చే గుణించగా,

$$3x^3 - 2x^2 - 1 = (Ax + B)(x^2 - x + 1) + (Cx + D)(x^2 + x + 1)$$

సరి పద గుణకాల్ని పోల్చగా

$$A + C = 3 \quad \dots (1)$$

$$\Rightarrow C = 3 - A$$

$$-A + B + C + D = -2 \quad \dots (2)$$

$$A - B + C + D = 0 \quad \dots (3)$$

$$B + D = -1 \quad \dots (4) \quad D = -1 - B$$

(2) నుండి

$$-A + B + 3 - A - 1 - B = -2$$

$$\Rightarrow -2A = -4 \Rightarrow A = 2$$

(3) నుండి

$$A - B + 3 - A - 1 - B = 0 \Rightarrow 2 = 2B \Rightarrow B = 1$$

$$\therefore C = 3 - 2 = 1, D = -1 - 1 = -2$$

$$Ax + B = 2x + 1, Cx + D = x - 2$$

$$\therefore \frac{3x^3 - 2x^2 - 1}{x^4 + x^2 + 1} = \frac{2x+1}{x^2+x+1} + \frac{x-2}{x^2-x+1}$$

3.
$$\frac{x^4 + 24x^2 + 28}{(x^2 + 1)^3}$$

Sol.
$$\frac{x^4 + 24x^2 + 28}{(x^2 + 1)^3}$$

$$x^2 + 1 = y \Rightarrow x^2 = y - 1$$

$$\frac{x^4 + 24x^2 + 28}{(x^2 + 1)^3} = \frac{(y-1)^2 + 24(y-1) + 28}{y^3}$$

$$= \frac{y^2 - 2y + 1 + 24y - 24 + 28}{y^3} = \frac{y^2 + 22y + 5}{y^3}$$

$$= \frac{y^2}{y^3} + \frac{22y}{y^3} + \frac{5}{y^3} = \frac{1}{y} + \frac{22}{y^2} + \frac{5}{y^3}$$

$$= \frac{1}{(x^2 + 1)} + \frac{22}{(x^2 + 1)^2} + \frac{5}{(x^2 + 1)^3}$$

$$\therefore \frac{x^4 + 24x^2 + 28}{(x^2 + 1)^3} = \frac{1}{(x^2 + 1)} + \frac{22}{(x^2 + 1)^2} + \frac{5}{(x^2 + 1)^3}$$

4.
$$\frac{x^3}{(2x-1)(x+2)(x-3)}$$

Sol.
$$\frac{x^3}{(2x-1)(x+2)(x-3)} = \frac{1}{2} + \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

$$2x^3 = (2x-1)(x+2)(x-3) + 2A(x+2)$$

$$(x-3) + 2B(2x-1)(x-3) + 2C(2x-1)(x+2)$$

$$x = \frac{1}{2}, 2\left(\frac{1}{8}\right) = 2A\left(\frac{5}{2}\right) \cdot \left(-\frac{5}{2}\right) \Rightarrow A = -\frac{1}{50}$$

$$x = -2, 2(-8) = 2B(-5)(-5) \Rightarrow B = \frac{-8}{25}$$

$$x = 3, 2(27) = 2C(5)(5) \Rightarrow C = \frac{27}{25}$$

$$\therefore \frac{x^3}{(2x-1)(x+2)(x-3)} = \frac{1}{2} - \frac{1}{50(2x-1)} - \frac{8}{25(x+2)} + \frac{27}{25(x-3)}$$

5. $\frac{x^4}{(x-1)(x-2)}$.

Sol. $\frac{x^4}{(x-1)(x-2)} = \frac{x^4}{x^2 - 3x + 2}$

$$= \frac{x^2(x^2 - 3x + 2) + 3x(x^2 - 3x + 2) + 7(x^2 - 3x + 2) + 15x - 14}{x^2 - 3x + 2}$$

$$= x^2 + 3x + 7 + \frac{15x - 14}{x^2 - 3x + 2}$$

$$\frac{15x - 14}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$15x - 14 = A(x-2) + B(x-1)$$

$$x = 1, 15 - 14 = A(-1) \Rightarrow A = -1$$

$$x = 2, 30 - 14 = B(1) \Rightarrow B = 16$$

$$\therefore \frac{x^4}{(x-1)(x-2)} = x^2 + 3x + 7 - \frac{1}{x-1} + \frac{16}{x-2}$$

6. $\frac{3x}{(x-2)(x+1)}$ విస్తరణ లో x^4 పద గుణకాన్ని కనుగొనుము.

Sol. $\frac{3x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$

$$3x = A(x+1) + B(x-2)$$

$$x = -1, -3 = B(-3) \Rightarrow B = 1$$

$$x = 2, 6 = A(3) \Rightarrow A = 2$$

$$\therefore \frac{3x}{(x-2)(x+1)} = \frac{2}{x-2} + \frac{1}{x+1}$$

$$= \frac{2}{-2\left(1-\frac{x}{2}\right)} + \frac{1}{1+x} = -\left(1-\frac{x}{2}\right)^{-1} + (1+x)^{-1}$$

$$= -\left[1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \frac{x^4}{16} + \dots\right] + (1 - x + x^2 - x^3 + x^4 \dots)$$

$$\therefore x^4 \text{ పద గుణకం} = -\frac{1}{16} + 1 = \frac{15}{16}$$

7. $\frac{x}{(x-1)^2(x-2)}$ విస్తరణ లో x^n పద గుణకాన్ని కనుగొనుము.

Sol. $\frac{x}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$

$$x = A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

$$x = 1, 1 = B(-1) \Rightarrow B = -1$$

$$x = 2, 2 = C(1) \Rightarrow C = 2$$

$$x = 0, 0 = 2A - 2B + C \Rightarrow 2A = 2B - C$$

$$= -2 - 2 = -4 \Rightarrow A = -2$$

$$\begin{aligned} \therefore \frac{x}{(x-1)^2(x-2)} &= \frac{-2}{x-1} - \frac{1}{(x-1)^2} + \frac{2}{x-2} \\ &= \frac{2}{1-x} - \frac{1}{(1-x)^2} + \frac{2}{-2\left(1-\frac{x}{2}\right)} \\ &= 2(1-x)^{-1} - (1-x)^{-2} - \left(1-\frac{x}{2}\right)^{-1} \\ &= 2[1+x+x^2+\dots+x^n+\dots] - [1+2x+3x^2+\dots+(n+1)x^n+\dots] \\ &\quad - \left[1+\frac{x^2}{2}+\frac{x^2}{4}+\dots+\frac{x^n}{2^n}+\dots\right] \end{aligned}$$

$$\begin{aligned} \therefore x^n \text{ పద గుణకం} &= 2(1) - (n+1) - \left(\frac{1}{2^n}\right) \\ &= 2 - n - 1 - \frac{1}{2^n} = 1 - n - \frac{1}{2^n}. \end{aligned}$$

8. $\frac{x+3}{(1-x)^2(1+x^2)}$.

Sol: $\frac{x+3}{(1-x)^2(1+x^2)} = \frac{A}{(1-x)} + \frac{B}{(1-x)^2} + \frac{Cx+D}{(1+x^2)}$

$$\Rightarrow x+3 = A(1-x)(1+x^2) + B(1+x^2) + (Cx+D)(1-x)^2$$

సరి పద గుణకాల్ని పోల్చగా

$$A + B + D = 3 \quad \dots (1)$$

$$-A + C - 2D = 1 \quad \dots (2)$$

$$A + B - 2C + D = 0 \quad \dots (3)$$

$$-A + C = 0 \quad \dots (4)$$

పై సమీకరణాల్ని సాధించగా

$$A = \frac{3}{2}, B = 2, C = \frac{3}{2}, D = -\frac{1}{2}$$

$$\therefore \frac{x+3}{(1-x)^2(1+x^2)} = \frac{3}{2(1-x)} + \frac{2}{(1-x)^2} + \frac{3x-1}{2(1+x^2)}$$

9. $\frac{x^2+5x+7}{(x-3)^3}$

Sol. $x-3 = y$ అనుకొనుము $\Rightarrow x = y+3$

$$\frac{x^2+5x+7}{(x-3)^3} = \frac{(y+3)^2+5(y+3)+7}{y^3}$$

$$= \frac{y^2+6y+9+5y+15+7}{y^3}$$

$$= \frac{y^2+11y+31}{y^3} = \frac{1}{y} + \frac{11}{y^2} + \frac{31}{y^3}$$

$$\therefore \frac{x^2+5x+7}{(x-3)^3} = \frac{1}{x-3} + \frac{11}{(x-3)^2} + \frac{31}{(x-3)^3}$$

10. $\frac{x^2+13x+15}{(2x+3)(x+3)^2}$

Sol. Let $\frac{x^2+13x+15}{(2x+3)(x+3)^2} = \frac{A}{2x+3} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

$$x^2+13x+15 = A(x+3)^2 + B(x+3)(2x+3) + C(2x+3)$$

Put $x = -\frac{3}{2}$, $\frac{9}{4} - \frac{39}{2} + 15 = A\left(\frac{9}{4}\right)$

$$\Rightarrow \frac{9A}{4} = \frac{9-78+60}{4} = -\frac{9}{4} \Rightarrow A = -1$$

$$x = -3, 9 - 39 + 15 = C(-3)$$

$$\Rightarrow -3C = -15 \Rightarrow C = 5$$

x^2 పద గుణకాల్ని పోల్చగా,

$$A + 2B = 1 \Rightarrow 2B = 1 - A = 1 + 1 = 2 \Rightarrow B = 1$$

$$\therefore \frac{x^2 + 13x + 15}{(2x + 3)(x + 3)^2} = \frac{-1}{2x + 3} + \frac{1}{x + 3} + \frac{5}{(x + 3)^2}$$

11. $\frac{1}{(x-1)^2(x-2)}$.

Sol. Let $\frac{1}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$

$$\begin{aligned} \Rightarrow \frac{1}{(x-1)^2(x-2)} \\ = \frac{A(x-1)(x-2) + B(x-2) + C(x-1)^2}{(x-1)^2(x-2)} \end{aligned}$$

$$\Rightarrow 1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2 \dots (1)$$

$$x = 1 \Rightarrow 1 = A(0) + B(1-2) + C(0)$$

$$\Rightarrow -B = 1 \Rightarrow B = -1$$

$$\text{Put } x = 2 \Rightarrow 1 = A(0) + B(0) + C(2-1)^2 \Rightarrow C = 1$$

x^2 పద గుణకాల్ని పోల్చగా,

$$0 = A + C \Rightarrow A = -C = -1 \Rightarrow A = -1$$

$$\therefore \frac{1}{(x-1)^2(x-2)} = \frac{-1}{x-1} - \frac{1}{(x-1)^2} + \frac{1}{x-2}$$

12. $\frac{x^2 - x + 1}{(x+1)(x-1)^2}$

Sol. Let $\frac{x^2 - x + 1}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

$$x^2 - x + 1 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$$x = -1, 1 + 1 + 1 = A(4) \Rightarrow A = \frac{3}{4}$$

$$x = 1, 1 - 1 + 1 = C(2) \Rightarrow C = \frac{1}{2}$$

x^2 పద గుణకాల్ని పోల్చగా,

$$A + B = 1 \Rightarrow B = 1 - A = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\therefore \frac{x^2 - x + 1}{(x+1)(x-1)^2} = \frac{3}{4(x+1)} + \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$$

13. $\frac{9}{(x-1)(x+2)^2}$

Sol. $\frac{9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

$$9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$$x = 1 \Rightarrow 9 = 9A \Rightarrow A = 1$$

$$x = -2 \Rightarrow 9 = -3C \Rightarrow C = -3$$

x^2 పద గుణకాల్ని పోల్చగా,

$$A + B = 0 \Rightarrow B = -A = -1$$

$$\therefore \frac{9}{(x-1)(x+2)^2} = \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$$

14. $\frac{x-1}{(x+1)(x-2)^2}$

Sol. Let $\frac{x-1}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$$\Rightarrow x - 1 =$$

$$A(x-2)^2 + B(x-2)(x+1) + C(x+1) \dots(1)$$

$$x = 2$$

$$2 - 1 = A(0) + B(0) + C(2 + 1)$$

$$\Rightarrow 1 = 3C \Rightarrow C = \frac{1}{3}$$

$$x = -1 \Rightarrow -1 - 1 = A(-1-2)^2 + B(0) + C(0)$$

$$\Rightarrow 9A = -2 \Rightarrow A = \frac{-2}{9}$$

x^2 పద గుణకాల్ని పోల్చగా,

$$0 = A + B \Rightarrow B = -A = \frac{2}{9}$$

$$\therefore \frac{x-1}{(x+1)(x-2)^2} = \frac{-2}{9(x+1)} + \frac{2}{9(x-2)} + \frac{1}{3(x-2)^2}$$

$$\frac{x-1}{(x+1)(x-2)^2} = \frac{-2}{9(x+1)} + \frac{2}{9(x-2)} + \frac{1}{3(x-2)^2}$$

15. $\frac{1}{(1-2x)^2(1-3x)}$

Sol.Let

$$\frac{1}{(1-2x)^2(1-3x)} = \frac{A}{1-3x} + \frac{B}{1-2x} + \frac{C}{(1-2x)^2}$$

$$1 = A(1-2x)^2 + B(1-3x)(1-2x) + C(1-3x)$$

$$x = \frac{1}{3} \Rightarrow 1 = A\left(1 - \frac{2}{3}\right)^2 = \frac{A}{9} \Rightarrow A = 9$$

$$x = \frac{1}{2} \Rightarrow 1 = C\left(1 - \frac{3}{2}\right) = -\frac{C}{2} \Rightarrow C = -2$$

x^2 పద గుణకాల్ని పోల్చగా,

$$0 = 4A + 6B$$

$$6B = -4A = -36$$

$$B = -6$$

$$\frac{1}{(1-2x)^2(1-3x)} = \frac{9}{1-3x} - \frac{6}{1-2x} - \frac{2}{(1-2x)^2}$$

16 $\frac{1}{x^3(x+a)}$

Sol.Let $\frac{1}{x^3(x+a)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+a}$

$$= \frac{A \cdot x^2(x+a) + B(a)(x+a) + C(x+a) + Dx^3}{x^3(x+a)}$$

$$\therefore 1 = A(x^2)(x+a) + Bx(x+a) + C(x+a) + Dx^3 \dots(1)$$

Put $x = 0$

$$1 = A(0) + B(0) + C(0 + a) + D(0)$$

$$\Rightarrow 1 = C(a) \Rightarrow C = \frac{1}{a}$$

Put $x = -a$

$$1 = A(0) + B(0) + C(0) + D(-a)^3$$

$$\Rightarrow 1 = -Da^3 \Rightarrow D = -\frac{1}{a^3}$$

x^3 పద గుణకాల్ని పోల్చగా,

$$0 = A + D$$

$$\Rightarrow A = -D = \frac{1}{a^3}, A = \frac{1}{a^3}$$

x^2 పద గుణకాల్ని పోల్చగా,

$$0 = Aa + B$$

$$\Rightarrow B = -aA = -a\left(\frac{1}{a^3}\right) = -\frac{1}{a^2}$$

$$\therefore B = -\frac{1}{a^2}$$

$$\frac{1}{x^3(x+a)} = \frac{\left(\frac{1}{a^3}\right)}{x} + \frac{\left(-\frac{1}{a^2}\right)}{x^2} + \frac{\left(\frac{1}{a}\right)}{x^3} + \frac{\left(-\frac{1}{a^3}\right)}{x+a}$$

$$\Rightarrow \frac{1}{x^3(x+a)} = \frac{1}{a^3x} - \frac{1}{a^2x^2} + \frac{1}{ax^3} - \frac{1}{a^3(x+a)}$$

$$17. \frac{3x^3 - 8x^2 + 10}{(x-1)^4}$$

Sol: $x - 1 = y$ అనుకోండి

$$\begin{aligned} \therefore \frac{3x^3 - 8x^2 + 10}{(x-1)^4} &= \frac{3(y+1)^3 - 8(y+1)^2 + 10}{y^4} \\ &= \frac{3(y^3 + 3y^2 + 3y + 1) - 8(y^2 + 2y + 1) + 10}{y^4} \\ &= \frac{3y^3 + y^2 - 7y + 5}{y^4} \\ &= \frac{3}{y} + \frac{1}{y^2} - \frac{7}{y^3} + \frac{5}{y^4} \\ &= \frac{3}{x-1} + \frac{1}{(x-1)^2} - \frac{7}{(x-1)^3} + \frac{5}{(x-1)^4} \end{aligned}$$

$$\begin{aligned} \therefore \frac{3x^3 - 8x^2 + 10}{(x-1)^4} &= \frac{3}{x-1} + \frac{1}{(x-1)^2} - \frac{7}{(x-1)^3} + \frac{5}{(x-1)^4} \end{aligned}$$

$$18. \frac{3x-18}{x^3(x+3)}$$

Sol. Let $\frac{3x-18}{x^3(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+3}$

$$\therefore \frac{3x-18}{x^3(x+3)} = \frac{Ax^2(x+3) + Bx(x+3) + C(x+3) + Dx^3}{x^3(x+3)}$$

$$\Rightarrow 3x - 18 = Ax^2(x+3) + Bx(x+3) + C(x+3) + Dx^3 \dots(1)$$

$$x = -3$$

$$3(-3) - 18 = A(0) + B(0) + C(0) + D(-3)^3$$

$$\Rightarrow -27D = -27 \Rightarrow D = 1$$

$$x = 0$$

$$3(0) - 18 = A(0) + B(0) + C(0 + 3) + D(0)$$

$$\Rightarrow 3C = -18 \Rightarrow C = -6$$

x^3 పద గుణకాల్ని పోల్చగా,

$$0 = A + D \Rightarrow A = -D = -1 \Rightarrow A = -1$$

x^2 పద గుణకాల్ని పోల్చగా,

$$0 = 3A + B \Rightarrow B = -3A = -3(-1) = 3 \Rightarrow B = 3$$

$$\frac{3x-18}{x^3(x+3)} = \frac{-1}{x} + \frac{3}{x^2} - \frac{6}{x^3} + \frac{1}{x+3}$$

19. $\frac{2x^2+1}{x^3-1}$.

Sol. $\frac{2x^2+1}{x^3-1} = \frac{2x^2+1}{(x-1)(x^2+x+1)}$... (1)

$$\frac{2x^2+1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$\Rightarrow \frac{2x^2+1}{(x-1)(x^2+x+1)} =$$

$$\frac{A(x^2+x+1) + (Bx+C)(x-1)}{(x-1)(x^2+x+1)}$$

$$\therefore 2x^2+1 = A(x^2+x+1) + (Bx+C)(x-1) \dots (1)$$

$$x = 1$$

$$2(1) + 1 = A(1 + 1 + 1) + (B + C)(0)$$

$$\Rightarrow 3A = 3 \Rightarrow A = 1$$

$$x = 0$$

$$0 + 1 = A(1) + (0 + C)(0 - 1)$$

$$\Rightarrow 1 = A - C \Rightarrow C = 0$$

x^2 పద గుణకాల్ని పోల్చగా,

$$2 = A + B \Rightarrow 2 = 1 + B \Rightarrow B = 1$$

$$\therefore \frac{2x^2+1}{x^3-1} = \frac{1}{x-1} + \frac{(1)(x)+0}{x^2+x+1} = \frac{1}{x-1} + \frac{x}{x^2+x+1}$$

20. $\frac{x^3+x^2+1}{(x^2+2)(x^2+3)}$.

Sol. Let $\frac{x^3+x^2+1}{(x^2+2)(x^2+3)} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{x^2+3}$

$$= \frac{(Ax+B)(x^2+3) + (Cx+D)(x^2+2)}{(x^2+2)(x^2+3)}$$

$$\therefore x^3+x^2+1 =$$

$$(Ax+B)(x^2+3) + (Cx+D)(x^2+2) \quad \dots(1)$$

$$\Rightarrow x^3+x^2+1 = (A+C)x^3 + (B+D)x^2 + (3A+2C)x + (3B+2D)$$

x^3, x^2, x , స్థిర పద గుణకాల్ని పోల్చగా,

$$A + C = 1, B + D = 1,$$

$$3A + 2C = 0, 3B + 2D = 1$$

$$A = -2, C = 3, B = -1, D = 2$$

$$\begin{aligned} \therefore \frac{x^3 + x^2 + 1}{(x^2 + 2)(x^2 + 3)} &= \frac{-2x - 1}{x^2 + 2} + \frac{3x + 2}{x^2 + 3} \\ &= \frac{3x + 2}{x^2 + 3} - \frac{2x + 1}{x^2 + 2} \end{aligned}$$

21. $\frac{3x^3 - 2x^2 - 1}{x^4 + x^2 + 1}$.

Sol. $x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2$

$$= (x^2 + 1)^2 - x^2 = (x^2 + x + 1)(x^2 - x + 1)$$

$$\begin{aligned} \frac{3x^3 - 2x^2 - 1}{x^4 + x^2 + 1} &= \frac{3x^3 - 2x^2 - 1}{(x^2 + x + 1)(x^2 - x + 1)} \\ &= \frac{Ax + B}{x^2 + x + 1} + \frac{Cx + D}{x^2 - x + 1} \end{aligned}$$

$$3x^3 - 2x^2 - 1 = (Ax + B)(x^2 - x + 1) + (Cx + D)(x^2 + x + 1)$$

ఇరువైపులా సరి పద గుణకాల్ని పోల్చగా,

$$A + C = 3 \quad \dots (1)$$

$$\Rightarrow C = 3 - A$$

$$-A + B + C + D = -2 \quad \dots (2)$$

$$A - B + C + D = 0 \quad \dots (3)$$

$$B + D = -1 \quad \dots (4) \quad D = -1 - B$$

(2)నుండి

$$-A + B + 3 - A - 1 - B = -2$$

$$\Rightarrow -2A = -4 \Rightarrow A = 2$$

(3)నుండి

$$A - B + 3 - A - 1 - B = 0 \Rightarrow 2 = 2B \Rightarrow B = 1$$

$$\therefore C = 3 - 2 = 1, D = -1 - 1 = -2$$

$$Ax + B = 2x + 1, Cx + D = x - 2$$

$$\therefore \frac{3x^3 - 2x^2 - 1}{x^4 + x^2 + 1} = \frac{2x + 1}{x^2 + x + 1} + \frac{x - 2}{x^2 - x + 1}$$

22. $\frac{x^3}{(2x-1)(x+2)(x-3)}$.

Sol. $\frac{x^3}{(2x-1)(x+2)(x-3)} = \frac{1}{2} + \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{x-3}$

$$2x^3 = (2x-1)(x+2)(x-3) + 2A(x+2)$$

$$(x-3) + 2B(2x-1)(x-3) + 2C(2x-1)(x+2)$$

$$x = \frac{1}{2} \Rightarrow 2\left(\frac{1}{8}\right) = 2A\left(\frac{5}{2}\right) \cdot \left(-\frac{5}{2}\right) \Rightarrow A = -\frac{1}{50}$$

$$x = -2 \Rightarrow 2(-8) = 2B(-5)(-5) \Rightarrow B = \frac{-8}{25}$$

$$x = 3 \Rightarrow 2(27) = 2C(5)(5) \Rightarrow C = \frac{27}{25}$$

$$\therefore \frac{x^3}{(2x-1)(x+2)(x-3)} = \frac{1}{2} - \frac{1}{50(2x-1)} - \frac{8}{25(x+2)} + \frac{27}{25(x-3)}$$

23. $\frac{x^4}{(x-1)(x-2)}$.

Sol. $\frac{x^4}{(x-1)(x-2)} = \frac{x^4}{x^2 - 3x + 2}$

$$= \frac{x^2(x^2 - 3x + 2) + 3x(x^2 - 3x + 2) + 7(x^2 - 3x + 2) + 15x - 14}{x^2 - 3x + 2}$$

$$= x^2 + 3x + 7 + \frac{15x - 14}{x^2 - 3x + 2}$$

$$\frac{15x - 14}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$15x - 14 = A(x-2) + B(x-1)$$

$$x = 1 \Rightarrow 15 - 14 = A(-1) \Rightarrow A = -1$$

$$x = 2 \Rightarrow 30 - 14 = B(1) \Rightarrow B = 16$$

$$\therefore \frac{x^4}{(x-1)(x-2)} = x^2 + 3x + 7 - \frac{1}{x-1} + \frac{16}{x-2}$$

24. $\frac{3x}{(x-2)(x+1)}$ విస్తరణ లో x^4 పద గుణకాన్ని కనుగొనుము.

$$\text{Sol. } \frac{3x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$3x = A(x+1) + B(x-2)$$

$$x = -1, -3 = B(-3) \Rightarrow B = 1$$

$$x = 2, 6 = A(3) \Rightarrow A = 2$$

$$\therefore \frac{3x}{(x-2)(x+1)} = \frac{2}{x-2} + \frac{1}{x+1}$$

$$= \frac{2}{-2\left(1-\frac{x}{2}\right)} + \frac{1}{1+x} = -\left(1-\frac{x}{2}\right)^{-1} + (1+x)^{-1}$$

$$= -\left[1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \frac{x^4}{16} + \dots\right] + (1-x+x^2-x^3+x^4 \dots)$$

$$\therefore x^4 \text{ పద గుణకం} = -\frac{1}{16} + 1 = \frac{15}{16}$$

25. $\frac{x}{(x-1)^2(x-2)}$ విస్తరణ లో x^n పద గుణకాన్ని కనుగొనుము.

Sol. $\frac{x}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$

$$x = A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

$$x = 1, 1 = B(-1) \Rightarrow B = -1$$

$$x = 2, 2 = C(1) \Rightarrow C = 2$$

$$x = 0, 0 = 2A - 2B + C \Rightarrow 2A = 2B - C$$

$$= -2 - 2 = -4 \Rightarrow A = -2$$

$$\therefore \frac{x}{(x-1)^2(x-2)} = \frac{-2}{x-1} - \frac{1}{(x-1)^2} + \frac{2}{x-2}$$

$$= \frac{2}{1-x} - \frac{1}{(1-x)^2} + \frac{2}{-2\left(1-\frac{x}{2}\right)}$$

$$= 2(1-x)^{-1} - (1-x)^{-2} - \left(1-\frac{x}{2}\right)^{-1}$$

$$= 2[1+x+x^2+\dots+x^n+\dots] - [1+2x+3x^2+\dots+(n+1)x^n+\dots] - \left[1+\frac{x^2}{2}+\frac{x^2}{4}+\dots+\frac{x^n}{2^n}+\dots\right]$$

$$\therefore x^n \text{ పద గుణకం} = 2(1) - (n+1) - \left(\frac{1}{2^n}\right)$$

$$= 2 - n - 1 - \frac{1}{2^n} = 1 - n - \frac{1}{2^n}$$

26. $\frac{5x+6}{(2+x)(1-x)}$ విస్తరణ లో x^3 పద గుణకాన్ని కనుగొనుము

Sol.Let $\frac{5x+6}{(2+x)(1-x)} = \frac{A}{2+x} + \frac{B}{1-x}$

$$5x + 6 = A(1 - x) + B(2 + x)$$

$$x = 1 \Rightarrow 11 = B(2+1) = 3B \Rightarrow B = \frac{11}{3}$$

$$x = -2 \Rightarrow -4 = A(1+2) = 3A \Rightarrow A = \frac{-4}{3}$$

$$\frac{5x+6}{(2+x)(1-x)} = \frac{-4}{3(2+x)} + \frac{11}{3(1-x)}$$

$$= \frac{-4}{3 \cdot 2 \left(1 + \frac{x}{2}\right)} + \frac{11}{3(1-x)}$$

$$= -\frac{2}{3} \left(1 + \frac{x}{2}\right)^{-1} + \frac{11}{3} (1-x)^{-1}$$

$$= -\frac{2}{3} \left(1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \dots\right) + \frac{11}{3} (1 + x + x^2 + x^3 \dots)$$

$$\therefore x^3 \text{ పద గుణకం} = -\frac{2}{3} \left(-\frac{1}{8}\right) + \frac{11}{3} (1)$$

$$= \frac{2+88}{24} = \frac{90}{24} = \frac{15}{4}$$

27. $\frac{3x^2+2x}{(x^2+2)(x-3)}$ విస్తరణ లో x^4 పద గుణకాన్ని కనుగొనుము..

Sol.Let $\frac{3x^2+2x}{(x^2+2)(x-3)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+2}$

$$3x^2 + 2x = A(x^2 + 2) + (Bx + C)(x - 3)$$

$$x = 3 \Rightarrow 27 + 6 = A(9 + 2)$$

$$33 = 11A \Rightarrow A = 3$$

x^2 పద గుణకాన్ని పోల్చగా,

$$3 = A + B \Rightarrow B = 3 - A = 3 - 3 = 0$$

స్థిర పదాన్ని పోల్చగా,

$$2A - 3C = 0 \Rightarrow 3C = 2A = 6 \Rightarrow C = 2$$

$$\frac{3x^2 + 2x}{(x^2 + 2)(x - 3)} = \frac{3}{x - 3} + \frac{2}{x^2 + 2}$$

$$= \frac{3}{-3\left(1 - \frac{x}{3}\right)} + \frac{2}{2\left(1 + \frac{x^2}{2}\right)}$$

$$= -\left(1 - \frac{x}{3}\right)^{-1} + \left(1 + \frac{x^2}{2}\right)^{-1}$$

$$= -\left(1 + \frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + \frac{x^4}{81} + \dots\right) + \left(1 - \frac{x^2}{2} + \frac{x^4}{4} - \dots\right)$$

$$\therefore x^4 \text{ పద గుణకం} = -\frac{1}{81} + \frac{1}{4}$$

$$= \frac{-4 + 81}{324} = \frac{77}{324}$$

28. $\frac{x-4}{x^2-5x+6}$ విస్తరణ లో x^n పద గుణకాన్ని కనుగొనుము.

$$\text{Sol. Let } \frac{x-4}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$x-4 = A(x-3) + B(x-2)$$

$$x=2 \Rightarrow -2 = A(2-3) = -A \Rightarrow A=2$$

$$x=3 \Rightarrow -1 = B(3-2) = B \Rightarrow B=-1$$

$$\begin{aligned} \frac{x-4}{x^2-5x+6} &= \frac{2}{x-2} - \frac{1}{x-3} \\ &= \frac{2}{-2\left(1-\frac{x}{2}\right)} + \frac{1}{3\left(1-\frac{x}{3}\right)} \\ &= -\left(1-\frac{x}{2}\right)^{-1} + \frac{1}{3}\left(1-\frac{x}{3}\right)^{-1} \\ &= -\left(1+\frac{x}{2}+\frac{x^2}{4}+\dots+\frac{x^n}{2^n}+\dots\right) + \frac{1}{3}\left(1+\frac{x}{3}+\frac{x^2}{9}+\dots+\frac{x^n}{3^n}+\dots\right) \end{aligned}$$

$$x^n \text{ పద గుణకం} = \frac{1}{3^{n+1}} - \frac{1}{2^n}$$

29. $\frac{3x}{(x-1)(x-2)^2}$ విస్తరణ లో x^n పద గుణకాన్ని కనుగొనుము.

Sol: Let

$$\frac{3x}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\Rightarrow A(x-2)^2 + B(x-1)(x-2) + C(x-1) = 3x$$

$$x = 1 \Rightarrow A = 3$$

$$x = 2 \Rightarrow C = 6$$

x^2 పద గుణకాల్ని పోల్చగా,

$$A + B = 0 \Rightarrow B = -A \Rightarrow B = -3$$

$$\therefore \frac{3x}{(x-1)(x-2)^2} = \frac{3}{x-1} - \frac{3}{x-2} + \frac{6}{(x-2)^2}$$

$$= -3\left(1-x\right)^{-1} + \frac{3}{2}\left(1-\frac{x}{2}\right)^{-1} + \frac{3}{2}\left(1-\frac{x}{2}\right)^{-2}$$

Now

$$(1-x)^{-1} = 1+x+x^2+\dots+x^n+\dots, |x| < 1$$

$$\left(1 - \frac{x}{2}\right)^{-1} = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots + \frac{x^n}{2^n} + \dots, \left|\frac{x}{2}\right| < 1$$

i.e. $|x| < 2$

$$\left(1 - \frac{x}{2}\right)^{-2} = 1 + 2\left(\frac{x}{2}\right) + 3\left(\frac{x}{2}\right)^2 + 4\left(\frac{x}{2}\right)^3 + \dots + (n+1)\left(\frac{x}{2}\right)^n + \dots \left|\frac{x}{2}\right| < 1 \text{ i.e., } |x| < 2.$$

$|x| < 1.$

$$\therefore \frac{3x}{(x-1)(x-2)^2} = -3(1+x+x^2+\dots+x^n+\dots) + \left(1 + \frac{x}{2} + \frac{x^2}{4} + \dots + \frac{x^n}{2^n} + \dots\right)$$

$$+ \frac{3}{2} \left(1 + 2\left(\frac{x}{2}\right) + 3\left(\frac{x}{2}\right)^2 + \dots + (n+1)\left(\frac{x}{2}\right)^n + \dots\right)$$

$$x^n \text{ పద గుణకం} = -3 + \frac{3}{2} \left(\frac{1}{2^n}\right) + \frac{3(n+1)}{2 \cdot 2^n}$$

$$= -3 + \frac{3}{2^{n+1}} + \frac{3(n+1)}{2^{n+1}}.$$