ద్విపద సిద్ధాంతం

అతిస్వల్ప సమాధాన ప్రశ్నలు

1. ద్విపద సిద్ధాంతం ఉపయోగించి $(4x + 5y)^7$ విస్తరించి వ్రాయండి.

Sol. $(4x+5y)^7 =$

$${}^7C_0(4x)^7(5y)^0 + {}^7C_1(4x)^6(5y)^1 + {}^7C_2(4x)^5(5y)^2 + {}^7C_3(4x)^4(5y)^3 +$$

$${}^7C_4(4x)^3(5y)^4 + {}^7C_5(4x)^2(5y)^5 + {}^7C_6(4x)^1(5y)^6 + {}^7C_7(4x)^0 + (5y)^7$$

$$= \sum_{r=0}^{7} {}^7C_r(4x)^{7-r}(5y)^r$$

2. $\left(\frac{2x}{3}+\frac{3y}{2}\right)^9$ లో6 వ పదాన్ని వ్రాసి సూక్ష్మీకరించండి

Sol. $\left(\frac{2x}{3}+\frac{3y}{2}\right)^9$ లో 6 వ పదం

$\left(\frac{2x}{3}+\frac{3y}{2}\right)^9$ లో సాధారణ పదం

$$T_{r+1} = {}^9C_r\left(\frac{2x}{3}\right)^{9-r}\left(\frac{3y}{2}\right)^r$$

r = 5ప్రతిక్షేపించగా

$$T_6 = {}^9C_5\left(\frac{2x}{3}\right)^4\left(\frac{3y}{2}\right)^5 = {}^9C_5\left(\frac{2}{3}\right)^4\left(\frac{3}{x}\right)^5 x^4y^5$$

$$= \frac{9\times8\times7\times6}{1\times2\times3\times4}\frac{(2^4)}{3^4}\frac{3^5}{2^5}x^4y^5 = 189x^4y^5$$

3. $(2x+3y+z)^7$ విస్తరణ లో పదాల సంఖ్య తెలపండి

Sol. $(a + b + c)^n$ విస్తరణ లో పదాల సంఖ్య $= \frac{(n+1)(n+2)}{2}$,

$(2x+3y+z)^7$ విస్తరణ లో పదాల సంఖ్య $= \frac{(7+1)(7+2)}{2} = \frac{8\times9}{2} = 36$

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4. i) $\left(3x-\frac{4}{x}\right)^{10}$ విస్తరణ లో x^{-6} యొక్క గుణకాన్ని తెలపండి.

ii) $\left(2x^2+\frac{3}{x^3}\right)^{13}$ విస్తరణ లో x^{11} యొక్క గుణకాన్ని తెలపండి.

i) $\left(3x-\frac{4}{x}\right)^{10}$

Sol. $\left(3x-\frac{4}{x}\right)^{10}$ లో సాధారణ పదం

$$T_{r+1}=(-1)^r\,{}^{10}C_r(3x)^{10-r}\left(\frac{4}{x}\right)^r$$

$$=(-1)^r\,{}^{10}C_r 3^{10-r}(4)^r x^{10-r-r}$$

$$=(-1)^r\,{}^{10}C_r 3^{10-r}(4)^r x^{10-2r} \quad ...(1)$$

x^{-6} యొక్క గుణకం కోసం, $10-2r=-6$

$\Rightarrow 2r = 10+6 = 16 \Rightarrow r = 8$

(1) లో $r = 8$ ప్రతిక్షేపించగా

$$T_{8+1}=(-1)^8\,{}^{10}C_8 3^{10-8}(4)^8 x^{10-16}={}^{10}C_8 3^2 4^8 x^{-6}$$

$\therefore$ $\left(3x-\frac{4}{x}\right)^{10}$ లో x^{-6} యొక్క గుణకం ${}^{10}C_8 3^2 4^8={}^{10}C_2 3^2 4^8=\frac{10\times 9}{1\times 2}\times 9\times 4^8=405\times 4^8$

ii) x^{11} in $\left(2x^2+\frac{3}{x^3}\right)^{13}$

Sol. $\left(2x^2+\frac{3}{x^3}\right)^{13}$ లో సాధారణ పదం

$$T_{r+1}={}^{13}C_r(2x^2)^{13-r}\left(\frac{3}{x^3}\right)^r$$

$$={}^{13}C_r(2)^{13-r}3^r x^{26-2r}x^{-3r}$$

$$={}^{13}C_r(2)^{13-r}(3)^r x^{26-5r} \quad ...(1)$$

x^{11} యొక్క గుణకం కోసం, $26-5r=11$

$\Rightarrow 5r = 15 \Rightarrow r = 3$

$r = 3$

$$T_{3+1}={}^{13}C_3(2)^{10}(3)^3x^{26-15}$$

$$T_4=\frac{13\times 12\times 11}{1\times 2\times 3}\cdot 2^{10}\cdot 3^3\cdot x^{11}$$

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$\therefore \left(2x^2 + \frac{3}{x^3}\right)^{13}$ లో x^{11} యొక్క గుణకం $=(286)(2^{10})(3^3)$

5. క్రింది విస్తరణలలో x లేని పదం (స్థిరపదం) కనుక్కోండి.

(i) $\left(\frac{3}{\sqrt[3]{x}} + 5\sqrt{x}\right)^{25}$ (ii) $\left(4x^3 + \frac{7}{x^2}\right)^{14}$

i) $\left(\frac{3}{\sqrt[3]{x}} + 5\sqrt{x}\right)^{25}$

Sol. $\left(\frac{3}{\sqrt[3]{x}} + 5\sqrt{x}\right)^{25}$ లో సాధారణ పదం

$$T_{r+1} = {}^{25}C_r \left(\frac{3}{\sqrt[3]{x}}\right)^{25-r} (5\sqrt{x})^r$$

$$= {}^{25}C_r (3)^{25-r} (5)^r \cdot x^{-1/3(25-r)} x^{r/2}$$

$$= {}^{25}C_r (3)^{25-r} (5)^r \cdot x^{-\frac{25}{3}+\frac{r}{3}+\frac{r}{2}}$$

$$= {}^{25}C_r (3)^{25-r} (5)^r \cdot x^{-\frac{50+2r+3r}{6}} \quad ...(1)$$

x లేని పదం (స్థిరపదం)కోసం $\frac{-50+5r}{6} = 0 \Rightarrow 5r = 50 \Rightarrow r = 10$

$r = 10$

$$T_{10+1} = {}^{25}C_{10} (3)^{15} (5)^{10} x^0$$

i.e. $T_{11} = {}^{25}C_{10} (3)^{15} (5)^{10}$

ii) $\left(4x^3 + \frac{7}{x^2}\right)^{14}$

Sol. $\left(4x^3 + \frac{7}{x^2}\right)^{14}$ లో సాధారణ పదం

$$T_{r+1} = {}^{14}C_r (4x^3)^{14-r} \left(\frac{7}{x^2}\right)^r$$

$$= {}^{14}C_r (4)^{14-r} (7)^r x^{42-3r} x^{-2r}$$

$$= {}^{14}C_r (4)^{14-r} (7)^r x^{42-5r} \quad ...(1)$$

x లేని పదం (స్థిరపదం)కోసం

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$4x - 5r = 0 \Rightarrow r = 42/5$, ఇది స్థిరరాశి కాదు. కావున ఇచ్చిన విస్తరణ లో స్థిరపదం ఉండదు.

6. విస్తరణ లో మధ్య పదం(పదాలు) కనుక్కోండి

(i) $\left(\frac{3x}{7} - 2y\right)^{10}$ (ii) $\left(4a + \frac{3}{2}b\right)^{11}$

Sol. nసరి సంఖ్య అయితే

$(x + a)^n$ విస్తరణ లో మధ్య పదం n= $T_{\left(\frac{n+1}{2}\right)}$, బేసి సంఖ్య అయితేమధ్య పదాలు =

. $T_{\left(\frac{n+1}{2}\right)}$, $T_{\left(\frac{n+3}{2}\right)}$.

i) $\left(\frac{3x}{7} - 2y\right)^{10}$

Sol. $n = 10$,సరి సంఖ్య

i.e. $\frac{10}{2} + 1 = 6$వ పదం

$\left(\frac{3x}{7} - 2y\right)^{10}$ లో 6వ పదం

$$= {}^{10}C_5\left(\frac{3x}{7}\right)^5(-2y)^5 = -({}^{10}C_5)\frac{3^5}{7^5}\cdot 2^5(xy)^5$$

$$= -{}^{10}C_5\left(\frac{6}{7}\right)^5 x^5y^5$$

ii) $\left(4a + \frac{3}{2}b\right)^{11}$

Sol. n = 11 బేసి సంఖ్య కావునమధ్య పదాలు = $\frac{n+1}{2}$, $\frac{n+3}{2}$, $= 6^{th}$, 7^{th} .

$\left(4a + \frac{3}{2}b\right)^{11}$ లో 6వ పదం

$$= {}^{11}C_5(4a)^6\left(\frac{3}{2}b\right)^5 = {}^{11}C_5(4)^6\frac{3^5}{2^5}a^6b^5$$

$$= \frac{11\times10\times9\times8\times7}{1\times2\times3\times4\times5}2^7\cdot3^5\cdot a^6b^5$$

$$= 77\times2^8\times3^6\times a^6b^5$$

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$\left(4a+\frac{3}{2}b\right)^{11}$ లో 7వ పదం

$$={}^{11}C_6(4a)^5\left(\frac{3}{2}b\right)^6={}^{11}C_5(4)^5\frac{3^6}{2^6}a^5b^6$$

$$=\frac{11\times10\times9\times8\times7}{1\times2\times3\times4\times5}2^4\cdot3^6\cdot a^5b^6$$

$$=77\times2^5\times3^7\times a^5b^6$$

7. క్రింది విస్తరణల లో సంఖ్య పరంగా గరిష్ట పదం కనుక్కోండి.

i) $(4+3x)^{15}$, $x=\frac{7}{2}$

ii) $(3x+5y)^{12}$, $x=\frac{1}{2}$ మరియు $y=\frac{4}{3}$

i) $(4+3x)^{15}$, $x=\frac{7}{2}$

Sol. $(4+3x)^{15}=\left[4\left(1+\frac{3}{4}x\right)\right]^{15}$

$$=4^{15}\left(1+\frac{3}{4}x\right)^{15}\qquad\ldots(1)$$

ముందుగా $\left(1+\frac{3}{4}x\right)^{15}$ విస్తరణల లో సంఖ్య పరంగా గరిష్ట పదం కనుక్కొందాం

$X=\frac{3}{4}x$ గా తీసుకుంటే

అప్పుడు $|X|=\left(\frac{3}{4}X\right)=\frac{3}{4}\times\frac{7}{2}=\frac{21}{8}$

మరియు $\frac{(n+1)|x|}{1+|x|}=\frac{15+1}{1+\frac{21}{8}}\cdot\frac{21}{8}$

$$=\frac{16\times21}{29}=\frac{336}{29}=11\frac{17}{29}$$

$$m=\left[11\frac{17}{29}\right]=11$$

T_{m+1} $\left(1+\frac{3}{4}x\right)^{15}$ విస్తరణల లో సంఖ్య పరంగా గరిష్ట పదం T_{m+1} అవుతుంది.

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$$T_{m+1} = T_{12} = {}^{15}C_{11}\left(\frac{3}{4}x\right)^4 = {}^{15}C_{11}\left(\frac{3}{4}\cdot\frac{7}{2}\right)^{11}$$

$$= 4^{15}\left[{}^{15}C_{11}\left(\frac{21}{8}\right)^{11}\right] = {}^{15}C_4\frac{(21)^{11}}{2^3}$$

ii) $(3x + 5y)^{12}$ $x = \frac{1}{2}$, $y = \frac{4}{3}$

Sol. $(3x + 5y)^{12} = \left[3x\left(1+\frac{5y}{3x}\right)\right]^{12}$

$$= 3^{12}x^{12}\left(1+\frac{5}{3}\frac{y}{x}\right)^{12}$$

$\left(1+\frac{5}{3}\frac{y}{x}\right)^{12}$ నుండి

$$n = 17,\ x = \frac{5}{3}\cdot\frac{y}{x} = \frac{5}{3}\frac{(4/3)}{(1/2)} = \frac{5}{3}\cdot\frac{8}{3} = \frac{40}{9}$$

$$\frac{(n+1)|x|}{1+|x|} = \frac{(12+1)\left(\frac{40}{9}\right)}{1+\frac{40}{9}} = \frac{13\times 40}{49} = \frac{520}{49} = 10\frac{30}{49}$$

$$\therefore\ m = \left[10\frac{30}{49}\right] = 10$$

$\left(1+\frac{5y}{3x}\right)^{12}$ విస్తరణల లో సంఖ్యా పరంగా గరిష్ట పదం $T_{m+1} = T_{11} = {}^{12}C_{10}\left(\frac{5}{3}\frac{y}{x}\right)^{10}$

$$= {}^{12}C_{10}\left(\frac{5}{3}\times\frac{(4/3)}{(1/2)}\right)^{10} = {}^{12}C_{10}\left(\frac{5}{3}\times\frac{8}{3}\right)^{10} = {}^{12}C_{10}\left(\frac{40}{9}\right)^{10}$$

$\therefore$ $(3x + 5y)^{12}$ విస్తరణల లో సంఖ్యా పరంగా గరిష్ట పదం

$$= 3^{12}\left(\frac{1}{2}\right)^{12}\ {}^{12}C_{10}\left(\frac{40}{9}\right)^{10}$$

$$= {}^{12}C_{10}\frac{3^{12}}{2^{12}}\frac{(2^2)^{10}\times(10)^{10}}{(3^2)^{10}} = {}^{12}C_{10}\left(\frac{3}{2}\right)^2\left(\frac{20}{3}\right)^{10}$$

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8. ఈ క్రింది వానిని నిరూపించండి

$= C_0 - 4\cdot C_1 + 7\cdot C_2 - 10\cdot C_3 + ... + (-1)^n (3n+1)C_n$

i) $2\cdot C_0 + 5\cdot C_1 + 8\cdot C_2 + ... + (3n+2)\cdot C_n \quad = (3n+4)\cdot 2^{n-1}$

Sol. $S = 2\cdot C_0 + 5\cdot C_1 + 8\cdot C_2 + ...$

$... + (3n-1)\cdot C_{n-1} + (3n+2)C_n$ అనుకోండి

$\because C_n = C_0, C_{n-1} = C_1 ...$

$S = (3n+2)C_0 + (3n-1)C_1 + (3n-4)C_2 + ... + 5C_{n-1} + 2\cdot C_n$ పై వాటిని కలుపగా

$2S = (3n+4)C_0 + (3n+4)C_1 + (3n+4)C_2 + ... + (3n+4)C_n$

$= (3n+4)(C_0 + C_1 + C_2 + ... + C_n) = (3n+4)2^n$

$\therefore S = (3n+4)\cdot 2^{n-1}$

ii) $C_0 - 4\cdot C_1 + 7\cdot C_2 - 10\cdot C_3 + ... = 0$

Sol. 1, 4, 7, 10 … are in A.P.

$T_{n+1} = a + nd = 1 + n(3) = 3n + 1$

$\therefore\ C_0 - 4\cdot C_1 + 7\cdot C_2 - 10\cdot C_3 + ...(n+1)\text{terms}$

$$= \sum_{r=0}^{n} (-1)^r (3r+1)C_r = \sum_{r=0}^{n} \left\{ (-1)^r (3r)C_r + (-1)^r C_r \right\}$$

$$= 3\cdot \sum_{r=0}^{n} (-1)^r r\cdot C_r + \sum_{r=0}^{n} (-1)^r \cdot C_r = 3(0) + 0 = 0$$

$\therefore C_0 - 4\cdot C_1 + 7\cdot C_2 - 10\cdot C_3 + ... = 0$

iii) $\dfrac{C_1}{2} + \dfrac{C_3}{4} + \dfrac{C_5}{6} + \dfrac{C_7}{8} + ... = \dfrac{2^n - 1}{n+1}$

$\dfrac{C_1}{2} + \dfrac{C_3}{4} + \dfrac{C_5}{6} + \dfrac{C_7}{8} + ...$

Sol.
$$= \frac{^nC_1}{2} + \frac{^nC_3}{4} + \frac{^nC_5}{6} + \frac{^nC_7}{8} + ...$$

$$= \frac{n}{2} + \frac{n(n-1)(n-2)}{4\times 3!} + \frac{n(n-1)(n-2)(n-3)(n-4)}{6\times 5!} + ...$$

$$= \frac{1}{n+1}\left[\frac{(n+1)n}{2!} + \frac{(n+1)n(n-1)(n-2)}{4!} + ... \quad \frac{(n+1)n(n-1)(n-2)(n-3)(n-4)}{6!} + ... \right]$$

$$= \frac{1}{n+1}\left[^{(n+1)}C_2 + {}^{(n+1)}C_4 + {}^{(n+1)}C_6 + ... \right]$$

$$= \frac{1}{n+1}\left[^{(n+1)}C_0 + {}^{(n+1)}C_2 + {}^{(n+1)}C_4 + ... - {}^{(n+1)}C_0 \right] = \frac{1}{n+1}\left[2^n - 1 \right] = \frac{2^n - 1}{n+1}$$

$$\therefore\ \frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \frac{C_7}{8} + ... = \frac{2^n - 1}{n+1}$$

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iv) $C_0+\frac{3}{2}C_1+\frac{9}{3}C_2+\frac{27}{4}C_3+\ldots+\frac{3^n}{n+1}C_n=\frac{4^{n+1}-1}{3(n+1)}$

Sol. $S=C_0+\frac{3}{2}C_1+\frac{3^2}{3}C_2+\frac{3^3}{4}C_3+\ldots+C_n\frac{3^n}{n+1}\ldots(1)$ అనుకోండి

$\Rightarrow 3S=C_0\cdot 3+\frac{3^2}{2}C_1+\frac{3^3}{3}C_2+\frac{3^4}{4}C_3+\ldots+C_n\frac{3^{n+1}}{n+1}\ldots(2$

$\Rightarrow (n+1)3\cdot S=(n+1)C_0\cdot 3+(n+1)C_1\cdot\frac{3^2}{2}+(n+1)C_2\cdot\frac{3^3}{3}\quad+(n+1)C_3\cdot\frac{3^4}{3}+\ldots+(n+1)C_n\cdot\frac{3^{n+1}}{n+1}$

$\Rightarrow (n+1)3\cdot S={}^{(n+1)}C_1\cdot 3+{}^{(n+1)}C_2\cdot 3^2+{}^{(n+1)}C_3\cdot 3^3+\ldots\ldots+{}^{(n+1)}C_{n+1}\cdot 3^{n+1}$

$=(1+3)^{n+1}-{}^{(n+1)}C_0=4^{n+1}-1$

$\therefore S=\frac{4^{n+1}-1}{3(n+1)}$

v) $C_0+2\cdot C_1+4\cdot C_2+8\cdot C_3+\ldots+2^n\cdot C_n=3^n$

Sol. L.H.S.= $C_0+2\cdot C_1+4\cdot C_2+8\cdot C_3+\ldots+2^n\cdot C_n$

$=C_0+C_1(2)+C_2(2^2)+C_3(2^3)+\ldots+C_n(2^n)$

$=(1+2)^n=3^n$

$[\ (1+x)^n=C_0+C_1\cdot x+C_2x^2+\ldots+C_nx^n\ ]$

9. ద్విపద సిద్ధాంతం ఉపయోగించి ప్రతి ధన పూర్ణాంకం n కు $50^n-49n-1$ ను 49^2 భాగిస్తుందని చూపండి.

Sol. $50^n-49n-1=(49+1)^n-49n-1$

$=[{}^nC_0(49)^n+{}^nC_1(49)^{n-1}+{}^nC_2(49)^{n-2}+\ldots+{}^nC_{n-2}(49)^2+{}^nC_{n-1}(49)+{}^nC_n(1)]-49n-1$

$=(49)^n+{}^nC_1(49)^{n-1}+{}^nC_2(49)^{n-2}+\ldots+{}^nC_{n-2}(49)^2+(n)(49)+1-49n-1$

$=49^2[(49)^{n-2}+{}^nC_1(49)^{n-3}+{}^nC_2(49)^{n-4}+\ldots+\ldots..+\ldots..+{}^nC_{n-2}]$

$=49^2$ [a positive integer]

$50^n-49n-1$ ను 49^2 భాగిస్తుంది.

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10 $(1 + x)^{21}$ విస్తరణ లో (2r + 4) (3r + 4) పదాల గుణకాలు సమానమయితే r విలువ కనుక్కోండి.

Sol. $(1 + x)^{21}$ లో $T_{2r+4} = {}^{21}C_{2r+3}(x)^{2r+3}$ ...(1)

$(1 + x)^{21}$ లో $T_{3r+4} = {}^{21}C_{3r+3}(x)^{3r+3}$ ...(2)

పదాల గుణకాలు సమానం

$\Rightarrow {}^{21}C_{2r+3} = {}^{21}C_{3r+3}$

$\Rightarrow 21 = (2r + 3) + (3r + 3)$ (or) $2r + 3 = 3r + 3$

$\Rightarrow 5r = 15 \Rightarrow r = 3$ (or) $r = 0$

$r = 0, 3.$

11. క్రింది సమాసాలకు ద్విపద విస్తరణ వ్యవస్థితం చే x ల సమితులను కనుక్కోండి.

(i) $(2 + 3x)^{-2/3}$ (ii) $(5 + x)^{3/2}$

Sol. (i) $(2 + 3x)^{-2/3} =$

$$\left[2\left(1+\frac{3}{2}x\right)\right]^{-2/3} = 2^{-2/3}\left(1+\frac{3}{2}x\right)^{-2/3}$$

$\therefore$ $(2 + 3x)^{-2/3}$ ద్విపద విస్తరణ వ్యవస్థితంకావాలంటే $\left|\frac{3}{2}x\right| < 1.$

i.e. $|x| < \frac{2}{3}$ i.e. $x \in \left(-\frac{2}{3}, \frac{2}{3}\right)$

ii) $$(5+x)^{3/2} = \left[5\left(1+\frac{x}{5}\right)\right]^{3/2} = 5^{3/2}\left(1+\frac{x}{5}\right)^{3/2}$$

$\therefore$ $(5 + x)^{3/2}$ ద్విపద విస్తరణ వ్యవస్థితంకావాలంటే $\left|\frac{x}{5}\right| < 1.$

i.e. $|x| < 5$

i.e. $x \in (-5, 5)$

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స్వల్ప సమాధాన ప్రశ్నలు

1. $(1 + x)^n$ విస్తరణ లో 3 వరస గుణకాలు36, 84, 126 అయితే n విలువ కనుగొనుము

Sol. $(1 + x)^n$ విస్తరణ లో 3 వరస గుణకాలు ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$ అనుకోండి. అప్పుడు

$${}^nC_{r-1} = 36, {}^nC_r = 84, {}^nC_{r+1} = 126$$

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{84}{36} \Rightarrow \frac{n-r+1}{r} = \frac{7}{3}$$

$$3n - 3r + 3 = 7r \Rightarrow 3n = 10r - 3$$

$$\Rightarrow \frac{3n+3}{10} = r \quad ...(1)$$

$$\Rightarrow \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{126}{84} \Rightarrow \frac{n-r}{r+1} = \frac{3}{2}$$

$$\Rightarrow 2n - 2r = 3r + 3 \Rightarrow 2n = 5r + 3 \quad ...(2)$$

$$\Rightarrow 2n = 5\left(\frac{3n+3}{10}\right) + 3$$, (1) నుండి.

$$\Rightarrow 2n = \frac{3n+3+6}{2} \Rightarrow 4n = 3n + 9 \Rightarrow n = 9$$

2. $(a + x)^n$ విస్తరణ లో 2, 3 , 4 పదాల గుణకాలు వరసగా240, 720, 1080 అయితే a, x, n.ల ను కనుక్కోండి.

Sol. $T_2 = 240 \Rightarrow {}^nC_1\, a^{n-1}\, x = 240 \quad ...(1)$

$T_3 = 720 \Rightarrow {}^nC_2\, a^{n-2}\, x^2 = 720 \quad ...(2)$

$T_4 = 1080 \Rightarrow {}^nC_3\, a^{n-3}\, x^3 = 1080 \quad ...(3)$

$$\frac{(2)}{(1)} \Rightarrow \frac{{}^nC_2 a^{n-2} x^2}{{}^nC_1 a^{n-1} x} = \frac{720}{240}$$

$$\Rightarrow \frac{n-1}{2}\frac{x}{a} = 3 \Rightarrow (n-1)x = 6a \; ...(4)$$

$$\frac{(3)}{(2)} \Rightarrow \frac{{}^nC_3 a^{n-3} x^3}{{}^nC_2 a^{n-2} x^2} = \frac{1080}{720} \Rightarrow \frac{n-2}{3}\frac{x}{a} = \frac{3}{2} \Rightarrow 2(n-2)x = 9a ...(5)$$

$$\frac{(4)}{(5)} \Rightarrow \frac{(n-1)x}{2(n-2)x} = \frac{6a}{9a} \Rightarrow \frac{n-1}{2n-4} = \frac{2}{3}$$

$$\Rightarrow 3n - 3 = 4n - 8 \Rightarrow n = 5$$

(4) నుండి, $(5 - 1)x = 6a \Rightarrow 4x = 6a$

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$\Rightarrow x = \frac{3}{2}a$

$x = \frac{3}{2}a, n = 5$ ల ను(1) లో ప్రతిక్షేపించగా

$^5C_1 \cdot a^4 \cdot \frac{3}{2}a = 240 \Rightarrow 5 \times \frac{3}{2}a^5 = 240$

$a^5 = \frac{480}{15} = 32 = 2^5$

$\therefore a = 2, x = \frac{3}{2}a = \frac{3}{2}(2) = 3 \quad \therefore a = 2, x = 3, n = 5$

3. $(1 + x)^{th}$ విస్తరణ లో r, (r+1), (r+2) పదాల గుణకాలు అంక శ్రేఢి లో ఉంటే $n^2 - (4r + 1)n + 4r^2 - 2 = 0$.అని చూపండి

Sol. T_r యొక్క గుణకం $= {}^nC_{r-1}$

T_{r+1} యొక్క గుణకం $= {}^nC_r$

T_{r+2} యొక్క గుణకం $= {}^nC_{r+1}$

${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$ లు అంక శ్రేఢి లోఉన్నాయి $\Rightarrow 2\,{}^nC_r = {}^nC_{r-1} + {}^nC_{r+1}$

$$\Rightarrow 2\frac{n!}{(n-r)!r!} = \frac{n!}{(n-r+1)!(r-1)!} + \frac{n!}{(n-r-1)!(r+1)!}$$

$$\Rightarrow \frac{2}{(n-r)r} = \frac{1}{(n-r+1)(n-r)} + \frac{1}{(r+1)r}$$

$$\Rightarrow \frac{1}{n-r}\left[\frac{2}{r} - \frac{1}{n-r+1}\right] = \frac{1}{(r+1)r}$$

$$\Rightarrow \frac{1}{n-r}\left[\frac{2n-2r+2-r}{r(n-r+1)}\right] = \frac{1}{r(r+1)}$$

$$\Rightarrow (2n-3r+2)(r+1) = (n-r)(n-r+1)$$

$$\Rightarrow 2nr + 2n - 3r^2 - 3r + 2r + 2 = n^2 - 2nr + r^2 + n - r$$

$$\Rightarrow n^2 - 4nr + 4r^2 - n - 2 = 0$$

$$\therefore n^2 - (4r+1)n + 4r^2 - 2 = 0$$

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4. $(x + a)^n$ ద్విపద విస్తరణ లో బేసి పదాల మొత్తం P, సరి పదాల మొత్తం Q అయితే

(i) $P^2 - Q^2 = (x^2 - a^2)^n$

(ii) $4PQ = (x + a)^{2n} - (x - a)^{2n}$ అని నిరూపించండి.

Sol. $(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + {}^nC_3x^{n-3}a^3 + \ldots + {}^nC_{n-1}xa^{n-1} + {}^nC_na^n$

$= ({}^nC_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4 + \ldots) + ({}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + {}^nC_5x^{n-5}a^5 + \ldots)$

$= P + Q$

$(x-a)^n = {}^nC_0x^n - {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 - {}^nC_3x^{n-3}a^3 + \ldots + {}^nC_n(-1)^na^n$

$= ({}^nC_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4 + \ldots) - ({}^nC_2x^{n-1}a + {}^nC_3x^{n-3}a^3 + {}^nC_5x^{n-5}a^5 + \ldots)$

$= P - Q$

i) $P^2 - Q^2 = (P+Q)(P-Q)$

$= (x + a)^n (x - a)^n$

$= [(x + a)(x - a)]^n = (x^2 - a^2)^n$

ii) $4PQ = (P + Q)^2 - (P - Q)^2$

$= [(x + a)^n]^2 - [(x - a)^n]^2$

$= (x + a)^{2n} - (x - a)^{2n}$

5. $(1 + x)^n$ విస్తరణ లో 4 వరుస పదాల గుణకాలు a_1, a_2, a_3, a_4 అయితే $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{2a_2}{a_2+a_3}$ అని చూపండి

Sol. $1 + x)^n$ విస్తరణ లో 4 వరుస పదాల గుణకాలు a_1, a_2, a_3, a_4 కావున

$a_1 = {}^nC_{r-1}$, $a_2 = {}^nC_r$, $a_3 = {}^nC_{r+1}$, $a_4 = {}^nC_{r+2}$

L.H.S. : $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{a_1}{a_1\left(1+\frac{a_2}{a_1}\right)} + \frac{a_3}{a_3\left(1+\frac{a_4}{a_3}\right)}$

$= \frac{1}{1+\frac{{}^nC_r}{{}^nC_{r-1}}} + \frac{1}{1+\frac{{}^nC_{r+2}}{{}^nC_{r+1}}} = \frac{1}{1+\frac{n-r+1}{r}} + \frac{1}{1+\frac{n-r-1}{r+2}}$

$= \frac{r}{n+1} + \frac{r+2}{r+2+n-r-1} = \frac{r+r+2}{n+1} = \frac{2(r+1)}{n+1}$

R.H.S : $\frac{2a_2}{a_2+a_3} = \frac{2a_2}{a_2\left(1+\frac{a_3}{a_2}\right)}$

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$$\frac{2}{1+\frac{{}^nC_{r+1}}{{}^nC_r}}=\frac{2}{1+\frac{n-r}{r+1}}=\frac{2(r+1)}{n+1}=\text{L.H.S}$$

$$\therefore \frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$$

6. $({}^{2n}C_0)^2-({}^{2n}C_1)^2+({}^{2n}C_2)^2-({}^{2n}C_3)^2+...+({}^{2n}C_{2n})^2=(-1)^n\ {}^{2n}C_n$ **అని చూపండి**

Sol. $(x+1)^{2n}={}^{2n}C_0x^{2n}+{}^{2n}C_1x^{2n-1}+{}^{2n}C_2x^{2n-2}+...+{}^{2n}C_{2n}$ ...(1)

$(x-1)^{2n}={}^{2n}C_0-{}^{2n}C_1x+{}^{2n}C_2x^2+...+{}^{2n}C_{2n}x^{2n}$ ...(2)

(1) , (2), లను గుణించగా

$({}^{2n}C_0x^{2n}+{}^{2n}C_1x^{2n-1}+{}^{2n}C_2x^{2n-2}+...+{}^{2n}C_{2n})({}^{2n}C_0-{}^{2n}C_1x+{}^{2n}C_2x^2+...+{}^{2n}C_{2n}x^{2n})$

$=(x+1)^{2n}(1-x)^{2n}=[(1+x)(1-x)]^{2n}$

$$=(1-x^2)^{2n}=\sum_{r=0}^{2n}{}^{2n}C_r(-x^2)^r$$

x^{2n} గుణకాలను పోల్చగా

$({}^{2n}C_0)^2-({}^{2n}C_1)^2+({}^{2n}C_2)^2-({}^{2n}C_3)^2+...+({}^{2n}C_{2n})^2=(-1)^n\ {}^{2n}C_n$

7. $(C_0+C_1)(C_1+C_2)(C_2+C_3)...(C_{n-1}+C_n)=\frac{(n+1)^n}{n!}\cdot C_0\cdot C_1\cdot C_2\cdot...C_n$ **అని చూపండి**

Sol. $(C_0+C_1)(C_1+C_2)(C_2+C_3)...(C_{n-1}+C_n)=$

$$=C_0\left(1+\frac{C_1}{C_0}\right)\cdot C_1\left(1+\frac{C_2}{C_1}\right)...C_{n-1}\left(1+\frac{C_n}{C_{n-1}}\right)$$

$$=\left(1+\frac{{}^nC_1}{{}^nC_0}\right)\left(1+\frac{{}^nC_2}{{}^nC_1}\right)......\left(1+\frac{{}^nC_n}{{}^nC_{n-1}}\right)C_0C_1C_2...C_{n-1}$$

$$=\left(1+\frac{n}{1}\right)\left(1+\frac{n-1}{2}\right)...\left(1+\frac{1}{n}\right)\ C_n\cdot C_1\cdot C_2\cdot...C_{n-1}[C_0=C_n]$$

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$$=\left(\frac{1+n}{1}\right)\left(\frac{1+n}{2}\right)\ldots\ldots\left(\frac{1+n}{n}\right)C_1\cdot C_2\cdot\ldots C_{n-1}\cdot C_n$$

$$=\frac{(1+n)^n}{n!}C_1C_2\ldots C_n$$

$$\therefore (C_0+C_1)(C_1+C_2)(C_2+C_3)\ldots(C_{n-1}+C_n)=\frac{(n+1)^n}{n!}\cdot C_0\cdot C_1\cdot C_2\cdot\ldots C_n$$

8. $(1+3x)^n\left(1+\frac{1}{3x}\right)^n$ **.విస్తరణ లో x లేని పదం (స్థిరపదం) కనుక్కోండి.**

Sol. $(1+3x)^n\left(1+\frac{1}{3x}\right)^n = (1+3x)^n\left(\frac{3x+1}{3x}\right)^n$

$$=\left(\frac{1}{3x}\right)^n(1+3x)^{2n}=\frac{1}{3^n\cdot x^n}\sum_{r=0}^{2n}({}^{2n}C_r)(3x)^r$$

$(1+3x)^n\left(1+\frac{1}{3x}\right)^n$ విస్తరణలలో x లేని పదం (స్థిరపదం) $==\frac{1}{3^n}({}^{2n}C_n)3^n={}^{2n}C_n$

9 . $(1+3x-2x^2)^{10}=a_0+a_1x+a_2x^2+\ldots+a_{20}x^{20}$ **అయితే**

i) $a_0+a_1+a_2+\ldots+a_{20}=2^{10}$

ii) $a_0-a_1+a_2-a_3+\ldots+a_{20}=4^{10}$ **అని చూపండి**

Sol. $(1+3x-2x^2)^{10}=a_0+a_1x+a_2x^2+\ldots+a_{20}x^{20}$

i) $x=1$ ప్రతిక్షేపించగా

$(1+3-2)^{10}=a_0+a_1+a_2+\ldots+a_{20}$

$\therefore a_0+a_1+a_2+\ldots+a_{20}=2^{10}$

ii) $x=-1$ ప్రతిక్షేపించగా

$(1-3-2)^{10}=a_0-a_1+a_2+\ldots+a_{20}$

$\therefore a_0-a_1+a_2-a_3+\ldots+a_{20}=(-4)^{10}=4^{10}$

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10. R, n లు ధన పూర్ణాంకాలు, n బేసి సంఖ్య, $0 < F < 1$, $(5\sqrt{5}+11)^n = R + F$ అయితే

i) R ఒక సరి పూర్ణాంకం ii) $(R + F)F = 4^n$ అని చూపండి

Sol. i) R, n లు ధన పూర్ణాంకాలు, $0 < F < 1$, $(5\sqrt{5}+11)^n = R + F$

$(5\sqrt{5}-11)^n = f$ అనుకోండి

అప్పుడు $11 < 5\sqrt{5} < 12 \Rightarrow 0 < 5\sqrt{5} - 11 < 1$

$\Rightarrow 0 < (5\sqrt{5}-11)^n < 1 \Rightarrow 0 < f < 1 \quad \Rightarrow 0 > -f > -1 \therefore -1 < -f < 0$

$R + F - f = (5\sqrt{5}+11)^n - (5\sqrt{5}-11)^n$

$= \left[{}^nC_0(5\sqrt{5})^n + {}^nC_1(5\sqrt{5})^{n-1}(11) + {}^nC_2(5\sqrt{5})^{n-2}(11)^2 + \ldots + {}^nC_n(11)^n \right]$

$- \left[{}^nC_0(5\sqrt{5})^n - {}^nC_1(5\sqrt{5})^{n-1}(11) + {}^nC_2(5\sqrt{5})^{n-2}(11)^2 - {}^nC_3(5\sqrt{5})^{n-3}(11)^3 + \ldots \right]$

$= 2\left[{}^nC_1(5\sqrt{5})^{n-1}(11) + \ldots + {}^nC_n(11)^n \right]$

$= 2k$, k ఒక స్థిరాంకం

$\therefore R + F - f$ సరి సంఖ్య

$\Rightarrow F - f$ పూర్ణాంకం

కానీ $0 < F < 1$, $-1 < -f < 0 \Rightarrow -1 < F - f < 1$

$\therefore F - f = 0 \Rightarrow F = f$

$\therefore$ R ఒక సరి పూర్ణాంకం

ii) $(R + F)F = (R + F)f, \quad \because F = f$

$= (5\sqrt{5}+11)^n(5\sqrt{5}-11)^n$

$= \left[(5\sqrt{5}+11)(5\sqrt{5}-11)\right]^n = (125-121)^n = 4^n$

$\therefore (R + F)F = 4^n$.

11. I, n లు ధన పూర్ణాంకాలు, $0 < f < 1$, $(7+4\sqrt{3})^n = I + f$ అయితే

(i) I బేసి పూర్ణాంకం

(ii) $(I + f)(I - f) = 1$. అని చూపండి.

Sol. I, n లు ధన పూర్ణాంకాలు

$(7+4\sqrt{3})^n = I + f$, $0 < f < 1$

$7 - 4\sqrt{3} = F$ అనుకోండి

Now $6 < 4\sqrt{3} < 7 \Rightarrow -6 > -4\sqrt{3} > -7$

$\Rightarrow 1 > 7 - 4\sqrt{3} > 0 \Rightarrow 0 < (7-4\sqrt{3})^n < 1$

$\therefore 0 < F < 1$

$1 + f + F = (7+4\sqrt{3})^n(7-4\sqrt{3})^n$

$= \left[{}^nC_0 7^n + {}^nC_1 7^{n-1}(4\sqrt{3}) + {}^nC_2 7^{n-2}(4\sqrt{3})^2 + \ldots + {}^nC_n(4\sqrt{3})^n \right]$

$= \left[{}^nC_0 7^n - {}^nC_1 7^{n-1}(4\sqrt{3}) + {}^nC_2 7^{n-2}(4\sqrt{3})^2 + \ldots + {}^nC_n(-4\sqrt{3})^n \right]$

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$$= 2\left[{}^{n}C_0 7^n + {}^{n}C_2 7^{n-2}(4\sqrt{3})^2......\right]$$

$= 2k$ k ఒక స్థిరాంకం

$\therefore\ 1 + f + F$ ఒక సరి పూర్ణాంకం

$\Rightarrow f + F$ పూర్ణాంకం .

$0 < f < 1\ ,\ 0 < F < 1 \Rightarrow f + F < 2$

$\therefore\ f + F = 1 \qquad ...(1)$

$\Rightarrow I + 1$ ఒక సరి పూర్ణాంకం

$\therefore$ I ఒక బేసి పూర్ణాంకం.

$(I + f)(I - f) = (I + f)F$, by (1)

$$= (7 + 4\sqrt{3})^n (7 - 4\sqrt{3})^n$$

$$= \left[(7 + 4\sqrt{3})(7 - 4\sqrt{3})\right]^n = (49 - 48)^n = 1$$

12. $\dfrac{1+2x}{(1-2x)^2}$ **.విస్తరణ లో x^{10} యొక్క గుణకాన్ని కనుక్కోండి.**

Sol. $\dfrac{1+2x}{(1-2x)^2} = (1+2x)(1-2x)^{-2}$

$$= (1+2x)[1 + 2(2x) + 3(2x)^2 + 4(2x)^3 + 5(2x)^4 + 6(2x)^5 + 7(2x)^6 + 8(2x)^7 + 9(2x)^8 + 10(2x)^9$$
$$+ 11(2x)^{10} + ... + (r+1)(2x)^r + ...]$$

$\therefore\ \dfrac{1+2x}{(1-2x)^2}$ విస్తరణ లో x^{10} యొక్క గుణకం $= (11)(2)^{10} + 10(2)(2^9) = 2^{10}(11+10) = 2 \times 1^{10}$

23. క్రింది అనంత శ్రేణుల మొత్తాలు కనుక్కోండి.

i) $1 + \dfrac{1}{3} + \dfrac{1\cdot 3}{3\cdot 6} + \dfrac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + ...$

Sol. ఇచ్చిన శ్రేణి ని క్రిందివిధంగా వ్రాయవచ్చు $S = 1 + \dfrac{1}{1}\cdot\dfrac{1}{3} + \dfrac{1\cdot 3}{1\cdot 2}\left(\dfrac{1}{3}\right)^2 + \dfrac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\left(\dfrac{1}{3}\right)^2 + ...$

కుడి వైపు ఉన్న శ్రేణి

$$1 + \frac{p}{1}\left(\frac{x}{q}\right) + \frac{p(p+q)}{1\cdot 2}\left(\frac{x}{q}\right)^2 + \frac{p(p+q)(p+2q)}{1\cdot 2\cdot 3}\left(\frac{x}{q}\right)^3 + ...$$

రూపంలో ఉంది. ఇక్కడ

$p = 1,\ q = 2,\ \dfrac{x}{q} = \dfrac{1}{3} \Rightarrow x = \dfrac{2}{3}$

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శ్రేణి మొత్తం $S = (1-x)^{-p/q} = \left(1-\frac{2}{3}\right)^{-1/2} = \left(\frac{1}{3}\right)^{-1/2} = \sqrt{3}$

ii) $\frac{3}{4}+\frac{3\cdot 5}{4\cdot 8}+\frac{3\cdot 5\cdot 7}{4\cdot 8\cdot 12}+\ldots\ldots$

Sol. $S = \frac{3}{4}+\frac{3\cdot 5}{4\cdot 8}+\frac{3\cdot 5\cdot 7}{4\cdot 8\cdot 12}+\ldots\ldots$

$$=\frac{3}{1}\cdot\frac{1}{4}+\frac{3\cdot 5}{1\cdot 2}\left(\frac{1}{4}\right)^2+\frac{3\cdot 5\cdot 7}{1\cdot 2\cdot 3}\left(\frac{1}{4}\right)^3+\ldots\ldots$$

$$\Rightarrow 1+S = 1+\frac{3}{1}\cdot\frac{1}{4}+\frac{3\cdot 5}{1\cdot 2}\left(\frac{1}{4}\right)^2+\ldots\ldots$$

(1 + S) ను

$$(1-x)^{-p/q} = 1+\frac{p}{1}\left(\frac{x}{q}\right)+\frac{p(p+q)}{1\cdot 2}\left(\frac{x}{q}\right)^2+\ldots$$ తో పోల్చగా

$p = 3, q = 2, \frac{x}{p} = \frac{1}{4} \Rightarrow x = \frac{1}{2}$

$$\therefore\ 1+S = (1-x)^{-p/q} = \left(1-\frac{1}{2}\right)^{-3/2}$$

$$=\left(\frac{1}{2}\right)^{-3/2} = 2^{3/2} = \sqrt{8}$$

$\therefore S = 2\sqrt{2}-1$

iii) $1-\frac{4}{5}+\frac{4\cdot 7}{5\cdot 10}-\frac{4\cdot 7\cdot 10}{5\cdot 10\cdot 15}+\ldots\ldots$

Sol. $S = 1-\frac{4}{5}+\frac{4\cdot 7}{5\cdot 10}-\frac{4\cdot 7\cdot 10}{5\cdot 10\cdot 15}+\ldots\ldots = 1+\frac{4}{1}\left(-\frac{1}{5}\right)+\frac{4\cdot 7}{1\cdot 2}\left(-\frac{1}{5}\right)^2+\frac{4\cdot 7\cdot 10}{1\cdot 2\cdot 3}\left(-\frac{1}{5}\right)^3+\ldots$

S ను $(1-x)^{-p/q} = 1+\frac{p}{1}\left(\frac{x}{q}\right)+\frac{p(p+q)}{1\cdot 2}\left(\frac{x}{q}\right)^2+\ldots$ తో పోల్చగా

$p = 4, q = 3, \frac{x}{q} = -\frac{1}{5} \Rightarrow x = \frac{-3}{5}$

$$\therefore S = (1-x)^{-p/q} = \left(1+\frac{3}{5}\right)^{-4/3} = \left(\frac{8}{5}\right)^{-4/3}$$

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$$=\left(\frac{5}{8}\right)^{4/3}=\frac{5^{4/3}}{8^{4/3}}=\frac{\sqrt[3]{5^4}}{2^4}=\frac{\sqrt[3]{625}}{16}$$

$$\therefore 1-\frac{4}{5}+\frac{4\cdot 7}{5\cdot 10}-\frac{4\cdot 7\cdot 10}{5\cdot 10\cdot 15}+....=\frac{\sqrt[3]{625}}{16}=\frac{5^{4/3}}{16}$$

iv) $\frac{3}{4\cdot 8}-\frac{3\cdot 5}{4\cdot 8\cdot 12}+\frac{3\cdot 5\cdot 7}{4\cdot 8\cdot 12\cdot 16}-....$

Sol. $S=\frac{3}{4\cdot 8}-\frac{3\cdot 5}{4\cdot 8\cdot 12}+\frac{3\cdot 5\cdot 7}{4\cdot 8\cdot 12\cdot 16}-....=\frac{1\cdot 3}{4\cdot 8}-\frac{1\cdot 3\cdot 5}{4\cdot 8\cdot 12}+\frac{1\cdot 3\cdot 5\cdot 7}{4\cdot 8\cdot 12\cdot 16}-....$

$1-\frac{1}{4}$ ఇరువైపులా కలుపగా

$$1-\frac{1}{4}+S=1-\frac{1}{4}+\frac{1\cdot 3}{4\cdot 8}-\frac{1\cdot 3\cdot 5}{4\cdot 8\cdot 12}+....$$

$$\Rightarrow \frac{3}{4}+S=1-\frac{1}{1}\cdot\frac{1}{4}+\frac{1\cdot 3}{1\cdot 2}\left(\frac{1}{4}\right)^2-\frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\left(\frac{1}{4}\right)^3+...$$

$$=1-\frac{p}{1}\frac{x}{q}+\frac{(p)(p+q)}{1\cdot 2}\left(\frac{x}{q}\right)^2-\frac{(p)(p+q)(p+2q)}{1\cdot 2\cdot 3}\left(\frac{x}{q}\right)^3+...$$

$p=1,\ q=2,\ \frac{x}{q}=\frac{1}{4}\Rightarrow x=\frac{1}{2}$

$$=(1+x)^{-p/q}=\left(1+\frac{1}{2}\right)^{-1/2}=\left(\frac{3}{2}\right)^{-1/2}=\sqrt{\frac{2}{3}}$$

$$\therefore\ S=\sqrt{\frac{2}{3}}-\frac{3}{4}$$

v). $\frac{7}{5}\left(1+\frac{1}{10^2}+\frac{1\cdot 3}{1\cdot 2}\frac{1}{10^4}+\frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\frac{1}{10^6}+...\right).$

Sol. $1+\frac{1}{10^2}+\frac{1\cdot 3}{1\cdot 2}\frac{1}{10^4}+\frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\frac{1}{10^6}+...=1+\frac{1}{1!}\left(\frac{1}{100}\right)+\frac{1\cdot 3}{2!}\left(\frac{1}{100}\right)^2+\frac{1\cdot 3\cdot 5}{3!}\left(\frac{1}{100}\right)^3+...$

$(1-x)^{-p/q}$ ను $=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2$ తో పోల్చగా

$p=1, p+q=3, q=2$

$$\frac{x}{q}=\frac{1}{100}\Rightarrow x=\frac{q}{100}=\frac{2}{100}=0.02$$

$$\therefore 1+\frac{1}{10^2}+\frac{1\cdot 3}{1\cdot 2}\cdot\frac{1}{10^4}+...=(1-x)^{-p/q}$$

$$=(1-0.02)^{-1/2}=(0.98)^{-1/2}=\left(\frac{49}{50}\right)^{-1/2}=\left(\frac{50}{49}\right)^{1/2}=\frac{5\sqrt{2}}{7}$$

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$$\therefore \frac{7}{5}\left[1+\frac{1}{10^2}+\frac{1\cdot3}{1\cdot2}\frac{1}{10^4}+\frac{1\cdot3\cdot5}{1\cdot2\cdot3}\frac{1}{10^6}+\ldots\right]=\frac{7}{5}\frac{5\sqrt{2}}{7}=\sqrt{2}$$

vi) $1+\frac{2}{3}\cdot\frac{1}{2}+\frac{2\cdot5}{3\cdot6}\left(\frac{1}{2}\right)^2+\frac{2\cdot5\cdot8}{3\cdot6\cdot9}\left(\frac{1}{2}\right)^3+\ldots\infty$

Sol.Let $S=1+\frac{2}{3}\cdot\frac{1}{2}+\frac{2\cdot5}{3\cdot6}\left(\frac{1}{2}\right)^2+\frac{2\cdot5\cdot8}{3\cdot6\cdot9}\left(\frac{1}{2}\right)^3+\ldots$

$$=1+\frac{2}{1}\cdot\frac{1}{6}+\frac{2\cdot5}{1\cdot2}\left(\frac{1}{6}\right)^2+\frac{2\cdot5\cdot8}{1\cdot2\cdot3}\left(\frac{1}{6}\right)^3+\ldots$$

$$\because 1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{1\cdot2}\left(\frac{x}{q}\right)^2+\ldots=(1-x)^{-p/q}$$

Here $p=2,\ q=3,\frac{x}{q}=\frac{1}{6}\Rightarrow x=\frac{3}{6}=\frac{1}{2}$

$$=(1-x)^{-p/q}=\left(1-\frac{1}{2}\right)^{-2/3}=2^{2/3}=\sqrt[3]{4}$$

vii. $\frac{3\cdot5}{5\cdot10}+\frac{3\cdot5\cdot7}{5\cdot10\cdot15}+\frac{3\cdot5\cdot7\cdot9}{5\cdot10\cdot15\cdot20}+\ldots\infty$

Sol. Let $S=\frac{3\cdot5}{5\cdot10}+\frac{3\cdot5\cdot7}{5\cdot10\cdot15}+\frac{3\cdot5\cdot7\cdot9}{5\cdot10\cdot15\cdot20}+\ldots$

$$==\frac{3\cdot5}{1\cdot2}\left(\frac{1}{5}\right)^2+\frac{3\cdot5\cdot7}{1\cdot2\cdot3}\left(\frac{1}{5}\right)^3+\frac{3\cdot5\cdot7\cdot9}{1\cdot2\cdot3\cdot4}\left(\frac{1}{5}\right)^4+\ldots$$

$1+3\cdot\frac{1}{5}$ ఇరువైపులా కలుపగా

$$1+\frac{3}{5}+S=1+\frac{3}{1}\left(\frac{1}{5}\right)+\frac{3\cdot5}{1\cdot2}\left(\frac{1}{5}\right)^2+\ldots..$$

$$=1+\frac{p}{1}\left(\frac{x}{q}\right)+\frac{p(p+q)}{1\cdot2}\left(\frac{x}{q}\right)^2+\ldots..\ p=3,q=2,\frac{x}{q}=\frac{1}{5}\Rightarrow x=\frac{2}{5}$$

$$=(1-x)^{-p/q}=\left(1-\frac{2}{5}\right)^{-3/2}=\left(\frac{5}{3}\right)^{3/2}=\frac{5\sqrt{5}}{3\sqrt{3}}$$

$$\Rightarrow\frac{8}{5}+S=\frac{5\sqrt{3}}{3\sqrt{3}}\Rightarrow S=\frac{5\sqrt{3}}{3\sqrt{3}}-\frac{8}{5}$$

$$\Rightarrow\frac{8}{5}+S=\frac{5\sqrt{3}}{3\sqrt{3}}\Rightarrow S=\frac{5\sqrt{3}}{3\sqrt{3}}-\frac{8}{5}$$

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viii. $\frac{5}{6\cdot 12}+\frac{5\cdot 8}{6\cdot 12\cdot 18}+\frac{5\cdot 8\cdot 11}{6\cdot 12\cdot 18\cdot 24}+\ldots\infty$

Sol. $S=\frac{5}{6\cdot 12}+\frac{5\cdot 8}{6\cdot 12\cdot 18}+\frac{5\cdot 8\cdot 11}{6\cdot 12\cdot 18\cdot 24}+\ldots$

$$\Rightarrow 2S=\frac{2\cdot 5}{1\cdot 2}\left(\frac{1}{6}\right)^2+\frac{2\cdot 5\cdot 8}{1\cdot 2\cdot 3}\left(\frac{1}{6}\right)^3+\frac{2\cdot 5\cdot 8\cdot 11}{1\cdot 2\cdot 3\cdot 4}\left(\frac{1}{6}\right)^4+\ldots\ldots$$

$$\Rightarrow 1+\frac{2}{1}\left(\frac{1}{6}\right)+2S=1+\frac{2}{1}\left(\frac{1}{6}\right)+\frac{2\cdot 5}{1\cdot 2}\left(\frac{1}{6}\right)^2+\frac{2\cdot 5\cdot 8}{1\cdot 2\cdot 3}\left(\frac{1}{6}\right)^3+\ldots\ldots$$

$$\Rightarrow \frac{4}{3}+2S=1+\frac{2}{1}\left(\frac{1}{6}\right)+\frac{2\cdot 5}{1\cdot 2}\left(\frac{1}{6}\right)^2+\frac{2\cdot 5\cdot 8}{1\cdot 2\cdot 3}\left(\frac{1}{6}\right)^3+\ldots\ldots$$

$\frac{4}{3}+2S$ ను

$(1-x)^{-p/q}$

$=1+\frac{p}{1}\left(\frac{x}{q}\right)+\frac{p(p+q)}{1\cdot 2}\left(\frac{x}{q}\right)^2+\ldots$ తో పోల్చగా

$p=2,\ q=3,\ \frac{x}{q}=\frac{1}{6}\Rightarrow x=\frac{q}{6}=\frac{3}{6}=\frac{1}{2}$

$$\therefore \frac{4}{3}+2S=(1-x)^{-p/q}=\left(1-\frac{1}{2}\right)^{-2/3}$$

$$=\left(\frac{1}{2}\right)^{-2/3}=(2)^{2/3}=\sqrt[3]{4}$$

$$\therefore 2S=\sqrt[3]{4}-\frac{4}{3}\Rightarrow S=\frac{\sqrt[3]{4}}{2}-\frac{2}{3}=\frac{1}{\sqrt[3]{2}}-\frac{2}{3}$$

$$\therefore \frac{5}{6\cdot 12}+\frac{5\cdot 8}{6\cdot 12\cdot 18}+\frac{5\cdot 8\cdot 11}{6\cdot 12\cdot 18\cdot 24}+\ldots=\frac{1}{\sqrt[3]{2}}-\frac{2}{3}$$

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13. $x = \frac{1\cdot3}{3\cdot6} + \frac{1\cdot3\cdot5}{3\cdot6\cdot9} + \frac{1\cdot3\cdot5\cdot7}{3\cdot6\cdot9\cdot12} + \ldots$ **ఐతే $9x^2 + 24x = 11$. ఐతే అని చూపుము**

1.

Sol. $x = \frac{1\cdot3}{3\cdot6} + \frac{1\cdot3\cdot5}{3\cdot6\cdot9} + \frac{1\cdot3\cdot5\cdot7}{3\cdot6\cdot9\cdot12} + \ldots$

$$= \frac{1\cdot3}{1\cdot2}\left(\frac{1}{3}\right)^2 + \frac{1\cdot3\cdot5}{1\cdot2\cdot3}\left(\frac{1}{3}\right)^2 + \ldots$$

$$= 1 + \frac{1}{1}\cdot\frac{1}{3} + \frac{1\cdot3}{1\cdot2}\left(\frac{1}{3}\right)^2 + \frac{1\cdot3\cdot5}{1\cdot2\cdot3}\left(\frac{1}{3}\right)^2 + \ldots - \left[1 + \frac{1}{3}\right]$$

$$p = 1,\ q = 2,\ \frac{x}{q} = \frac{1}{3} \Rightarrow x = \frac{2}{3}$$

$$= (1-x)^{-p/q} - \frac{4}{3} = \left(1 - \frac{2}{3}\right)^{-1/2} - \frac{4}{3}$$

$$= \left(\frac{1}{3}\right)^{-1/2} - \frac{4}{3} = \sqrt{3} - \frac{4}{3}$$

$$\Rightarrow 3x + 4 = 3\sqrt{3}$$

ఇరువైపులా వర్గం చేయగా

$$(3x+4)^2 = (3\sqrt{3})^2 \Rightarrow 9x^2 + 24x + 16 = 27$$

$$\Rightarrow 9x^2 + 24x = 11$$

14. $x = \frac{5}{(2!)\cdot3} + \frac{5\cdot7}{(3!)\cdot3^2} + \frac{5\cdot7\cdot9}{(4!)3^3} + \ldots$, **ఐతే $x^2 + 4x$. విలవ కనుక్కోండి.**

Sol. $x = \frac{5}{(2!)\cdot3} + \frac{5\cdot7}{(3!)\cdot3^2} + \frac{5\cdot7\cdot9}{(4!)3^3} + \ldots$

$$= \frac{3\cdot5}{2!3^2} + \frac{3\cdot5\cdot7}{3!3^3} + \frac{3\cdot5\cdot7\cdot9}{4!3^4} + \ldots$$

$$= \frac{3\cdot5}{2!}\left(\frac{1}{3}\right)^2 + \frac{3\cdot5\cdot7}{3!}\left(\frac{1}{3}\right)^3 + \frac{3\cdot5\cdot7\cdot9}{4!}\left(\frac{1}{3}\right)^4 \ldots = 1 + \frac{3}{1}\left(\frac{1}{3}\right) + x$$

$$= 1 + \frac{3}{1}\left(\frac{1}{3}\right) + \frac{3\cdot5}{2!}\left(\frac{1}{3}\right)^2 + \frac{3\cdot5\cdot7}{3!}\left(\frac{1}{3}\right)^3 + \ldots$$

$$\Rightarrow 2 + x = 1 + \frac{3}{1}\left(\frac{1}{3}\right) + \frac{3\cdot5}{2!}\left(\frac{1}{3}\right)^2 + \ldots$$

$x + 2$ ను $(1-y)^{-p/q}$ తో పోల్చగా

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$$=1+\frac{p}{1}\left(\frac{y}{q}\right)+\frac{p(p+q)}{1\cdot 2}\left(\frac{y}{q}\right)^2+\ldots$$

$$p = 3,\ q = 2,\ \frac{y}{q}=\frac{1}{3}\Rightarrow y=\frac{q}{3}=\frac{2}{3}$$

$$\therefore x+2=(1-y)^{-p/q}=\left(1-\frac{2}{3}\right)^{-3/2}=\left(\frac{1}{3}\right)^{-3/2}=(3)^{3/2}=\sqrt{27}$$ ఇరుపైపులా వర్గం చేయగా

$$x^2+4x+4=27 \quad \Rightarrow x^2+4x=23$$

15. $x=\frac{1}{5}+\frac{1\cdot 3}{5\cdot 10}+\frac{1\cdot 3\cdot 5}{5\cdot 10\cdot 15}+\ldots\infty$ **ఐతే $3x^2 + 6x$ ను కనుగొనుము.**

Sol. $$x=\frac{1}{5}+\frac{1\cdot 3}{5\cdot 10}+\frac{1\cdot 3\cdot 5}{5\cdot 10\cdot 15}+\ldots\ldots$$

$$=\frac{1}{5}+\frac{1\cdot 3}{1\cdot 2}\left(\frac{1}{5}\right)^2+\frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\left(\frac{1}{5}\right)^3+\ldots\ldots$$

$$\Rightarrow 1+x=1+1\cdot\frac{1}{5}+\frac{1\cdot 3}{1\cdot 2}\left(\frac{1}{5}\right)^2+\frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3}\left(\frac{1}{5}\right)^3+\ldots=1+\frac{p}{1!}\frac{1}{5}+\frac{p(p+q)}{2!}\left(\frac{1}{5}\right)^2$$

$$+\frac{p(p+q)(p+2q)}{3!}\left(\frac{1}{5}\right)^3+\ldots.=(1-x)^{-p/q}$$

$$p=1,q=2,\frac{x}{q}=\frac{1}{5}\Rightarrow x=\frac{2}{5}$$

$$=\left(1-\frac{2}{5}\right)^{-1/2}=\left(\frac{3}{5}\right)^{-1/2}=\sqrt{\frac{5}{3}}$$

$$\Rightarrow 1+x=\sqrt{\frac{5}{3}}\Rightarrow 3(1+x)^2=5$$

$$\Rightarrow 3x^2+6x+3=5\Rightarrow 3x^2+6x=2$$

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16. $1+\frac{x}{2}+\frac{x(x-1)}{2\cdot4}+\frac{x(x-1)(x-2)}{2\cdot4\cdot6}+\ldots=1+\frac{x}{3}+\frac{x(x+1)}{3\cdot6}+\frac{x(x+1)(x+2)}{3\cdot6\cdot9}+\ldots$ **అని చూపండి**

Sol. L.H.S. $=1+\frac{x}{2}+\frac{x(x-1)}{2\cdot4}+\frac{x(x-1)(x-2)}{2\cdot4\cdot6}+\ldots$

$$=1+x\left(\frac{1}{2}\right)\frac{(x)(x-1)}{1\cdot2}\left(\frac{1}{2}\right)^2+\frac{x(x-1)(x-2)}{1\cdot2\cdot3}\left(\frac{1}{2}\right)^3+\ldots$$

$(1+x)^n=1+{}^nC_1\cdot x+{}^nC_2x^2+\ldots\ldots.\quad =1+\frac{n}{1!}\cdot x+\frac{n(n-1)}{1\cdot2}x^2+\ldots.$ తో పోల్చగా

$$x=\frac{1}{2},n=x=\left(1+\frac{1}{2}\right)^x=\left(\frac{3}{2}\right)^x$$

R.H.S. $=1+\frac{x}{3}+\frac{x(x+1)}{3\cdot6}+\frac{x(x+1)(x+2)}{3\cdot6\cdot9}+\ldots$

$$=1+\frac{x}{1}\left(\frac{x}{3}\right)+\frac{(x)(x+1)}{1\cdot2}\left(\frac{1}{3}\right)^2+\frac{(x)(x+1)(x+2)}{1\cdot2\cdot3}\left(\frac{1}{3}\right)^3+\ldots$$

$(1-x)^{-n}\quad =1+n(x)+\frac{n(n+1)}{1\cdot2}x^2+\ldots\ldots$ తో పోల్చగా

$$x=\frac{1}{3},n=x$$

$$=\left(1-\frac{1}{3}\right)^{-x}=\left(\frac{2}{3}\right)^{-x}=\left(\frac{3}{2}\right)^x$$

$\therefore$ L.H.S. = R.H.S.

17. క్రింది వానిని నిరూపించండి

i) $C_0+3C_1+3^2C_2+\ldots+3^nC_n=4^n$

ii) $\frac{C_1}{C_0}+2\cdot\frac{C_2}{C_1}+3\cdot\frac{C_3}{C_2}+\ldots+n\frac{C_n}{C_{n-1}}=\frac{n(n+1)}{2}$

Sol. (i)

$(1+x)^n=C_0+C_1x+C_2x^2+\ldots+C_nx^n$

$x=3$ ప్రతిక్షేపించగా

$(1+3)^n=C_0+C_1\cdot3+C_23^2+\ldots+C_n3^n$

$\therefore C_0+3C_1+3^2C_2+\ldots+3^nC_n=4^n$

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(ii) $\frac{C_1}{C_0}+2\cdot\frac{C_2}{C_1}+3\cdot\frac{C_3}{C_2}+\ldots+n\frac{C_n}{C_{n-1}}$

$=\frac{{}^nC_1}{{}^nC_0}+2\left(\frac{{}^nC_2}{{}^nC_1}\right)+3\left(\frac{{}^nC_3}{{}^nC_2}\right)+\ldots+n\left(\frac{{}^nC_n}{{}^nC_1}\right)$

$=\frac{n}{1}+2\frac{n-1}{2}+3\frac{n-2}{3}+\ldots+n\frac{1}{n}$

$=n+(n-1)+(n-2)+\ldots+3+2+1$

$=1+2+3+\ldots+n=\frac{n(n+1)}{2}$

18. n = 0, 1, 2, 3, … n ఐతే $C_0\cdot C_r+C_1\cdot C_{r+1}+C_2\cdot C_{r+2}+\ldots+C_{n-r}\cdot C_n={}^{2n}C_{n+r}$ **అని చూపి దీని ద్వారా క్రింది వానిని రాబట్టండి.**

i) $C_0^2+C_1^2+C_2^2+\ldots+C_n^2={}^{2n}C_n$

ii) $C_0\cdot C_1+C_1\cdot C_2+C_2\cdot C_3+\ldots+C_{n-1}C_n={}^{2n}C_{n+1}$

Sol.

$(1+x)^n=C_0+C_1x+C_2x^2+\ldots+C_nx^n\ \ldots(1)$

x కు బదులుగ 1/x వ్రాయగా $\left(1+\frac{1}{x}\right)^n=C_0+\frac{C_1}{x}+\frac{C_2}{x^2}+\ldots+\frac{C_n}{x^n}\ldots(2)$

(1) , (2) ల నుండి

$\left(1+\frac{1}{x}\right)^n(1+x)^n=\left(C_0+\frac{C_1}{x}+\frac{C_2}{x^2}+\ldots+\frac{C_n}{x^n}\right)$

$(C_0+C_1x+C_2x^2+\ldots+C_nx^n)\ldots(3)$

(3) నుండి R.H.S. లో x^r యొక్క గుణకం

$=C_0C_r+C_1C_{r+1}+C_2C_{r+2}+\ldots+C_{n-r}C_n$

(3) నుండి L.H.S.. లో x^r యొక్క

$=\frac{(1+x)^{2n}}{x^n}$ విస్తరణ లో x^r యొక్క గుణకం

$=$ $(1+x)^{2n}$ విస్తరణ లో x^{n+r} యొక్క గుణకం

$={}^{2n}C_{n+r}$

(3) , (4), ల నుండి

$C_0\cdot C_1+C_1\cdot C_2+C_2\cdot C_3+\ldots+C_{n-1}C_n={}^{2n}C_{n+1}$

i) (i), లో $r=0$ ప్రతిక్షేపించగా

$C_0^2+C_1^2+C_2^2+\ldots+C_n^2={}^{2n}C_n$

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ii) (i) లో $r = 1$ ప్రతిక్షేపించగా $C_0 \cdot C_1 + C_1 \cdot C_2 + C_2 \cdot C_3 + \ldots + C_{n-1}C_n = {}^{2n}C_{n+1}$

30. $3 \cdot C_0^2 + 7 \cdot C_1^2 + 11 \cdot C_2^2 + \ldots + (4n+3)C_n^2 = (2n+3)\,{}^{2n}C_n$ **అని నిరూపించండి**

Sol. $S = 3 \cdot C_0^2 + 7 \cdot C_1^2 + 11 \cdot C_2^2 + \ldots$

$+(4n-1)C_{n-1}^2 + (4n+3)C_n^2 \quad \ldots(1)$

పై సమికరణం లోని పదాలను వ్యతిరేక వరుసక్రమంలో వ్రాయగా

$S = (4n+3)C_0^2 + (4n-1)C_1^2 + \ldots + 7C_{n-1}^2 + 3C_n^2 \ldots\ldots(2)$

(1) + (2)

$2S = (4n+6)C_0^2 + (4n+6)C_1^2 + \ldots + (4n+6)C_n^2$

$\Rightarrow 2S = (4n+6)(C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2)$

$= 2(2n+3)\,{}^{2n}C_n$

$\therefore S = (2n+3)\,{}^{2n}C_n$

19. n ధ న పూర్ణాంకం అయితే $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$. **అని నిరూపించండి**

Sol. $S = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1}$

$S = {}^nC_0 + \frac{1}{2} \cdot {}^nC_1 + \frac{1}{3} \cdot {}^nC_2 + \ldots + \frac{1}{n+1} \cdot {}^nC_n$

$\therefore (n+1)S = \frac{n+1}{1} \cdot {}^nC_0 + \frac{n+1}{2} \cdot {}^nC_1$

$+ \frac{n+1}{3} \cdot {}^nC_2 + \ldots + \frac{n+1}{n+1} \cdot {}^nC_n$

$\therefore (n+1)S = {}^{(n+1)}C_1 + {}^{(n+1)}C_2 + {}^{(n+1)}C_3 + \ldots + {}^{(n+1)}C_{n+1}$ $\quad S = \frac{2^{n+1}-1}{n+1}$

$\left(\because \frac{n+1}{r+1} \cdot {}^nC_r = {}^{n+1}C_{r+1} \right)$

$= 2^{n+1} - 1$

$\therefore C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$

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20. **n ధన పూర్ణాంకం x శూన్యేతర వాస్తవసంఖ్య అయితే**

$$C_0 + C_1\frac{x}{2} + C_2\cdot\frac{x^2}{3} + C_3\cdot\frac{x^3}{4} + \ldots + C_n\cdot\frac{x^n}{n+1} = \frac{(1+x)^{n+1}-1}{(n+1)x}$$ అని నిరూపించండి

Sol. $$C_0 + C_1\frac{x}{2} + C_2\cdot\frac{x^2}{3} + C_3\cdot\frac{x^3}{4} + \ldots + C_n\cdot\frac{x^n}{n+1}$$

$$= {}^nC_0 + \frac{1}{2}\,{}^nC_1x + \frac{1}{3}\,{}^nC_2x^2 + \ldots + \frac{1}{n+1}\,{}^nC_nx^n$$

$$= 1 + \frac{n}{1!}\frac{x}{2} + \frac{n(n-1)}{2!}\frac{x^2}{3} + \ldots\ldots$$

$$= 1 + \frac{n}{2!}x^1 + \frac{n(n-1)}{3!}x^2 + \ldots\ldots$$

$$= \frac{1}{(n+1)x}\left[\frac{(n+1)x^1}{1!} + \frac{(n+1)n}{2!}x^2 + \frac{(n+1)n(n-1)}{3!}x^3 + \ldots\right]$$

$$= \frac{1}{(n+1)x}\left[{}^{(n+1)}C_1x + {}^{(n+1)}C_2x^2 + {}^{(n+1)}C_3x^3 + \ldots\right]$$

$$= \frac{1}{(n+1)x}\left[1 + {}^{n+1}C_1x + {}^{n+1}C_2x^2 + \ldots + {}^{n+1}C_{n+1}x^{n+1} - 1\right]$$

$$= \frac{1}{(n+1)x}\left[(1+x)^{n+1} - 1\right]$$

$$C_0 + C_1\frac{x}{2} + C_2\cdot\frac{x^2}{3} + C_3\cdot\frac{x^3}{4} + \ldots + C_n\cdot\frac{x^n}{n+1} = \frac{(1+x)^{n+1}-1}{(n+1)x}$$

21. $$C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots + (-1)^nC_n^2 = \begin{cases}(-1)^{n/2}\,{}^nC_{n/2}, & \text{if n is even}\\ 0, & \text{if n is odd}\end{cases}$$ అని నిరూపించండి

Sol. $$(1-x)^n\left(1+\frac{1}{x}\right)^n$$

$$= (C_0 - C_1x + C_2x^2 - C_3x^3 + \ldots + (-1)^n\cdot C_nx^n)\left(C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \ldots + \frac{C_n}{x^n}\right) \quad \ldots(1)$$

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R.H.S. లో x లేని పదం (స్థిరపదం)

$$= C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots + (-1)^n C_n^2$$

L.H.S. లో x లేని పదం (స్థిరపదం)

$$\text{L.H.S. of (1)} = (1-x)^n\left(1+\frac{1}{x}\right)^n$$

$$= (1-x)^n\left(\frac{1+x}{x}\right)^n = \frac{(1-x^2)^n}{x^n}$$

$$= \sum_{r=0}^{n} {}^nC_r(-x^2)^r \qquad \ldots(2)$$

n ధన పూర్ణాంకాలు అయితే

n = 2k.

(2) నుండి

$$(1-x)^n\left(1+\frac{1}{x}\right)^n = \frac{\sum_{r=0}^{n} {}^nC_r(-x^2)^r}{x^n}$$

$$= \frac{\sum_{r=0}^{2k} {}^{2k}C_r(-x^2)^r}{x^{2k}} = \sum_{r=0}^{2k} {}^{2k}C_r(-1)^r x^{2r-2k} \ldots(3)$$

(3), నుండి x లేని పదంకావాలంటే

$2r - 2k = 0 \Rightarrow r = k$

x లేని పదం (స్థిరపదం) $(1-x)^n\left(1+\frac{1}{x}\right)^n$ is ${}^{2k}C_k(-1)^k = {}^nC_{(n/2)}(-1)^{n/2}$

n బేసి పూర్ణాంకం అయితే **:** (2) లోని పదాలు x యొక్క సరిఘాత పదాలు కావున x లేని పదం సున్నా అవుతుంది x in $(1-x)^n\left(1+\frac{1}{x}\right)^n$ విస్తరణ లో x లేని పదం సున్నా.

$$\therefore\ C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots + (-1)^n C_n^2$$

$$= \begin{cases} (-1)^{n/2}\, {}^nC_{n/2}, & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$$

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