# అవిచ్ఛిన్నత

**1.** $\underset{x \to a}{Lt} \dfrac{x^2 - a^2}{x - a}$

**సాధన :** ఇచ్చిన అవధి

$$= \underset{x \to a}{Lt} \frac{x^2 - a^2}{x - a} = \underset{x \to a}{Lt} \frac{(x+a)(x-a)}{x - a}$$

$$= \underset{x \to a}{Lt} (x + a) \quad = a + a \quad = 2a$$

**2.** $f(x) = \begin{cases} 1 - x & \text{if} \quad x \le 1 \\ 1 + x & \text{if} \quad x > 1 \end{cases}$ ప్రమేయానికి అవధి **a=1** వద్ద కనుగొనుము

**Sol :** $Left \lim it\ at$ x=1 is $\underset{x \to 1-}{Lt} 1 - x = 1 - 1 = 0$

$Right$ limit at x=1 is $\underset{x \to 1+}{Lt} (1 + x) = 1 + 1 = 2$

$$\underset{x \to 1-}{Lt} f(x) \ne \underset{x \to 1+}{Lt} f(x)$$

$\therefore \underset{x \to 1}{Lt} f(x)$ వ్యవస్థితంకాదు

**3.** $f(x) = \begin{cases} x + 2 & \text{if} \quad -1 < x \le 3 \\ x^2 & \text{if} \quad 3 < x < 5 \end{cases}$. ప్రమేయానికి $a = 3$ వద్ద అవధి కనుగొనుము

**Sol :** $L.L = \underset{x \to 3-}{Lt} (x + 2) = 3 + 2 = 5$

$$R.L = \underset{x \to 3+}{Lt} f(x) = \underset{x \to 3+}{Lt} x^2 = 3^2 = 9$$

$$\underset{x \to 3-}{Lt} f(x) \ne \underset{x \to 3+}{Lt} f(x)$$

$\underset{x\to 3}{Lt} f(x)$ వ్యవస్థితం కాదు

**4.** $f(x)=\begin{cases} 1 & \text{if} \quad x<0 \\ 2x+1 & \text{if} \quad 0\le x<1 \\ 3x & \text{if} \quad x>1 \end{cases}$ . ప్రమేయానికి $a=1$ వద్ద అవధి కనుగొనుము

సాధన :

$$L.L = \underset{x\to 1-}{Lt} f(x) = \underset{x\to 1-}{Lt} 2x+1 = 2(1)+1 = 3$$

$$R.L = \underset{x\to 1+}{Lt} f(x) = \underset{x\to 1+}{Lt} 3x = 3(1) = 3$$

$$\underset{x\to 1-}{Lt} f(x) = \underset{x\to 1+}{Lt} f(x) = 3$$

$$\therefore \underset{x\to 1}{Lt} f(x) = 3$$

**5.** $\underset{x\to 2-}{Lt} \dfrac{|x-2|}{x-2} = -1$ అని చూపుము

**Sol :** $x \to 2- \Rightarrow x<2 \Rightarrow \quad x-2<0$

**Then,** $|x-2| = -(x-2)$

$$\underset{x\to 2-}{Lt} \frac{|x-2|}{x-2} = \underset{x\to 2-}{Lt} \frac{-(x-2)}{(x-2)} = -1$$

**6.** $\underset{x\to 0+}{Lt}\left(\dfrac{2|x|}{x}+x+1\right) = 3.$ అని చూపుము

సాధన : $x \to 0+ \Rightarrow x>0 \Rightarrow |x| = x$

$$\therefore \underset{x\to 0+}{Lt}\left(\frac{2|x|}{x}+x+1\right)$$

$$= \underset{x\to 0+}{Lt}\left(\frac{2x}{x}+x+1\right) = \underset{x\to 0+}{Lt}(2+x+1) = \underset{x\to 0+}{Lt}(2+0+1) = 3$$

**7.** $\underset{x\to 2+}{Lt}([x]+x)$ మరియు $\underset{x\to 2-}{Lt}([x]+x).$ లను కనుగొనుము

సాధన: $\underset{x\to 2^+}{Lt}\{[x]+x\}. = \underset{h\to 0+}{Lt}\{[2+h]+(2+h)\}$

$= [2+0]+2+0 \quad \left(\because \left[2^+\right]=2\right)$

$= 2+2 = 4$

$\underset{x\to 2-}{Lt}\{[x]+x\} = \left[2^-\right]+2$

$= 1+2 = 3$

**8.** $\underset{x\to \frac{\pi}{2}}{Lt}\dfrac{\cos x}{\left[x-\frac{\pi}{2}\right]}$ ను కనుగొనుము

సాధన: $y = x-\frac{\pi}{2}$ అనుకొనుము

అప్పుడు $x\to\frac{\pi}{2}, y\to 0$ **and** $x = y+\frac{\pi}{2}$

$$\therefore \underset{x\to\frac{\pi}{2}}{Lt}\frac{\cos x}{x-\frac{\pi}{2}} = \underset{y\to 0}{Lt}\frac{\cos\left(\frac{\pi}{2}+y\right)}{y}$$

$$\underset{y\to 0}{Lt}\frac{-\sin y}{y} = -1$$

**9.** $\underset{x \to 0}{Lt} \dfrac{\sin ax}{x \cos x}$

సాధన: $\underset{x \to 0}{Lt} \dfrac{\sin ax}{x.\cos x} = \dfrac{\underset{x \to 0}{Lt} \dfrac{\sin ax}{ax}.a}{\underset{x \to 0}{Lt} \cos x} = \dfrac{a}{1} = a$

**10.** $\underset{x \to 1}{Lt} \dfrac{\sin(x-1)}{(x^2-1)}$

సాధన. $\underset{x \to 1}{Lt} \dfrac{\sin(x-1)}{x^2-1} = \underset{x \to 1}{Lt} \dfrac{\sin(x-1)}{(x-1)(x+1)} = \underset{x \to 1}{Lt} \dfrac{\sin(x-1)}{(x-1)}.\underset{x \to 1}{Lt} \dfrac{1}{x+1}$

$y = x - 1$ అనుకొనుము, అప్పుడూ

$x \to 1,\ y \to 0$

$= \underset{y \to 0}{Lt} \dfrac{\sin y}{y}.\dfrac{1}{1+1} = 1.\dfrac{1}{2} = \dfrac{1}{2}$

**11.** $\underset{x \to 0}{Lt} \dfrac{\sin(a+bx) - \sin(a-bx)}{x}$

సాధన: $\underset{x \to 0}{Lt} \dfrac{\sin(a+bx) - \sin(a-bx)}{x}$

$= \underset{x \to 0}{Lt} \dfrac{2\cos\left(\dfrac{a+bx+a-bx}{2}\right).\sin\left(\dfrac{a+bx-a+bx}{2}\right)}{x} = \underset{x \to 1}{Lt}\ 2\cos a.\ \underset{x \to 0}{Lt} \dfrac{\sin bx}{bx}.b$

$= 2\cos a.\ \ b = 2b.\cos a.$

**12.** $\underset{x \to a}{Lt} \dfrac{\tan(x-a)}{x^2-a^2} (a \neq 0)$

సాధన:

$\underset{x \to a}{Lt} \dfrac{\tan(x-a)}{x^2-a^2} = \underset{x \to a}{Lt} \dfrac{\tan(x-a)}{(x-a)(x+a)}$

$$= \underset{x \to a}{Lt} \frac{\tan(x-a)}{(x-a)} . \underset{x \to a}{Lt} \frac{1}{(x+a)}$$

**In the first limit Put** $x - a = h$ **so that as** $x \to a,\ h \to 0$

**G. L** $= \underset{h \to 0}{Lt} \frac{\tan h}{h} . \left(\frac{1}{a+a}\right)$

$$= 1 . \frac{1}{2a} = \frac{1}{2a}$$

**13.** $\underset{x \to a}{Lt} \left[\frac{x \sin a - a \sin x}{x - a}\right]$

సాధన : $\underset{x \to a}{Lt} \frac{x \sin a - a \sin x}{x - a}$

$$= \underset{x \to a}{Lt} \frac{(x \sin a - a \sin a) - (a \sin x - a \sin a)}{(x-a)}$$

$$= \underset{x \to a}{Lt} \frac{(x-a)\sin a - a(\sin x - \sin a)}{x-a} = \underset{x \to a}{Lt} \frac{(x-a)\sin a}{x-a} - \underset{x \to a}{Lt}\, a\left(\frac{\sin x - \sin a}{x-a}\right)$$

$$= \sin a - a . \underset{x \to a}{Lt} \frac{2\cos\frac{x+a}{2} . \sin\frac{x-a}{2}}{x-a}$$

$$= \sin a - a . \underset{x \to a}{Lt} \frac{\cos(x+a)}{2} \underset{x \to a}{Lt} \frac{\sin\left(\frac{x-a}{2}\right)}{\left(\frac{x-a}{2}\right)}$$

$$= \sin a - a \cos a - 1 = \sin a - a \cos a$$

**14.** $\underset{x\to 0}{Lt}\left[\dfrac{\cos ax-\cos bx}{x^2}\right]$ కనుగొనుము

సాధన: $\underset{x\to 0}{Lt}\dfrac{\cos ax-\cos bx}{x^2}$

$$=\underset{x\to 0}{Lt}\frac{2\sin\frac{(a+b)x}{2}.\sin\frac{(b-a)x}{2}}{x^2}$$

$$=2.\underset{x\to 0}{Lt}\frac{\sin(b+a)\frac{x}{2}}{x}.\underset{x\to 0}{Lt}\frac{\sin(b-a)\frac{x}{2}}{x}$$

$$=2.\underset{x\to 0}{Lt}\frac{\sin(b+a)\frac{x}{2}}{(b+a)\frac{x}{2}}\times\frac{(b+a)}{2}.\underset{x\to 0}{Lt}\frac{\sin(b-a)\frac{x}{2}}{(b-a)\frac{x}{2}}\times\frac{(b-a)}{2}$$

$$=2.\left(\frac{b+a}{2}\right)\left(\frac{b-a}{2}\right)\quad=\frac{1}{2}\left(b^2-a^2\right)$$

**15.** $\underset{x\to 0}{Lt}\left[\dfrac{(1+x)^{\frac{1}{8}}-(1-x)^{\frac{1}{8}}}{x}\right]$ కనుగొనుము

సాధన: $\underset{x\to 0}{Lt}\dfrac{(1+x)^{\frac{1}{8}}-(1-x)^{\frac{1}{8}}}{x}=\underset{x\to 0}{Lt}\dfrac{(1+x)^{\frac{1}{8}}-1+1-(1-x)^{\frac{1}{8}}}{x}$

$$=\underset{(1+x)\to 1}{Lt}\frac{(1+x)^{\frac{1}{8}}-1}{(1+x)-1}+\underset{(1-x)\to 1}{Lt}\frac{(1-x)^{\frac{1}{8}}-1^{8}}{(1-x)-1}$$

$$=\frac{1}{8}1^{-7/8}+\frac{1}{8}1^{-7/8}=\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$$

**16.** $\underset{x\to 0}{Lt}\left[\dfrac{3^x-1}{\sqrt{1+x}-1}\right]$ కనుగొనుము

సాధన: $\underset{x\to 0}{Lt}\dfrac{3^x-1}{\sqrt{1+x}-1}=\underset{x\to 0}{Lt}\dfrac{3^x-1}{\sqrt{1+x}-1}\times\dfrac{\sqrt{1+x}+1}{\sqrt{1+x}+1}(rationalise\ \text{Dr.})$

$$=\underset{x\to 0}{Lt}\frac{(3^x-1)(\sqrt{1+x}+1)}{1+x-1}=\underset{x\to 0}{Lt}\frac{3^x-1}{x}.\underset{x\to 0}{Lt}(\sqrt{1+x}+1)$$

$$=(\log 3)(\sqrt{1+0}+1)=2.\log 3$$

**17.** $\underset{x\to 0}{Lt}\dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$ కనుగొనుము

సాధన: $\underset{x\to 0}{Lt}\dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}=\underset{x\to 0}{Lt}\dfrac{(1+x)^{\frac{1}{3}}-(1-x)^{\frac{1}{3}}}{x}=\underset{x\to 0}{Lt}\dfrac{(1+x)^{\frac{1}{3}}-1+1-(1-x)^{\frac{1}{3}}}{x}$

$$=\underset{x\to 0}{Lt}\frac{(1+x)^{\frac{1}{3}}-1}{(1+x)-1}+\underset{x\to 0}{Lt}\frac{(1-x)^{\frac{1}{3}}-1}{(1-x)-1}$$

$$=\frac{1}{3}.1^{-2/3}+\frac{1}{3}.1^{-2/3}=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$$

**18.** $\underset{x\to a}{Lt}\dfrac{\sin(x-a)\tan^2(x-a)}{(x^2-a^2)^2}$ కనుగొనుము

సాధన: $\underset{x\to a}{Lt}\dfrac{\sin(x-a)\tan^2(x-a)}{(x^2-a^2)^2}$

$$=\underset{x\to a}{Lt}\frac{\sin(x-a)}{x-a}\left(\underset{x\to a}{Lt}\frac{\tan(x-a)}{(x-a)}\right)^2+\underset{x\to a}{Lt}\frac{(x-a)}{(x+a)^2}\quad=1.1\frac{0}{(2a)^2}=0$$

**19.** $\underset{x\to 0}{Lt}\dfrac{1-\cos 2mx}{\sin^2 nx}(m,n\in -2)$ కనుగొనుము

సాధన: $\underset{x\to 0}{Lt}\dfrac{1-\cos 2mx}{\sin^2 nx}=\underset{x\to 0}{Lt}\dfrac{2\sin^2 mx}{\sin^2 nx}$

$$=\underset{x\to 0}{Lt}\frac{2\sin^2 mx}{\sin^2 nx}\times\frac{x^2}{x^2}$$

$$=\underset{x\to 0}{Lt}\frac{2\dfrac{\sin^2 mx}{x^2}}{\dfrac{\sin^2 nx}{x^2}}=2\frac{\left(\underset{x\to 0}{Lt}\dfrac{\sin mx}{mx}\right)^2}{\left(\underset{x\to 0}{Lt}\dfrac{\sin nx}{nx}\right)^2}\times\frac{m^2}{n^2}=\frac{2m^2}{n^2}$$

**20.** $\underset{x\to\infty}{Lt}\dfrac{3x^2+4x+5}{2x^3+3x-7}$ కనుగొనుము

సాధన: $\underset{x\to\infty}{Lt}\dfrac{3x^2+4x+5}{2x^3+3x-7}$

$$=\underset{x\to\infty}{Lt}\frac{\left(3+\dfrac{4}{x}+\dfrac{5}{x^2}\right)x^2}{\left(2+\dfrac{3}{x^2}-\dfrac{7}{x^3}\right)x^3}=\underset{x\to\infty}{Lt}\frac{\left(3+\dfrac{4}{x}+\dfrac{5}{x^2}\right)}{2+\dfrac{3}{x^2}-\dfrac{7}{x^3}}.\frac{1}{x}$$

**As** $x\to\infty,\dfrac{1}{x},\dfrac{1}{x^2},\dfrac{1}{x^3}\to 0$

$$=\underset{x\to\infty}{Lt}\frac{3x^2+4x+5}{2x^3+3x-7}=\frac{(3+0+0)}{2+0-0}.0=0$$

**21.** $\underset{x\to\infty}{Lt}\dfrac{6x^2-x+7}{x+3}$ కనుగొనుము

**Sol :** $\underset{x\to\infty}{Lt}\dfrac{6x^2-x+7}{x+3}$ $\underset{x\to\infty}{Lt}\dfrac{x^2\left(6-\frac{1}{x}+\frac{7}{x^2}\right)}{x\left(1+\frac{3}{x}\right)}$

$$\underset{x\to\infty}{Lt}\frac{6-\frac{1}{x}+\frac{7}{x^2}}{1+\frac{3}{x}}.\underset{x\to\infty}{Lt}\,x=\frac{6-0+0}{1+0}\infty=\infty$$

**22.** $\underset{x\to 0}{Lt}\dfrac{8|x|+3x}{3|x|-2x}$ కనుగొనుము

సాధన : $as\ \ x\to\infty\Rightarrow|x|=x$

ఇక్కడ $x$ ధనాత్మకమ్

$$\underset{x\to\infty}{Lt}\frac{8|x|+3x}{3|x|-2x}=\underset{x\to\infty}{Lt}\frac{8x+3x}{3x-2x}$$

$$=\underset{x\to\infty}{Lt}\frac{11x}{x}=11$$

**23.** $\underset{x\to\infty}{Lt}\dfrac{x^2+5x+2}{2x^2-5x+1}$ కనుగొనుము

సాధన : $=\underset{x\to\infty}{Lt}\dfrac{x^2+5x+2}{2x^2-5x+1}=\underset{x\to\infty}{Lt}\dfrac{x^2\left[1+\frac{5}{x}+\frac{2}{x^2}\right]}{x^2\left(2\frac{-5}{x}+\frac{1}{x^2}\right)}$

**As** $x\to\infty.\dfrac{1}{x}\,and\,\dfrac{1}{x^2}\to 0$

$$\underset{x\to\infty}{Lt}\frac{x^2+5x+2}{2x^2-5x+1}=\underset{x\to\infty}{Lt}\frac{1+\frac{5}{x}+\frac{2}{x^2}}{2-\frac{5}{x}+\frac{1}{x^2}}$$

$$=\frac{1+0+0}{2-0+0}=\frac{1}{2}$$

**24.** $\underset{x\to 2}{Lt}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)$ కనుగొనుము

సాధన : $\underset{x\to 2}{Lt}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)=\underset{x\to 2}{Lt}\frac{x+2-4}{x^2-4}$

$$=\underset{x\to 2}{Lt}\frac{x-2}{x^2-4}=\underset{x\to 2}{Lt}\frac{x-2}{(x-2)(x+2)}$$

$$=\underset{x\to 2}{Lt}\frac{x+2-4}{x^2-4}$$

$$=\frac{1}{4}$$

**25.** $\underset{x\to-\infty}{Lt}\frac{5x^3+4}{\sqrt{2x^4+1}}$ కనుగొనుము

సాధన : $\underset{x\to-\infty}{Lt}\frac{5x^3+4}{\sqrt{2x^4+1}}=\underset{x\to-\alpha}{Lt}\frac{x^3\left(5+\frac{4}{x^3}\right)}{x^2\sqrt{2+\frac{1}{x^4}}}$

$$=\underset{x\to-\infty}{Lt}\ x.\frac{5+\frac{4}{x^3}}{\sqrt{2}+\frac{1}{x^4}}$$

**As** $x\to-\infty,\frac{1}{x},\frac{1}{x^2}.\frac{1}{x^3},\frac{1}{x^4}\to 0$

$$\underset{x\to-\infty}{Lt}\frac{5x^3+4}{\sqrt{2x^4}+1}(-\infty)=(-\infty).\frac{5}{\sqrt{1}}=-\infty.$$

**26.** $\underset{x\to\infty}{Lt}\left(\sqrt{x^2+x}-x\right)$

సాధన: $= \underset{x\to\infty}{Lt}\sqrt{x^2+x}-x$

$$= \underset{x\to\infty}{Lt}\frac{\left(\sqrt{x^2+x}-x\right)\left(\sqrt{x^2+x}+\sqrt{x}\right)}{\sqrt{x^2+x}+x} = \underset{x\to\infty}{Lt}\frac{x^2+x-x^2}{x\left(\sqrt{1+\frac{1}{x}}+1\right)} = \underset{x\to\infty}{Lt}\frac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)}$$

$$= \frac{1}{\sqrt{1+0}+1} = \frac{1}{1+1} = \frac{1}{2}$$

**27.** వాస్తవ సమితి పై $f(x)=\begin{cases} x^2 & \text{if} \quad x\le 1 \\ x & \text{if} \quad x>1 \end{cases}$ అవిచ్ఛిన్న ప్రమేయము అని చూపుము

సాధన: $\underset{x\to 1-}{Lt} f(x) = \underset{x\to 1-}{Lt} x^2 = 1^2 = 1$

$\underset{x\to 1+}{Lt} f(x) = \underset{x\to 1+}{Lt} x = 1$

$\therefore \underset{x\to 1-}{Lt} f(x) = \underset{x\to 1+}{Lt} f(x) = 1 \quad f(1) = 1^2 = 1$

$\underset{x\to 1}{Lt} f(x) = f(1)$

f, $x=1$ విచ్ఛిన్న ప్రమేయము

కావున వాస్తవ సమితి పై

**28.** $'\sin x'$ మరియు $'\cos x'$ లు **R** లు అవిచ్ఛిన్న ప్రమేయములు అని చూపుము

**Sol :** **i)** $a \in R$

$$\underset{x \to a}{Lt} f(x) = \underset{x \to a}{Lt} \sin x = \sin a = f(a)$$

**a** వద్ద $f(x)$ అవిచ్ఛిన్నం

**ii)** $a \in R$

$$\underset{x \to a}{Lt} f(x) \underset{x \to a}{Lt} \cos x - \cos a = f(a)$$

**a** వద్ద $f(x)$ అవిచ్ఛిన్నం

**29.** $f(x) = \begin{cases} \dfrac{\sin 2x}{x} & \text{if} \quad x \neq 0 \\ 1 & \text{if} \quad x = 0 \end{cases}$ అనే ప్రమేయము 0 వద్ద అవిచ్ఛినమా

సాధన : $\underset{x \to 0}{Lt} f(x) = \underset{x \to 0}{Lt} \dfrac{\sin 2x}{x} = 2$

కాని $f(0) = 1$

$$\therefore \underset{x \to 0}{Lt} f(x) \neq f(0)$$

**f**ప్రమేయము 0 వద్ద అవిచ్ఛిన్నమ్ కాదు.

**30.** $f(x) = \begin{cases} \dfrac{1}{2}\left(x^2 - 4\right) & \text{if} \quad 0 < x < 2 \\ 2 - 8x^{-3} & \text{if} \quad x > 2 \end{cases}$ అనే ప్రమేయానికి **2** వద్ద అవిచ్ఛిన్నతను పరిశీలించంది

**Sol :** $l.l = \underset{x \to 2-}{Lt} f(x) = \underset{x \to 2-}{Lt} \dfrac{1}{2}\left(x^2 - 4\right)$

$$= \frac{1}{2}(4 - 4) = 0$$

$$R.L = \underset{x\to 2+}{Lt} f(x) = \underset{x\to 2+}{Lt}\left(2 - \frac{8}{x^3}\right) = 2\frac{-8}{8} = 2 - 1 = 1$$

$$\underset{x\to 2-}{Lt} f(x) \neq \underset{x\to 2+}{Lt} f(x)$$

$\therefore \underset{x\to 2}{Lt} f(x)$ వ్యవస్థితమ్ కాదు

2 వద్ద $f(x)$ అవిచ్ఛిన్నం కాదు

**31.** $f(x) = \begin{cases} \dfrac{x^2 - 9}{x^2 - 2x - 3} & \text{if} \quad 0 < x < 5, x \neq 3 \\ 1.5 & \text{if} \quad x = 3 \end{cases}$ అనే ప్రమేయానికి 3 వద్ద అవిచ్ఛిన్నతను పరిశీలించంది

సాధన **f(3) =1.5.**

$$\underset{x\to 3}{Lt} f(x) = \underset{x\to 3}{Lt} \frac{x^2 - 9}{x^2 - 2x - 3}$$

$$= \underset{x\to 3}{Lt} \frac{(x-3)(x+3)}{(x-3)(x+1)} = \frac{3+3}{3+1} = \frac{6}{4} = 1.5 = f(3)$$

3 వద్ద $f(x)$ అవిచ్ఛిన్నం ప్రమేయము.

**32.** $f(x) = \begin{cases} k^2 x - k & \text{if} \quad x \geq 1 \\ 2 & \text{if} \quad x < 1 \end{cases}$ అనే ప్రమేయం R పై అవిచ్ఛిన్నం ఐతే k ను కనుగొనుము

సాధన: $\underset{x\to 1-}{Lt} f(x) = \underset{x\to 1-}{Lt} 2 = 2$

$$\underset{x\to 1+}{Lt} f(x) = \underset{x\to 1+}{Lt}\left(k x^2 - k\right) = k^2 - k$$

$f(x)$ $x = 0$ వద్ద $f(x)$ అవిచ్ఛిన్నం కావున

$$\underset{x\to 1-}{Lt} f(x) = \underset{x\to 1+}{Lt} f(x) \quad 2 = k^2 - k$$

$$k^2 - k - 2 = 0$$

$$(k-2)(k+1) = 0 \qquad k = 2 \; or \; -1$$

**33.** $$f(x)\begin{cases} \dfrac{\cos ax - \cos bx}{x^2} & \text{if} \quad x \neq 0 \\ \dfrac{1}{2}\left(b^2 - a^2\right) & \text{if} \quad x = 0 \end{cases}$$

**a , b** వాస్తవాలు ఐతే ం వద్ద $f(x)$ అవిచ్ఛిన్న ప్రమేయములు అని చూపుము

సాధన $$\underset{x\to 0}{Lt} f(x) = \underset{x\to 0}{Lt} \frac{\cos ax - \cos bx}{x^2}$$

$$= \underset{x\to 0}{Lt} \frac{2\sin\frac{(a+b)x}{x}\sin\frac{(b-a)x}{2}}{x^2}$$

$$= 2 \underset{x\to 0}{Lt} \frac{\sin(a+b)\frac{x}{2}}{x} \underset{x\to 0}{Lt} \frac{\sin(b-a)\frac{x}{2}}{x}$$

$$= \frac{2(b+a)}{2}\frac{(b-a)}{2} = \frac{b^2 - a^2}{2}$$

**Given** $f(0) = \dfrac{b^2 - a^2}{2}$.

$$\therefore \underset{x\to 0}{Lt} f(x) = f(0)$$

కావున ం వద్ద $f(x)$ అవిచ్ఛిన్న ప్రమేయము.