అవధులు మరియు అవిచ్ఛిన్నత

Standard limits.

n అకరణీయసంఖ్య అయి a > 0 అయితే $\underset{x\to a}{Lt}\frac{x^n-a^n}{x-a}=n.a^{n-1}$

$\underset{x\to 0}{Lt}\frac{\sin x}{x}=1$,

$\underset{x\to 0}{Lt}\frac{\tan x}{x}=1$,

$\underset{x\to 0}{Lt}\frac{e^x-1}{x}=1$,

$\underset{x\to 0}{Lt}\frac{a^x-1}{x}=\log_e a$,

$\underset{x\to 0}{Lt}(1+x)^{\frac{1}{x}}=e$,

$\underset{x\to\infty}{Lt}\left(1+\frac{1}{x}\right)^x=e$ **అయితే** $\underset{x\to-\infty}{Lt}\left(1+\frac{1}{x}\right)^x=e$

అతిస్వల్ప సమాధాన ప్రశ్నలు

1 $\underset{x\to a}{Lt}\frac{x^2-a^2}{x-a}$ **నుకనుగొనుము** .

Sol : $\underset{x\to a}{Lt}\frac{x^2-a^2}{x-a}=\underset{x\to a}{Lt}\frac{(x+a)(x-a)}{x-a}=\underset{x\to a}{Lt}(x+a)\;=a+a\;=2a$

2 $\underset{x\to 0}{Lt}x^2\cos\frac{2}{x}$ **నుకనుగొనుము**

Sol : $\underset{x\to 0}{Lt}x^2.\underset{x\to 0}{Lt}\cos\frac{2}{x}=0.k$ **Where** $|k|\le 1=0$

3. $\underset{x\to 0}{Lt}\left(\sqrt{x}+x^{5/2}\right)(x>0)$

Sol : $\underset{x\to 0}{Lt}\left(\sqrt{x}+x^{5/2}\right)=\sqrt{0}+0^{5/2}=0+0=0$

4. $\underset{x\to 1}{Lt}\left(\frac{2}{x+1}-\frac{3}{x}\right)$

Sol : G.L.= $\underset{x\to 2}{Lt}\left(\frac{2}{x+1}-\frac{3}{x}\right)=\frac{2}{2+1}-\frac{3}{2}=\frac{2}{3}-\frac{3}{2}=\frac{4-9}{6}=\frac{-5}{6}$

5. క్రింది ఇచ్చిన ప్రమేయాలకు ఎడమ,కుడి అవధులను కనుగొనుము మరియు ఇచ్చిన బిందువు వద్ద అవధి వ్యవస్థితం అవుతుందో లేదో తెలపండి.

A. $f(x)=\begin{cases}1-x & \text{if} \quad x\le 1\\ 1+x & \text{if} \quad x>1\end{cases}\ ;\ a=1.$

Sol : $LL\ at$ x=1 is $\underset{x\to 1-}{Lt}\ 1-x=1-1=0$

RLat x=1 is $\underset{x\to 1+}{Lt}\ (1+x)=1+1=2$

$\underset{x\to 1-}{Lt}\ f(x)\ne \underset{x\to 1+}{Lt}\ f(x)$

$\therefore\ \underset{x\to 1}{Lt}\ f(x)$ **అవధి వ్యవస్థితం కాదు.**

B. $f(x)=\begin{cases}x+2 & \text{if} \quad -1<x\le 3\\ x^2 & \text{if} \quad 3<x<5\end{cases}\ ;\ a=3.$

Sol : $L.L=\underset{x\to 3-}{Lt}\ (x+2)=3+2=5$

$R.L=\underset{x\to 3+}{Lt}\ f(x)=\underset{x\to 3+}{Lt}\ x^2=3^2=9$

$\underset{x\to 3-}{Lt}\ f(x)\ne \underset{x\to 3+}{Lt}\ f(x)$

$\underset{x\to 3}{Lt}\ f(x)$ **అవధి వ్యవస్థితం కాదు**

C. $f(x)=\begin{cases}1 & \text{if} \quad x<0\\ 2x+1 & \text{if} \quad 0\le x<1\\ 3x & \text{if} \quad x>1\end{cases}\ ;a=1.$

Sol :

$L.L=\underset{x\to 1-}{Lt}\ f(x)=\underset{x\to 1-}{Lt}\ 2x+1=2(1)+1=3$

$R.L=\underset{x\to 1+}{Lt}\ f(x)=\underset{x\to 1+}{Lt}\ 3x=3(1)=3$

$\underset{x\to1-}{Lt}\ f(x) = \underset{x\to1+}{Lt}\ f(x) = 3 \quad \therefore \underset{x\to1}{Lt}\ f(x) = 3$

D. $f(x) = \begin{cases} x^2 & \text{if} \quad x \le 1 \\ x & \text{if} \quad 1 < x \le 2 \\ x-3 & \text{if} \quad x > 2 \end{cases}; \ a = 2.$

$L.L = \underset{x\to2-}{Lt}\ f(x) = \underset{x\to2-}{Lt}\ x = 20$

$R.L = \underset{x\to2+}{Lt}\ f(x) = \underset{x\to2+}{Lt}\ (x-3) \qquad = 2-3 = -1$

$\underset{x\to2-}{Lt}\ f(x) \ne \underset{x\to2+}{Lt}\ f(x)$.

$\underset{x\to2}{Lt}\ f(x)$ **అవధి వ్యవస్థితం కాదు**

6.. $\underset{x\to2-}{Lt} \frac{|x-2|}{x-2} = -1$ **అని చూపుము**

Sol : $x \to 2- \Rightarrow x < 2 \Rightarrow \quad x-2 < 0$

$\Rightarrow |x-2| = -(x-2)$

$\underset{x\to2-}{Lt} \frac{|x-2|}{x-2} = \underset{x\to2-}{Lt} \frac{-(x-2)}{(x-2)} = -1$

7. $\underset{x\to0+}{Lt} \left(\frac{2|x|}{x} + x + 1\right) = 3.$ **అని చూపుము**

Sol : $x \to 0+ \Rightarrow x > 0 \Rightarrow \quad |x| = x$

$\therefore \underset{x\to0+}{Lt} \left(\frac{2|x|}{x} + x + 1\right)$

$= \underset{x\to0+}{Lt} \left(\frac{2x}{x} + x + 1\right) = \underset{x\to0+}{Lt}\ (2 + x + 1) = \underset{x\to0+}{Lt}\ (2 + 0 + 1) = 3$

8. $\underset{x\to2+}{Lt}\ ([x] + x)$ **మరియు** $\underset{x\to2-}{Lt}\ ([x] + x).$ **లనుకనుగొనుము**

Sol : $\underset{x\to2^+}{Lt}\ \{[x] + x\}. = \underset{h\to0+}{Lt}\ \{[2+h] + (2+h)\} \ = [2+0] + 2 + 0 \quad \left(\because \left[2^+\right] = 2\right) \qquad = 2 + 2 = 4$

$\underset{x\to2-}{Lt}\ \{[x] + x\} = \left[2^-\right] + 2 \quad = 1 + 2 = 3$

9. $\underset{x\to 2-}{Lt}\sqrt{2-x}\ (x<2)$. **కనుగొనుము** $\underset{x\to 2}{Lt}\sqrt{2-x}$? **విలువ ఎంత**

Sol : $\underset{x\to 2-}{Lt}\sqrt{2-x} = \underset{h\to 0^+}{Lt}\sqrt{2-(2-h)}\ \left(\because \underset{x\to a-}{Lt} f(x) = \underset{h\to o^+}{Lt} f(x)\right)$

$= \underset{h\to 0+}{Lt}\sqrt{h} = 0$

$x \to 2^+ \Rightarrow x>2 \Rightarrow x-2<0$

అప్పుడు $\sqrt{x-2}$ **వాస్తవం కాదు**

కావున ప్రమేయము $x>2$ **అయినప్పుడు నిర్వచింపబడదు. కావున** $x>2$ **అయినప్పుడు కుడి అవధిని పరిశీలించము.**

కావున $\underset{x\to 2}{Lt}\sqrt{2-x} = \underset{x\to 2-}{Lt}\sqrt{2-x}$ $\quad\therefore \underset{x\to 2}{Lt}\sqrt{2-x}=0$

10. $\underset{x\to\left(-\frac{1}{2}\right)+}{Lt}\sqrt{1+2x}$. **కనుగొనుము** $\underset{x\to -\frac{1}{2}}{Lt}\sqrt{1+2x}$. **విలువ ఎంత**

Sol : $\underset{x\to -\frac{1}{2}+}{Lt}\sqrt{1+2x} = \underset{h\to 0+}{Lt}\sqrt{1+2\left\{\left(-\frac{1}{2}\right)+h\right\}} = \underset{h\to 0+}{Lt}\sqrt{1-1+2h}=0$

$x<\frac{-1}{2}$, **అయినప్పుడు** $\sqrt{1+2x}$ **వాస్తవం కాదు. కావున ప్రమేయము** $x<\frac{-1}{2}$ **అయినప్పుడు నిర్వచింపబడదు. కావున** $x<\frac{-1}{2}$ **అయినప్పుడు left limit పరిశీలించము.**

కావున $\underset{x\to -\frac{1}{2}}{Lt}\sqrt{1+2x} = \underset{x\to -\frac{1}{2}+}{Lt}\sqrt{1+2x}=0$

11. $\underset{x \to \frac{\pi}{2}}{Lt} \dfrac{\cos x}{\left[x - \frac{\pi}{2}\right]}$ ను కనుగొనుము

Sol : $y = x - \frac{\pi}{2}$ అనుకొనుము అప్పుడు $x \to \frac{\pi}{2}, y \to 0$ and

$$x = y + \frac{\pi}{2} \therefore \underset{x \to \frac{\pi}{2}}{Lt} \frac{\cos x}{x - \frac{\pi}{2}} = \underset{y \to 0}{Lt} \frac{\cos\left(\frac{\pi}{2} + y\right)}{y} \underset{y \to 0}{Lt} \frac{-\sin y}{y} = -1$$

12. $\underset{x \to 0}{Lt} \dfrac{\sin ax}{x \cos x}$ ను కనుగొనుము

Sol : $$\underset{x \to 0}{Lt} \frac{\sin a x}{x.\cos x} = \frac{\underset{x \to 0}{Lt} \frac{\sin a x}{a x}.a}{\underset{x \to 0}{Lt} \cos x} = \frac{a}{1} = a$$

13. $\underset{x \to 2}{Lt} \dfrac{\left(2x^2 - 7x - 4\right)}{(2x-1)\left(\sqrt{x} - 2\right)}$

Sol $$\underset{x \to 2}{Lt} \frac{2x^2 - 7x - 4}{(2x-1)\left(\sqrt{x} - 2\right)} = \underset{x \to 2}{Lt} \frac{(x-4)(2x+1)}{(2x-1)\left(\sqrt{x} - 2\right)} = \underset{x \to 2}{Lt} \frac{\left(\sqrt{x} + 2\right)(2x+1)}{(2x-1)}$$

$$= \frac{\left(\sqrt{2} + 2\right)(4+1)}{(4-1)} = \frac{5\left(2 + \sqrt{2}\right)}{3}$$

14. $\underset{x \to 1}{Lt} \dfrac{\sin(x-1)}{\left(x^2 - 1\right)}$ ను కనుగొనుము

Sol. $$\underset{x \to 1}{Lt} \frac{\sin(x-1)}{x^2 - 1} = \underset{x \to 1}{Lt} \frac{\sin(x-1)}{(x-1)(x+1)} = \underset{x \to 1}{Lt} \frac{\sin(x-1)}{(x-1)}. \underset{x \to 1}{Lt} \frac{1}{x+1}$$

$y = x - 1$ అనుకొనుము అప్పుడు $x \to 1,\ y \to 0$

$$= \underset{y \to 0}{Lt} \frac{\sin y}{y}.\frac{1}{1+1} = 1.\frac{1}{2} = \frac{1}{2}$$

15. $\underset{x\to 0}{Lt}\dfrac{\sin(a+bx)-\sin(a-bx)}{x}$ నుకనుగొనుము

Sol : $\underset{x\to 0}{Lt}\dfrac{\sin(a+bx)-\sin(a-bx)}{x}=\underset{x\to 0}{Lt}\dfrac{2\cos a.\sin bx}{x}$

$=\underset{x\to 1}{Lt}\,2\cos a.\underset{x\to 0}{Lt}\dfrac{\sin bx}{bx}.b=2\cos a.\quad b=2b.\cos a.$

16. $\underset{x\to a}{Lt}\dfrac{\tan(x-a)}{x^2-a^2}(a\neq 0)$ నుకనుగొనుము

Sol : $\underset{x\to a}{Lt}\dfrac{\tan(x-a)}{x^2-a^2}=\underset{x\to a}{Lt}\dfrac{\tan(x-a)}{(x-a)(x+a)}=\underset{x\to a}{Lt}\dfrac{\tan(x-a)}{(x-a)}.\underset{x\to a}{Lt}\dfrac{1}{(x+a)}$

$x-a=h$ అనుకొనుము. అప్పుడు $x\to a,\ h\to 0$

G. L $=\underset{h\to 0}{Lt}\dfrac{\tan h}{h}.\left(\dfrac{1}{a+a}\right)=1.\dfrac{1}{2a}=\dfrac{1}{2a}$

17. $\underset{x\to 1}{Lt}\dfrac{(2x-1)\left(\sqrt{x}-1\right)}{\left(2x^2+x-3\right)}$ కనుగొనుము

Sol : $\underset{x\to 1}{Lt}\dfrac{(2x-1)\left(\sqrt{x}-1\right)}{2x^2+x-3}=\underset{x\to 1}{Lt}\dfrac{(2x-1)\left(\sqrt{x}-1\right)}{(x-1)(2x+3)}$

$=\underset{x\to 1}{Lt}\dfrac{(2x-1)\left(\sqrt{x}-1\right)}{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)(2x+3)}$

$=\dfrac{2.1-1}{\left(\sqrt{1}+1\right)(2.1+3)}=\dfrac{1}{(1-1)5}=\dfrac{1}{10}$

18. $\underset{x\to 3}{Lt}\dfrac{x^2+3x+2}{x^2-6x+9}$ నుకనుగొనుము

Sol : $\underset{x\to 3}{Lt}\dfrac{x^2+3x+2}{x^2-6x+9}=\underset{x\to 3}{Lt}\dfrac{x^2+3x+2}{(x-3)^2}=\dfrac{9+9+2}{0}=\infty$

19. $\underset{x\to\infty}{Lt}\dfrac{3x^2+4x+5}{2x^3+3x-7}$ నుకనుగొనుము

Sol : $\underset{x\to\infty}{Lt}\dfrac{3x^2+4x+5}{2x^3+3x-7}=\underset{x\to\infty}{Lt}\dfrac{\left(3+\dfrac{4}{x}+\dfrac{5}{x^2}\right)x^2}{\left(2+\dfrac{3}{x^2}-\dfrac{7}{x^3}\right)x^3}=\underset{x\to\infty}{Lt}\dfrac{\left(3+\dfrac{4}{x}+\dfrac{5}{x^2}\right)}{2+\dfrac{3}{x^2}-\dfrac{7}{x^3}}.\dfrac{1}{x}$

$x\to\infty,\dfrac{1}{x},\dfrac{1}{x^2},\dfrac{1}{x^3}\to 0$

$=\underset{x\to\infty}{Lt}\dfrac{3x^2+4x+5}{2x^3+3x-7}=\dfrac{(3+0+0)}{2+0-0}.0=0$

20. $\underset{x\to\infty}{Lt}\dfrac{6x^2-x+7}{x+3}$ నుకనుగొనుము

Sol : $\underset{x\to\infty}{Lt}\dfrac{6x^2-x+7}{x+3}\quad\underset{x\to\infty}{Lt}\dfrac{x^2\left(6-\dfrac{1}{x}+\dfrac{7}{x^2}\right)}{x\left(1+\dfrac{3}{x}\right)}$

$\underset{x\to\infty}{Lt}\dfrac{6-\dfrac{1}{x}+\dfrac{7}{x^2}}{1+\dfrac{3}{x}}.\underset{x\to\infty}{Lt}\ x=\dfrac{6-0+0}{1+0}\infty=\infty$

21. $\underset{x\to 0}{Lt}\dfrac{8|x|+3x}{3|x|-2x}$ నుకనుగొనుము

Sol : $as\ x\to\infty\Rightarrow|x|=x$

$\underset{x\to\infty}{Lt}\dfrac{8|x|+3x}{3|x|-2x}=\underset{x\to\infty}{Lt}\dfrac{8x+3x}{3x-2x}=\underset{x\to\infty}{Lt}\dfrac{11x}{x}=11$

22. $\underset{x\to\infty}{Lt}\dfrac{x^2+5x+2}{2x^2-5x+1}$ నుకనుగొనుము

Sol : $\underset{x\to\infty}{Lt}\dfrac{x^2+5x+2}{2x^2-5x+1}=\underset{x\to\infty}{Lt}\dfrac{x^2\left[1+\dfrac{5}{x}+\dfrac{2}{x^2}\right]}{x^2\left(2\dfrac{-5}{x}+\dfrac{1}{x^2}\right)}$

As $x \to \infty . \frac{1}{x}$ and $\frac{1}{x^2} \to 0$

$$\underset{x\to\infty}{Lt} \frac{x^2+5x+2}{2x^2-5x+1} = \underset{x\to\infty}{Lt} \frac{1+\frac{5}{x}+\frac{2}{x^2}}{2-\frac{5}{x}+\frac{1}{x^2}} = \frac{1+0+0}{2-0+0} = \frac{1}{2}$$

23. $\underset{x\to 2}{Lt}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)$ ను కనుగొనుము

Sol : $\underset{x\to 2}{Lt}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right) = \underset{x\to 2}{Lt} \frac{x+2-4}{x^2-4}$

$$= \underset{x\to 2}{Lt} \frac{x-2}{x^2-4} = \underset{x\to 2}{Lt} \frac{x-2}{(x-2)(x+2)}$$

$$= \underset{x\to 2}{Lt} \frac{x+2-4}{x^2-4} = \frac{1}{4}$$

24. $\underset{x\to -\infty}{Lt} \frac{5x^3+4}{\sqrt{2x^4+1}}$ నికనుగొనుము

Sol : $\underset{x\to -\infty}{Lt} \frac{5x^3+4}{\sqrt{2x^4+1}} = \underset{x\to -\infty}{Lt} \frac{x^3\left(5+\frac{4}{x^3}\right)}{x^2\sqrt{2+\frac{1}{x^4}}}$

$$= \underset{x\to -\infty}{Lt} x.\frac{5+\frac{4}{x^3}}{\sqrt{2}+\frac{1}{x^4}}$$

As $x \to -\infty, \frac{1}{x}, \frac{1}{x^2}.\frac{1}{x^3}, \frac{1}{x^4} \to 0$

$$\underset{x\to -\infty}{Lt} \frac{5x^3+4}{\sqrt{2x^4+1}}(-\infty) = (-\infty).\frac{5}{\sqrt{1}} = -\infty.$$

25. $\underset{x\to\infty}{Lt}\left(\sqrt{x+1}-\sqrt{x}\right)$ కనుగొనుము

Sol : $\underset{x\to\infty}{Lt}\left(\sqrt{x+1}-\sqrt{x}\right) = \underset{x\to\infty}{Lt}\dfrac{\left(\sqrt{x+1}-\sqrt{x}\right)\left(\sqrt{x+1}+\sqrt{x}\right)}{\sqrt{x+1}+\sqrt{x}}$

$$= \underset{x\to\infty}{Lt}\frac{x+1-x}{\sqrt{x+1}+\sqrt{x}} = \underset{x\to\infty}{Lt}\frac{1}{\sqrt{x}\left(\sqrt{1+\frac{1}{x}}+1\right)} = \underset{x\to\infty}{Lt}\frac{\frac{1}{\sqrt{x}}}{\sqrt{1+\frac{1}{x}}+1} = \frac{0}{1+1} = 0$$

26. $\underset{x\to\infty}{Lt}\dfrac{2+\sin x}{x^2+3}$ కనుగొనుము

Sol : $\underset{x\to\infty}{Lt}\dfrac{2+\sin x}{x^2+3} = \underset{x\to\infty}{Lt}\dfrac{2+\sin x}{x^2\left(1+\frac{3}{x^2}\right)} = \underset{x\to\infty}{Lt}\dfrac{2+\sin x}{x^2\left(1+\frac{3}{x^2}\right)} = \underset{x\to\infty}{Lt}\dfrac{\frac{2}{x^2}+\frac{\sin x}{x^2}}{\left(1+\frac{3}{x^2}\right)}$

$as\ \ \mathrm{x}\to\infty, \dfrac{1}{x^2}\ and\ \dfrac{\mathrm{sinx}}{x^2}\to 0.(\because -1\le \sin x\le 1) = \dfrac{0+0}{1+0} = 0$

27. $\underset{x\to 0}{Lt}\dfrac{x-3}{\sqrt{|x^2-9|}} = 0$ అని చూపుము.

Sol : For $x^2 \neq 9, \left|\dfrac{x-3}{\sqrt{x^2-9}}\right| = \sqrt{\left|\dfrac{x-3}{x+3}\right|}$ .....(1)

$\underset{x\to 3}{Lt}\sqrt{|x-3|} = 0,\ \underset{x\to 3}{Lt}\sqrt{|x+3|} = \sqrt{6}$

$\therefore \underset{x\to 3}{Lt}\dfrac{x-3}{\sqrt{x^2-9}} = 0$

28. $\underset{x\to 0}{Lt}\dfrac{a^x-1}{b^x-1}(a>0, b>0, b\neq 1)$. కనుగొనుము

Sol : For $x\neq 0, \dfrac{a^x-1}{b^x-1} = \dfrac{\left[\dfrac{a^x-1}{x}\right]}{\left[\dfrac{b^x-1}{x}\right]}$

$$\underset{x\to 0}{Lt}\frac{a^x-1}{b^x-1}=\frac{\underset{x\to 0}{Lt}\frac{a^x-1}{x}}{\underset{x\to 0}{Lt}\frac{b^x-1}{x}}=\frac{\log_e^a}{\log_e^b}$$

29. $\underset{x\to\infty}{Lt}\frac{2+\cos^2 x}{x+2007}$ నుకనుగొనుము

Sol. $\underset{x\to\infty}{Lt}\frac{2+\cos^2 x}{x+2007}$

$$=\underset{x\to\infty}{Lt}\frac{\frac{2+\cos^2 x}{x}}{\frac{x+2007}{x}}$$

$$=\underset{x\to\infty}{Lt}\frac{\frac{2}{x}+\frac{\cos^2 x}{x}}{1+\frac{2007}{x}}=\frac{0+0}{1+0}=0$$

30. $\underset{x\to 0}{Lt}\frac{e^x-\sin x-1}{x}$ నుకనుగొనుము

Sol. $\underset{x\to 0}{Lt}\frac{e^x-\sin x-1}{x}=\underset{x\to 0}{Lt}\left[\frac{e^x-1}{x}-\frac{\sin x}{x}\right]$

$$=\underset{x\to 0}{Lt}\left(\frac{e^x-1}{x}\right)-\underset{x\to 0}{Lt}\frac{\sin x}{x}$$

$$=1-1=0$$

31. $\lim\limits_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x}$

Sol. $\lim\limits_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x}=\lim\limits_{x\to 0}\frac{\frac{\log(1+x^3)}{x^3}}{\frac{\sin^3 x}{x^3}}$

$$=\frac{\lim\limits_{x\to 0}\log(1+x^3)^{\frac{1}{x^3}}}{\lim\limits_{x\to 0}\frac{\sin^3 x}{x^3}}=\frac{\log e}{1^3}=1$$

32. $\lim\limits_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}$

Sol. $\lim\limits_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}$

$=\lim\limits_{x\to 0}\frac{\sin\left(\pi\left(1-\mathrm{s}\,in^2 x\right)\right)}{x^2}$

$=\lim\limits_{x\to 0}\frac{\sin\pi-\sin\left(\pi\,\mathrm{s}\,in^2 x\right)}{x^2}$

$\lim\limits_{x\to 0}\frac{0-\sin\left(\pi\,\mathrm{s}\,in^2 x\right)}{\pi\,\mathrm{s}\,in^2 x}*\frac{\pi\,\mathrm{s}\,in^2 x}{x^2}$

$=1X\pi=\pi$

33.

$\lim\limits_{x\to 0}\frac{\log\left(1+5x\right)}{x}$

$sol.\lim\limits_{x\to 0}\frac{\log\left(1+5x\right)}{x}$

$=\lim\limits_{x\to 0}\log\left(1+5x\right)^{\frac{1}{x}}$

$=\lim\limits_{x\to 0}e^5$

$=5X1=5$

స్వల్ప సమాధాన ప్రశ్నలు

1. $\underset{x \to a}{Lt}\left[\frac{x\sin a - a\sin x}{x-a}\right]$ **కనుగొనుము**

Sol : $\underset{x \to a}{Lt}\frac{x\sin a - a\sin x}{x-a} = \underset{x \to a}{Lt}\frac{(x\sin a - a\sin a)-(a\sin x - a\sin a)}{(x-a)}$

(Adding and subtactin a sina in Nr.)

$$= \underset{x \to a}{Lt}\frac{(x-a)\sin a - a(\sin x - \sin a)}{x-a} = \underset{x \to a}{Lt}\frac{(x-a)\sin a}{x-a} - \underset{x \to a}{Lt}\, a\left(\frac{\sin x - \sin a}{x-a}\right)$$

$$= \sin a - a.\ \underset{x \to a}{Lt}\frac{2\cos\frac{x+a}{2}.\sin\frac{x-a}{2}}{x-a} = \sin a - a.\ \underset{x \to a}{Lt}\frac{\cos(x+a)}{2}\ \underset{x \to a}{Lt}\frac{\sin\left(\frac{x-a}{2}\right)}{\left(\frac{x-a}{2}\right)}$$

$$= \sin a - a\cos a - 1 = \sin a - a\cos a$$

2. $\underset{x \to 0}{Lt}\left[\frac{\cos ax - \cos bx}{x^2}\right]$ **కనుగొనుము**

Sol : $\underset{x \to 0}{Lt}\frac{\cos ax - \cos bx}{x^2} = \underset{x \to 0}{Lt}\frac{2\sin\frac{(a+b)x}{2}.\sin\frac{(b-a)x}{2}}{x^2}$

$$= 2.\ \underset{x \to 0}{Lt}\frac{\sin(b+a)\frac{x}{2}}{x}.\ \underset{x \to 0}{Lt}\frac{\sin(b-a)\frac{x}{2}}{x}$$

$$= 2.\ \underset{x \to 0}{Lt}\frac{\sin(b+a)\frac{x}{2}}{(b+a)\frac{x}{2}} \times \frac{(b+a)}{2}.\ \underset{x \to 0}{Lt}\frac{\sin(b-a)\frac{x}{2}}{(b-a)\frac{x}{2}} \times \frac{(b-a)}{2}$$

$$= 2.\left(\frac{b+a}{2}\right)\left(\frac{b-a}{2}\right) \quad = \frac{1}{2}\left(b^2 - a^2\right)$$

3. $\underset{x\to 0}{Lt}\left[\frac{(1+x)^{\frac{1}{8}}-(1-x)^{\frac{1}{8}}}{x}\right]$ **కనుగొనుము**

Sol : $\underset{x\to 0}{Lt}\frac{(1+x)^{\frac{1}{8}}-(1-x)^{\frac{1}{8}}}{x}$

$$=\underset{x\to 0}{Lt}\frac{(1+x)^{\frac{1}{8}}-1+1-(1-x)^{\frac{1}{8}}}{x}$$

$$=\underset{(1+x)\to 1}{Lt}\frac{(1+x)^{\frac{1}{8}}-1}{(1+x)-1}+\underset{(1-x)\to 1}{Lt}\frac{(1-x)^{\frac{1}{8}}-1^{8}}{(1-x)-1}$$

$$=\frac{1}{8}1^{-7/8}+\frac{1}{8}1^{-7/8}=\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$$

4. $\underset{x\to 0}{Lt}\left[\frac{(1+x)^{\frac{1}{3}}-(1-x)^{\frac{1}{3}}}{x}\right]=\frac{2}{3}$ **అని చూపుము.**

5. $\underset{x\to 0}{Lt}\left[\frac{3^{x}-1}{\sqrt{1+x}-1}\right]$ **కనుగొనుము**

Sol : $\underset{x\to 0}{Lt}\frac{3^{x}-1}{\sqrt{1+x}-1}$

$$=\underset{x\to 0}{Lt}\frac{3^{x}-1}{\sqrt{1+x}-1}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

(*rationalise* Dr.)

$$=\underset{x\to 0}{Lt}\frac{\left(3^{x}-1\right)\left(\sqrt{1+x}+1\right)}{1+x-1}$$

$$=\underset{x\to 0}{Lt}\frac{3^{x}-1}{x}.\underset{x\to 0}{Lt}\left(\sqrt{1+x}+1\right)$$

$$=(\log 3)\left(\sqrt{1+0}+1\right)=2.\log 3$$

6. $\underset{x\to a}{Lt}\left[\frac{\left(\sqrt{a+2x}-\sqrt{3x}\right)}{\left(\sqrt{3a+x}-2\sqrt{x}\right)}\right]$ **నుకనుగొనుము**

Sol. $\underset{x\to a}{Lt}\left[\frac{\left(\sqrt{a+2x}-\sqrt{3x}\right)}{\left(\sqrt{3a+x}-2\sqrt{x}\right)}\right]=$

$$\underset{x\to a}{Lt}\frac{\left(\sqrt{a+2x}-\sqrt{3x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)}{\left(\sqrt{a+2x}+\sqrt{3x}\right)}\times\frac{\left(\sqrt{3a+x}+\sqrt{4x}\right)}{\left(\sqrt{3a+x}-\sqrt{4x}\right)\left(\sqrt{3a}+x+\sqrt{4x}\right)}$$

$$=\underset{x\to a}{Lt}\frac{a+2x-3x}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+\sqrt{4x}}{3a+x-4x}=\underset{x\to a}{Lt}\frac{(a-x)\left(\sqrt{3a+x}+\sqrt{4x}\right)}{\left(\sqrt{a+2x}+\sqrt{3x}\right)3(a-x)}=\frac{2(2a)}{2\left(\sqrt{3a}\right)3}=\frac{2}{3\sqrt{3}}$$

7. $\underset{x\to 0}{Lt}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$ నుకనుగొనుము

Sol : $\underset{x\to 0}{Lt}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}=\underset{x\to 0}{Lt}\frac{(1+x)^{\frac{1}{3}}-(1-x)^{\frac{1}{3}}}{x}$

$$=\underset{x\to 0}{Lt}\frac{(1+x)^{\frac{1}{3}}-1+1-(1-x)^{\frac{1}{3}}}{x}=\underset{x\to 0}{Lt}\frac{(1+x)^{\frac{1}{3}}-1}{(1+x)-1}+\underset{x\to 0}{Lt}\frac{(1-x)^{\frac{1}{3}}-1}{(1-x)-1}=\frac{1}{3}.1^{-2/3}+\frac{1}{3}.1^{-2/3}$$

$$=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$$

8. $\underset{x\to a}{Lt}\frac{\sin(x-a)\tan^2(x-a)}{\left(x^2-a^2\right)^2}$ నుకనుగొనుము

Sol : $\underset{x\to a}{Lt}\frac{\sin(x-a)\tan^2(x-a)}{\left(x^2-a^2\right)^2}=\underset{x\to a}{Lt}\frac{\sin(x-a)}{x-a}\left(\underset{x\to a}{Lt}\frac{\tan(x-a)}{(x-a)}\right)^2+\underset{x\to a}{Lt}\frac{(x-a)}{(x+a)^2}$

$$=1.1\frac{0}{(2a)^2}=0$$

9. $\underset{x\to 0}{Lt}\frac{1-\cos 2mx}{\sin^2 nx}(m,n\in -2)$ నుకనుగొనుము

Sol : $\underset{x\to 0}{Lt}\frac{1-\cos 2mx}{\sin^2 nx}=\underset{x\to 0}{Lt}\frac{2\sin^2 mx}{\sin^2 nx}$

$$=\underset{x\to 0}{Lt}\frac{2\sin^2 mx}{\sin^2 nx}\times\frac{x^2}{x^2}=\underset{x\to 0}{Lt}\frac{2\frac{\sin^2 mx}{x^2}}{\frac{\sin^2 nx}{x^2}}=2\frac{\left(\underset{x\to 0}{Lt}\frac{\sin mx}{mx}\right)^2}{\left(\underset{x\to 0}{Lt}\frac{\sin nx}{nx}\right)^2}\times\frac{m^2}{n^2}=\frac{2m^2}{n^2}$$

10. $\underset{x\to\infty}{Lt}\left(\sqrt{x^2+x}-x\right)$ **కనుగొనుము**

Sol : $\underset{x\to\infty}{Lt}\ \sqrt{x^2+x}-x$

$$=\underset{x\to\infty}{Lt}\frac{\left(\sqrt{x^2+x}-x\right)\left(\sqrt{x^2+x}+\sqrt{x}\right)}{\sqrt{x^2+x}+x}=\underset{x\to\infty}{Lt}\frac{x^2+x-x^2}{x\left(\sqrt{1+\frac{1}{x}}+1\right)}=\underset{x\to\infty}{Lt}\frac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)}$$

$$=\frac{1}{\sqrt{1+0}+1}=\frac{1}{1+1}=\frac{1}{2}$$

11. $\underset{x\to 0}{Lt}\left[\frac{e^x-1}{\sqrt{1+x}-1}\right]$. **కనుగొనుము**

Sol : **For** $0<|x|<1.$

$$\frac{e^x-1}{\sqrt{1+x}-1}=\frac{e^x-1}{\sqrt{1+x}-1}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}=\frac{e^x-1\left(\sqrt{1+x}+1\right)}{14x-y}=\frac{e^x-1}{x}\left(\sqrt{1+x}+1\right)$$

$$=\underset{x\to 0}{Lt}\frac{e^x-1}{\sqrt{1+x}-1}=\underset{x\to 0}{Lt}\frac{e^x-1}{x}$$

$$=\underset{x\to 0}{Lt}\sqrt{1+x}+1=1.\left(\sqrt{1+0}+1\right)=(1+1)=2$$

12. $\underset{x\to 0}{\text{Lt}}\frac{1-\cos mx}{1-\cos nx}, n\neq 0$ **ను కనుగొనుము**

Sol. $$\underset{x\to 0}{\text{Lt}}\frac{2\sin^2\frac{mx}{2}}{2\sin^2\frac{nx}{2}}=\underset{x\to 0}{\text{Lt}}\frac{\dfrac{\sin^2\frac{mx}{2}}{\left(\frac{mx}{2}\right)^2}\left(\frac{mx}{2}\right)^2}{\dfrac{\sin^2\frac{nx}{2}}{\left(\frac{nx}{2}\right)^2}\left(\frac{nx}{2}\right)^2}$$

As $x\to 0$, $\frac{mx}{2}\to 0,\ \frac{nx}{2}\to 0$

$$= \frac{\lim\limits_{\frac{mx}{2}\to 0}\left(\frac{\sin\frac{mx}{2}}{\frac{mx}{2}}\right)^2}{\lim\limits_{\frac{nx}{2}\to 0}\left(\frac{\sin\frac{nx}{2}}{\frac{nx}{2}}\right)^2} \times \frac{m^2}{n^2} = (1) = \frac{m^2}{n^2}$$

13. $\underset{x\to 0}{Lt}\frac{x(e^x-1)}{1-\cos x}$ **నుకనుగొనుము**

Sol. $\underset{x\to 0}{Lt}\frac{x^2(e^x-1)}{x\cdot 2\sin^2\frac{x}{2}}$

$$= \underset{x\to 0}{Lt}\frac{x^2}{2\sin^2\frac{x}{2}} \times \underset{x\to 0}{Lt}\frac{e^x-1}{x}$$

$$\underset{\frac{x}{2}\to 0}{Lt}\frac{\frac{x^2}{4}\times 4}{2\sin^2\frac{x}{4}}\left(\underset{x\to 0}{Lt}\frac{e^x-1}{x}\right)$$

$$= \frac{4}{2}\underset{\frac{x}{2}\to 0}{Lt}\left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\right)\cdot \underset{x\to 0}{Lt}\frac{e^x-1}{x}$$

$$= 2(1)(1) = 2$$

14.. $f(x) = \frac{a_n x^n + \ldots + a_1 x + a_0}{b_m x^m + \ldots + b_1 x + b_0}$ **ఐతే** $a_n > 0, b_m > 0$ **అయినపుడు n>m. కు** $\underset{x\to\infty}{Lt} f(x) = \infty$ **అనిచూపుము**

Sol. $f(x) = \frac{a_n x^n + \ldots + a_1 x + a_0}{b_m x^m + \ldots + b_1 x + b_0}$

$$= \frac{x^n\left(a_n + \frac{a_n - 1}{x} + \ldots + \frac{a_1}{x^{n-1}} + \frac{a_0}{x^n}\right)}{x^m\left(b_m + \frac{b_m - 1}{x} + \ldots + \frac{b_1}{x^{m-1}} + \frac{b_0}{x^m}\right)}$$

$$= x^{n-m}\left(\frac{a_n + \frac{a_n - 1}{x} + \ldots + \frac{a_1}{x^{n-1}} + \frac{a_0}{x^n}}{b_m + \frac{b_m - 1}{x} + \ldots + \frac{b_1}{x^{m-1}} + \frac{b_0}{x^m}}\right)$$

$x \to \infty$ ఐతే, $\frac{a_{n-i}}{x^i}, \frac{b_{m-i}}{x^i}$ =0

కాని $\underset{x\to\infty}{Lt}\ x^{n-m} = \infty$

$\therefore \underset{x\to\infty}{Lt}\ f(x) = \infty$

15. $\underset{x\to 0}{Lt} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ నుకనుగొనుము

Sol. $\underset{x\to 0}{Lt} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$

$$= \underset{x\to 0}{Lt} \frac{x\frac{\tan x}{1-\tan^2 x} - 2x\tan x}{(2\sin^2 x)^2}$$

$$= \underset{x\to 0}{Lt} \frac{2x\tan\left[\frac{1}{1-\tan^2 x} - 1\right]}{4\sin^2 x}$$

$$= \underset{x\to 0}{Lt} \frac{2x\tan x}{4\sin^2 x}\left[\frac{1-1+\tan^2 x}{1-\tan^4 x}\right]$$

$$= \underset{x\to 0}{Lt} \frac{2x\tan^3 x}{4\sin^4 x}$$

$$= \underset{x\to 0}{Lt} \frac{2x^4\tan^3 x}{x^3 4\sin^4 x} dx$$

$$= \frac{2}{4} \underset{x\to 0}{Lt} \frac{x^4}{\sin^4 x} \cdot \underset{x\to 0}{Lt} \frac{\tan^3 x}{x^3}$$

$$= \frac{1}{2}(1)(1) = \frac{1}{2}$$

16. $\underset{x\to 0}{Lt}\left[\frac{e^x - 1}{\sqrt{1+x} - 1}\right].$

సాధన

$$\lim_{x\to 0} \frac{e^x - 1}{\sqrt{1+x} - 1} = \lim_{x\to 0} \frac{e^x - 1}{\sqrt{1+x} - 1} \times \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1}$$

$$= \lim_{x\to 0} \frac{(e^x - 1)(\sqrt{1+x} + 1)}{1 + x - 1} = \lim_{x\to 0} \frac{e^x - 1}{x}(\sqrt{1+x} + 1)$$

$$= 1.\left(\sqrt{1+0}+1\right) = (1+1) = 2$$

17. $\underset{x \to 0}{Lt} \frac{a^x - 1}{b^x - 1} (a > 0, b > 0, b \neq 1).$ కనుగొనుము

సాధన: $\underset{x \to 0}{Lt} \frac{a^x - 1}{b^x - 1} = \frac{\underset{x \to 0}{Lt} \frac{a^x - 1}{x}}{\underset{x \to 0}{Lt} \frac{b^x - 1}{x}} = \frac{\log_e^a}{\log_e^b}$