TRIGONOMETRY

1. త్రికోణమితీయ నిష్పత్తులు

1. i) $\sin\theta \operatorname{cosec}\theta = 1$

ii) $\cos\theta \sec\theta = 1$ iii) $\tan\theta \cot\theta = 1$

iv) $\sin^2\theta + \cos^2\theta = 1$

v) $1 + \tan^2\theta = \sec^2\theta \quad \rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1.$

$\rightarrow \sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta} = 1$

vi) $1 + \cot^2\theta = \operatorname{cosec}^2\theta \rightarrow (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

$\rightarrow \operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$

vii) $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \,.\, \operatorname{cosec}^2\theta$

viii) $\tan^2\theta - \sin^2\theta = \tan^2\theta \,.\, \sin^2\theta;$

$\cot^2\theta - \cos^2\theta = \cot^2\theta \,.\, \cos^2\theta$

ix) $\sin^2\theta + \cos^4\theta = 1 - \sin^2\theta\cos^2\theta$

$= \sin^4\theta + \cos^2\theta$

x) $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

xi) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

xii) $\sin^2 x + \operatorname{cosec}^2 x \geq 2$

xiii) $\cos^2 x + \sec^2 x \geq 2$

xiv) $\tan^2 x + \cot^2 x \geq 2.$

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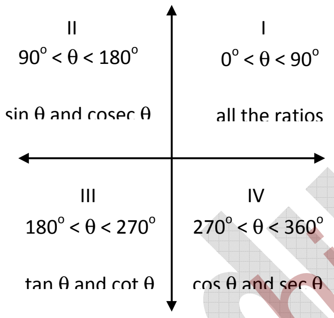
2. Values of trigonometric ratios of certain angles

angle ↓	0^o	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined
cot	undefined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
cosec	undefined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
sec	1	$2/\sqrt{3}$	$\sqrt{2}$	2	undefined

3. Signs of Trigonometric ratios



Fn	$90 \mp \theta$	$180 \mp \theta$	$270 \mp \theta$	$360 \mp \theta$
$\sin\theta$	$\cos\theta$	$\pm\sin\theta$	$-\cos\theta$	$\mp\sin\theta$
$\cos\theta$	$\pm\sin\theta$	$-\cos\theta$	$\mp\sin\theta$	$\cos\theta$
$\tan\theta$	$\pm\cot\theta$	$\mp\tan\theta$	$\pm\cot\theta$	$\mp\tan\theta$
$\text{cosec}\,\theta$	$\sec\theta$	$\pm\text{cosec}\,\theta$	$-\sec\theta$	$\mp\text{cosec}\,\theta$
$\sec\theta$	$\pm\text{cosec}\,\theta$	$-\sec\theta$	$\mp\text{cosec}\,\theta$	$\sec\theta$
$\cot\theta$	$\pm\tan\theta$	$\mp\cot\theta$	$\pm\tan\theta$	$\mp\cot\theta$

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అతి స్వల్ప సమాధాన ప్రశ్నలు

ఈ క్రింది వాటివిలువను కనుగొనుము

1. $\tan(\theta - 14\pi)$

సాధన: $-\tan(14\pi - \theta)$

$= \tan[7 \cdot 2\pi - \theta] = -[-\tan\theta] = \tan\theta$

2. $\operatorname{cosec}(5\pi + \theta)$

సాధన: $\operatorname{cosec}(5\pi + \theta) = -\operatorname{cosec}\theta$

$(\because \operatorname{cosec}(n\pi + \theta) = (-1)^n \operatorname{cosec}\theta)$

3. $\cos\left(-\frac{7\pi}{2}\right)$ విలువను కనుగొనుము

సాధన:

$\cos\left(-\frac{7\pi}{2}\right) = \cos\frac{7\pi}{2} = 0$

$(\because \cos(2n+1)\frac{\pi}{2} = 0)$

4. Find the value of cot(315°)

సాధన:

$\cot(315°) = -\cot 315°$

$= -[\cot 360° - 45°]$

$= -[-\cot 45°]$

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= cot 45° = 1

5. $\cos^2 45° + \cos^2 135° + \cos^2 225° + \cos^2 315°$ విలువను కనుగొనుము

సాధన:

$$\cos 45° = \frac{1}{\sqrt{2}}, \cos 135° = \cos(180° - 45°)$$

$$= \cos 45° \qquad = -\frac{1}{\sqrt{2}}$$

$$\cos 225° = \cos(180° + 45°)$$

$$= \cos 45° \qquad = -\frac{1}{\sqrt{2}}$$

$$\cos 135° = \cos(360° - 45°)$$

$$= \cos 45° \qquad = \frac{1}{\sqrt{2}}$$

Given expression

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$$

6. cos 225° – sin 225° + tan 495° – cot 495° ని గణించండి

సాధన:

cos 225° = cos(180° + 45°)

$$= \cos 45° = -\frac{1}{\sqrt{2}}$$

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$\sin 225° = \sin(180° + 45°)$

$= -\sin 45° = \dfrac{-1}{\sqrt{2}}$

$\tan 495° = \tan[5(90°) + 45]$

$= -\cot 45° = -1$

$\cot 495° = \cot[5(90°) + 45°]$

$= -\tan 45° = -1$

G.E. $= \dfrac{-1}{\sqrt{2}} - \left(-\dfrac{1}{\sqrt{2}}\right) - 1 + 1$

$= \dfrac{-1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} - 1 + 1 = 0$

6. $\sin 330^0 . \cos 120^0 + \cos 210^0 . \sin 300^0$ విలువ కనుక్కోండి

సాధన:

$\sin 330^0 = \sin\left(360^0 - 30^0\right) = -\sin 30^0 = -\dfrac{1}{2}$

$\cos 120^0 = -\dfrac{1}{2} \cos 210^0 = -\dfrac{\sqrt{3}}{2}; \sin 300^0 = \sin\left(360^0 - 60^0\right) = -\dfrac{\sqrt{3}}{2}$

$\sin 330^0 \cos 120^0 + \cos 210^0 . \sin 300^0 = -\dfrac{1}{2} \times -\dfrac{1}{2} + \left(-\dfrac{\sqrt{3}}{2}\right)\left(-\dfrac{\sqrt{3}}{2}\right) = 1$

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స్వల్ప సమాధాన ప్రశ్నలు

8. cosec θ + cot θ = $\frac{1}{3}$, ఐతే $\cos\theta$ కనుగొనుము మరియు θ ఏ పాదం లో వుంటుందో తెలుపుము.

సాధన:

$\text{cosec}^2\,\theta - \cot^2\theta = 1$

$(\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 1$

$\text{cosec}\,\theta - \cot\theta = 3$ ...(1)

and $\text{cosec}\,\theta + \cot\theta = \frac{1}{3}$ ...(2)

From (1) + (2)

$\text{cosec}\,\theta - \cot\theta = 3$

$\text{cosec}\,\theta + \cot\theta = \frac{1}{3}$

$2\text{cosec}\,\theta = 3+\frac{1}{3}=\frac{10}{3}$

$\text{cosec}\,\theta = \frac{10}{6}$

$\sin\theta = \frac{6}{16}$

From (1) – (2)

$\text{cosec}\,\theta - \cot\theta = 3$

$\text{cosec}\,\theta + \cot\theta = \frac{1}{3}$

$-2\cot\theta = 3-\frac{1}{3}=\frac{8}{3}$

$\cot\theta = -\frac{8}{2\times 3}=\frac{-8}{6}$

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$$\therefore \cos\theta = \cot\theta \cdot \sin\theta = \frac{-8}{6} \times \frac{6}{10} = \frac{-4}{5}$$

sin θ +ve and cos θ is –ve

కావునకోణం θ రెండవ పాదంలోఉంటుంది.

9. sec θ + tan θ = 5 ఐతే θ ఏ పాదం లో ఉంటుందో తెలిపి sin θ కనుగొనుము .

సాధన:

sec θ + tan θ = 5 ...(1)

$\sec^2\theta - \tan^2\theta = 1$

(sec θ + tan θ)(sec θ – tan θ) = 1

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

$$\sec\theta - \tan\theta = \frac{1}{5} \quad ...(2)$$

Adding (1), (2)

$$\sec\theta + \tan\theta = 5$$

$$\sec\theta - \tan\theta = \frac{1}{5}$$

$$2\sec\theta = 5 + \frac{1}{5}$$

$$2\sec\theta = \frac{25+1}{5}$$

$$\sec\theta = \frac{26}{5} \times \frac{1}{2}$$

$$\sec\theta = \frac{13}{5}$$

$$\tan\theta = \frac{12}{5}, \sin\theta = \frac{12}{13}$$

∴ sin θ , sec θ , tan θ ఋధనాత్మకం

కావున θ మొదటి పాదం లో వుంటుంది.

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10. A రెండో పాదం లో లేని కోణం, B మూడో పాదం లో లేని కోణం, $\cos A = \cos B = -\frac{1}{2}$:

అయితే $\frac{4\sin B - 4\tan A}{\tan B + \sin A}$ విలువను కనుగొనుము

సాధన:

$\cos A = \cos B = -\frac{1}{2}$; A రెండో పాదం లో లేని కోణం

$\therefore$ A మూడో పాదం లోని కోణం

$$\sin A = -\frac{\sqrt{3}}{2}$$

$\cos B = -\frac{1}{2}$ B మూడో పాదం లో లేని కోణం

$\therefore$ B రెండో పాదం లోని కోణం

$\sin B = \frac{\sqrt{3}}{2}$ $\tan A = \sqrt{3}$ $\tan B = -\sqrt{3}$

$$\frac{4\sin B - 3\tan A}{\tan B + \sin A} = \frac{4\times\frac{\sqrt{3}}{2} - 3\times\sqrt{3}}{-\sqrt{3} - \frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{\frac{-3\sqrt{3}}{2}} = \frac{2}{3}$$

11. $\frac{\sin(3\pi - A)\cos\left(A - \frac{\pi}{2}\right)\tan\left(\frac{3\pi}{2} - A\right)}{\operatorname{cosec}\left(\frac{13\pi}{2} + A\right)\sec(3\pi + A)\cot\left(A - \frac{\pi}{2}\right)} = -\cos^4 A$ విలువను కనుగొనుము

సాధన:

. $\sin(3\pi - A) = \sin A$

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$$\cos\left(A-\frac{\pi}{2}\right)=\cos\left(\frac{\pi}{2}-A\right)=\sin A$$

$$\tan\left(\frac{3\pi}{2}-A\right)=\cot A$$

$$\operatorname{cosec}\left(\frac{13\pi}{2}+A\right)=-\sec A$$

$$\sec(3\pi+A)=-\sec A$$

$$\cot\left(A-\frac{\pi}{2}\right)=-\cot\left(\frac{\pi}{2}-A\right)=-\tan A$$

$$\text{L.H.S.}\quad=\frac{\sin A\cdot\sin A\cdot\cot A}{-\sec A\cdot-\sec A\cdot-\tan A}$$

$$=\frac{\sin^2 A\times\dfrac{\cos A}{\sin A}}{-\dfrac{1}{\cos^2 A}\times\dfrac{\sin A}{\cos A}}$$

$$=\sin A\cos A\times\frac{\cos^3 A}{-\sin A}=-\cos^4 A$$

12. $\cot\left(\frac{\pi}{20}\right)\cot\left(\frac{3\pi}{20}\right)\cot\left(\frac{5\pi}{20}\right)\cot\left(\frac{7\pi}{20}\right)\cot\left(\frac{9\pi}{20}\right)=1$ అని చూపుము

Sol. $\cot\left(\frac{\pi}{20}\right)=\cot 9^\circ=\frac{1}{\tan 9^\circ}$

$$\cot\left(\frac{3\pi}{20}\right)=\cot 27^\circ=\frac{1}{\tan 27^\circ}$$

$$\cot\left(\frac{5\pi}{20}\right)=\cot 45^\circ=1$$

$$\cot\left(\frac{7\pi}{20}\right)=\cot 63^\circ=\cot(90^\circ-27^\circ)=\tan 27^\circ$$

$$\cot\left(\frac{9\pi}{20}\right)=\cot 81^\circ=\cot(90^\circ-9^\circ)=\tan 9^\circ$$

$$\therefore \cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$$

$$=\frac{1}{\tan 9^\circ}\frac{1}{\tan 27^\circ}\cdot 1\cdot\tan 27^\circ\cdot\tan 9^\circ=1$$

13. $\dfrac{\sin\left(-\frac{11\pi}{3}\right)\tan\left(\frac{35\pi}{6}\right)\sec\left(-\frac{7\pi}{3}\right)}{\cos\left(\frac{5\pi}{4}\right)\operatorname{cosec}\left(\frac{7\pi}{4}\right)\cos\left(\frac{17\pi}{6}\right)}$ విలువ కనుగొనుము .

Sol. $\sin\left(-\frac{11\pi}{3}\right) = -\sin\frac{11\pi}{3} = -\sin 660°$

$= -\sin(2\cdot 360° - 60°)$

$= \sin 60° = \frac{\sqrt{3}}{2}$

$\tan\left(\frac{35\pi}{6}\right) = \tan 1050° = \tan(3\cdot 360° - 30°) = -\tan 30° = -\frac{1}{\sqrt{3}}$

$\sec\left(-\frac{7\pi}{3}\right) = \sec(-420°) = \sec 420° = \sec(360° + 60°) = \sec 60° = 2$

$\cos\left(\frac{5\pi}{4}\right) = \cos(225°) = \cos(180° + 45°) = \cos 45° = -\frac{1}{\sqrt{2}}$

$\csc\left(\frac{7\pi}{4}\right) = \csc 315° = \csc(360° - 45°) = -\csc 45° = -\sqrt{2}$

$\cos\left(\frac{17\pi}{6}\right) = \cos 510° = \cos(360° + 150°)$

$= \cos 100° = \cos(180° - 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

$$\text{L.H.S.} = \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{-1}{\sqrt{3}}\right)(2)}{\left(\frac{-1}{\sqrt{2}}\right)(-\sqrt{2})\left(-\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{-1}{\frac{-\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \text{R.H.S.}$$

14. tan 20° = p అయితే $\dfrac{\tan 610° + \tan 700°}{\tan 560° - \tan 470°} = \dfrac{1-p^2}{1+p^2}$ అని చూపుము

Sol. $\dfrac{\tan 610° + \tan 700°}{\tan 560° - \tan 470°}$

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$$= \frac{\tan(360° + 250°) + \tan(360° + 340°)}{\tan(360° + 200°) - \tan(360° + 110°)}$$

$$= \frac{\tan 250° - \tan 340°}{\tan 200° - \tan 110°}$$

$$= \frac{\tan(270° - 20°) - \tan(360° - 20°)}{\tan(180° + 20°) + \tan(90° + 20°)}$$

$$= \frac{\cot 20° - \tan 20°}{\tan 20° + \cot 20°}$$

$$= \frac{\frac{1}{\tan 20°} - \tan 20°}{\tan 20° + \frac{1}{\tan 20°}}$$

$$= \frac{\frac{1}{p} - p}{p + \frac{1}{p}} = \frac{\frac{1-p^2}{p}}{\frac{p^2+1}{p}} = \frac{1-p^2}{p^2+1}$$

$$\therefore \frac{\tan 610° + \tan 700°}{\tan 560° - \tan 470°} = \frac{1-p^2}{1+p^2}$$

15. α, β లు పూరకకోణాలు అయి $b \sin \alpha = a$, అయితే $(\sin \alpha \cos \beta - \cos \alpha \sin \beta)$.విలువ కనుక్కోండి

Sol. α, β లు పూరకకోణాలు కావున $\alpha + \beta = 90°$

$\beta = 90° - \alpha$

$$b \sin \alpha = a \Rightarrow \sin \alpha = \frac{a}{b}$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \left(\frac{a}{b}\right)^2 = \frac{b^2 - a^2}{b^2}$$

$$\cos \alpha = \frac{\sqrt{b^2 - a^2}}{b}$$

కానీ $\beta = 90° - \alpha$

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$\sin\beta = \sin(90-\alpha) = \cos\alpha$

$\therefore \cos\alpha = \sin\beta$

and $\alpha = 90° - \beta$

$\sin\alpha = \sin(90° - \beta) = \cos\beta$

$\therefore \cos\beta = \sin\alpha = \dfrac{a}{b}$

$\therefore \cos\beta = \dfrac{a}{b}$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \dfrac{a}{b}\times\dfrac{a}{b} - \dfrac{\sqrt{b^2-a^2}}{b}\times\dfrac{\sqrt{b^2-a^2}}{b}$

$= \dfrac{a^2}{b^2} - \dfrac{(b^2-a^2)}{b^2} = \dfrac{a^2-b^2+a^2}{b^2} = \dfrac{2a^2-b^2}{b^2}$

16. $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ ఐతే $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ అని చూపుము

Solution:

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ -------- (1)

Let $\cos\theta - \sin\theta = x$ --------(2)

$(1)^2 + (2)^2 \Rightarrow (\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = \left(\sqrt{2}\cos\theta\right)^2 + x^2$

$\Rightarrow\ 2 - 2\cos^2\theta = x^2 \Rightarrow x = \sqrt{2}\sin\theta \quad \therefore \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

17. $\sin^2\dfrac{\pi}{10} + \sin^2\dfrac{4\pi}{10} + \sin^2\dfrac{6\pi}{10} + \sin^2\dfrac{9\pi}{10} = 2$ అని చూపుము

Solution:

$\sin^2\dfrac{\pi}{10} + \sin^2\dfrac{4\pi}{10} + \sin^2\dfrac{6\pi}{10} + \sin^2\dfrac{9\pi}{10}$

$\sin^2\dfrac{\pi}{10} + \sin^2\left(\dfrac{\pi}{2} - \dfrac{\pi}{10}\right) + \sin^2\left(\dfrac{\pi}{2} + \dfrac{\pi}{10}\right) + \sin^2\left(\pi - \dfrac{\pi}{10}\right)$

$\sin^2\dfrac{\pi}{10} + \cos^2\dfrac{\pi}{10} + \cos^2\dfrac{\pi}{10} + \sin^2\dfrac{\pi}{10} = 2\left\{\sin^2\dfrac{\pi}{10} + \cos^2\dfrac{\pi}{10}\right\} = 2$

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$$= \sin^2 \frac{\pi}{10} + \cos^2 \frac{\pi}{10} = 1$$

18. $\tan 20^0 = \lambda$ అయితే $\frac{\tan 160^0 - \tan 110^0}{1 + \tan 160^0 \tan 110^0} = \frac{1-\lambda^2}{2\lambda}$ అని చూపుము

19. $\sec\theta + \tan\theta = \frac{2}{3}$ ఐతే $\sin\theta$ కనుగొనుము మరియు θ ఏ పాదం లో వుంటుందో తెలుపుము.

సాధన:– $\sec\theta + \tan = \frac{2}{3} \sec\theta + \tan\theta = \frac{3}{2}$

$$2\sec\theta = \frac{2}{3} + \frac{3}{2} \Rightarrow \sec\theta = 13/12 \quad \sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{-15}{13}$$

$\therefore \theta$ నాల్గవ పాదం లో వుంటుంది

20. A, B, C, D లు ఒక చక్రీయ చతుర్భుజం లోని కోణాలు అయితే

i) sin A – sin C = sin D – sin B

ii) cos A + cos B + cos C + cos D = 0 అని చూపుము

Sol. A, B, C, D are the angles of a cyclic quadrilateral.

A + C = 180°; B + D = 180°

A = 180° – C, B = 180° – D

i) sin A – sin C = sin D – sin B

sin A + sin B = sin C + sin D

we have

A = 180° – C $\Rightarrow$ Sin A = sin C ...(1)

B = 180° – D $\Rightarrow$ sin B = sin D ...(2)

Adding (1) and (2)

Sin A + sin B = sin C + sin D

ii) cos A = cos (180° – C) = – cos C

$\Rightarrow$ cos A + cos C = 0 ...(1)

cos B = cos (180° – D) = – cos D

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$\Rightarrow$ cos B + cos D = 0 ...(2)

Adding (1) and (2)

cos A + cos B + cos C + cos D = 0.

21. a cos θ – b sin θ = c, అయితే $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$ అని చూపుము

Sol. a cos θ – b sin θ = c ...(1)

Let a sin θ + b cos θ = k ...(2)

Squaring and adding

22. 3 sin A + 5 cos A = 5, అయితే 5 sin A – 3 cos A = ±3 అని చూపుము

Sol. 3 sin A + 5 cos A = 5

Let 5 sin A – 3 cos A = k

Squaring and adding

$(3\sin A + 5\cos A)^2 + (5\sin A - 3\cos A)^2 = 25 + k^2$

$9\sin^2 A + 25\cos^2 A + 30\sin A\cos A + 25\sin^2 A$
$+9\cos^2 A - 30\sin A\cos A = 25 + k^2$

$34\sin^2 A + 34\cos^2 A = 25 + k^2$

$34(\sin^2 A + \cos^2 A) = 25 + k^2$

$34(1) = 25 + k^2$

$34 = 25 + k^2$

$k^2 = 34 - 26 = 9$

$k = \pm 3$

23. $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$ అని చూపుము

Sol. We have $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$= \frac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$

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$$= \frac{[\tan\theta + \sec\theta][1 - \sec\theta + \tan\theta]}{\tan\theta - \sec\theta + 1}$$

$= \tan\theta + \sec\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$

$$\therefore \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$$

24. $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4(\sin^6\theta + \cos^6\theta) = 13$ అని చూపుము

Sol. $(\sin\theta - \cos\theta)^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= 1 - 2\sin\theta\cos\theta$

$(\sin\theta - \cos\theta)^4 = [(\sin\theta - \cos\theta)^2]^2$

$= [1 - 2\sin\theta\cos\theta]^2$

$= 1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta$

$\therefore (\sin\theta - \cos\theta)^4 = 1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta$

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta$

$\therefore (\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

$\sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$= 1 - 3\sin^2\theta\cos^2\theta \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$\therefore \sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

$\therefore 3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4(\sin^6\theta + \cos^6\theta)$

$= 3[1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta] + 6[1 + 2\sin\theta\cos\theta] + 4[1 - 3\sin^2\theta\cos^2\theta]$

$= 3 + 12\sin^2\theta\cos^2\theta - 12\sin\theta\cos\theta + 6 + 12\sin\theta\cos\theta + 4 - 12\sin^2\theta\cos^2\theta$

$= 3 + 6 + 4 = 13$

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25. $\cos^4\alpha + 2\cos^2\alpha\left[1-\dfrac{1}{\sec^2\alpha}\right] = 1-\sin^4\alpha$ అని నిరూపించండి

Sol. L.H.S.= $\cos^4\alpha + 2\cos^2\alpha\left[1-\dfrac{1}{\sec^2\alpha}\right]$

$= \cos^4\alpha + 2\cos^2\alpha\sin^2\alpha$

$(\because 1-\cos^2\alpha = \sin^2\alpha)$

$= \cos^2\alpha[\cos^2\alpha + 2\sin^2\alpha]$

$= (1-\sin^2\alpha)[(1-\sin^2\alpha)+2\sin^2\alpha]$

$= (1-\sin^2\alpha)(1+\sin^2\alpha)$

$= 1-\sin^4\alpha = \text{R.H.S.}$

26. $\dfrac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta+\cos\theta)^2} = \dfrac{1-\cos\theta}{1+\cos\theta}$ అని నిరూపించండి

Sol.

$1+\sin\theta-\cos\theta = (1-\cos\theta)+\sin\theta = 2\sin^2\dfrac{\theta}{2}+2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} = 2\sin\dfrac{\theta}{2}\left[\sin\dfrac{\theta}{2}+\cos\dfrac{\theta}{2}\right]$

$1+\sin\theta+\cos\theta = (1+\cos\theta)+\sin\theta = 2\cos^2\dfrac{\theta}{2}+2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

$= 2\cos\dfrac{\theta}{2}\left[\cos\dfrac{\theta}{2}+\sin\dfrac{\theta}{2}\right]$

$\text{L.H.S.}: \left[\dfrac{2\sin\frac{\theta}{2}\left[\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right]}{2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right]}\right]^2$

$= \dfrac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}} = \dfrac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \dfrac{1-\cos\theta}{1+\cos\theta} = \text{R.H.S.}$

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27. $\frac{2\sin\theta}{1+\cos\theta+\sin\theta} = x$ అయితే $\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}$ విలువ కనుక్కోండి.

Sol. $\frac{2\sin\theta}{1+\cos\theta+\sin\theta} = x$

$$\Rightarrow \frac{2\cdot\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = x$$

$$\Rightarrow \frac{4\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right]} = x$$

$$\Rightarrow \frac{2\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}} = x \qquad ...(1)$$

$$\frac{1-\cos\theta+\sin\theta}{1+\sin\theta} = \frac{2\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\left[\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right]}{\left[\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right]^2} = \frac{2\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}+\cos\frac{\theta}{2}} = x \ (\because \text{from}(1))$$

ఈక్రింది వానినుండి θ తొలగించండి

28. x = a cos$^3\theta$; y = b sin^3 θ

Sol. x = a cos$^3\theta$; y = b sin^3 θ

$$\frac{x}{a} = \cos^3\theta \Rightarrow \cos\theta = \left(\frac{x}{a}\right)^{1/3}$$

$$\frac{y}{b} = \sin^3\theta \Rightarrow \sin\theta = \left(\frac{y}{b}\right)^{1/3}$$

We have cos^2 θ + sin^2 θ = 1

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$$\Rightarrow\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$$

29. x = a cos^4 θ = b sin^4 θ.

Sol. $x=a\cos^4\theta\Rightarrow\cos^4\theta=\frac{x}{a}$

$$\Rightarrow\cos^2\theta=\sqrt{\frac{x}{a}}$$

$$y=b\sin^4\theta\Rightarrow\sin^4\theta=\frac{y}{b}$$

$$\Rightarrow\sin^2\theta=\sqrt{\frac{y}{b}}$$

$$\cos^2\theta+\sin^2\theta=1$$

$$\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1\Rightarrow\left(\frac{x}{a}\right)^{1/2}+\left(\frac{y}{b}\right)^{1/2}=1$$

30. x = a(sec θ + tan θ) ; y = b (sec θ – tan θ)

Sol. x = a(sec θ + tan θ) ; y = b (sec θ – tan θ)

xy = ab(sec^2 θ – tan^2 θ) = ab (1) = ab

xy = ab.

31. $\frac{\cos(\pi-A)\cot\left(\frac{\pi}{2}+A\right)\cos(-A)}{\tan(\pi+A)\tan\left(3\frac{\pi}{2}+A\right)\sin(2\pi-A)}=\cos A$ అని చూపుము

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$$l.h.s = \frac{\cos(\pi - A)\cot\left(\frac{\pi}{2} + A\right)\cos(-A)}{\tan(\pi + A)\tan\left(3\frac{\pi}{2} + A\right)\sin(2\pi - A)}$$

$$= \frac{-\cos A.(-\tan A)\cos A}{\tan A.(-\cot A)(-sin2A)}$$

$$= \frac{\cos^2 A}{2\sin A\cos A} X \frac{\sin A}{\cos A}$$

$$= \frac{1}{2}$$

32. కోణం θ ఒకటవ పాదం లో లేదు cos θ = t (0 < t < 1) అయితే (a) sin θ (b) tan θ విలువలు కనుక్కోండి

Sol. cos θ = t ధనాత్మకం కావున నాల్గవ పాదం లో ఉంటుంది.

$$x^2 = AC^2 - BC^2 = 1 - t^2$$

$$x = \sqrt{1 - t^2}$$

$$a)\ \sin\theta = \frac{AB}{AC} = -\sqrt{1 - t^2}$$

$$b)\ \tan\theta = \frac{AB}{BC} = -\frac{\sqrt{1 - t^2}}{t}$$

33.కోణం θ మూడవ పాదం లో లేదు. $\sin\theta = -\frac{1}{3}$ అయితే

(i) $\cos\theta$ (ii) $\cot\theta$ విలువలు కనుక్కోండి

సాధన:

$\sin\theta = -\frac{1}{3}$ కోణం θ మూడవ పాదం లో లేదు

కావునకోణంθనాల్గవపాదంలోఉంటుంది $\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$

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$$\cot\theta = \frac{\cos\theta}{\sin\theta} = 2\sqrt{2}$$

34. 8 tan A = 15 , 25 sin B = –7 మరియు A , B నాల్గవ పాదం లోలేని కోణాలు అయితే sin A cos B + cos A sin B = $-\frac{304}{425}$ అని చూపుము

Sol. 8 tan A = –15

$$\tan A = -\frac{15}{8}$$

A , II పాదం లోని కోణం $\sin A = \frac{15}{17}, \cos A = \frac{-8}{17}$

$$25\sin B = -7$$

$$\sin B = -\frac{7}{25}$$

B , III పాదం లోని కోణం $\sin B = -\frac{7}{25}, \cos B = \frac{-24}{25}$

L.H.S. : sin A cos B + cos A sin B

$$= \frac{15}{17}\times\frac{-24}{25} + \frac{-8}{17}\times\frac{-7}{25} = \frac{-3\times 24}{17\times 5} + \frac{-8\times -7}{17\times 25}$$

$$= \frac{-360}{425} + \frac{56}{425} = \frac{-304}{425}$$

35. $(1+\cot\theta-\csc\theta)(1+\tan\theta+\sec\theta) = 2$ అని చూపుము

Sol. L.H.S. : $(1+\cot\theta-\csc\theta)(1+\tan\theta+\sec\theta)$

$$= \left[1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right]\left[1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right]$$

$$= \left[\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right]\left[\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right]$$

$$= \frac{(\sin\theta+\cos\theta)^2-1^2}{\sin\theta\cos\theta} = \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$= \frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta} = 2\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = \text{R.H.S.}$$

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ఆవర్తనం మరియు అంత్య విలువలు

అతి స్వల్ప సమాధాన ప్రశ్నలు

1-4 ప్రమేయాలకి ఆవర్తనం కనుగొనుము.

1. cos (3x + 5) + 7

Sol. f(x) = cos(3x + 5) + 7

$$\text{ఆవర్తనం} = \frac{\text{period of cos x}}{[\text{coefficient of x}]} = \frac{2\pi}{3}$$

2. tan 5x

Sol. f(x) = tan 5x

$$\text{Period} = \frac{\pi}{5}$$

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3. |sin x|

Sol. f(x) = |sin x|

Period = π

$\left[\because |\sin(\pi + x)| = ((-\sin x) = (\sin x))\right]$

4. tan (x + 4x + 9x + ... + n^2x) (n any positive integer).

Sol. f(x) = tan (x + 4x + 9x + ... + n^2x)

$= \tan(1 + 4 + 9 + ... + n^2)x$

$= \tan(1^2 + 2^2 + 3^2 + ... + n^2)x$

$= \tan\left[\frac{n(n+1)(2n+1)}{6}\right]x$

$\text{Period} = \frac{\pi}{\frac{n(n+1)(2n+1)}{6}} = \frac{6\pi}{n(n+1)(2n+1)}$

5. ఆవర్తనం2/3 గా గల sine ప్రమేయాన్ని కనుగొనుము 2/3.

Sol. $f(x) = \sin\left(\frac{2\pi}{2/3}\right)x = \sin(3\pi x)$

6. [0, π] అంతరంలో cos 2x యొక్క గ్రాఫ్ గీయుము

Sol.

x	0	30°	45°	60°	90°	135°	180°
cos 2x	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	0	1

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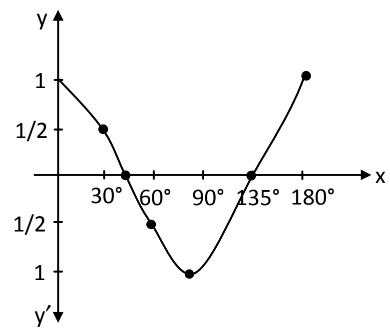
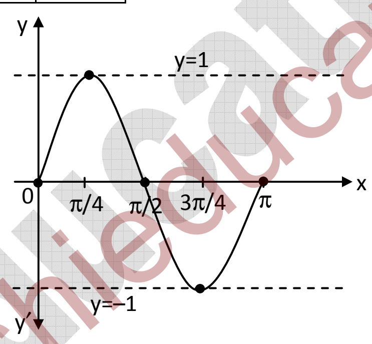


7. $(0, \pi)$ అంతరం లో $y = \sin 2x$ యొక్క రేఖాచిత్రం గీయండి.

Sol.

x	0	30°	60°	90°	135°	180°
sin 2x	0	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	0	–1	0



8. [–π, +π]అంతరంలో sin x యొక్క రేఖాచిత్రం గీయండి.

Sol.

x	–180° (–π)	–90° (–π/2)	0 (0)	90° (π/2)	180° (π)
sin x	0	–1	0	1	0

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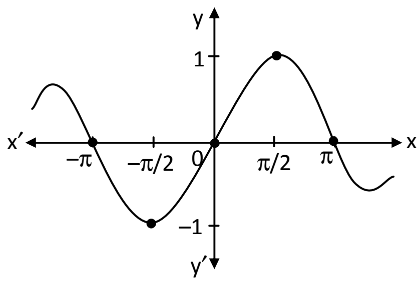


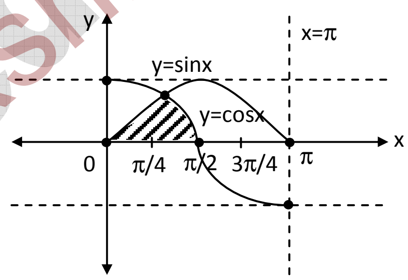
9. [0, π] అంతరంలో y = sin x, y = cos x , x-axis యొక్క రేఖాచిత్రం గీయండి.

Sol. y = sin x

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	(π)
y	0	$\frac{1}{\sqrt{2}}$	1	$\frac{1}{\sqrt{2}}$	0

y = cos x

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	(π)
y	1	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{2}$	−1



10. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ అంతరంలో $y = \tan x$ యొక్క గ్రాఫ్ గీయుము

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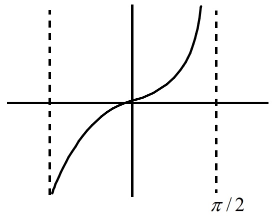
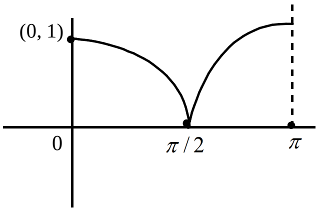


11. $(0, \pi)$ అంతరం లో $y = \cos^2 x$ యొక్కరేఖాచిత్రం గీయండి.



12. $f(x) = 2\sin\frac{\pi x}{4} + 3\cos\frac{\pi x}{3}$ యొక్క ఆవర్తనం కనుగొనుము

యొక్క ఆవర్తనం $\sin\frac{\pi x}{4}$ is $\frac{2\pi}{\pi/4} = 8$

$\cos\frac{\pi x}{3}$ యొక్క ఆవర్తనం is $\frac{2\pi}{\pi/3} = 6$

L.C.M of 8, 6

$\therefore$ $f(x) = 2\sin\frac{\pi x}{4} + 3\cos\frac{\pi x}{3}$ ఆవర్తనం 24

13. $a \le \cos\theta + 3\sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) + 6 \le b$, అయితే a యొక్క గరిష్ఠ విలువ and smallest value of b యొక్క కనిష్ఠవిలువ కనుగొనుము.

Sol. f = $\cos\theta + 3\sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) + 6$

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$= \cos\theta + 3\sqrt{2}\left[\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right] + 6$

$= \cos\theta + 3\sqrt{2}\left[\frac{\sin\theta}{\sqrt{2}} + \frac{\cos\theta}{\sqrt{2}}\right] + 6$

$= \cos\theta + \frac{3\sqrt{2}}{\sqrt{2}}[\sin\theta + \cos\theta] + 6$

$= \cos\theta + 3\sin\theta + 3\cos\theta + 6$

$= 3\sin\theta + 4\cos\theta + 6$

కనిష్ఠవిలువ= $C - \sqrt{a^2 + b^2}$

$= 6 - \sqrt{3^2 + 4^2} = 6 - \sqrt{25} = 6 - 5 = 1$

గరిష్ఠ విలువ $C + \sqrt{a^2 + b^2}$ = $6 + 5 = 11$

∴ a = 1, b = 11.

14. క్రింది ప్రమేయాలకి ఆవర్తనం కనుగొనుము.

1. $\cos^4 x$.

Sol. Let $f(x) = \cos^4 x = (\cos^2 x)^2$

$= \left[\frac{1 + \cos 2x}{2}\right]^2$

$= \frac{1 + 2\cos 2x + \cos^2 2x}{4}$

$= \frac{1}{4}\left[1 + 2\cos 2x + \frac{1 + \cos 4x}{2}\right]$

$= \frac{1}{8}[1 + 4\cos 2x + 1 + \cos 4x]$

$= \frac{1}{8}[3 + 4\cos 2x + \cos 4x]$

$\cos 2x$ యొక్క ఆవర్తనం $\frac{2\pi}{2} = \pi$

$\cos 4x$ యొక్క ఆవర్తనం $= \frac{2\pi}{4} = \frac{\pi}{2}$

L.C.M. of $\left(\pi, \frac{\pi}{2}\right) = \pi$

∴ Period of f(x) = π.

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15. $\frac{5\sin x+3\cos x}{4\sin 2x+5\cos x}$

Sol. Let f(x) = $\frac{5\sin x+3\cos x}{4\sin 2x+5\cos x}$

sin x యొక్క ఆవర్తనం = 2π

cos x యొక్క ఆవర్తనం = 2π

sin 2xయొక్క ఆవర్తనం = $\frac{2\pi}{2}=\pi$

cos 2x యొక్క ఆవర్తనం = $\frac{2\pi}{2}=\pi$

f(x) యొక్క ఆవర్తనం =

L.C.M. of {2π, 2π, π, π}= 2π

16.క్రింది ప్రమేయాలకు కనిష్ఠ, గరిష్ఠ విలువలను కనుగొనుము

1. $4\sin^2 x+5\cos^2 x$

$4\sin^2 x+5\cos^2 x=4\left(1-\cos^2 x\right)+5\cos^2 x$

$=\cos^2 x+4$

$\cos^2 x$ గరిష్ఠ విలువ 1

$\cos^2 x+4$ గరిష్ఠ విలువ$1+4=5$

$\cos^2 x$ కనిష్ఠవిలువ= 0

$\therefore$ $\cos^2 x+4$ కనిష్ఠవిలువ= $0+4=4$

2. 3 cos x + 4 sin x.

Sol. f(x) = 3 cos x + 4 sin x

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గరిష్ఠ విలువ $C+\sqrt{a^2+b^2}$

$=0+\sqrt{4^2+3^2}=\sqrt{16+9}=\sqrt{25}=5$

కనిష్ఠవిలువ= $=C-\sqrt{a^2+b^2}$

$=-\sqrt{4^2+3^2}=-\sqrt{16+9}=-\sqrt{25}=-5$

17. 7 cos x – 24 sin x + 5 యొక్క వ్యాప్తిని కనుగొనుము.

Sol. కనిష్ఠవిలువ= $=C-\sqrt{a^2+b^2}$

$=5-\sqrt{(-24)^2+7^2}$

$=5-\sqrt{576+49}$

$=5-\sqrt{625}=5-25=-20$

గరిష్ఠ విలువ $C+\sqrt{a^2+b^2}$ $=5+\sqrt{625}=5+25=30$

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సంయుక్త కోణాలు

* If A and B are any two angles then

i) Sin(A + B) = sin A cos B + cos A sin B.

ii) Sin(A – B) = sin A cos B – cos A sin B

iii) cos(A + B) = cos A cos B – sin A sin B

iv) cos(A – B) = cos A cos B + sin A sin B.

* If A, B, A + B, A – B are not odd multiples of $\pi/2$ then

i) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

ii) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$.

* If A, B, A + B and A – B are not integral multiples of π, then

i) $\cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

ii) $\cot(A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$.

* i) sin(A + B) + sin(A – B) = 2 sin A cos B

ii) sin(A + B) – sin(A – B) = 2 cos A sin B

iii) cos(A + B) + cos(A – B) = 2 cos A cos B

iv) cos(A + B) – cos(A – B) = –2 sin A sin B

v) cos(A – B) – cos(A + B) = 2sin A sin B.

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* i) $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$

$= \cos^2 B - \cos^2 A$

ii) $\cos(A + B)\cos(A - B) = \cos^2 - \sin^2 B$

$= \cos^2 B - \sin^2 A$

iii) $\tan(A + B)\tan(A - B) = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$

iv) $\cot(A + B)\cot(A - B) = \frac{\cot^2 A \cot^2 B - 1}{\cot^2 B - \cot^2 A}$

v) $\tan(45^o + \theta) = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$

$= \cot(45^o - \theta) = \frac{1 + \tan\theta}{1 - \tan\theta}$

vi) $\tan(45^o - \theta) = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$= \cot(45^o + \theta) = \frac{1 - \tan\theta}{1 + \tan\theta}$

and $\tan(45^o + \theta) \,.\, \tan(45^o - \theta) = 1$.

* i) $\sin(A + B + C)$

$= \sum(\cos A \cos B \cos C) - \sin A \sin B \sin C$

ii) $\cos(A + B + C)$

$= \cos A \cos B \cos C - \sum(\sin A \sin B \sin C)$

iii) $\tan(A + B + C) = \frac{\sum(\tan A) - \pi(\tan A)}{1 - \sum(\tan A \tan B)}$

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అతి స్వల్ప సమాధాన ప్రశ్నలు

Simplify the following

1. cos 100° · cos 40° + sin 100° · sin 40°

Sol. L.H.S. =

= cos 100° · cos 40° + sin 100° · sin 40°

= cos (100° – 40°) = cos 60° = $\frac{1}{2}$ = R.H.S.

2. $\tan\left(\frac{\pi}{4}+\theta\right)\cdot\tan\left(\frac{\pi}{4}-\theta\right)$

Sol. $\dfrac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\cdot\dfrac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$

$=\dfrac{(1+\tan\theta)}{(1-\tan\theta)}\cdot\dfrac{(1-\tan\theta)}{(1+\tan\theta)}=1$

3. tan 75° + cot 75°

Sol. $\tan 75° = 2+\sqrt{3}$

$\cot 75° = 2-\sqrt{3}$

$\therefore \tan 75° + \cot 75° = 2+\sqrt{3}+2-\sqrt{3} = 4$

4. $\frac{(\sqrt{3}\cos 25° + \sin 25°)}{2}$ sine కోణం లో వ్రాయుము.

సాధన: $\frac{(\sqrt{3}\cos 25° + \sin 25°)}{2}$

$$= \frac{\sqrt{3}}{2}\cos 25° + \frac{1}{2}\sin 25°$$
$$= \sin 60° \cos 25° + \cos 60° \sin 25°$$
$$= \sin(60° + 25°) = \sin 85°$$

5. sin(θ + α) = cos (θ + α) అయితే tan θ ను tan α లో వ్రాయుము.

Sol. sin(θ + α) = cos (θ + α)

$$\frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)} = 1$$
$$\tan(\theta+\alpha) = 1$$
$$\frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha} = 1$$
$$\tan\theta + \tan\alpha = 1 - \tan\theta\tan\alpha$$
$$\tan\theta + \tan\theta\tan\alpha = 1 - \tan\alpha$$
$$\tan\theta[1 + \tan\alpha] = 1 - \tan\alpha$$
$$\therefore \tan\theta = \frac{1 - \tan\alpha}{1 + \tan\alpha}$$

6. $\tan\theta = \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$ θ మూడవ పాదం లోని కోణం, $\tan\theta = \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$ అయితే θ కనుగొనుము

.

Sol. $\tan\theta = \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$

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$$= \frac{\cos 11^\circ \left[1 + \frac{\sin 11^\circ}{\cos 11^\circ}\right]}{\cos 11^\circ \left[1 - \frac{\sin 11^\circ}{\cos 11^\circ}\right]}$$

$$= \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$$

$$= \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \tan 11^\circ} \quad (\because \tan 45^\circ = 1)$$

$$\tan\theta = \tan(45^\circ + 11^\circ)$$

$$= \tan 56^\circ$$

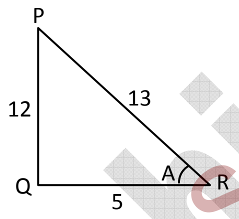
$$= \tan(180^\circ + 56^\circ)$$

$$= \tan 236^\circ$$

$$\theta = 236^\circ$$

7. $0^\circ < A, B < 90^\circ$, $\cos A = \frac{5}{13}$, $\sin B = \frac{4}{5}$ అయితే $\sin(A - B)$ కనుగొనుము.

Sol. $\cos A = \frac{5}{13}$ and $\sin B = \frac{4}{5}$

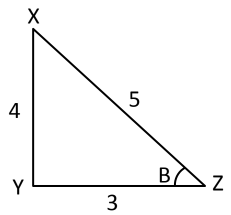


$$PQ^2 = PR^2 - QR^2$$

$$= (13)^2 - 5^2 = 169 - 25 = 144$$

$$PQ = 12$$

$$\cos A = \frac{5}{13}, \sin A = \frac{12}{13}$$



$$YZ^2 = XZ^2 - XY^2$$
$$= 5^2 - 4^2 = 24 - 16 = 9$$
$$YZ = 3$$

$$\sin B = \frac{4}{5}, \cos B = \frac{3}{5}$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$= \frac{12}{13} \times \frac{3}{5} - \frac{5}{13} \times \frac{4}{5} = \frac{36}{65} - \frac{20}{65} = \frac{36 - 20}{65} = \frac{16}{65}$$

8. $\tan 20° + \tan 40° + \sqrt{3} \tan 20° \tan 40°$ కనుగొనుము

Sol. $\tan 20° + \tan 40° + \sqrt{3} \tan 20° \tan 40°$

Consider $20° + 40° = 60°$

$\text{Tan}(20° + 40°) = \tan 60°$

$$\frac{\tan 20° + \tan 40°}{1 - \tan 20° \tan 40°} = \sqrt{3}$$

$$\tan 20° + \tan 40° = \sqrt{3}(1 - \tan 20° \tan 40°)$$

$$\tan 20° + \tan 40° = \sqrt{3} - \sqrt{3} \tan 20° \tan 40°$$

$$\tan 20° + \tan 40° + \sqrt{3} \tan 20° \tan 40° = \sqrt{3}$$

9. $\Sigma \frac{\sin(C - A)}{\cos C \sin A}$ కనుగొనుము

Sol. $\Sigma \frac{\sin(C - A)}{\sin C \sin A} = \Sigma \frac{\sin C \cos A - \cos C \sin A}{\sin C \sin A}$

$$= \Sigma \frac{\sin C \cos A}{\sin C \sin A} - \frac{\cos C \sin A}{\sin C \sin A}$$

$$= \Sigma \cot A - \cot C$$

$$= \cot A - \cot C + \cot B - \cot A + \cot C - \cot B = 0$$

10. $\tan 72° = \tan 18° + 2 \tan 54°$ అని చూపుము

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Sol. $72° - 18° = 54°$

Take tan on both sides

$$\tan(72° - 18°) = \tan 54°$$

$$\frac{\tan 72° - \tan 18°}{1 + \tan 72° \tan 18°} = \tan 54°$$

$$\frac{\tan 72° - \tan 18°}{1 + \tan(90 - 18)\tan 18°} = \tan 54°$$

$$\frac{\tan 72° - \tan 18°}{1 + \tan 18° \tan 18°} = \tan 54°$$

$$\frac{\tan 72° - \tan 18°}{1 + 1} = \tan 54°$$

$$\frac{\tan 72° - \tan 18°}{2} = \tan 54°$$

$$\tan 72° - \tan 18° = 2\tan 54°$$

$$\tan 72° = \tan 18° + 2\tan 54°$$

11. $\sin 750° \cos 480° + \cos 120° \cos 60° = \frac{-1}{2}$ అని చూపుము

Sol. $\sin 750° = \sin(2\cdot 360° + 30°) = \sin 30° = \frac{1}{2}$

$\cos 480° = \cos(360° + 120°) = \cos 120°$

$= 120° = \cos(180° - 60°)$

$= -\cos 60° = -\frac{1}{2}$

$\cos 120° = -\frac{1}{2}$

$\cos 60° = \frac{1}{2}$

$\therefore$ L.H.S. =

$= \sin 750° \cos 480° + \cos 120° \cos 60°$

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$$=\frac{1}{2}\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\frac{1}{2}$$

$$=-\frac{1}{4}-\frac{1}{4}=\frac{-2}{4}=\frac{-1}{2}=\text{R.H.S.}$$

12. $\cos A+\cos\left(\frac{4\pi}{3}-A\right)+\cos\left(\frac{4\pi}{3}+A\right)=0$ అని చూపుము

Sol. Consider $\cos\left(\frac{4\pi}{3}-A\right)+\cos\left(\frac{4\pi}{3}+A\right)$

$$=\cos(240°+A)+\cos(240°-A)$$

$$=\cos 240°\cos A-\sin 240°\sin A$$

$$+\cos 240°\cos A+\sin 240°\sin A$$

$$=2\cos 240°\cos A$$

$$=2\cos(180°+60°)\cos A$$

$$=-2\cos 60°\cos A$$

$$=-2\times\frac{1}{2}\cos A=-\cos A$$

13. $\cos^2\theta+\cos^2\left(\frac{2\pi}{3}+\theta\right)+\cos^2\left(\frac{2\pi}{3}-\theta\right)=\frac{3}{2}$ అని చూపుము

Sol. $\cos^2\left(\frac{2\pi}{3}+\theta\right)+\cos^2\left(\frac{2\pi}{3}-\theta\right)$

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$= \cos^2(60° + \theta) + \cos^2(60° - \theta)$

$= [\cos 60° \cos\theta - \sin 60° \sin\theta]^2$

$\quad + [\cos 60° \cos\theta + \sin 60° \sin\theta]^2$

$= 2(\cos^2 60° \cos^2\theta + \sin^2 60° \sin^2\theta)$

$\quad [\because (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)]$

$$= 2\left[\left(\frac{1}{2}\right)^2 \cos^2\theta + \left(\frac{\sqrt{3}}{2}\right)^2 \sin^2\theta\right]$$

$$= 2\left[\frac{1}{4}\cos^2\theta + \frac{3}{4}\sin^2\theta\right]$$

$$= \frac{2}{4}[\cos^2\theta + 3\sin^2\theta]$$

$$= \frac{1}{2}\cos^2\theta + \frac{3}{2}\sin^2\theta$$

$$\therefore \text{L.H.S.} = \cos^2\theta + \frac{1}{2}\cos^2\theta + \frac{3}{2}\sin^2\theta$$

$$= \frac{3}{2}\cos^2\theta + \frac{3}{2}\sin^2\theta$$

$$= \frac{3}{2}\left[\cos^2\theta + \sin^2\theta\right]$$

$$= \frac{3}{2} \quad (\because \cos^2\theta + \sin^2\theta = 1)$$

$= \text{R.H.S.}$

14. $\sin^2 82\frac{1°}{2} - \sin^2 22\frac{1°}{2}$ కనుగొనుము

Sol. $A = \sin^2 82\frac{1°}{2}$ and $B = \sin^2 22\frac{1°}{2}$, then

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$\sin^2 82\frac{1^\circ}{2} - \sin^2 22\frac{1^\circ}{2}$

$= \sin^2 A - \sin^2 B$

$= \sin(A+B)\sin(A-B)$

$= \sin 105^\circ \sin 60^\circ$

$= \sin(90^\circ + 15^\circ)\sin 60^\circ$

$= \cos 15^\circ \sin 60^\circ$

$= \frac{1+\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{3}}{2} = \frac{3+\sqrt{3}}{4\sqrt{2}}$

15. $\sin^2 52\frac{1}{2}^0 - \sin^2 22\frac{1}{2}^0 = \frac{\sqrt{3}+1}{4\sqrt{2}}$ అని చూపుము

Sol. $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)$

$[\because \sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)]$

$= \sin\left(\frac{\pi}{8}+\frac{A}{2}+\frac{\pi}{8}-\frac{A}{2}\right)\sin\left(\frac{\pi}{8}+\frac{A}{2}-\frac{\pi}{8}+\frac{A}{2}\right)$

$= \sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{2A}{2}\right)$

$= \sin\frac{\pi}{4}\sin A = \sin 45^\circ \sin A = \frac{1}{\sqrt{2}}\sin A$

16. $\cos^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$ కనుగొనుము

Sol. $\cos^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$

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$[\because \cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)]$

$= \cos\left(52\frac{1}{2}° + 22\frac{1}{2}°\right)\cos\left(52\frac{1}{2}° - 22\frac{1}{2}°\right)$

$= \cos 75° \cos 30°$

$= \frac{\sqrt{3}}{2}(\cos 75°)$

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = \frac{3-\sqrt{3}}{4\sqrt{2}}$

17. $\cos^2 112\frac{1}{2}^0 - \sin^2 52\frac{1}{2}^0$ కనుగొనుము

$\cos^2 112\frac{1}{2}^0 - \sin^2 52\frac{1}{2}^0 = \cos\left(112\frac{1}{2}^0 + 52\frac{1}{2}^0\right)\cos\left(112\frac{1}{2}^0 - 52\frac{1}{2}^0\right)$

$\left(\because \cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)\right)$

$= \cos 165^0 . \cos 60^0 = \frac{1}{2}\cos\left(180^0 - 15^0\right) = -\frac{1}{2}\cos 15^0$

$= -\frac{1}{2}\left\{\frac{\sqrt{3}+1}{2\sqrt{2}}\right\}$

స్వల్ప సమాధాన ప్రశ్నలు

18. $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8} = \frac{3}{2}$ అని చూపుము

Solution:

$\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$

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$$\left(\sin\frac{\pi}{8}\right)^4+\left\{\sin^2\frac{\pi}{2}-\frac{\pi}{8}\right\}^4+\left\{\sin^2\left(\frac{\pi}{2}+\frac{\pi}{8}\right)\right\}^2+\left\{\sin\left(\pi-\frac{\pi}{8}\right)\right\}^4$$

$$\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}+\sin^4\frac{\pi}{8}$$

$$2\left\{\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}\right\}=2\left\{\left(\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}\right)^2-2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right\}$$

$$=2-4\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$$

$$=2-\left(2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)^2=2-\sin^2\frac{\pi}{4}=2-\frac{1}{2}=\frac{3}{2}$$

19. $\tan 3A\tan 2A\tan A=\tan 3A-\tan 2A-\tan A$ అని చూపుము

Solution:

$$\tan 3A=\tan(2A+A)$$

$$\tan 3A=\frac{\tan 2A+\tan A}{1-\tan 2A\tan A}$$

$$\tan 3A-\tan 2A\tan A\tan 3=\tan 2A+\tan A$$

$$\tan 3A-\tan 2-\tan A=\tan A\tan 2A\tan 3A$$

20. $\cos^4\frac{\pi}{8}+\cos^4 3\frac{\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=\frac{3}{2}$ అని చూపుము

21. (i) $\sin A\sin(60-A)\sin(60^0+A)=\frac{1}{4}\sin 3A$

(ii) $\cos A\cos(60+A)\cos(60^0-A)=\frac{1}{4}\cos 3A$

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(iii) $\tan A \tan\left(60^0 + A\right)\tan\left(60^0 - A\right) = \tan 3A$

(iv) $\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}$

(v) $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{16}$ అని చూపుము

22. $\tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta = \cot\theta$ అని చూపుము

23. $\cos\alpha = \frac{-3}{5}, \frac{\pi}{2} < \alpha < \pi$, $\sin\beta = \frac{7}{25}$, $0 < \beta < \frac{\pi}{2}$, అయితే

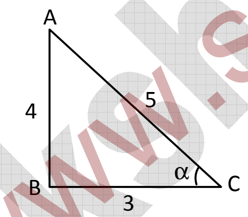
$\tan(\alpha + \beta)$ and $\sin(\alpha + \beta)$ కనుగొనుము

Sol. $\cos\alpha = \frac{-3}{5}$, where $\frac{\pi}{2} < \alpha < \pi$

α in II quadrant

$\sin\beta = \frac{7}{25}$, where $0 < \beta < \frac{\pi}{2}$

β in I Quadrant



$AB^2 = AC^2 - BC^2$

$= 5^2 - 3^2 = 25 - 9 = 16$

$\therefore AB = 4$

$\cos\alpha = -\frac{3}{5}, \sin\alpha = \frac{4}{5}, \tan\alpha = -\frac{4}{3}$

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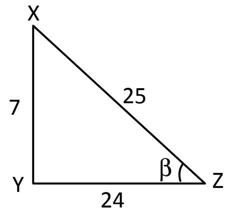


$$YZ^2 = XZ^2 - XY^2$$
$$= 25^2 - 7^2$$
$$= 625 - 49 = 576$$
$$YZ = 24$$

$$\sin\beta = \frac{7}{25}, \cos\beta = \frac{24}{25}, \tan\beta = \frac{7}{24}$$

$$\therefore \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$= \frac{\frac{-4}{3} + \frac{7}{24}}{1 + \frac{4}{3} \times \frac{7}{24}} = \frac{\frac{-32+7}{4}}{1 + \frac{7}{18}}$$

$$= \frac{\frac{-25}{24}}{\frac{18+7}{18}} = \frac{-25}{24} \times \frac{18}{25} = -\frac{3}{4}$$

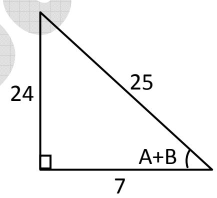
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{4}{5} \times \frac{24}{25} + \frac{-3}{5} \times \frac{7}{25}$$

$$= \frac{96}{125} - \frac{21}{125} = \frac{75}{125} = \frac{3}{5}$$

24. $0 < A < B < \frac{\pi}{4}$ $\sin(A+B) = \frac{7}{25}$ $\cos(A-B) = \frac{12}{13}$ ఐతే tan2A కనుగొనుము .

Sol. $\sin(A+B) = \frac{24}{25}$



$$\tan(A+B) = \frac{24}{7}$$

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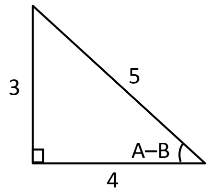
$\cos(A-B)=\frac{4}{5}$



$\tan(A-B)=\frac{3}{4}$

Now 2A = (A + B) + (A – B)

$$\tan 2A=\tan[(A+B)+(A-B)]$$

$$=\frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\tan(A-B)}$$

$$=\frac{\frac{24}{7}+\frac{3}{4}}{1-\frac{24}{7}\times\frac{3}{4}}=\frac{96+21}{28-72}=\frac{-117}{44}$$

25. $A+B, A$ లు లఘు కోణాలు $\sin(A+B)=\frac{24}{25}$ మరియు $\tan A=\frac{3}{4}$ ఐతే $\cos B$ కనుగొనుము

Solution:

$\sin(A+B)=\frac{24}{25}$ $\quad\therefore \cos(A+B)=\frac{7}{25}$

$\tan A=\frac{3}{4}$ $\quad\therefore \sin A=\frac{3}{5}$ $\quad \cos A=\frac{4}{5}$

$$\cos B=\cos(A+B-A)=\cos(A+B)\cos A+\sin(A+B)\sin A$$

$$\frac{7}{25}\times\frac{4}{5}+\frac{24}{25}\times\frac{3}{5}=\frac{100}{125}=\frac{4}{5}$$

26. $\tan\alpha-\tan\beta=m$, $\cot\alpha-\cot\beta=n$ ఐతే $\cot(\alpha-\beta)=\frac{1}{m}-\frac{1}{n}$ అని చూపుము.

Sol. We have $\tan\alpha-\tan\beta=m$

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$$\frac{1}{\cot\alpha} - \frac{1}{\cot\beta} = m$$

$$\frac{\cot\beta - \cot\alpha}{\cot\alpha\cot\beta} = m$$

$$\therefore \frac{\cot\alpha\cot\beta}{\cot\beta - \cot\alpha} = \frac{1}{m} \quad ...(1)$$

$$\cot\alpha - \cot\beta = n$$

$$-(\cot\beta - \cot\alpha) = n$$

$$\cot\beta - \cot\alpha = -n$$

$$\frac{1}{\cot\beta - \cot\alpha} = -\frac{1}{n} \quad ...(2)$$

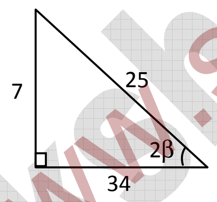
$$\text{L.H.S.} = \cot(\alpha - \beta) = \frac{\cot\alpha\cot\beta}{\cot\beta - \cot\alpha}$$

$$= \frac{\cot\alpha\cot\beta}{\cot\beta - \cot\alpha} + \frac{1}{\cot\beta - \cot\alpha}$$

$$= \frac{1}{m} - \frac{1}{n} \ (\because \text{from}(1)\ \&\ (2)) = \text{R.H.S.}$$

27. $\tan(\alpha - \beta) = \frac{7}{24}$, $\tan\alpha = \frac{4}{3}$, అయి α, β లు ప్రథమ పాదం లోని కోణాలు అయితే $\alpha + \beta = \pi/2$ అని చూపుము.

Sol. $\tan(\alpha - \beta) = \frac{7}{24}$ and $\tan\alpha = \frac{4}{3}$



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$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\Rightarrow \frac{\frac{4}{3}-\tan\beta}{1+\frac{4}{3}\tan\beta}=\frac{7}{24}$$

$$\Rightarrow \frac{\frac{4-3\tan\beta}{3}}{\frac{3+4\tan\beta}{3}}=\frac{7}{24}$$

$$\Rightarrow \frac{4-3\tan\beta}{3+4\tan\beta}=\frac{7}{24}$$

$$\Rightarrow 24[4-3\tan\beta]=7(3+4\tan\beta)$$

$$\Rightarrow 96-72\tan\beta=21+28\tan\beta$$

$$\Rightarrow 96-21=28\tan\beta+72\tan\beta$$

$$\Rightarrow 100\tan\beta=75$$

$$\therefore \tan\beta=\frac{75}{100}=\frac{3}{4}$$

$$\therefore \tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$=\frac{\frac{4}{3}+\frac{3}{4}}{1-\frac{4}{3}\cdot\frac{3}{4}}=\alpha$$

$$\tan(\alpha+\beta)=\tan\frac{\pi}{2}$$

$$\therefore \alpha+\beta=\frac{\pi}{2}$$

28. క్రింది వానిని విస్తరించిం వ్రాయండి.

1. sin(A + B + C).

Sol. sin(A + B + C)

$$=\sin[(A+B)-C]$$

$$=\sin(A+B)\cos C-\cos(A-B)\sin C$$

$$=(\sin A\cos B+\cos A\sin B)\cos C-[\cos(A-B)\cos B-\sin A\sin B]\sin c$$

$$=\sin A\cos B\cos C+\cos A\sin B\cos C-\cos A\cos B\sin C+\sin A\sin B\sin C$$

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2. cos (A – B – C).

Sol. cos (A – B – C) = cos[(A – B) – C]

$= \cos(A-B)\cos C + \sin(A-B)\sin C$

$= (\cos A\cos B + \sin A\sin B)\cos C + (\sin A\cos B - \cos A\sin B]\sin C$

$= \cos A\cos B\cos C + \sin A\sin B\cos C + \sin A\cos B\sin C - \cos A\sin B\sin C$

29. $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{a+b}{a-b}$ అయితే $a\tan\beta = b\tan\alpha$ అని చూపుము

Sol. $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{a+b}{a-b}$

కాంపొనెన్డో, డివిడెన్డో సిద్ధాంతం నుండి

$\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = \frac{a+b+a-b}{a+b-a+b} = \frac{2a}{2b} = \frac{a}{b}$

$\Rightarrow \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = \frac{a}{b}$

$\Rightarrow \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta+\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta-\sin\alpha\cos\beta+\cos\alpha\sin\beta} = \frac{a}{b}$

$\Rightarrow \frac{2\sin\alpha\cos\beta}{2\cos\alpha\sin\beta} = \frac{a}{b}$

$\Rightarrow \tan\alpha\cot\beta = \frac{a}{b}$

$\Rightarrow b\tan\alpha = a\tan\beta$

hence, $a\tan\beta = b\tan\alpha$

30. $A-B=\frac{3\pi}{4}$ ఐతే (1 – tan A) (1 + tan B) = 2 అని చూపుము.

Sol.

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$A - B = 135°$

$\tan(A - B) = \tan 135°$

$= \tan(90° + 45)° = -\cot 45° = -1$

$\therefore \frac{\tan A - \tan B}{1 + \tan A \tan B} = -1$

$\tan A - \tan B = -(1 + \tan A \tan B)$

$\tan A - \tan B = -1 - \tan A \tan B$

$\tan A - \tan B + \tan A \tan B = -1$

$\tan B - \tan A - \tan A \tan B = 1$ ...(1)

$L.H.S. = (1 - \tan A)(1 + \tan B)$

$= 1 + (\tan B - \tan A - \tan A \tan B)$

$= 1 + 1$ ($\because$ from(1))

$= 2 = R.H.S.$

31. $A + B + C = \frac{\pi}{2}$ అయిA, B, C లు $\frac{\pi}{2}$ బేసి గుణకాలు కాకపోతే cot A + cot B + cot C = cotA cotB cotC అని చూపుము.

Sol. $A + B + C = \frac{\pi}{2}$

$A + B = \frac{\pi}{2} - C$

$\cot(A + B) = \cot\left(\frac{\pi}{2} - C\right)$

$\frac{\cot A \cot B - 1}{\cot B + \cot A} = \tan C$

$\frac{\cot A \cot B - 1}{\cot B + \cot A} = \frac{1}{\cot C}$

$\cot C[\cot A \cot B - 1] = \cot B + \cot A$

$\cot A \cot B \cot C - \cot C \cot A + \cot B$

$\cot A \cot B \cot C = \cot A + \cot B + \cot C$

$\therefore \cot A + \cot B + \cot C = \cot A \cot B \cot C$

32. $A + B + C = \frac{\pi}{2}$ అయిA, B, C లు $\frac{\pi}{2}$ బేసి గుణకాలు కా tan A tan B + tan B tan C + tan C tan A = 1 అని చూపుము.

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Sol. $A+B+C=\frac{\pi}{2}$

$A+B=\frac{\pi}{2}-C$

$\tan(A+B)=\tan\left(\frac{\pi}{2}-C\right)$

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=\cot C$

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{1}{\tan C}$

$\Rightarrow \tan C[\tan A+\tan B]=1-\tan A\tan B$

$\Rightarrow \tan C\tan A+\tan C\tan B=1-\tan A\tan B$

$\Rightarrow \tan A\tan B+\tan B\tan C+\tan C\tan A=1$

33. $\Sigma\frac{\cos(B+C)}{\cos B\cos C}=2$ అని చూపుము.

.

Sol. L.H.S. = $\Sigma\frac{\cos(B+C)}{\cos B\cos C}$

$=\Sigma\frac{\cos B\cos C-\sin B\sin C}{\cos B\cos C}$

$=\Sigma\frac{\cos B\cos C}{\cos B\cos C}-\frac{\sin B\sin C}{\cos B\cos C}$

$=\Sigma(1-\tan B\tan C)$

$=1-\tan B\tan C+1-\tan C\tan A+\quad 1-\tan A\tan B$

$=3-(\tan A\tan B+\tan B\tan C+\tan C\tan A)$

$=3-1\ (\because \text{from(b)})$

$=2=\text{R.H.S.}$

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34. $\sin^2 \alpha + \cos^2 (\alpha + \beta) + 2 \sin \alpha \sin \beta \cos(\alpha + \beta)$ అనేది α మీద ఆదార పడదని చూపుము.

Sol. $\sin^2\alpha + \cos^2 (\alpha+\beta) + 2 \sin\alpha \sin\beta \cos(\alpha+\beta)$

$$= \sin^2 \alpha + 1 - \sin^2(\alpha+\beta) + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$$
$$= 1 + [\sin^2 \alpha - \sin^2(\alpha+\beta)] + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$$
$$= 1 + \sin(\alpha+\alpha+\beta)\sin(\alpha-\alpha-\beta) + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$$
$$= 1 + \sin(2\alpha+\beta)\sin(-\beta) + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$$
$$= 1 - \sin(2\alpha+\beta)\sin\beta + [2\sin\alpha\cos(\alpha+\beta)]\sin\beta$$
$$= 1 - \sin(2\alpha+\beta)\sin\alpha + [\sin(\alpha+\alpha+\beta) + \sin(\alpha-\alpha-\beta)]\sin\beta$$
$$= 1 - \sin(2\alpha+\beta)\sin\alpha + [\sin(2\alpha+\beta) - \sin\beta]\sin\beta$$
$$= 1 - \sin(2\alpha+\beta)\sin\alpha + \sin(2\alpha+\beta)\sin\beta - \sin^2\beta$$
$$= 1 - \sin^2\beta = \cos^2\beta$$

$\sin^2 \alpha + \cos^2 (\alpha + \beta) + 2 \sin \alpha \sin \beta \cos(\alpha + \beta)$ అనేది α మీద ఆదార పడదు.

35. $\cot\frac{\pi}{16}\cdot\cot\frac{2\pi}{16}\cdot\cot\frac{3\pi}{16}\ldots\cot\frac{7\pi}{16} = 1$ అని చూపుము.

Sol. $\cot\frac{\pi}{16}\cdot\cot\frac{2\pi}{16}\cdot\cot\frac{3\pi}{16}\ldots\cot\frac{7\pi}{16}$

$$= \left(\cot\frac{\pi}{16}\cdot\cot\frac{7\pi}{16}\right)\left(\cot\frac{2\pi}{16}\cdot\cot\frac{6\pi}{16}\right)\left(\cot\frac{3\pi}{16}\cdot\cot\frac{5\pi}{16}\right)\cdot\cot\frac{4\pi}{16}$$
$$= \left[\cot\frac{\pi}{16}\cdot\cot\left(\frac{\pi}{2}-\frac{\pi}{16}\right)\right]\left[\cot\frac{2\pi}{16}\cdot\cot\left(\frac{\pi}{2}-\frac{2\pi}{16}\right)\right]\left[\cot\frac{3\pi}{16}\cdot\cot\left(\frac{\pi}{2}-\frac{3\pi}{16}\right)\right]\cdot\cot\frac{\pi}{4}$$
$$= \left(\cot\frac{\pi}{16}\cdot\tan\frac{\pi}{16}\right)\left(\cot\frac{2\pi}{16}\cdot\tan\frac{2\pi}{16}\right)\left(\cot\frac{3\pi}{16}\cdot\tan\frac{3\pi}{16}\right)\cdot 1$$
$$= 1\times1\times1\times1 = 1$$

36. $\tan 70° - \tan 20° = 2 \tan 50°$ అని చూపుము.

Sol. $\tan 50° = \tan(70° - 20°)$

$$= \frac{\tan 70^\circ - \tan 20^\circ}{1 + \tan 70^\circ \tan 20^\circ}$$

$\Rightarrow \tan 70^\circ - \tan 20^\circ$

$= \tan 50^\circ (1 + \tan 70^\circ \cdot \tan 20^\circ)$

$= \tan 50^\circ (1 + \tan 70^\circ \cdot \tan(90^\circ - 70^\circ)]$

$= \tan 50^\circ [1 + \tan 70^\circ \cdot \cot 70^\circ]$

$= \tan 50^\circ [1 + 1]$

$= 2 \tan 50^\circ$

$\therefore \tan 70^\circ - \tan 20^\circ = 2 \tan 50^\circ$

37. A + B = 45° ఐతే

i) (1 + tan A) (1 + tan B) = 2

ii) (cot A – 1)(cot B – 1) = 2 అని చూపుము.

Sol. i) A + B = 45°

$\Rightarrow \tan(A + B) = \tan 45^\circ = 1$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$\Rightarrow \tan A + \tan B = 1 - \tan A \tan B$

$\Rightarrow \tan A + \tan B + \tan A \tan B = 1 \ldots (1)$

Now, $(1 + \tan A)(1 + \tan B) = 1 + \tan A + \tan B + \tan A \tan B = 2$ (from(1))

ii) A + B = 45° $\Rightarrow$ cot(A + B) = cot 45° = 1

$$\Rightarrow \frac{\cot A \cot B - 1}{\cot B + \cot A} = 1$$

$\Rightarrow \cot A \cot B - 1 = \cot A + \cot B$

$\Rightarrow \cot A \cot B - \cot A - \cot B = 1 \ldots (2)$

Now, $(\cot A - 1)(\cot B - 1) = \cot A \cot B - \cot A - \cot B + 1 = 2$ (from(2))

38. A, B, C లు త్రిభుజంలోని కోణాలు మరియు ఏది కూడా π/2 కు సమం కాక పోతే చూపుము i) tan A + tan B + tan C = tan A tan B tan C

ii) cotA cotB + cotB cotC + cot C cot A =1 అని చూపుము.

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Sol. i) $A + B + C = \pi$

$\Rightarrow A+B=\pi-C$

$\Rightarrow \tan(A+B)=\tan(\pi-C)$

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$

$\Rightarrow \tan A+\tan B=-\tan C(1-\tan A\tan B)$

$\Rightarrow \tan A+\tan B=-\tan C+\tan A\tan B\tan C$

$\Rightarrow \tan A+\tan B+\tan C=\tan A\tan B\tan C$

ii) $A + B + C = \pi$

$\Rightarrow A+B=\pi-C$

$\Rightarrow \cot(A+B)=\cot(\pi-C)$

$\Rightarrow \frac{\cot A\cot B-1}{\cot A+\cot B}=-\cot C$

$\Rightarrow \cot A\cot B-1=-\cot A\cot C-\cot B\cot C$

$\Rightarrow \cot A\cot B+\cot A\cot C+\cot B\cot C=1$

39. ABC అత్రిభుజంలో $\cot A + \cot B + \cot C = \sqrt{3}$ ఐతే ABC సమభాహు త్రిభుజం అని చూపుము

Sol. $A + B + C = 180°$

$\Rightarrow \Sigma \cot A \cot B = 1$

Now, $\Sigma(\cot A - \cot B)^2 =$

$\Sigma\cot^2 A + \cot^2 B - 2\cot A\cot B$

$= 2\cot^2 A+2\cot^2 B+2\cot^2 C-2\cot A\cot B-2\cot B\cot C-2\cot C\cot A$

(on expanding)

$= 2\{(\cot A+\cot B+\cot C)^2-2(\cot A\cot B)-2\cot B\cot C-2\cot C\cot A\}$

$-2(\cot A\cot B+\cot B\cot C+\cot C\cot A)$

$= 2(\cot A+\cot B+\cot C)^2-6(\cot A\cot B+\cot B\cot C+\cot C\cot A)$

$= 2\cdot 3-6=0$

$\Rightarrow \cot A=\cot B=\cot C$

$\Rightarrow \cot A=\cot B=\cot C=\frac{\sqrt{3}}{3}=\frac{1}{\sqrt{3}}$

(since $\cot A+\cot B+\cot C=\sqrt{3}$)

$\Rightarrow A+B+C=60°$

40. సమబాహు ప్రథమ పాదం x లోని యొక్క ఏ విలువ $\frac{2\tan x}{1-\tan^2 x}$ ధనాత్మకం?

Sol. $\frac{2\tan x}{1-\tan^2 x} > 0 \Rightarrow \tan 2x > 0$

$\Rightarrow 0 < 2x < \frac{\pi}{2}$ (ప్రథమ పాదం)

$\Rightarrow 0 < x < \frac{\pi}{4}$

41. tan 56° – tan 11° – tan 56° tan 11° కనుగొనుము

Sol. We have 56° – 11° = 45°

$\tan(56° - 11°) = \tan 45°$

$\frac{\tan 56° - \tan 11°}{1 + \tan 56° \tan 11°} = 1$

$\tan 56° - \tan 11° = 1 + \tan 56° \tan 11°$

$\tan 56° - \tan 11° - \tan 56° \tan 11° = 1$

MULTIPLE AND SUB MULTIPLE ANGLES

Formulae :

I. i) Sin 2A = 2 sin A cos A = $\frac{2\tan A}{1+\tan^2 A}$.

ii) sin A = $2\sin\frac{A}{2}\cos\frac{A}{2} = \frac{2\tan A/2}{1+\tan^2 A/2}$.

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iii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$

$$= 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

iv) $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} = 2\cos^2\frac{A}{2} - 1$

$$= 1 - 2\sin^2\frac{A}{2} = \frac{1-\tan^2 A}{1+\tan^2 A}$$

v) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

vi) $\tan A = \dfrac{2\tan A/2}{1-\tan^2 A/2} = \operatorname{cosec} 2A - \cot 2A$

vii) $\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}$

viii) $\cot A = \dfrac{\cot^2 (A/2) - 1}{2\cot A/2} = \operatorname{cosec} 2A + \cot 2A$

ix) $\tan\left(\frac{\pi}{4} + \frac{A}{2}\right) = \dfrac{1+\tan A/2}{1-\tan A/2}$

$$= \frac{\cos A/2 + \sin A/2}{\cos A/2 - \sin A/2} = \frac{1+\sin A}{\cos A} = \frac{\cos A}{1-\sin A}$$

$$= \sec A + \tan A = \cot\left(\frac{\pi}{4} - \frac{A}{2}\right).$$

x) $\tan\left(\frac{\pi}{4} - \frac{A}{2}\right) = \dfrac{1-\tan A/2}{1+\tan A/2}$

$$= \frac{\cos A/2 - \sin A/2}{\cos A/2 + \sin A/2} = \sqrt{\frac{1-\sin A}{1+\sin A}} = \frac{1-\sin A}{\cos A}$$

$$= \sec A - \tan A = \cot\left(\frac{\pi}{4} + \frac{A}{2}\right)$$

II. i) $\sin 3A = 3\sin A - 4\sin^3 A$

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ii) $\cos 3A = 4\cos^3 A - 3\cos A$

iii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

iv) $\sin^3 A = \frac{1}{4}[3\sin A - \sin 3A]$

v) $\cos^3 A = \frac{1}{4}[\cos 3A + 3\cos A]$

III. $\sin^2 A = \dfrac{1-\cos 2A}{2}$; $\sin A = \pm\sqrt{\dfrac{1-\cos 2A}{2}}$

$\cos^2 A = \dfrac{1+\cos 2A}{2}$; $\cos A = \pm\sqrt{\dfrac{1+\cos 2A}{2}}$

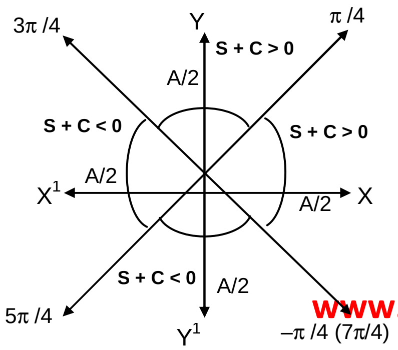
$\tan^2 A = \dfrac{1-\cos 2A}{1+\cos 2A}$; $\tan A = \pm\sqrt{\dfrac{1-\cos 2A}{1+\cos 2A}}$

$\sin^2 \dfrac{A}{2} = \dfrac{1-\cos A}{2}$; $\sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$

$\cos^2 \dfrac{A}{2} = \dfrac{1+\cos A}{2}$; $\cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$

$\tan^2 \dfrac{A}{2} = \dfrac{1-\cos A}{1+\cos A}$; $\tan\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}$

IV. $S = \sin\dfrac{A}{2}$ and $C = \cos\dfrac{A}{2}$ ఐతే



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i) S + C = $\pm\sqrt{1+\sin A}$

ii) S – C = $\pm\sqrt{1-\sin A}$

iii) 2sin$\frac{A}{2}$ = $\pm\sqrt{1+\sin A}\pm\sqrt{1-\sin A}$

iv) 2cos$\frac{A}{2}$ = $\pm\sqrt{1+\sin A}\pm\sqrt{1-\sin A}$

v) a) S + C > 0, S – C > 0 if

$\frac{\pi}{4} < \frac{A}{2} < \frac{3\pi}{4}$

b) S + C < 0, S – C > 0 if

$\frac{3\pi}{4} < \frac{A}{2} < \frac{5\pi}{4}$

c) S + C < 0, S – C < 0 if

$\frac{5\pi}{4} < \frac{A}{2} < \frac{7\pi}{4}$

d) S + C > 0, S – C < 0 if

$\frac{-\pi}{4} < \frac{A}{2} < \frac{\pi}{4}$

VII.

	18^o	36^o	54^o	72^o
Sin	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$

Cos $\frac{\sqrt{10+2\sqrt{5}}}{4}$ $\frac{\sqrt{5}+1}{4}$ $\frac{\sqrt{10-2\sqrt{5}}}{4}$ $\frac{\sqrt{5}-1}{4}$

అతి స్వల్ప సమాధాన ప్రశ్నలు

1. $\frac{\sin 2\theta}{1+\cos 2\theta}$ సూక్ష్మీకరించుము

Sol. $\frac{\sin 2\theta}{1+\cos 2\theta}=\frac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$

$=\frac{2\sin\theta\cos\theta}{2\cos^2\theta}=\frac{\sin\theta}{\cos\theta}=\tan\theta$

2. $\sin^2 42°-\sin^2 12°$ కనుగొనుము

Sol. $\sin(42°+12°)\sin(42°-12°)$

$=\sin 54°\sin 30°$

$=\frac{\sqrt{5}+1}{4}\cdot\frac{1}{2}=\frac{\sqrt{5}+1}{8}$

3. $\frac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}$ ను $\tan\frac{\theta}{2}$ లో వ్రాయండి.

Sol. $\frac{1-\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}=\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}$

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$$= \frac{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}-\left(1-2\sin^2\frac{\theta}{2}\right)}{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+2\cos^2\frac{\theta}{2}-1}$$

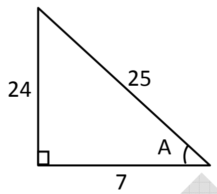
$$= \frac{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+2\sin^2\frac{\theta}{2}-1}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+2\cos^2\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}+2\cos^2\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)} = \tan\frac{\theta}{2}$$

4. If $\cos A = \frac{7}{25}$ and $\frac{3\pi}{2} < A < 2\pi$, then find the value of $\cot\frac{A}{2}$.

Sol. $\cos A = \frac{7}{25}$, where $\frac{3\pi}{2} < A < 2\pi$



$$\sin A = -\frac{24}{25}, \tan A = -\frac{24}{7}, \cos A = \frac{7}{25}$$

$$\cot\frac{A}{2} = \frac{\sin A}{1-\cos A} = \frac{-\frac{24}{25}}{1-\frac{7}{25}}$$

$$= \frac{-24}{25}\times\frac{25}{18} = \frac{-24}{18}\times\frac{-4}{3} = \frac{16}{9}$$

5. $0<\theta<\frac{\pi}{8}$ ఐతే $\sqrt{2+\sqrt{2+2+2\cos 4\theta}}=2\cos\frac{\theta}{2}$ అని చూపుము.

Sol. $\sqrt{2+\sqrt{2+2+2\cos 4\theta}}$

$1+\cos 4\theta = 2\cos^2 2\theta$

$2(1+\cos 4\theta) = 4\cos^2 2\theta$

$\sqrt{2(1+\cos 4\theta)} = 2\cos 2\theta$

$2+\sqrt{2(1+\cos 4\theta)} = 2+2\cos 2\theta$

$= 2(1+\cos 2\theta) = 2(2\cos^2\theta) = 4\cos^2\theta$

$\sqrt{2+\sqrt{2(1+\cos 4\theta)}} = \sqrt{\cos^2\theta} = 2\cos\theta$

$2+\sqrt{2+\sqrt{2(1+\cos 4\theta)}} = 2+2\cos\theta$

$= 2(1+\cos\theta) = 2\left(2\cos^2\frac{\theta}{2}\right) = 4\cos^2\frac{\theta}{2}$

6. $\frac{\cos 3A+\sin 3A}{\cos A-\sin A} = 1+2\sin 2A$ అని చూపుము.

Sol. L.H.S. = $\frac{\cos 3A+\sin 3A}{\cos A-\sin A}$

$= \frac{4\cos^3 A-3\cos A+3\sin A-4\sin^3 A}{\cos A-\sin A}$

$= \frac{4(\cos^3 A-\sin^3 A)-3(\cos A-\sin A)}{\cos A-\sin A}$

$= \frac{4[(\cos A-\sin A)(\cos^2 A+\cos A\sin A+\sin^2 A)]-3(\cos A-\sin A)}{\cos A-\sin A}$

$= \frac{(\cos A-\sin A)[(4+4\sin A\cos A)-3]}{(\cos A-\sin A)}$

$= 1+4\sin A\cos A$

$= 1+2\sin 2A = \text{R.H.S.}$

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7. $\cot\left(\frac{\pi}{4}-\theta\right)=\frac{\cos 2\theta}{1-\sin 2\theta}$ అని చూపుము తద్వార cot 15° కనుగొనుము

Sol. R.H.S. = $\frac{\cos 2\theta}{1-\sin 2\theta}$

$$=\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta}$$

$$=\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta-\sin\theta)^2}$$

$$=\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$$

$$=\frac{\sin\theta\left[\frac{\cos\theta}{\sin\theta}+1\right]}{\sin\theta\left[\frac{\cos\theta}{\sin\theta}-1\right]}$$

$$=\frac{\cot\theta+1}{\cot\theta-1}$$

$$=\frac{\cot\theta\cdot\cot\frac{\pi}{4}+1}{\cot\theta-\cot\frac{\pi}{4}}$$

$$=\cot\left(\frac{\pi}{4}-\theta\right)=\text{L.H.S.}$$

Put θ = 30° ⇒ cot 15° = $\frac{\cos 60^\circ}{1-\sin 60^\circ}$

$$=\frac{\frac{1}{2}}{1-\frac{\sqrt{3}}{2}}=\frac{1}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$=\frac{2+\sqrt{3}}{4-3}=2+\sqrt{3}$$

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8. $\dfrac{1}{\cos 290^\circ}+\dfrac{1}{\sqrt{3}\sin 250^\circ}=\dfrac{4}{\sqrt{3}}$ అని చూపుము..

SOL. $\cos 290^\circ=\cos(270^\circ+20^\circ)=\sin 20^\circ$

$\sin 250^\circ=\sin(270^\circ-20^\circ)=\cos 20^\circ$

$$\text{L.H.S.}=\frac{1}{\sin 20^\circ}-\frac{1}{\sqrt{3}\cos 20^\circ}=\frac{\sqrt{3}\cos 20^\circ-\sin 20^\circ}{\sqrt{3}\sin 20^\circ\cos 20^\circ}$$

$$=\frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ\right)}{\frac{\sqrt{3}}{2}(2\sin 20^\circ\cos 20^\circ)}=\frac{4}{\sqrt{3}}\left[\frac{\sin 60^\circ\cos 20^\circ-\cos 60^\circ\sin 20^\circ}{\sin 40^\circ}\right]$$

$$=\frac{4}{\sqrt{3}}\frac{\sin(60^\circ-20^\circ)}{\sin 40^\circ}=\frac{4}{\sqrt{3}}=\text{R.H.S.}$$

స్వల్ప సమాధాన ప్రశ్నలు

9. $\dfrac{\sin 2x}{(\sec x+1)}\cdot\dfrac{\sec 2x}{(\sec 2x+1)}=\tan\left(\dfrac{x}{2}\right)$ అని చూపుము..

Sol. L.H.S. = $\dfrac{\sin 2x}{(\sec x+1)}\cdot\dfrac{\sec 2x}{(\sec 2x+1)}$

$$=\frac{\sin 2x}{\frac{1}{\cos x}+1}\times\frac{\frac{1}{\cos 2x}}{\frac{1}{\cos x}+1}$$

$$=\frac{\sin 2x}{\frac{1+\cos x}{\cos x}}\times\frac{1}{\cos 2x}\times\frac{\cos 2x}{1+\cos 2x}$$

$$=\frac{\sin 2x\cdot\cos x}{1+\cos x}\cdot\frac{1}{1+\cos 2x}$$

$$=\frac{2\sin x\cos^2 x}{1+\cos x}=\frac{1}{2\cos^2 x}$$

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$$= \frac{\sin x}{1+\cos x} = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1+2\cos^2\frac{x}{2}-1}$$

$$= \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \tan\frac{x}{2} = \text{R.H.S.}$$

10. $\cos\alpha = \frac{3}{5}, \cos\beta = \frac{5}{13}$ మరియు α, β లఘు కోణాలు అయితే

(i) $\sin^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{65}$ (ii) $\cos^2\left(\frac{\alpha+\beta}{2}\right) = \frac{16}{65}$ అని చూపుము

Solution:

$$\cos\alpha = \frac{3}{5} \quad \cos\beta = \frac{5}{13}$$

$$\sin\alpha = \frac{3}{5} \quad \sin\beta = \frac{5}{13}$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{3}{4}\times\frac{5}{13} + \frac{4}{5}\times\frac{12}{13} = \frac{63}{65}$$

$$2\sin^2\left(\frac{\alpha-\beta}{2}\right) - 1 - \cos(\alpha-\beta) = 1 - \frac{63}{65} \Rightarrow 2\sin^2\left(\frac{\alpha-\beta}{2}\right) = \frac{2}{65}$$

$$\therefore \sin^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{65}$$

$$\cos^2\left(\frac{\alpha+\beta}{2}\right) = \frac{1+\cos(\alpha+\beta)}{2} \Rightarrow 2\cos^2\left(\frac{\alpha+\beta}{2}\right) = 1 - \frac{33}{65}$$

$$2\cos^2\left(\frac{\alpha+\beta}{2}\right) = \frac{32}{65} \Rightarrow \cos^2\left(\frac{\alpha+\beta}{2}\right) = \frac{16}{65}$$

11. $\cos A = \frac{\cos 3A}{(2\cos 2A - 1)}$ అని చూపుము దీని నుండి $\cos 15°$ కనుగొనుము

Sol. R.H.S. = $\frac{\cos 3A}{(2\cos 2A - 1)}$

$$= \frac{4\cos^2 A - 3\cos A}{2(2\cos^2 A - 1) - 1}$$

$$= \frac{\cos(4\cos^2 A - 3)}{(4\cos^2 A - 3)} = \cos A = \text{L.H.S.}$$

$\cot 15° = \cot(45° - 30°)$

$$= \frac{\cot 45° \cdot \cot 30° + 1}{\cot 30° - \cot 45°}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2}{3-1}$$

$$= \frac{3+1+2\sqrt{3}}{2} = \frac{4+2\sqrt{3}}{2}$$

$$= \frac{2[2+\sqrt{3}]}{2} = 2+\sqrt{3}$$

$\therefore \cot 15° = 2+\sqrt{3}$

12. $\cos A = \frac{\sin 3A}{1+2\cos 2A}$ అని చూపుముదీని నుండి sin 15°.కనుగొనుము.

Sol. R.H.S. = $\frac{\sin 3A}{1+2\cos 2A}$

$$= \frac{3\sin A - 4\sin^3 A}{1+2\left(1-2\sin^2 A\right)}$$

$$= \frac{\sin[3-4\sin^2 A]}{[1+2-4\sin^2 A]}$$

$$= \frac{\sin A[3-4\sin^2 A]}{[3-4\sin^2 A]}$$

$= \sin A = \text{L.H.S.}$

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$$\sin 15° = \frac{\sin 45°}{1+2\cos 30°}$$

$$= \frac{\frac{1}{\sqrt{2}}}{1+2\cdot\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{2}(1+\sqrt{3})} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{2}(3-1)} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

12. $\tan\alpha = \frac{\sin 2\alpha}{1+\cos 2\alpha}$ అని చూపుము దీని నుండి $\tan 15°$, $\tan 22\frac{1}{2}°$ కనుగొనుము.

Sol. R.H.S. $= \frac{\sin 2\alpha}{1+\cos 2\alpha}$

$$= \frac{2\sin\alpha\cos\alpha}{1+2\cos^2\alpha - 1} = \frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha}$$

$$= \frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \text{L.H.S.}$$

$$\alpha = 15° \Rightarrow \tan 15° = \frac{\sin 30°}{1+\cos 30°}$$

$$= \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{1}{2} \times \frac{2}{2+\sqrt{3}}$$

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

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$$\alpha = 22\frac{1}{2}° \Rightarrow \tan 22\frac{1}{2}° = \frac{\sin 45°}{1+\cos 45°}$$

$$= \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{1+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

13. $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = 4$ అని చూపుము.

Sol. L.H.S. $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°}$

$$= \frac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10° \cos 10°}$$

$$= \frac{2\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°}{\frac{1}{2}(2\sin 10° \cos 10°)}$$

$$= 4\frac{[\sin 30° \cos 10° - \cos 30° \sin 10°]}{\sin 20°}$$

$$= 4\frac{\sin(30° - 10°)}{\sin 20°}$$

$$= 4\frac{\sin 20°}{\sin 20°} = 4 = \text{R.H.S.}$$

14. $\sqrt{3}\csc 20° - \sec 20° = 4$ అని చూపుము

Sol. L.H.S. = $\sqrt{3}\csc 20° - \sec 20°$

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$$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$$

$$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°}$$

$$= \frac{2 \cdot \frac{\sqrt{3}}{2} \sin 20° - \frac{1}{2} \sin 20°}{\frac{1}{2}(2\sin 20° \cos 20°)}$$

$$= \frac{4(\sin 60° \cos 20° - \cos 60° \sin 20°)}{\sin 40°}$$

$$= 4\frac{\sin(60° - 20°)}{\sin 40°} = 4 \cdot \frac{\sin 40°}{\sin 40°}$$

$$= 4 = \text{R.H.S.}$$

15. $\tan 9° - \tan 27° - \cot 27° + \cot 9° = 4$ అని చూపుము.

Sol. Consider,

$$\tan A + \cot A = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{1}{\sin A \cos A}$$

$$= \frac{2}{\sin 2A} = 2\csc 2A$$

$\tan 81° = \tan(90° - 9°) = \cot 9°$

$\tan 63° = \tan(90° - 27°) = \cot 27°$

$A = 9° \Rightarrow \tan 9° + \cot 9° = 2\csc 18°$

$A = 27° \Rightarrow \tan 27° + \cot 27° = 2\csc 54°$

$\text{L.H.S.} = 2(\csc 17° - \csc 54°)$

$$= 2\left(\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right)$$

$$= 2 \times 4\left(\frac{1}{\sqrt{5}-1} - \frac{1}{\sqrt{5}+1}\right)$$

$$= 8\left(\frac{\sqrt{5}+1-\sqrt{5}+1}{5-1}\right)$$

$$= \frac{8 \times 2}{4} = 4 = \text{R.H.S.}$$

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16. $\frac{\sin\alpha}{a} = \frac{\cos\alpha}{b}$ అయితే $a\sec 2\alpha + b\cos 2\alpha = b$ అని చూపుము.

Sol. Given that $\frac{\sin\alpha}{a} = \frac{\cos\alpha}{b}$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{a}{b}$$

$$\therefore \tan\alpha = \frac{a}{b}$$

L.H.S. = $a\sec 2\alpha + b\cos 2\alpha$

$$= a\left[\frac{2\tan\alpha}{1+\tan^2\alpha}\right] + b\left[\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\right]$$

$$= a\left[\frac{2\times\frac{a}{b}}{1+\left(\frac{a}{b}\right)^2}\right] + b\left[\frac{1-\left(\frac{a}{b}\right)^2}{1+\left(\frac{a}{b}\right)^2}\right]$$

$$= \left[\frac{\frac{2a^2}{b}}{\frac{b^2+a^2}{b^2}}\right] + b\left[\frac{\frac{b^2-a^2}{b^2}}{\frac{b^2+a^2}{b^2}}\right]$$

$$= \frac{2a^2b}{a^2+b^2} + \frac{b(b^2-a^2)}{a^2+b^2}$$

$$= \frac{2a^2b+b^3-ba^2}{a^2+b^2}$$

$$= \frac{b^3+a^2b}{a^2+b^2} = \frac{b(b^2+a^2)}{a^2+b^2} = b = \text{R.H.S.}$$

17. త్రిభుజం ΔABC, $\tan\frac{A}{2} = \frac{5}{6}$, $\tan\frac{B}{2} = \frac{20}{37}$ ఐతే $\tan\frac{C}{2} = \frac{2}{5}$ అని చూపుము.

Sol. A + B + C = 180°

$$\frac{A+B}{2} = \frac{180^\circ - C}{2}$$

$$\tan\left(\frac{A+B}{2}\right) = \tan\left(90^\circ - \frac{C}{2}\right)$$

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$$\Rightarrow \tan\left(\frac{A}{2}+\frac{B}{2}\right)=\cot\frac{C}{2}$$

$$\Rightarrow \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\cot\frac{C}{2}$$

$$\Rightarrow \frac{\frac{5}{6}+\frac{20}{37}}{1-\frac{5}{6}\cdot\frac{20}{37}}=\cot\frac{C}{2}$$

$$\Rightarrow \frac{\frac{185+120}{222}}{\frac{222-100}{222}}=\cot\frac{C}{2}$$

$$\Rightarrow \frac{305}{122}=\frac{1}{\tan(C/2)}$$

$$\Rightarrow \tan\frac{C}{2}=\frac{122}{305}$$

$$\Rightarrow \tan\frac{C}{2}=\frac{2\times 61}{5\times 61}$$

$$\therefore \tan\frac{C}{2}=\frac{2}{5}$$

18. α, β లు $a\cos\theta + b\sin\theta = c$ యొక్క సాధనలైతే $\cos\alpha\cos\beta = \frac{2ac}{a^2+b^2}$ (ii) $\cos\alpha\cos\beta = \frac{c^2-b^2}{a^2+b^2}$ అని చూపుము.

Solution:

$$b\sin\theta = c - a\cos\theta \Rightarrow b^2\sin^2\theta = c^2 + a^2\cos^2\theta - 2ac\cos\theta + \left(c^2-b^2\right)=0$$

α, β లు యొక్క సాధనలు కావున $\cos\alpha, \cos\beta$ లు మూలాలు అనుకోండి.

$$\therefore \cos\alpha\cos\beta = \frac{2ac}{a^2+b^2} \qquad \cos\alpha.\cos\beta = \frac{c^2-b^2}{a^2+b^2}$$

19. $\sin\alpha + \sin\beta = \frac{2bc}{a^2+b^2}$ $\sin\alpha\sin\beta = \frac{c^2-a^2}{a^2+b^2}$ అని చూపుము.

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20. $\cos\theta = \frac{5}{13}$, 270° < θ < 360° అయితే sin(θ/2) cos(θ/2) లను కనుగొనుము

Sol. $\cos\theta = \frac{5}{13}$ where 270° < θ < 360°

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{5}{13}}{2}}$$

$$= \sqrt{\frac{13-5}{2\times 13}} = \sqrt{\frac{8}{2\times 13}} = \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\frac{5}{13}}{2}}$$

$$= \sqrt{\frac{13+5}{2\times 13}} = \sqrt{\frac{18}{2\times 13}} = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}}$$

Since θ lies in IV quadrant and θ/2 lies in II quadrant.

$$\sin\frac{\theta}{2} = \frac{2}{\sqrt{13}} \text{ and } \cos\frac{\theta}{2} = -\frac{3}{\sqrt{13}}.$$

21. $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8} = 2$ అని చూపుము.

Sol. L.H.S. = $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8}$

$$= \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\left(\pi - \frac{3\pi}{8}\right) + \cos^2\left(\pi - \frac{\pi}{8}\right)$$

$$= \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{\pi}{8}$$

$$= 2\left(\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8}\right)$$

$$= 2\left(\cos^2\frac{\pi}{8} + \cos^2\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right)$$

$$= 2\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right) = 2(1) = 2 = \text{R.H.S.}$$

22. $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$ అయితే tan 3x = 1 అని చూపుము.

Sol. $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$

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$$\tan x+\frac{\tan x+\tan\frac{\pi}{3}}{1-\tan x\tan\frac{\pi}{3}}+\frac{\tan x+\tan\frac{2\pi}{3}}{1-\tan x\tan\frac{2\pi}{3}}=3$$

$$\Rightarrow \tan x+\frac{\tan x+\sqrt{3}}{1-\sqrt{3}\tan x}+\frac{\tan x-\sqrt{3}}{1+\sqrt{3}\tan x}=3$$

$$\Rightarrow \tan x+\frac{(\tan x+\sqrt{3})(1+\sqrt{3}\tan x)+(\tan x-\sqrt{3})(1-\sqrt{3}\tan x)}{1-3\tan^2 x}=3$$

$$\Rightarrow \tan x+\frac{\tan x+\sqrt{3}\tan^2 x+\sqrt{3}+3\tan x+\tan x-\sqrt{3}\tan^2 x-\sqrt{3}+3\tan x}{1-3\tan^2 x}=3$$

$$\Rightarrow \tan x+\frac{8\tan x}{1-3\tan^2 x}=3$$

$$\Rightarrow \frac{\tan x(1-3\tan^2 x)+8\tan x}{1-3\tan^2 x}=3$$

$$\Rightarrow \frac{\tan x-3\tan^3 x+8\tan x}{1-3\tan^2 x}=3$$

$$\Rightarrow \frac{9\tan x-3\tan^3 x}{1-3\tan^2 x}=3$$

$$\Rightarrow \frac{3(3\tan x-\tan^3 x)}{1-3\tan^2 x}=3$$

$$\Rightarrow \frac{3\tan x-\tan^3 x}{1-3\tan^2 x}=1\Rightarrow \tan 3x=1$$

23. $\sin\frac{\pi}{5}\cdot\sin\frac{2\pi}{5}\cdot\sin\frac{3\pi}{5}\cdot\sin\frac{4\pi}{5}=\frac{5}{16}$ అని చూపుము

Sol. L.H.S. = $\sin\frac{\pi}{5}\cdot\sin\frac{2\pi}{5}\cdot\sin\frac{3\pi}{5}\cdot\sin\frac{4\pi}{5}$

$= \sin 36° \cdot \sin 72° \cdot \sin 108° \cdot \sin 144°$

$= \sin 36° \cdot \sin(90° - 18°) \sin(90° + 18°)$

$\sin(180° - 36°)$

$= \sin 36° \cdot \cos 18° \cdot \cos 18° \cdot \sin 36°$

$= \sin^2 36° \cdot \cos^2 18°$

$= \frac{10 - 2\sqrt{5}}{16} \cdot \frac{10 + 2\sqrt{5}}{16}$

$= \frac{100 - 20}{16 \times 16} = \frac{80}{16 \times 16} = \frac{5}{16} = \text{R.H.S.}$

24. $\cos^2\left(\frac{\pi}{10}\right) + \cos^2\left(\frac{5\pi}{5}\right) + \cos^2\left(\frac{3\pi}{5}\right) + \cos^2\left(\frac{9\pi}{10}\right) = 2$ అని చూపుము

Sol. L.H.S. = $\cos^2\left(\frac{\pi}{10}\right) + \cos^2\left(\frac{5\pi}{5}\right) + \cos^2\left(\frac{3\pi}{5}\right) + \cos^2\left(\frac{9\pi}{10}\right)$

$= \cos^2 \frac{\pi}{10} + \cos^2 \frac{2\pi}{5} + \cos^2\left(\pi - \frac{2\pi}{5}\right) + \cos^2\left(\pi - \frac{\pi}{10}\right)$

$= \cos^2 \frac{\pi}{10} + \cos^2 \frac{2\pi}{5} + \cos^2 \frac{2\pi}{5} + \cos^2 \frac{\pi}{10}$

$= 2\left(\cos^2 \frac{\pi}{10} + \cos^2 \frac{2\pi}{5}\right)$

$= 2(\cos^2 18° + \cos^2 72°)$

$= 2[\cos^2 18° + \cos^2(90° - 18°)]$

$= 2[\cos^2 18° + \sin^2 18°]$

$= 2(1) = 2$

25. $\frac{1 - \sec 8\alpha}{1 - \sec 4\alpha} = \frac{\tan 8\alpha}{\tan 2\alpha}$ అని చూపుము

Sol. L.H.S. = $\frac{1 - \sec 8\alpha}{1 - \sec 4\alpha}$

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$$= \frac{1 - \frac{1}{\cos 8\alpha}}{1 - \frac{1}{\cos 4\alpha}} = \frac{\frac{\cos 8\alpha - 1}{\cos 8\alpha}}{\frac{\cos 4\alpha - 1}{\cos 4\alpha}}$$

$$= \frac{\cos 8\alpha - 1}{\cos 8\alpha} \times \frac{\cos 4\alpha}{\cos 4\alpha - 1}$$

$$= \frac{-2\sin^2 4\alpha \cos 4\alpha}{-2\sin^2 2\alpha \cos 8\alpha}$$

$$= \frac{2\sin 4\alpha \cos 4\alpha \sin 4\alpha}{2\sin^2 2\alpha \cos 8\alpha}$$

$$= \frac{\sin 8\alpha \cdot \sin 4\alpha}{(2\sin^2 2\alpha)\cos 8\alpha}$$

$$= \frac{\sin 8\alpha}{\cos 8\alpha} \cdot \frac{2\sin 2\alpha \cos 2\alpha}{2\sin^2 2\alpha}$$

$$= \tan 8\alpha \cdot \frac{\cos 2\alpha}{\sin 2\alpha}$$

$$= \tan 8\alpha \cdot \cot 2\alpha$$

$$= \frac{\tan 8\alpha}{\tan 2\alpha} = \text{R.H.S.}$$

26. $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right) = \frac{1}{16}$ అని చూపుము.

Sol. L.H.S. =

$$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)$$

$$= \left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left[1+\cos\left(\pi-\frac{3\pi}{10}\right)\right]\left[1+\cos\left(\pi-\frac{\pi}{10}\right)\right]$$

$$= \left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)$$

$$= \left(1-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right)$$

$$= \sin^2\frac{\pi}{10}\sin^2\frac{3\pi}{10} = \left[\sin\frac{\pi}{10}\right]^2\left[\sin\frac{3\pi}{10}\right]^2$$

$$= \sin^2 18^\circ \sin^2 54^\circ$$

$$= \left[\frac{\sqrt{5}-1}{4}\right]^2 \left[\frac{\sqrt{5}+1}{4}\right]^2 = \frac{(\sqrt{5}-1)^2}{16} \times \frac{(\sqrt{5}+1)^2}{16}$$

$$= \frac{[(\sqrt{5}-1)(\sqrt{5}+1)]^2}{16\times16} = \frac{(5-1)^2}{16\times16} = \frac{4^2}{16\times16} = \frac{16}{16\times16} = \frac{1}{16}$$

27. $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11} = \frac{1}{32}$ అని చూపుము.

Sol. Let C = $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}$

$$S = \sin\frac{\pi}{11}\sin\frac{2\pi}{11}\sin\frac{3\pi}{11}\sin\frac{4\pi}{11}\sin\frac{5\pi}{11}$$

$$C\cdot S = \left(\sin\frac{\pi}{11}\cos\frac{\pi}{11}\right)\left(\sin\frac{2\pi}{11}\cos\frac{2\pi}{11}\right)\left(\sin\frac{3\pi}{11}\cos\frac{3\pi}{11}\right)\left(\sin\frac{4\pi}{11}\cos\frac{4\pi}{11}\right)\left(\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}\right)$$

$$= \frac{1}{32}\left(2\sin\frac{\pi}{11}\cos\frac{\pi}{11}\right)\left(2\sin\frac{2\pi}{11}\cos\frac{2\pi}{11}\right)\left(2\sin\frac{3\pi}{11}\cos\frac{3\pi}{11}\right)\left(2\sin\frac{4\pi}{11}\cos\frac{4\pi}{11}\right)\left(2\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}\right)$$

$$C\cdot S = \frac{1}{32}\sin\frac{2\pi}{11}\sin\frac{4\pi}{11}\sin\frac{6\pi}{11}\sin\frac{8\pi}{11}\sin\frac{10\pi}{11}$$

$$= \frac{1}{32}\sin\frac{2\pi}{11}\sin\frac{4\pi}{11}\sin\left(\pi-\frac{5\pi}{11}\right)\sin\left(\pi-\frac{3\pi}{11}\right)\sin\left(\pi-\frac{\pi}{11}\right)$$

$$= \frac{1}{32}\sin\frac{\pi}{11}\sin\frac{2\pi}{11}\sin\frac{3\pi}{11}\sin\frac{4\pi}{11}\sin\frac{5\pi}{11}$$

$$= \frac{1}{32}\cdot S$$

$$C = \frac{1}{32}$$

$$\therefore \cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11} = \frac{1}{32}$$

28. If A is not an integral multiple of π, prove that

(i) $\cos A\cdot\cos 2A\cdot\cos 4A\cdot\cos 8A = \frac{\sin 16A}{16\sin A}$ and hence deduce that

$\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{16\pi}{15} = \frac{1}{16}$.

Sol. (i) L.H.S. : cos A cos 2A cos 4A cos 8A

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$$=\frac{1}{2\sin A}(2\sin A\cos A)\cos 2A\cos 4A\cos 8A$$

$$=\frac{1}{2\sin A}\sin 2A\cos 2A\cos 4A\cos 8A$$

$$=\frac{1}{2^2\sin A}(2\sin 2A\cos 2A)\cos 4A\cos 8A$$

$$=\frac{1}{2^2\sin A}\sin 4A\cos 4A\cos 8A$$

$$=\frac{1}{2^3\sin A}2\sin 4A\cos 4A\cos 8A$$

$$=\frac{1}{2^3\sin A}\sin 8A\cos 8A$$

$$=\frac{1}{2^4\sin A}2\sin 8A\cos 8A$$

$$=\frac{\sin 16A}{16\sin A}=\text{R.H.S.}$$

ii) Put $n=\frac{2\pi}{15}$ in above result,

$$\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$$

$$=\frac{\sin\frac{32\pi}{15}}{16\sin\frac{2\pi}{15}}=\frac{\sin\left(2\pi+\frac{2\pi}{15}\right)}{16\sin\frac{2\pi}{15}}$$

$$=\frac{\sin\frac{2\pi}{15}}{16\sin\frac{2\pi}{15}}=\frac{1}{16}=\text{R.H.S.}$$

TRANSFORMATIONS

1. sin C + sin D = 2sin $\frac{C+D}{2}$. cos $\frac{C-D}{2}$.
2. sin C – sin D = 2cos $\frac{C+D}{2}$. sin $\frac{C-D}{2}$.
3. cos C + cos D = 2cos$\frac{C+D}{2}$. cos $\frac{C-D}{2}$.
4. cos C – cos D = 2sin $\frac{C+D}{2}$. sin $\frac{D-C}{2}$.
5. 2sin A cos B = sin(A + B) + sin(A – B)
6. 2cos A sin B = sin(A + B) – sin(A – B)
7. 2cos A cos B = cos(A + B) + cos(A – B)
8. 2sin A sin B = cos(A – B) – cos(A + B)

   (or)

   cos(A – B) – cos(A + B) = 2 sin A sin B.
9. $\frac{\sin A+\sin B}{\sin A-\sin B}$ = tan($\frac{A+B}{2}$).
10. If sin A + sin B = x, and cos A + cos B = y. Then

    i) tan($\frac{A+B}{2}$) = $\frac{x}{y}$

    ii) sin(A + B) = $\frac{2xy}{y^2+x^2}$

    iii) cos(A + B) = $\frac{y^2-x^2}{y^2+x^2}$

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iv) tan(A + B) = $\frac{2xy}{y^2 - x^2}$

1. $\sin 50^0 - \sin 70^0 + \sin 10^0 = 0$ అని చూపుము.

Solution:

$$\sin 50^0 - \sin 70^0 + \sin 10^0 = 2\cos\left(\frac{50^0 + 70^0}{2}\right)\sin\left(\frac{50^0 - 70^0}{2}\right) + \sin 10^0$$

$$= 2\cos 60^0 \sin\left(-10^0\right) + \sin 10^0$$

$$= -2 \times \frac{1}{2}\sin 10^0 + \sin 10^0 = -\sin 10^0 + \sin 10^0 = 0$$

2. $\frac{\sin 70^0 - \cos 40^0}{\cos 50^0 - \sin 20^0} = \frac{1}{\sqrt{3}}$ అని చూపుము.

$$\frac{\sin 70^0 - \cos 40^0}{\cos 50^0 - \sin 20^0} = \frac{\sin 70^0 - \sin 50^0}{\cos 50^0 - \cos 70^0} = \frac{2\cos 60^0 . \sin 10^0}{2\sin 60^0 . \sin 10^0} = \frac{1}{\sqrt{3}}$$

3. cos 55° + cos 65° + cos 175° = 0 అని చూపుము.

Sol. cos 55° + cos 65° + cos 175°

$$= 2\cos\left(\frac{55° + 65°}{2}\right)\cos\left(\frac{55° - 65°}{2}\right) + \cos(180° - 5°)$$

$$= 2\cos 60° \cos(-5°) - \cos 5°$$

$$= 2 \times \frac{1}{2}\cos 5° - \cos 5° = 0$$

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4. $\cos 20^\circ \cos 40^\circ - \sin 5^\circ \sin 25^\circ = \frac{\sqrt{3}+1}{4}$ అని చూపుము.

Sol. $\cos 20^\circ \cos 40^\circ - \sin 5^\circ \sin 25^\circ$

$= \frac{1}{2}[2\cos 40^\circ \cos 20^\circ - 2\sin 25^\circ \sin 5^\circ]$

$= \frac{1}{2}[\cos(40^\circ + 20^\circ) + \cos(40^\circ - 20^\circ)$

$+ \cos(25^\circ + 5^\circ) - \cos(25^\circ - 5^\circ)]$

$= \frac{1}{2}[\cos 60^\circ + \cos 20^\circ + \cos 30^\circ - \cos 20^\circ]$

$= \frac{1}{2}[\cos 60^\circ + \cos 30^\circ]$

$= \frac{1}{2}\left[\frac{1}{2} + \frac{\sqrt{3}}{2}\right] = \frac{\sqrt{3}+1}{4}$

5. $4\left\{\cos 66^0 + \sin 84^0\right\} = \sqrt{3} + \sqrt{5}$ అని చూపుము.

Solution:

$4\left\{\cos 66^0 + \sin 80^0\right\} = 4\left\{\cos 66^0 + \cos 66^0\right\}\left\{\because \sin 84^0 = \cos 66^0\right\}$

$= 4\left\{2\cos 36^0 . \cos 30^0\right\} = 8\left(\frac{\sqrt{5}+1}{4}\right)\frac{\sqrt{3}}{2} = \sqrt{3} + \sqrt{15}$

6. $\cos 48^0 \cos 12^0 = \frac{3+\sqrt{5}}{8}$ అని చూపుము.

Solution:

$\cos 48^0 \cos 12^0 = \frac{1}{2}\left\{2\cos 48^0 + \cos 12^0\right\} = \frac{1}{2}\left\{\cos 60^0 + \cos 36^0\right\}$

$\frac{1}{2}\left\{\frac{1}{2} + \frac{\sqrt{5}+1}{4}\right\} = \frac{2+\sqrt{5}+1}{8} = \frac{\sqrt{5}+3}{8}$

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7. $\sin^2(\alpha-45°)+\sin^2(\alpha+15°)-\sin^2(\alpha-15°)=\frac{1}{2}$ అని చూపుము.

Sol. $\sin^2(\alpha-45°)+\sin^2(\alpha+15°)-\sin^2(\alpha-15°)$

$=\sin^2(\alpha-45°)+\sin^2(\alpha+15°+\alpha-15°)\cdot$
$\sin(\alpha+15-\alpha+15)$

$=\sin^2(\alpha-45°)+\sin 2\alpha\cdot\sin 30°$

$=\frac{1-\cos(2\alpha-90°)}{2}+\frac{\sin 2\alpha}{2}$

$=\frac{1-\sin 2\alpha+\sin 2\alpha}{2}=\frac{1}{2}$

8. $\cos\theta+\cos\left(120^0+\theta\right)+\cos\left(240^0+\theta\right)=0$ అని చూపుము.

Solution:

$\cos\theta+\cos\left(120^0-\theta\right)+\cos\left(240^0-\theta\right)=\cos\theta+2\cos\left(\frac{120^0\theta+240^0+\theta}{2}\right)$

$\cos\left(\frac{120^0+\theta-240^0\theta}{2}\right)$

$=\cos\theta+2\cos\left(180^0+\theta\right)-\cos\left(60^0\right)=\cos\theta-2\cos\theta\times\frac{1}{2}$

$=\cos\theta-\cos\theta=0$

9. $\sin x+\sin y=\frac{1}{4}$, $\cos x+\cos y=\frac{1}{3}$ అయితే

(i) $\tan\left(\frac{x+y}{2}\right)=\frac{3}{4}$, (ii) $\cot(x+y)=\frac{7}{24}$ అని చూపుము.

Sol. i) $\sin x+\sin y=\frac{1}{4}$ ...(1)

$\cos x+\cos y=\frac{1}{3}$ ...(2)

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$$(1) \Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{1}{4} \ldots (3)$$

$$(2) \Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{1}{3} \ldots (4)$$

Dividing $\frac{(3)}{(4)}$, we get

$$\frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{1}{4} \times \frac{3}{1}$$

$$\tan\left(\frac{x+y}{2}\right) = \frac{3}{4}$$

ii) Let $\tan\left(\frac{x+y}{2}\right) = \frac{3}{4} = t$

$$\tan(x+y) = \frac{2t}{1-t^2} = \frac{2\left(\frac{3}{4}\right)}{1-\frac{9}{16}} = \frac{24}{7}$$

$$\therefore \cot(x+y) = \frac{1}{\tan(x+y)} = \frac{7}{24}$$

10. $\sin x + \sin y = a$, $\cos x + \cos y = b$ అయితే (i) $\tan\left(\frac{x+y}{2}\right)$ (ii) $\sin\left(\frac{x-y}{2}\right)$ లను కనుగొనుము.

Solution:

$$\frac{\sin x + \sin y}{\cos x + \cos y} = \frac{a}{b} \Rightarrow \frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{a}{b}$$

$$\tan\left(\frac{x+y}{2}\right) = \frac{a}{b}$$

$$(\sin x + \sin y)^2 + (\cos x + \cos y)^2 = a^2 + b^2$$

$$\sin^2 x + \sin^2 y + 2\sin x \sin y + \cos^2 x + \cos^2 y + 2\cos x \cos y = a^2 + b^2$$

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$$\left(\sin^2 x + \cos^2 x\right) + \left(\sin^2 y + \cos^2 y\right) + 2\{\cos x \cos y + \sin x + \sin y\} = a^2 + b^2$$

$$1 + 1 + 2\cos(x - y) = a^2 + b^2 \Rightarrow \cos(x - y) = \frac{a^2 + b^2 - 2}{2}$$

$$\sin^2\left(\frac{x - y}{2}\right) = \sqrt{1 \frac{-\cos(x - y)}{2}} = \pm\sqrt{\left(4 - a^2 - b^2\right)/4}$$

11. $\dfrac{1 - \cos A + \cos B - \cos(A + B)}{1 + \cos A - \cos B - \cos(A + B)} = \tan\dfrac{A}{2}\cot\dfrac{B}{2}$ అని చూపుము.

Solution:

$$\frac{1 - \cos A + \cos B - \cos(A + B)}{1 + \cos A - \cos B - \cos(A + B)} = \frac{\{1 - \cos(A + B) - \{\cos A - \cos B\}\}}{\{1 - \cos(A + B) + \{\cos A - \cos B\}\}}$$

$$\frac{2\sin^2\left(\frac{A + B}{2}\right) + 2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{A - B}{2}\right)}{2\sin^2\left(\frac{A + B}{2}\right) - 2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{A - B}{2}\right)} = \frac{2\sin\left(\frac{A + B}{2}\right)\left\{\sin\left(\frac{A + B}{2}\right) + \sin^2\left(\frac{A + B}{2}\right)\right\}}{2\sin\left(\frac{A + B}{2}\right)\left\{\sin\left(\frac{A + B}{2}\right) - \sin\left(\frac{A - B}{2}\right)\right\}}$$

$$\frac{2\sin\frac{A}{2}\cos\frac{B}{2}}{2\cos\frac{A}{2}\sin\frac{B}{2}} = \tan\frac{A}{2}\cot\frac{B}{2}$$

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12. $(A - 15°)$, $(A - 75°)$ లు $180°$ యొక్క పూర్ణాంక గుణిజాలు కాకపోతే $\cot(15° - A) + \tan(15° + A) = \dfrac{4\cos 2A}{1-2\sin 2A}$ అని చూపుము.

.

Sol. $\cot(15° - A) + \tan(15° + A)$

$$= \frac{\cos(15°-A)}{\sin(15°-A)} + \frac{\sin(15°+A)}{\cos(15°+A)}$$

$$= \frac{\cos(15°-A)\cos(15°+A) + \sin(15°+A)\sin(15°-A)}{\sin(15°-A)\cos(15°+A)}$$

$$= \frac{2\left[(\cos^2 A - \sin^2 15°) + \sin^2(15° - \sin^2 A)\right]}{\left[2\cos(15°+A)\sin(15°-A)\right]}$$

$$= \frac{2(\cos^2 A - \sin^2 A)}{\sin(15°+A+15°-A) - \sin(15°+A-15°+A)}$$

$$= \frac{2(\cos^2 A - \sin^2 A)}{\sin 30° - \sin 2A}$$

$$= \frac{2\cos 2A}{\frac{1}{2} - \sin 2A} = \frac{4\cos 2A}{1-2\sin 2A}$$

13. $4\cos 12° \cos 48° \cos 72° = \cos 36°$ అని చూపుము.

Sol. $4\cos 12° \cos 48° \cos 72°$

$$= (2\cos 48° \cos 12°)(2\cos 72°)$$

$$= [\cos(48+12) + \cos(48-12)]2\cos 72°$$

$$= [\cos 60° + \cos 36°]2\cos 72°$$

$$= 2\cos 60° \cos 72° + 2\cos 36° \cos 72°$$

$$= 2\times\frac{1}{2}\cos 72° + \cos(72°+36°) + \cos(72°-36°)$$

$$= \cos 72° + \cos 108° + \cos 36°$$

$$= \cos(90°-18°) + \cos(90°+18°) + \cos 36°$$

$$= \sin 18° - \sin 18° + \cos 36°$$

$$= \cos 36°$$

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14. $\sin 10° + \sin 20° + \sin 40° + \sin 50° = \sin 70° + \sin 80°$ అని చూపుము.

Sol. $\sin 10° + \sin 20° + \sin 40° + \sin 50°$

$$= 2\sin\left(\frac{10°+20°}{2}\right)\cos\left(\frac{10°-20°}{2}\right) + 2\sin\left(\frac{40°+50°}{2}\right)\cos\left(\frac{40°-50°}{2}\right)$$

$$= 2\sin 15° \cos 5° + 2\sin 45° \cos 5°$$

$$= 2\cos 5°[\sin 15° + \cos 45°]$$

$$= 2\cos 5°\left[2\sin\left(\frac{15°+45°}{2}\right)\cos\left(\frac{15°-45°}{2}\right)\right]$$

$$= 2\cos 5°[2\sin 30° \cos 15°]$$

$$= 4\cos 5° \cdot \frac{1}{2}\cos 15°$$

$$= 2\cos 15° \cos 5°$$

$$= \cos(15+5) + \cos(15-5)$$

$$= \cos 20° + \cos 10°$$

$$= \cos(90° - 70°) + \cos(90° - 10°)$$

$$= \sin 70° + \sin 80°$$

15. $\sin A - \sin B \neq 0, \cos A - \cos B \neq 0$ అయితే $\left[\frac{\cos A + \cos B}{\sin A - \sin B}\right]^n + \left[\frac{\sin A + \sin B}{\cos A - \cos B}\right]^n$

$$= \begin{cases} 2 \cdot \cot^n\left(\frac{A-B}{2}\right), \text{ if n is even} \\ 0, \text{ if n is odd} \end{cases}$$ అని చూపుము.

Sol. $\left[\frac{\cos A + \cos B}{\sin A - \sin B}\right]^n + \left[\frac{\sin A + \sin B}{\cos A - \cos B}\right]^n$

$$= \left[\frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right]^n + \left[\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right]$$

$$= \cot^n\left(\frac{A-B}{2}\right) + (-1)^n \cot^n\left(\frac{A-B}{2}\right) = 0$$

if n is odd, since $(-1)^n = -1$

$= 2\cot^{n}\left(\frac{A-B}{2}\right)$ if n is even, since $(-1)^n=1$

15. $\cos x + \cos y = \frac{4}{5}$ *and* $\cos x - \cos y = \frac{2}{7}$ అయితే $14\tan\left(\frac{x-y}{2}\right) + 5\cot\left(\frac{x+y}{2}\right)$ కనుగొనుము.

Solution:

$$\frac{\cos x + \cos y}{\cos x - \cos y} = \frac{\left(\frac{4}{5}\right)}{\left(\frac{2}{5}\right)} \Rightarrow \frac{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)} = \frac{\cancel{4}^{2}}{5} \times \frac{7}{\cancel{2}}$$

$$\frac{-\cot\left(\frac{x+y}{2}\right)}{\tan\left(\frac{x-y}{2}\right)} = \frac{14}{5} \Rightarrow -5\cot\left(\frac{x-y}{2}\right) = 14\tan\left(\frac{x-y}{2}\right)$$

$$14\tan\left(\frac{x-y}{2}\right) + 5\cot\left(\frac{x+y}{2}\right) = 0$$

16. $\sec(\theta+\alpha) + \sec(\theta-\alpha) = 2\sec\theta$ *and* $\cos\alpha \neq 1$ అయితే $\cos\theta = \pm\sqrt{2}\cos\frac{\alpha}{2}$ అని చూపుము.

Solution:

$$\sec(\theta+\alpha) + \sec(\theta-\alpha) = 2\sec\theta$$

$$\frac{1}{\cos(\theta+\alpha)} + \frac{1}{\cos(\theta-\alpha)} = \frac{2}{\cos\theta} \Rightarrow \frac{\cos(\theta-\alpha) + \cos(\theta+\alpha)}{\cos(\theta-\alpha)\cos(\theta+\alpha)} = \frac{2}{\cos\theta}$$

$$(\cancel{2}\cos\theta\cos\alpha)\cos\theta = \cancel{2}\{\cos^2\theta - \sin^2\alpha\}$$

$$\cos^2\theta\cos\alpha = \cos^2\theta - \sin^2\alpha \Rightarrow \sin^2\alpha = \cos^2\theta(1-\cos\alpha)$$

$$\cos^2\theta = \frac{\cancel{(1-\cos\alpha)}(1+\cos\alpha)}{\cancel{(1-\cos\alpha)}} \Rightarrow \cos\theta = \pm\sqrt{2}\cos\frac{\alpha}{2}$$

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17. sin A = sin B , cos A = cos B అయితే A = 2nπ + B, $\forall n \in N$ అని చూపుము.

Sol. sin A = sin B and cos A = cos B

$$\Rightarrow \sin A - \sin B = 0 \text{ and } \cos A - \cos B = 0$$

$$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) = 0 \text{ and}$$

$$-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) = 0$$

$$\Rightarrow \sin\left(\frac{A-B}{2}\right) = 0 \text{ and } \sin\left(\frac{A-B}{2}\right) = 0$$

$$\Rightarrow \frac{A-B}{2} = n\pi$$

$$\Rightarrow A - B = 2n\pi \Rightarrow A = 2n\pi + B \ (n \in Z)$$

18. cos nα ≠ 0 , $\cos\frac{\alpha}{2} \neq 0$, అయితే $\frac{\sin(n+1)\alpha - \sin(n-1)\alpha}{\cos(n+1)\alpha + 2\cos n\alpha + \cos(n-1)\alpha} = \tan\frac{\alpha}{2}$ అని చూపుము.

Sol. Let cos nα ≠ 0 and $\cos\frac{\alpha}{2} \neq 0$ then

$$\cos(n+1)\alpha + 2\cos n\alpha + \cos(n-1)\alpha$$

$$= \cos(n\alpha + \alpha) + \cos(n\alpha - \alpha) + 2\cos n\alpha$$

$$= 2\cos n\alpha \cos\alpha + 2\cos n\alpha$$

$$= 2\cos n\alpha[1 + \cos\alpha]$$

$$= 4\cos^2\frac{\alpha}{2}\cos n\alpha \neq 0$$

$$\sin(n+1)\alpha - \sin(n-1)\alpha$$

$$= \sin(n\alpha + \alpha) - \sin(n\alpha - \alpha)$$

$$= 2\cos n\alpha \sin\alpha$$

$$= 4\cos n\alpha \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

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$$\therefore \frac{\sin(n+1)\alpha - \sin(n-1)\alpha}{\cos(n+1)\alpha + 2\cos n\alpha + \cos(n-1)\alpha}$$

$$= \frac{4\cos n\alpha \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{4\cos^2\frac{\alpha}{2}\cos n\alpha}$$

$$= \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \tan\frac{\alpha}{2}$$

19. x, y, z లు π/2 యొక్క బేసి గుణకాలు కాకుండా sin(y + z – x), sin(z + x – y), sin(x + y – z) లు A.P ఉంటే tan x, tan y, tan లు కూడా A.P ఉంటాయనిచూపండి.

Sol. sin(y + z – x), sin(z + x – y), sin(x + y – z) లు A.P లో ఉన్నాయి.

$$\Rightarrow \sin(z+x-y) - \sin(y+z-x)$$
$$= \sin(x+y-z) - \sin(z+x-y)$$

$$\Rightarrow 2\cos\left(\frac{z+x-y+y+z-x}{2}\right) \sin\left(\frac{z+x-y-y-z+x}{2}\right)$$

$$= 2\cos\left(\frac{x+y-z+z+x-y}{2}\right) \sin\left(\frac{x+y-z-z-x+y}{2}\right)$$

$$\Rightarrow 2\cos z\sin(x-y) = 2\cos x\sin(y-z)$$

$$\Rightarrow 2\cos z[\sin x\cos y - \cos x\sin y]$$
$$= 2\cos x[\sin y\cos z - \cos y\sin z]$$

cosx cosy cosz చేభాగించగా

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$$\Rightarrow \frac{2\cos z[\sin x\cos y-\cos x\sin y]}{\cos x\cos y\cos z}$$

$$=\frac{2\cos x[\sin y\cos z-\cos y\sin z]}{\cos x\cos y\cos z}$$

$$\Rightarrow \frac{\sin x\cos y}{\cos x\cos y}-\frac{\cos x\sin y}{\cos x\cos y}$$

$$=\frac{\sin y\cos z}{\cos y\cos z}-\frac{\cos y\sin z}{\cos y\cos z}$$

$$\Rightarrow \frac{\sin x}{\cos x}-\frac{\sin y}{\cos y}=\frac{\sin y}{\cos y}-\frac{\sin z}{\cos z}$$

$$\Rightarrow \tan x-\tan y=\tan y-\tan z$$

$$\Rightarrow \tan x+\tan z=2\tan y$$

$\Rightarrow$ tan x, tan y, tan z are in A.P.

20. $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right)$, $\theta\in R$ అయితే $xy+yz+zx=0$ అని చూపుము.

Solution:

Let $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right)=k$

$$\cos\theta=\frac{k}{x}\quad \cos\left(\theta+\frac{2\pi}{3}\right)=\frac{k}{y}\quad :\cos\left(\theta+\frac{4\pi}{3}\right)=\frac{k}{z}$$

$$\frac{k}{x}+\frac{k}{y}+\frac{k}{2}=\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{4\pi}{3}\right)$$

$$\frac{k}{x}+\frac{k}{4}+\frac{k}{2}=0$$

$$xy+yz+zx=0$$

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21. A లేదా A + B లు π/2 అగుణిజాలు అయి m sin B = n sin(2A + B) అయితే (m+n) tan A = (m–n) tan (A+B) అని చూపుము..

Sol. Given m sin B = n sin(2A + B)

$$\frac{m}{n}=\frac{\sin(2A+B)}{\sin B}$$

By componendo and dividendo, we get

$$\frac{m+n}{m-n}=\frac{\sin(2A+B)+\sin B}{\sin(2A+B)-\sin B}$$

$$=\frac{2\sin(A+B)\cos A}{2\cos(A+B)\sin A}$$

$$\frac{m+n}{m-n}=\tan(A+B)\cot A$$

$$\frac{(m+n)}{\cot A}=(m-n)\tan(A+B)$$

$$(m+n)\tan A=(m-n)\tan(A+B)$$

22. $\tan(A+B)=\lambda\tan(A-B)$ అయితే $(\lambda+1)\sin 2B=(\lambda-1)\sin 2A$ అని చూపుము.

Solution:

$$\frac{\tan(A+B)}{\tan(A-B)}=\frac{\lambda}{1}\Rightarrow\frac{\sin(A+B)}{\cos(A+B)}\times\frac{\cos(A-B)}{\sin(A+B)}=\frac{\lambda}{1}$$

Using componedo and dividendo

$$\frac{\sin(A+B)\cos(A-B)+\cos(A+B).\sin(A-B)}{\sin(A+B)\cos(A-B)-\cos(A+B)\sin(A-B)}=\frac{\lambda+1}{\lambda-1}$$

$$\frac{\sin 2A}{\sin 2B}=\frac{\lambda+1}{\lambda-1}\Rightarrow(\lambda-1)\sin 2A=(\lambda+1)\sin 2B$$

7marks questions

1. A, B, C లు త్రిభుజ కోణాలు ఐతే క్రింది వానిని నిరూపించండి

(i) $\sin 2A - \sin 2B + \sin 2C = 4\cos A \sin B \cos C$

(ii) $\cos 2A - \cos 2B + \cos 2C = 1 - 4\sin A \cos B \sin C$

Solution :

(i) $A + B + C = 180^0 \Rightarrow A + B = 180^0 - C \quad \sin(A+B) = \sin C$

$C = 180^0 - (A+B) \quad \sin C = \sin(A+B)$

$\cos(A+B) = -\cos C$

$\sin 2A - \sin 2B + \sin 2C = 2\cos(A+B)\sin(A-B) + 2\sin C \cos C$

$= -2\cos C \sin(A+B) + 2\sin C \cos C$

$= +2\cos C[-\sin(A-B) + \sin(A+B)]$

$= 2\cos C[2\cos A \sin B] = 4\cos A \sin B \cos C$

(ii) $\cos 2A - \cos 2B + \cos 2C = -2\sin(A+B).\sin(A-B) + 1 - 2\sin^2 C$

$= 1 - 2\sin C.\sin(A-B) - 2\sin^2 C$

$= 1 - 2\sin C\{\sin(A-B) + \sin C\}$

$= 1 - 2\sin C\{\sin(A+B) + \sin(A+B)\}$

$= 1 - 2\sin C\{2\sin A \cos B\} = 1 - 2\sin A \cos B \sin C$

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2. A, B, C లు త్రిభుజ కోణాలు ఐతే క్రింది వానిని నిరూపించండి

(i) $\sin A + \sin B - \sin C = 4\sin A/2 \sin B/2 \cos C/2$

(ii) $\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Solution :

(i) $\sin A + \sin B - \sin C = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$$= \sin\left(90^0 - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) - 2\sin\frac{C}{2}\cos\frac{C}{2}$$

$$= 2\cos\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \sin\frac{C}{2}\right\}$$

$$= 2\cos\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right\}$$

$$= 2\cos\frac{C}{2}\left\{2\sin\frac{A}{2}\sin\frac{B}{2}\right\}$$

$$= 4\sin A/2 \sin B/2 \cos C/2$$

Solution

(ii) $\cos A + \cos B - coC = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) - \cos C$

$$= 2\cos\left(90^0 - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) - \left\{1 - 2\sin^2\frac{C}{2}\right\}$$

$$= 2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - 1 + 2\sin^2\frac{C}{2}$$

$$-1 + 2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) + \sin\frac{C}{2}\right\}$$

$$= -1 + 2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) + \sin\left(90^0 - \frac{A+B}{2}\right)\right\}$$

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$$=-1+2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)+\cos\left(\frac{A+B}{2}\right)\right\}$$

$$=-1+2\sin\frac{C}{2}\left\{2\cos\frac{A}{2}\cos\frac{B}{2}\right\}$$

$$=-1+4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

3. A, B, C లు త్రిభుజ కోణాలు ఐతే క్రింది వానిని నిరూపించండి

(i) $\sin^2 A+\sin^2 B-\sin^2 C=2\sin A\sin B\cos C$

(ii) $\cos^2 A+\cos^2 B-\cos^2 C=1-2\sin A\sin B\cos C$

Solution:

(i) $A+B+C=180^0$

$$\sin^2 A+\sin^2 B-\sin^2 C=\sin^2 A+\sin(B+C)\sin(B-C)$$
$$\{\because \sin^2 B-\sin^2 C=\sin(B+C)\sin(B-C)\}$$

$$=\sin^2 A+\sin(180^0-A)\sin(B-C)$$

$$=\sin^2 A+\sin A\sin(B-C)$$

$$=\sin A\{\sin A+\sin(B-C)\}$$

$$=\sin A\{\sin(180^0-B+C)+\sin(B-C)\}$$

$$=\sin A\{\sin(B+C)+\sin(B-C)\}$$

$$=\sin A\{2\sin B\cos C\}=2\sin A\sin B\cos C$$

Solution :

(ii) $\cos^2 A+\cos^2 B-\cos^2 C=\cos^2 A=\{\cos^2 C-\cos^2 B\}$

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$= \cos^2 A - \sin(B+C)\sin(B-C)$

$= 1 - \sin^2 A - \sin(180^0 - A)\sin(B-C)$

$= 1 - \sin^2 A - \sin A \sin(B-C)$

$= 1 - \sin A\{\sin A + \sin(B-C)\}$

$= 1 - \sin A\{\sin(180^0 - B + C) + \sin(B-C)\}$

$= 1 - \sin A\{\sin(B+C) + \sin(B-C)\}$

$= 1 - \sin A\{2\sin B \cos C\}$

$= 1 - 2\sin A \sin B \cos C$

4. A, B, C లు త్రిభుజ కోణాలు ఐతే క్రింది వానిని నిరూపించండి

(i) $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2\left\{1 + \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right\}$

(ii) $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} - \cos^2\frac{C}{2} = 2\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Solution:

$A + B + C = 180^0 = A/2 + B/2 + C/2 = 90^0$

(i) $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = \cos^2\frac{A}{2} + 1 - \sin^2 B/2 + \cos^2 C/2$

$= 1 + \left\{\cos^2\frac{A}{2} - \sin^2\frac{B}{2}\right\} + \cos^2\frac{C}{2}$

$= 1 + \cos\left(\frac{A}{2} + \frac{B}{2}\right)\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \cos^2\frac{C}{2}$

$= 1 + \cos\left(90^0 - \frac{C}{2}\right)\cos\left(\frac{A}{2} - \frac{B}{2}\right) + 1 - \sin^2\frac{C}{2}$

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$$= 2 + \sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - \sin^2\frac{C}{2}$$

$$= 2 + \sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \sin\frac{C}{2}\right\}$$

$$= 2 + \sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \sin\left(90^0 - \frac{A+B}{2}\right)\right\}$$

$$= 2 + \sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right\}$$

$$2 + 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$= 2 + 2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

(ii) $$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} - \cos^2\frac{C}{2} = \cos^2\frac{A}{2} - \left\{\cos^2\frac{C}{2} - \cos^2\frac{B}{2}\right\}$$

$$= \cos^2\frac{A}{2} - \sin\left(\frac{B+C}{2}\right).\sin\left(\frac{B-C}{2}\right)$$

$$= \cos^2\frac{A}{2} - \sin\left(90^0 - \frac{A}{2}\right)\sin\left(\frac{B-C}{2}\right)$$

$$= \cos^2\frac{A}{2} - \cos\frac{A}{2}\sin^2\left(\frac{B-C}{2}\right)$$

$$= \cos\frac{A}{2}\left\{\cos\frac{A}{2} - \sin\frac{B-C}{2}\right\}$$

$$= \cos\frac{A}{2}\left\{\cos\left(90^0 - \frac{B+C}{2}\right) - \sin\left(\frac{B-C}{2}\right)\right\}$$

$$= \cos\frac{A}{2}\left\{\sin\left(\frac{B+C}{2}\right) - \sin\left(\frac{B-C}{2}\right)\right\} = 2\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

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5. త్రిభుజం ABC లో క్రింది వానిని నిరూపించండి

(i) $\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}=4\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\cos\left(\frac{\pi-C}{4}\right)$

(ii) $\cos\frac{A}{2}+\cos\frac{B}{2}-\cos\frac{C}{2}=4\cos\left(\frac{\pi+A}{4}\right)\cos\left(\frac{\pi+B}{4}\right)\cos\left(\frac{\pi-C}{4}\right)$

(iii) $\sin\frac{A}{2}+\sin\frac{B}{2}-\sin\frac{C}{2}=-1+4\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$

Solution:

(i) $A+B+C=\pi$

R.H..S $4\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\cos\left(\frac{\pi-C}{4}\right)=2\left\{\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\right\}\left\{2\cos\left(\frac{\pi-C}{4}\right)\right\}$

$$=\left\{\cos\left(\frac{\pi-A+\pi-B}{4}\right)+\cos\left(\frac{\pi-A-\pi+B}{4}\right)\right\}\left\{2\cos\left(\frac{\pi-C}{4}\right)\right\}$$

$$\{\because 2\cos A\cos B=\cos(A+B)+\cos(A+B)\}$$

$$=\left\{\cos\left\{\frac{\pi}{2}-\left(\frac{A+B}{4}\right)\right\}+\cos\left(\frac{A-B}{4}\right)\right\}2\cos\left(\frac{\pi-C}{4}\right)$$

$$=2\cos\frac{\pi-C}{4}\sin\left(\frac{A+B}{4}\right)+2\cos\left(\frac{\pi-C}{4}\right)\cos\left(\frac{A-B}{4}\right)\quad\because\ \cos\left(\frac{\pi}{2}-\frac{A+B}{4}\right)=\sin\left(\frac{A+B}{4}\right)$$

$$=\sin\left(\frac{\pi-C+A+B}{4}\right)-\sin\left(\frac{\pi-C-A-B}{4}\right)+\cos\left(\frac{\pi-C+A-B}{4}\right)+\cos\left(\frac{\pi-C-A+B}{4}\right)$$

$$\left\{\begin{array}{l}\because 2\cos A\sin B=\sin(A+B)-\sin(A-B)\\ 2\cos A\cos B=\cos(A+B)+\cos(A-B)\end{array}\right\}$$

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$$\therefore \sin\left(\frac{\pi - C + \pi - C}{4}\right) - \sin\left\{\frac{A + B + C - C - A - B}{4}\right\} + \cos\left(\frac{A + B + C - C + A - B}{4}\right)$$

$$+ \cos\left\{\frac{A + B + C - C - A + B}{4}\right\} \left\{\begin{matrix}\because \pi = A + B + C \\ ad\ A + B = \pi - C\end{matrix}\right\}$$

$$= \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) + \cos\frac{A}{2} + \cos\frac{B}{2}$$

$$= \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}$$

Solution :

(ii) R.H.S

$$= 4\cos\left(\frac{\pi + A}{4}\right)\cos\left(\frac{\pi + B}{4}\right)\cos\left(\frac{\pi - C}{4}\right)$$

$$= \left\{2\cos\left(\frac{\pi + A}{4}\right)\cos\left(\frac{\pi + B}{4}\right)\right\} 2\cos\left(\frac{\pi - C}{4}\right)$$

$$= \left[\cos\left(\frac{\pi + A + \pi + B}{4}\right) + \cos\left(\frac{\pi + A - \pi - B}{4}\right)\right] 2\cos\left(\frac{\pi - C}{4}\right)$$

$$= \left[\cos\left(\frac{\pi}{2} + \frac{A + B}{4}\right) + \cos\left(\frac{A - B}{4}\right)\right] 2\cos\left(\frac{\pi - C}{4}\right)$$

$$= -2\cos\left(\frac{\pi - C}{4}\right)\sin\left(\frac{A + B}{4}\right) + 2\cos\left(\frac{\pi - C}{4}\right)\cos\left(\frac{A - B}{4}\right)$$

$$= -2\cos\left(\frac{\pi - C}{4}\right)\sin\left(\frac{\pi - C}{4}\right) + \cos\left(\frac{\pi - C + A - B}{4}\right) + \cos\left(\frac{\pi - C - A + B}{4}\right)$$

$$= -\sin 2\left(\frac{\pi - C}{4}\right) + \cos\left(\frac{A + B + C - A - B}{4}\right) + \cos\left(\frac{A + B + C - C - A + B}{4}\right)$$

$$= -\sin\left(\frac{\pi}{2} - \frac{C}{2}\right) + \cos\frac{A}{2} + \cos\frac{B}{2}$$

$$= \cos\frac{A}{2} + \cos\frac{B}{2} - \cos\frac{C}{2}$$

Solution :

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(iii) R.H.S $= -1 + 4\cos\left(\frac{\pi - A}{4}\right)\cos\left(\frac{\pi - B}{4}\right).\sin\left(\frac{\pi - C}{4}\right)$

$$= -1 + \left\{2\cos\left(\frac{\pi - A}{4}\right)\cos\left(\frac{\pi - B}{4}\right)\right\} 2\sin\left(\frac{\pi - C}{4}\right)$$

$$= -1 + \left\{\cos\left(\frac{\pi}{2} - \left(\frac{A+B}{4}\right)\right) + \cos\frac{A-B}{4}\right\} 2\sin\left(\frac{\pi - C}{4}\right)$$

$$= -1 + 2\sin\left(\frac{A+B}{4}\right)\sin\left(\frac{\pi - C}{4}\right) + 2\sin\left(\frac{\pi - C}{4}\right)\cos\left(\frac{A-B}{4}\right)$$

$$= -1 + 2\sin\left(\frac{\pi - C}{4}\right) + \sin\left(\frac{A+B+C-C+A-B}{4}\right) + \sin\left\{\frac{A+B+C-C-A+B}{4}\right\}$$

$$= -\left\{1 - 2\sin^2\frac{\pi - C}{4}\right\} + \sin\frac{A}{2} + \sin\frac{B}{2}$$

$$= -\cos\left(\frac{\pi - C}{2}\right) + \sin\frac{A}{2} + \sin\frac{B}{2}$$

$$= \sin\frac{A}{2} + \sin\frac{B}{2} - \sin\frac{C}{2}$$

6. $A + B + C = 90^0$ ఐతే $\cos 2A + \cos 2B + \cos 2C = 1 + 4\sin A \sin B \sin C$ అని చూపుము

Solution :

$$\cos 2A + \cos 3B + \cos 2C = 2\cos(A+B)\cos(A-B) + \cos 2C$$

$$= 2\cos(90^0 - C)\cos(A-B) + \cos 2C \left\{\because A + B = 90^0 - C\right\}$$

$$= 2\sin C\cos(A-B) + 1 - 2\sin^2 C$$

$$= 1 + 2\sin C\left\{\cos(A-B) - \sin C\right\}$$

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$$= 1 + 2\sin\left\{\cos(A-B) - \sin\left(90^0 - A + B\right)\right\}$$

$$= 1 + 2\sin C\left\{\cos(A-B) - \cos(A+B)\right\}$$

$$= 1 + 2\sin C\left\{2\sin A\sin B\right\}$$

$$= 1 + 4\sin A\sin B\sin C$$

7. $A + B + C = 270^0$ ఐతే

(i) $\cos^2 A + \cos^2 B - \cos^2 C = -2\cos A\cos B\sin C$

(ii) $\sin 2A + \sin 2B - \sin 2C = -4\sin A\sin B\cos C$ అని చూపుము

Solution:

(i) $\cos^2 A + \cos^2 B - \cos^2 C = \cos^2 A - \left\{\cos^2 C - \cos^2 B\right\}$

$$= \cos^2 A - \sin(B+C)\sin(B-C)$$

$$= \cos^2 A - \sin\left(270^0 - A\right)\sin(B-C)$$

$$= \cos^2 A + \cos A\sin(B-C)$$

$$= \cos A\left\{\cos A + \sin(B-C)\right\}$$

$$= \cos A\left\{\cos\left(270^0 - \overline{B+C}\right) + \sin(B-C)\right\}$$

$$\cos A\left\{-\sin(B-C) + \sin(B-C)\right\}$$

$$-\cos A\left(2\cos B\sin C\right) = -2\cos A\cos B\sin C$$

(ii) $\sin 2A + \sin 2B - \sin 2C = -4\sin A\sin B\cos C$

$$2\sin(A+B)\cos(A-B) - \sin 2C$$

$$2\sin\left(270^0 - C\right)\cos(A-B) - \sin 2C$$

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$-2\cos C\cos(A-B)-2\sin C\cos C$

$-2\cos\left[\cos(A-B)+\sin(220^0-A+B)\right]$

$-2\cos C\left[\cos(A-B)-\cos(A+B)\right]$

$-4\sin A\sin B\cos C$

1. If $A+B+C=0^0$ then prove that

(i) $\sin 2A+\sin 2B+\sin 2C=-4\sin A\sin B\sin C$

(ii) $\sin A+\sin B-\sin C=-4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Solution :

(i) $A+B+C=0^0 \Rightarrow \frac{A}{2}+\frac{B}{2}+\frac{C}{2}=0^0 \Rightarrow \frac{A}{2}+\frac{B}{2}=-\frac{C}{2}$

$\sin 2A+\sin 2B+\sin 2C=2\sin(A+B)\cos(A-B)+\sin 2C$

$=2\sin(-C)\cos(A-B)+\sin 2C$

$=-2\sin C\cos(A-B)+2\sin C\cos C$

$=-2\sin C\left[\cos(A-B)-\cos C\right]$

$=-2\sin C\left[\cos(A-B)-\cos(A+B)\right]$ $\quad\left[\begin{array}{l}\because C=-(A+B)\\ \cos C=\cos(A+B)\end{array}\right]$

$=-2\sin C\{2\sin A\sin B\}$

$=-4\sin A\sin B\sin C$

Solution :

(ii) $\sin A + \sin B - \sin C = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = \sin C$

$= 2\sin\left(-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= -2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) + \cos\frac{C}{2}\right]$

$= -2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right]$

$= -2\sin\frac{C}{2}\left[2\cos\frac{A}{2}\cos\frac{B}{2}\right]$

$= -4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

8. $A + B + C + D = 360^0$ అయితే

(i) $\sin A - \sin B + \sin C - \sin D = -4\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A+D}{2}\right)$

(ii) $\cos 2A + \cos 2B + \cos 2C + \cos 2D = 4\cos(A+B)\cos(A+C)\cos(A+D)$ అని చూపుము

Solution:

(i) $A + B + C + D = 360^0 \Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} + \frac{D}{2} = 180^0$

$\therefore \frac{A+B}{2} = 180^0 - \left(\frac{C+D}{2}\right)$

$\sin A - \sin B + \sin C - \sin D = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) + 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

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$$= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) + 2\cos\left\{180^0 + \overline{\frac{A+B}{2}}\right\}\sin\left(\frac{C-D}{2}\right)$$

$$= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) - 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$2\cos\left(\frac{A+B}{2}\right)\left\{\sin\left(\frac{A-B}{2}\right) - \sin\left(\frac{C-D}{2}\right)\right\}$$

$$2\cos\left(\frac{A+B}{2}\right)\left\{2\cos\left(\frac{A+B+C-D}{4}\right).\sin\left(\frac{A-B-C+D}{4}\right)\right\}$$

$$4\cos\left(\frac{A+B}{2}\right)\cos\left\{\frac{A+C-360^0+A+C}{4}\right\}\sin\left\{\frac{A+D-360^0+A+D}{4}\right\}$$

$$4\cos\left(\frac{A+B}{2}\right)\cos\left\{\frac{A+C}{2} - 90^0\right\}\sin\left\{\frac{A+D}{2} - 90^0\right\}$$

$$4\cos\left(\frac{A+B}{2}\right)\cos\left\{\frac{A+C}{2} - 90^0\right\}\sin\left\{\frac{A+D}{2} - 90^0\right\}$$

$$-4\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A+D}{2}\right)$$

(ii) $\cos 2A + \cos 2B + \cos 2C + \cos 2D = 2\cos(A+B)\cos(A-B) + 2\cos(C+D).\cos(C-D)$

$$= 2\cos(A+B)\cos(A-B) + 2\cos\left(360^0 - \overline{A+B}\right).\cos(C-D)$$

$$= 2\cos(A+B)\cos(A-B) + 2\cos(A+B)\cos(C-D)$$

$$= 2\cos(A+B)\{\cos(A-B) + \cos(C-D)\}$$

$$= 2\cos(A+B)\left\{+2\cos\left(\frac{A+B+C-D}{2}\right)\cos\left(\frac{A-B-C+D}{2}\right)\right\}$$

$$4\cos(A+B)\cos\left\{\frac{(B+D)-(A+C)}{2}\right\}\cos\left\{\frac{(B+C)-(A+D)}{2}\right\}$$

$$4\cos(A+B)\cos\left\{\frac{360^0 - 2(A+C)}{2}\right\}\cos\left\{\frac{360^0 - 2(A+D)}{2}\right\}$$

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$4\cos(A+B)\cos\left[180^0 - \overline{A+C}\right]\cos\left(180^0 - \overline{A+D}\right)$

$\{4\cos(A+B)\}\{-\cos(A+D)\}\{-\cos(A+D)\}$

$4\cos(A+B)\cos(A+C)\cos(A+D)$

9. If $A+B+C=2S$ అయితే

(i) $\sin(s-A)+\sin(s-B)+\sin C = 4\cos\left(\frac{S-A}{2}\right)\cos\left(\frac{S-B}{2}\right)\sin\frac{C}{2}$

(ii) $\cos(s-A)+\cos(s-B)+\cos C = -1+4\cos\left(\frac{s-A}{2}\right)\cos\left(\frac{s-B}{2}\right)\cos\frac{C}{2}$ అని చూపుము

Solution :

(i) $\sin(s-A)+\sin(s-B)+\sin C$

$= 2\cos\left(\frac{2s-A-B}{2}\right)\cos\left(\frac{B-A}{2}\right)+\sin C$

$= 2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)+2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)+\cos\frac{C}{2}\right\}$

$= 2\sin\frac{C}{2}\left\{2\cos\left(\frac{A-B+C}{4}\right).\cos\left(\frac{A-B-C}{4}\right)\right\}$

$= 4\sin\frac{C}{2}\left\{\cos\left(\frac{2s-B-B}{4}\right)\cos\left(\frac{2s-A-A}{4}\right)\right\}$

$4\cos\left(\frac{s-A}{2}\right)\cos\left(\frac{s-B}{2}\right)\sin\frac{C}{2}$

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Solution (ii)

$$\cos(s-A)+\cos(s-B)+\cos C$$

$$=2\cos\left(\frac{2s-A-B}{2}\right)\cos\left(\frac{B-A}{2}\right)+\cos C$$

$$=2\cos\frac{C}{2}\cos\left(\frac{B-A}{2}\right)+2\cos^2\frac{C}{2}-1$$

$$=-1+2\cos\frac{C}{2}\left[\cos\left(\frac{B-A}{2}\right)+\cos\frac{C}{2}\right]$$

$$=-1+2\cos\frac{C}{2}\left[2\cos\left(\frac{B-A+C}{4}\right)\cos\left(\frac{B-A-C}{4}\right)\right]$$

$$=-1+4\cos\frac{C}{2}\cos\left(\frac{B+C-A}{4}\right)\cos\left(\frac{A+C-B}{4}\right)$$

$$=-1+4\cos\frac{C}{2}\cos\left\{\frac{2s-A-A}{4}\right\}\cos\left(\frac{2s-B-B}{4}\right)$$

$$=-1+4\cos\frac{C}{2}\cos\left(\frac{S-A}{2}\right)\cos\left(\frac{s-B}{2}\right)$$

$$=-1+4\cos\left(\frac{S-A}{2}\right)\cos\left(\frac{S-B}{2}\right)\cos\left(\frac{S-B}{2}\right)\cos\frac{C}{2}$$

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10. A, B, C లు త్రిభుజ కోణాలు ఐతే $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ అని చూపుము

SOL.

A+B+C = 180^0

$$LHS=\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}$$

$$=\sin^2\frac{A}{2}+\sin\left(\frac{B}{2}+\frac{C}{2}\right).\sin\left(\frac{B}{2}-\frac{C}{2}\right)$$

$$=\sin^2\frac{A}{2}+\sin\left(90-\frac{A}{2}\right).\sin\left(\frac{B}{2}-\frac{C}{2}\right)$$

$$=1\text{-}\cos^2\frac{A}{2}+\cos\frac{A}{2}.\sin\left(\frac{B}{2}-\frac{C}{2}\right)$$

$$=1-\cos\frac{A}{2}\left(\cos\frac{A}{2}-\sin\left(\frac{B}{2}-\frac{C}{2}\right)\right)$$

$$=1-\cos\frac{A}{2}\left(\cos\left(90-\left(\frac{B}{2}+\frac{C}{2}\right)\right)-\sin\left(\frac{B}{2}-\frac{C}{2}\right)\right)$$

$$=1-\cos\frac{A}{2}\left(\sin\left(\frac{B}{2}+\frac{C}{2}\right)-\sin\left(\frac{B}{2}-\frac{C}{2}\right)\right)$$

$$=1-\cos\frac{A}{2}\left(2\cos\frac{B}{2}\sin\frac{C}{2}\right)$$

$$=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}=RHS$$

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11. A+B+C =3π/2 అయితే cos2A+cos2B+cos2C =1- 4sinA.sinB.sinC అని చూపుము

12. A, B, C లు త్రిభుజ కోణాలు ఐతే

$\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}=\ 1+4\sin\frac{\pi-A}{4}+\sin\frac{\pi-B}{4}+\sin\frac{\pi-C}{4}$ అని చూపుము

13. A, B, C లు త్రిభుజ కోణాలు ఐతే cos2A+cos2B+cos2C =

-4cosAcosBcosC-1 అని చూపుము

cos2A+cos2B+cos2C =

$$
\begin{aligned}
&=2\cos\frac{2A+2B}{2}\cos\frac{2A-2B}{2}+\cos 2C\\
&=2\cos(A+B)\cos(A-B)+2\cos^2 C-1\\
&=2\cos(\pi-c)\cos(A-B)+2\cos^2 C-1\\
&=-2\cos C\cos(A-B)+2\cos^2 C-1\\
&=2\cos C\left(-\cos(A-B)+\cos C\right)-1\\
&=2\cos C\left(-\cos(A-B)+\cos(\pi-(A+B))\right)-1\\
&=2\cos C\left(-\cos(A-B)-\cos(A+B)\right)-1\\
&=2\cos C\left(-2\cos A\cos B\right)-1\\
&=-4\cos A\cos B\cos C-1
\end{aligned}
$$

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