విలోమ త్రికోణమితీయ ప్రమేయాలు

1. ఈ క్రింది వానిని గణించండి.

(i) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(ii) $\cos^{-1}\left(\frac{1}{2}\right)$

(iii) $\sec^{-1}\left(-\sqrt{2}\right)$

(iv) $\cot^{-1}\left(-\sqrt{3}\right)$

(v) $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right\}$

(vi) $\sin\left\{\frac{\pi}{2}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}$

(vii) $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$

(iv) $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$

(i) సాధన:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=-\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=-\frac{\pi}{3}\{\because \sin^{-1}(-x)=\sin^{-1}x\}$$

(ii)సాధన:

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$$

(iii)సాధన:

$$\sec^{-1}\left(-\sqrt{2}\right)=\pi-\sec^{-1}\sqrt{2}=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$$

(iv)సాధన:

$$\cot^{-1}\left(-\sqrt{3}\right)=\pi-\cot^{-1}\sqrt{3}=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$$

(v)సాధన:

$$\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right\}=\sin\left\{\frac{\pi}{3}+\sin^{-1}\frac{1}{2}\right\}=\sin\left\{\frac{\pi}{3}+\frac{\pi}{6}\right\}=\sin\frac{\pi}{2}=1$$

(vi)సాధన:

$$\sin\left\{\frac{\pi}{2}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}=\sin\left\{\frac{\pi}{2}+\sin^{-1}\frac{\sqrt{3}}{2}\right\}=\sin\left\{\frac{\pi}{2}+\frac{\pi}{6}\right\}=\sin 120^{0}=\frac{\sqrt{3}}{2}$$

(vii)సాధన:

$$\sin^{-1}\left(\sin\frac{5\pi}{6}\right)=\sin^{-1}\left(\frac{1}{2}\right)\left\{\because \sin\frac{5\pi}{6}=\frac{1}{2}\right\}$$

$$=\frac{\pi}{6}\because \sin\frac{\pi}{6}=\frac{1}{2}$$

(iv)సాధన:

$$\cos^{-1}\left(\cos\frac{5\pi}{4}\right)=\cos^{-1}\left\{-\frac{1}{\sqrt{2}}\right\}=\pi-\cos^{-1}\frac{1}{\sqrt{2}}=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$$

2. ఈ క్రింది వానిని కనుగొనుము

(i) $\sin\left\{\cos^{-1}\frac{3}{2}\right\}$

(ii) $\tan\left\{\operatorname{cos}ec^{-1}\frac{65}{63}\right\}$

(iii) $\sin\left\{2\sin^{-1}\frac{4}{5}\right\}$

(iv) $\sin^{-1}\sin\left(\frac{33\pi}{7}\right)$

(v) $\cos^{-1}\cos\left(\frac{17\pi}{6}\right)$

(i)సాధన:

$$\sin\left\{\cos^{-1}\frac{3}{5}\right\} \quad let\ \cos^{-1}\frac{3}{5}=\alpha \Rightarrow \cos\alpha=\frac{3}{5}$$

$$\sin\left\{\cos^{-1}\frac{3}{5}\right\} = \sin\alpha=\frac{4}{5} \quad \left\{\because \cos\alpha=\frac{3}{5}\right\}$$

(ii)సాధన:

$$\tan\left\{\cos ec^{-1}\frac{65}{63}\right\} \quad let\ \cos ec^{-1}\frac{65}{63}=\alpha \Rightarrow \cos ec\,\alpha=\frac{65}{63}$$

(iii)సాధన:

$$\sin\left\{2\sin^{-1}\frac{4}{5}\right\} \quad let\ \sin^{-1}\frac{4}{5}=\alpha \Rightarrow \sin\alpha=\frac{4}{5}:\cos\alpha=\frac{3}{5}$$

$$\sin\left\{2\sin^{-1}\frac{4}{5}\right\} = \sin^2\alpha=2\sin\alpha\cos\alpha=2\times\frac{4}{5}\times\frac{3}{5}=\frac{24}{25}$$

(iv)సాధన:

$$\sin^{-1}\sin\left(\frac{33\pi}{7}\right)=\sin^{-1}\left\{\sin\left\{5\pi-\frac{2\pi}{7}\right\}\right\}=\sin^{-1}\left\{+\sin\left(\frac{2\pi}{7}\right)\right\}=\frac{2\pi}{7}$$

(v)సాధన:

$$\cos^{-1}\left\{\cos\frac{17\pi}{6}\right\}=\cos^{-1}\left\{\cos 3\pi-\frac{\pi}{6}\right\}=\cos^{-1}\left\{-\cos\frac{\pi}{6}\right\}=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$$

3) క్రింది వాటిని సూక్ష్మీకరించండి.

i) $\tan^{-1}(\sec x+\tan x)$.

సాధన:

$$\tan^{-1}\left[\frac{1+\sin x}{\cos x}\right]=\tan^{-1}\left[\frac{1+\frac{2\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}}{\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}}\right]$$

$$=\tan^{-1}\left(\frac{1+\tan\frac{x}{2}+2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}\right)$$

$$=\tan^{-1}\left(\frac{\left(1+\tan\frac{x}{2}\right)^2}{\left(1+\tan\frac{x}{2}\right)\left(1-\tan\frac{x}{2}\right)}\right)$$

ii) $\sin^{-1}\left(2\cos^2\theta-1\right)+\cos^{-1}\left(1-2\sin^2\theta\right)$

సాధన: $\sin^{-1}(\cos 2\theta)+\cos^{-1}(\cos 2\theta)$

$=\sin^{-1}\left[\sin\left(90^0-20^0\right)\right]+\cos^{-1}(\cos 2\theta)$

$=90^0-2\theta+2\theta=90^0$

iii) $\tan^{-1}\left(x+\sqrt{1+x^2}\right), x\in R$

సాధన: $x=\tan\theta\Rightarrow\theta=\tan^{-1}x$

$\tan^{-1}\left(\tan\theta+\sqrt{1+\tan^2\theta}\right)$

$\tan^{-1}(\tan\theta+\sec\theta)$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right) = \tan^{-1}\left[\frac{1 + \dfrac{2\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}}{\dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}}\right]$$

$$= \tan^{-1}\left(\frac{1+\tan\frac{\theta}{2} + 2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\left(1+\tan\frac{\theta}{2}\right)^2}{\left(1+\tan\frac{\theta}{2}\right)\left(1-\tan\frac{\theta}{2}\right)}\right)$$

$$= \tan^{-1}\left(\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{2}+\frac{\theta}{2}\right)\right)$$

$$= \frac{\pi}{4}+\frac{\theta}{2}$$

$$= \frac{\pi}{4}+\frac{1}{2}\tan^{-1}x = \frac{\pi}{4}+\frac{1}{2}\tan^{-1}x$$

5. $\sec^2(Tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}2) = 10$ **నిరూపించండి.**

సాధన: $Tan^{-1}2 = \alpha$ *and* $\cot^{-1}2 = \beta$ అనుకొనుము

$Tan\alpha = 2$ *and* $\cot\beta = 2$

LHS = $\sec^2\alpha + \operatorname{cosec}^2\beta = 1 + Tan^2\alpha + 1 + \cot^2\beta = 44 + 1 + 4 = 10$

స్వల్ప సమాధాన ప్రశ్నలు..

ఈ క్రింది వానిని నిరూపించండి.

1(i) $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{36}{85}$

(ii) $\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

iii) $\tan\left[\cot^{-1}9+\operatorname{cosec}^{-1}\frac{\sqrt{41}}{4}\right]=1$

సాధన:

(i) సాధన:

L.H.S $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}$

Let $\sin^{-1}\frac{3}{5}=\alpha$ $\quad\sin^{-1}\frac{8}{17}=\beta$

$\sin\alpha=\frac{3}{5}$ $\quad\sin\beta=\frac{8}{17}$

$\cos\alpha=\frac{4}{5}$ $\quad\cos\beta=\frac{15}{17}$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=\frac{4}{5}\times\frac{15}{17}-\frac{3}{5}\times\frac{8}{17}=\frac{36}{85}$$

$\therefore\ \alpha+\beta=\cos^{-1}\frac{36}{85}\Rightarrow\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{36}{85}$

(ii) సాధన:

$\sin^{-1}\frac{3}{5}=\alpha$ $\quad\cos^{-1}\frac{12}{13}=\beta$ అనుకొనుము.

$\sin\alpha=\frac{3}{5}$ $\quad\cos\beta=\frac{12}{13}$

$$\cos\alpha\frac{4}{5} \qquad \sin\beta=\frac{5}{13}$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

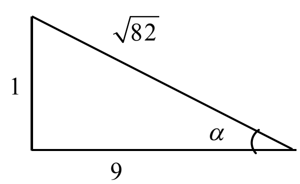
$$\cos(\alpha+\beta)=\frac{4}{5}\times\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}=\frac{33}{65}$$

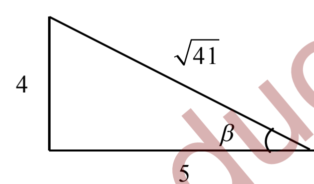
$$\therefore\alpha+\beta=\cos^{-1}\frac{33}{65}$$

$$\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$$

iii) $\alpha=\cot^{-1}9, \beta=\operatorname{cos}ec^{-1}\frac{\sqrt{41}}{4}$

$\cot\alpha=9$, $\operatorname{cos}ec\beta=\frac{\sqrt{41}}{4}$





$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{1}{9}+\frac{4}{5}}{1-\frac{1}{9}.\frac{4}{5}}$$

$$=\frac{\frac{5+36}{45}}{\frac{45-4}{45}}=\frac{41}{41}=1$$

$$\tan(\alpha+\beta)=1$$

$$\tan\left(\cos^{-1}9+\operatorname{cos}ec\frac{41}{4}\right)=1$$

ఈ క్రింది వానిని కనుగొనుము.

2(i) $\sin\left\{\cos^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13}\right\}$

(ii) $Tan\left\{\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{3}{\sqrt{34}}\right\}$

(iii) $\cos\left\{\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right\}$

(i)సాధన:

$$\text{Let } \cos^{-1}\frac{3}{5}=\alpha \quad and \quad \cos^{-1}\frac{12}{13}=\beta$$

$$\cos\alpha=\frac{3}{5} \ and \ \cos\beta=\frac{12}{13}$$

$$\sin\left\{\cos^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13}\right\}=\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$=\frac{4}{5}\times\frac{12}{13}+\frac{3}{5}\times\frac{5}{13}$$

$$=\frac{63}{65}$$

(ii)సాధన:

$$\text{Let } \sin^{-1}\frac{3}{5}=\alpha \ and \cos^{-1}\frac{5}{\sqrt{34}}=\beta$$

$$\sin\alpha=\frac{3}{5} \quad \cos\beta=\frac{5}{\sqrt{34}}$$

$$\cos\alpha=\frac{4}{5} \quad \sin\beta=\frac{3}{\sqrt{34}}$$

$$Tan(\alpha+\beta)=\frac{Tan\alpha+Tan\beta}{1-Tan\alpha Tan\beta}=\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{9}{20}}=\frac{\frac{27}{20}}{\frac{20-9}{20}}=\frac{27}{11}$$

(iii)సాధన:

Let $\sin^{-1}\frac{3}{5}=\alpha\ and\ \sin^{-1}\frac{5}{13}=\beta$

$$\sin\alpha=\frac{3}{5}\quad and\ \ \sin\beta=\frac{5}{13}$$

$$\cos\alpha\frac{4}{5}\quad and\ \cos\beta=\frac{12}{13}$$

$$\cos\left\{\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right\}=\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$=\frac{4}{5}\times\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}=\frac{33}{65}$$

3. ఈ క్రిందివానిని నిరూపించండి.

i) $\cos\left[2\tan^{-1}\frac{1}{7}\right]=\sin\left[2\tan^{-1}\frac{3}{4}\right]$

సాధన: $\tan^{-1}\frac{1}{7}=\alpha\Rightarrow\tan\alpha=\frac{1}{7}$

$$\cos 2\alpha=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}=\frac{1-\frac{1}{49}}{1+\frac{1}{49}}=\frac{48}{50}=\frac{24}{25}$$

$$\tan^{-1}\frac{3}{4}=\beta\Rightarrow\tan\beta=\frac{3}{4}$$

$$\sin 2\beta=\frac{2\tan\beta}{1+\tan^2\beta}=\frac{2\left[\frac{3}{4}\right]}{1+\frac{9}{16}}$$

$$=\frac{6}{4}\times\frac{16}{25}=\frac{24}{25}$$

$\therefore \cos 2\alpha = \sin 2\beta$

$\therefore \cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(2\tan^{-1}\frac{3}{4}\right)$

ii) $\tan\left(2\tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right)\right) = 2$

సాధన:

$$\alpha = \tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right) \Rightarrow \tan\alpha = \frac{\sqrt{5}-1}{2}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$= \frac{2\left(\frac{\sqrt{5}-1}{2}\right)}{1-\frac{\left(\sqrt{5}-1\right)}{4}} = \frac{\sqrt{5}-1}{\frac{4-(5+1)-2\sqrt{5}}{4}}$$

$$= \frac{4\left(\sqrt{5}-1\right)}{4-6+2\sqrt{5}} = \frac{4\left(\sqrt{5}-1\right)}{2\sqrt{5}-2}$$

$$= \frac{4\left(\sqrt{5}-1\right)}{2\left(\sqrt{5}-1\right)} = 2$$

iii) $\cos\left\{2\left(\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)\right)\right\} = \frac{3}{5}$

$$\tan^{-1}\frac{1}{4} = \alpha = \tan\alpha = \frac{1}{4}$$

$$\cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{1-\frac{1}{16}}{1+\frac{1}{16}} = \frac{15}{17}$$

$$\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha} = \frac{2\left(\frac{1}{4}\right)}{1+\frac{1}{16}}$$

$$= \frac{2}{4}\times\frac{16}{17} = \frac{8}{17}$$

$$\cos 2\beta = \frac{2\tan\beta}{1+\tan^2\beta} = \frac{2\left(\frac{2}{9}\right)}{1+\frac{4}{81}}$$

$$= \frac{4}{9}\times\frac{\overset{9}{81}}{85} = \frac{36}{85}$$

$$\cos(2\alpha+2\beta) = \cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta$$

$$= \left(\frac{15}{17}\right)\left(\frac{77}{85}\right) - \left(\frac{8}{17}\right)\left(\frac{36}{85}\right)$$

$$= \frac{1155}{1445} - \frac{288}{1445}$$

$$= \frac{867}{1445} = \frac{3}{5}$$

$$\therefore \cos\left\{2^{-}\left[\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}\right]\right\} = \frac{3}{5}$$

4. ఈ క్రింది వానిని నిరూపించండి.

(i) $Tan^{-1}\frac{1}{7}+Tan^{-1}\frac{1}{13}+Tan^{-1}\frac{2}{4}=0$

(ii) $Tan^{-1}\frac{1}{2}+Tan^{-1}\frac{1}{5}+Tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

(iii) $Tan^{-1}\frac{3}{4}+Tan^{-1}\frac{3}{5}-Tan^{-1}\frac{8}{19}=\frac{\pi}{4}$

(iv) $Tan^{-1}\frac{1}{7}+Tan^{-1}\frac{1}{8}=\cot^{-1}\frac{201}{43}+\cot^{-1}(18)$

(i) సాధన:

Let $Tan^{-1}\frac{1}{7}=\alpha \quad Tan^{-1}\frac{1}{13}=\beta \quad Tan^{-1}\frac{2}{9}=\gamma$

$$Tan\alpha=\frac{1}{7} \quad Tan\beta\frac{1}{13} \quad Tan\gamma=\frac{2}{9}$$

$$Tan(\alpha+\beta)=\frac{Tan\alpha+Tan\beta}{1-Tan\alpha Tan\beta}=\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{91}}=\frac{\frac{20}{91}}{\frac{90}{91}}=\frac{2}{9}$$

$$Tan(\alpha+\beta-\gamma)=\frac{Tan(\alpha+\beta)-Tan\gamma}{1+Tan(\alpha+\beta)Tan\gamma}=\frac{\frac{2}{9}-\frac{2}{9}}{1+\frac{4}{8}}=0$$

$$\because \alpha+\beta-\gamma=0 \Rightarrow Tan^{-1}\frac{1}{7}+Tan^{-1}\frac{1}{13}-Tan^{-1}\frac{2}{9}=0$$

(ii)సాధన:

$$Tan^{-1}\frac{1}{2}+Tan^{-1}\frac{1}{5}+Tan^{-1}\left(\frac{1}{8}\right)$$

Let $Tan^{-1}\frac{1}{2}=\alpha \quad Tan^{-1}\frac{1}{5}=\beta \quad Tan^{-1}\frac{1}{8}=\delta$

$$Tan\alpha=\frac{1}{2} \quad Tan\beta=\frac{1}{5} \quad Tan\gamma=\frac{1}{8}$$

$$Tan(\alpha+\beta)=\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{10}}=\frac{7}{9}$$

$$Tan(\alpha+\beta+\gamma)=\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}\times\frac{1}{8}}=\frac{56+9}{72}\times\frac{72}{72-7}=1$$

$$\alpha+\beta+\delta=\frac{\pi}{4}\Rightarrow Tan^{-1}\frac{1}{2}+Tan^{-1}\frac{1}{5}+Tan^{-1}\frac{1}{8}=\frac{\pi}{4}$$

(iii) సాధన:

$$Tan^{-1}\frac{3}{4}=\alpha \quad Tan^{-1}\frac{3}{5}=\beta \quad and\ Tan^{-1}\frac{8}{19}=\delta$$

$$Tan\alpha=\frac{3}{4} \quad Tan\beta=\frac{3}{5} \quad and\ Tan\delta=\frac{8}{19}$$

$$Tan(\alpha+\beta)=\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{9}{20}}=27 \Rightarrow Tan\left(\alpha+\beta-\delta\right)=\frac{Tan\left(\alpha+\beta\right)+Tan\delta}{1-Tan\left(\alpha+\beta\right)Tan\delta}$$

$$=\frac{\frac{27}{11}+\frac{8}{19}}{1+\frac{27}{11}\times\frac{8}{19}}=\frac{\frac{513-88}{209}}{\frac{209+216}{209}}=1$$

$$\because \alpha+\beta-\delta=\frac{\pi}{4}$$

(iv) సాధన:

$$Tan^{-1}\frac{1}{7}=\alpha \quad Tan^{-1}\frac{1}{8}=\beta$$ అనుకొనుము

$$Tan\alpha=\frac{1}{7} \quad and\ Tan\beta=\frac{1}{8}$$

$$Tan(\alpha+\beta)=\frac{Tan\alpha+Tan\beta}{1-Tan\alpha Tan\beta}=\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{50}}=\frac{15}{55}=\frac{3}{11}$$

LHS $\alpha+\beta=Tan^{-1}\frac{3}{11}=\cot^{-1}\left(\frac{11}{3}\right)$

Let $\cot^{-1}\frac{201}{43}=\delta \quad \cot^{-1}18=\delta$

$$\cot\delta=\frac{201}{43} \quad \cot\delta=18$$

$$\cot(\gamma+\delta)=\frac{\cot\gamma\cot\delta-1}{\cot\delta+\cot\gamma}=\frac{\frac{201\times18}{43}-1}{\frac{201}{43}}=\frac{\frac{3618-43}{43}}{\frac{201+774}{43}}$$

$$=\frac{3575}{975}=\frac{11}{3}$$

$$\gamma+\delta=\cot^{-1}\frac{11}{3} \quad \because \text{LHS=RHS}$$

5. $Tan\left\{\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right\}$ **విలువను కనుగొనుము.**

సాధన: $\cos^{-1}\frac{4}{5}=\alpha \quad and \quad \tan^{-1}\frac{2}{3}=\beta$

$$\cos\alpha=\frac{4}{5} \quad and \quad \tan\beta=\frac{2}{3}$$

$$Tan\alpha=\frac{3}{4} \quad and \quad \tan\beta=\frac{2}{3}$$

$$Tan\left\{\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right\}=Tan(\alpha+\beta)=\frac{Tan\alpha+Tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}=\frac{\frac{17}{12}}{\frac{6}{12}}=\frac{17}{6}$$

6. $Tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+Tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$ **అని నిరూపించండి.**

సాధన: $\cos^{-1}\frac{a}{b}=\alpha\Rightarrow\cos\alpha=\frac{a}{b}$

L.H.S $\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$

$$\frac{1+\tan\frac{\alpha}{2}}{1-\tan\frac{\alpha}{2}}+\frac{1-\tan\frac{\alpha}{2}}{1+\tan\frac{\alpha}{2}}=\frac{\left(1+\tan\frac{\alpha}{2}\right)^2+\left(1-\tan\frac{\alpha}{2}\right)^2}{1-\tan^2\frac{\alpha}{2}}$$

$$\frac{2\left\{1+\tan^2\frac{\alpha}{2}\right\}}{1-\tan^2\frac{\alpha}{2}}=2\sec\alpha=\frac{2b}{a}$$

iv) $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$ **అయితే x విలువను కనుక్కోండి.**

సాధన. $\alpha = \sin^{-1}x \Rightarrow \sin\alpha = x$

$\beta = \cos^{-1}x \Rightarrow \cos\beta = x$

$$\sin(\alpha - \beta) = \sin\frac{\pi}{6}$$

$$\sin\alpha\cos\beta - \cos\alpha\sin\beta = \frac{1}{2}$$

$$(x)(x) - \sqrt{1-x^2}\sqrt{1-x^2} = \frac{1}{2}$$

$$x^2 - \frac{1}{2} = \sqrt{(1-x^2)(1-x^2)}$$

ఇరువైపుల వర్గం చేయగా

$$x^4 + \frac{1}{4} - x^2 = 1 - x^2 - x^2 + x^4$$

$$\frac{1}{4} = 1 - x^2$$

$$x^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$x = \frac{\sqrt{3}}{2}$$

7. ఈ క్రింది వానిని సాధించండి.

1) $\tan\left\{2\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)\right\} = x$ **అని చూపండి.**

సాధన. $\tan\left\{2\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)\right\}$

$x = \tan\theta$

$$= \tan\left\{2\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)\right\}$$

$$= \tan\left\{2\tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)\right\}$$

$$= \tan\left\{2\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)\right\}$$

$$= \tan\left\{2\tan^{-1}\left(\frac{2\sin^2\theta/2}{\sin\theta/2\cos\theta/2}\right)\right\}$$

$$= \text{ta}\left\{2\tan^{-1}\left(\tan\theta/2\right)\right\}$$

$$= \tan\left(2.\frac{\theta}{2}\right) = \tan\theta = x$$

2) $\sin\left[\cot^{-1}\frac{2x}{1-x^2} + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] = 1$ **అని చూపండి?**

$$\sin\left[\cot^{-1}\frac{2x}{1-x^2} + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

$$x = \tan\theta$$

$$= \sin\left[\cot^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right]$$

$$= \sin\left[\cot^{-1}2\theta\left(\tan 2\theta\right) + \cos^{-1}\left(\cos w\theta\right)\right]$$

$$= \sin\left[\cot^{-1}\left[\cot 90 - 2\theta\right] + \cos^{-1}\left(\cos 2\alpha\right)\right]$$

3) $\sin^{-1}\left[\frac{2p}{1+p^2}\right] - \cos^{-1}\left(\frac{1-q^2}{1+q^2}\right) = \tan^{-1}\left[\frac{2x}{1-x^2}\right],$

అయితే $x = \frac{p-q}{1+pq}$ **అని చూపండి.**

$$\because \sin^{-1}\left(\frac{2p}{1+p^2}\right) - \cos^{-1}\left(\frac{1-q^2}{1+q^2}\right)$$

$$= \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$p = \tan A, q = \tan B, x = \tan C$ **అనుకుంటే**

$$\Rightarrow \sin^{-1}\left(\frac{2\tan A}{1+\tan^2 A}\right) - \cos^{-1}\left(\frac{1-\tan^2 B}{1+\tan^2 B}\right)$$

$$= \tan^{-1}\left(\frac{2\tan C}{1-\tan^2 C}\right)$$

$$\Rightarrow \sin^{-1}\left(\sin 2A\right) - \cos^{-1}\left(\cos 2B\right)$$

$$= \tan^{-1}\left(\tan 2C\right)$$

$\Rightarrow 2A-2B=2C$

$\Rightarrow A-B=C$

$\Rightarrow \tan^{-1} p-\tan^{-1} q=\tan^{-1}(x)$

$\Rightarrow \tan^{-1}\left(\frac{p-q}{1+pq}\right)=\tan^{-1}(x)$

$\Rightarrow x=\frac{p-q}{1+pq}$

(4) $\sin^{-1}(1-x)+\sin^{-1} x=\cos^{-1} x$

సాధన: $\sin^{-1}(1-x)+\sin^{-1} x=\cos^{-1} x$

$\therefore \sin^{-1}(1-x)=\cos^{-1} x-\sin^{-1} x$

but $\cos^{-1} x=\frac{\pi}{2}-\sin^{-1} x$

$\sin^{-1}(1-x)=\frac{\pi}{2}-\sin^{-1} x-\sin^{-1} x$

$(1-x)=\sin\left\{\frac{\pi}{2}-2\sin^{-1} x\right\}$

$\therefore (1-x)=\cos\left(2\sin^{-1} x\right)$

Let $\sin^{-1} x=\alpha \Rightarrow \sin\alpha=x$

$\therefore 1-x=\cos^2\alpha$

$1-x=1-2\sin^2\alpha \Rightarrow 1-x=1-2x^2$

$2x^2-x=0 \Rightarrow x=0=\frac{1}{2}$

8. $\alpha=Tan^{-1}\frac{\sqrt{1-x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$ **ఐతే** $x^2=\sin^2\alpha$ **అని చూపుము.**

సాధన:

$\tan\alpha\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}} \quad \Rightarrow \quad \frac{1}{\tan\alpha}=\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}$

$\frac{1+\tan\alpha}{1-\tan\beta}=\frac{\sqrt{1+x^2}+\sqrt{1-x^2}+\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}-\sqrt{1+x^2}+\sqrt{1-x^2}}$

$\frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}=\frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\Rightarrow$ ఇరుపైపులా వర్గం చేయగా

$$\frac{\cos^2+\sin^2\alpha+2\cos\alpha\sin\alpha}{\cos^2+\sin^2\alpha-2\cos\alpha\sin\alpha}+\frac{1+x^2}{1-x^2}\Rightarrow\frac{1+\sin 2\alpha}{1-\sin 2\alpha}=\frac{1+x^2}{1-x^2}$$

$$\frac{1+\sin 2\alpha+1-\sin 2\alpha}{1+\sin 2\alpha-1+\sin 2\alpha}=\frac{1+x^2+1-x^2}{1+x^2-1+x^2}\Rightarrow\frac{2}{2\sin^2\alpha}=\frac{2}{2\sin^2}$$

$$\Rightarrow x^2=\sin 2\alpha$$

9. $2\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{5}{13}=\cos^{-1}\left(\frac{323}{325}\right)$ **అని నిరూపించండి.**

Let $\sin^{-1}\frac{3}{5}=\alpha \qquad \cos^{-1}\frac{5}{13}=\beta$

$$\sin\alpha=\frac{3}{5}\qquad \cos\beta=\frac{5}{13}$$

$$\cos\alpha=\frac{4}{5}\qquad \sin\beta=\frac{12}{13}$$

$$\sin^2\alpha=2\sin\alpha\cos\alpha=\frac{24}{25}\quad \cos^2\alpha=1-2\sin^2\alpha$$

$$=1-2\left(\frac{9}{25}\right)=\frac{7}{25}$$

$$\cos(2\alpha-\beta)=\cos^2\alpha\cos\beta+\sin^2\alpha\sin\beta$$

$$=\frac{7}{25}\times\frac{5}{13}+\frac{24}{25}\times\frac{12}{13}=\frac{35+288}{325}=\frac{323}{325}$$

(ii) సాధన:

$$\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$$

Let $\sin^{-1}\frac{4}{5}=\alpha \quad \tan^{-1}\frac{1}{3}=\beta$

$$\sin\alpha=\frac{4}{5}\qquad \tan\beta=\frac{1}{3}$$

$\cos\alpha = \frac{3}{5}$ $\quad\quad$ $\sin^2\beta = \frac{2\tan\beta}{1+\tan^2\beta} = \frac{\frac{2}{3}}{1+\frac{1}{9}} = \frac{2}{3}\times\frac{9}{10} = \frac{3}{5}$

$\cos^2\beta = \frac{4}{5}$

$\cos(\alpha+2\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{3}{4}\times\frac{4}{5} - \frac{4}{5}\times\frac{3}{5} = 0$

$\alpha + 2\beta = \frac{\pi}{2}$

(iii) $4\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{99} - \tan^{-1}\frac{1}{70} = \frac{\pi}{4}$ **అని నిరూపించండి.**

సాధన: $\tan^{-1}\frac{1}{5} = \alpha$ $\quad$ $\tan^{-1}\frac{1}{99} = \beta$ $\quad$ $\tan^{-1}\frac{1}{70} = \delta$

$\tan\alpha = \frac{1}{5}$ $\quad$ $\tan\beta = \frac{1}{99}$ $\quad$ $\tan\delta = \frac{1}{70}$

$Tan2\alpha = \frac{\frac{2}{5}}{1-\frac{1}{25}} = \frac{2}{5}\times\frac{25}{24} = \frac{5}{12}$ $\quad\quad$ $Tan4\alpha = \frac{2Tan2\alpha}{1-Tan^2 2\alpha} = \frac{2\times\frac{5}{12}}{1-\frac{25}{144}} = \frac{120}{119}$

$Tan(4\alpha+\beta) = \frac{Tan4\alpha + Tan\beta}{1 - Tan4\alpha Tan\beta} = \frac{\frac{120}{119} - \frac{1}{99}}{1-\frac{120}{119}\times\frac{1}{94}} = \frac{\frac{11880+119}{119\times 99}}{11781-120} = \frac{11999}{11661}$

$Tan(4\alpha+\beta-\delta) = \frac{Tan(4\alpha+\beta) - Tan\delta}{1+\tan(4\alpha+\beta)Tan\delta} = \frac{\frac{11999}{11661} - \frac{1}{70}}{1+\frac{11999}{11661}\times\frac{1}{70}} = \frac{828269}{828269} = 1$

$\therefore 4\alpha + \beta - \delta = \frac{\pi}{4}$ $\quad\quad$ $\therefore 4\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{99} - \tan^{-1}\frac{1}{70} = \frac{\pi}{4}$

10. $\cos^{-1}p+\cos^{-1}q+\cos^{-1}r=\pi$ **ఐతే** $p^2+q^2+r^2+2pqr=1$ **అని చూపుము.**

సాధన: $\cos^{-1}p=\alpha \quad \cos^{-1}q=\beta \quad \cos^{-1}r=\delta$

$p=\cos\alpha \quad q=\cos\beta \quad r=\cos\delta$

Given $\alpha+\beta+\gamma=\pi \quad \cos(\alpha+\beta)=\cos(\pi-\gamma)$

$\cos\alpha\cos\beta-\sin\alpha\sin\gamma=\cos\gamma$

$pq=-r+\sqrt{1-p^2}\ \sqrt{1-q^2} \Rightarrow pq+r=\sqrt{1-p^2}\sqrt{1-q^2}$

$p^2q^2+r^2+2pqr=1-p^2-q^2+p^2q^2 \Rightarrow p^2+q^2+r^2+2pqr=1$

11) a,b,c లు ఒకే గుర్తు గల విభిన్న శూన్యేతర వాస్తవ సంఖ్యలు

$\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)=\pi$ **లేదా** 2π **అని చూపండి.**

$\because (a-b)+(b-c)+(c-a)=0$

$(a-b),(b-c),(c-a)$లన్నింటికీ ఒకే గుర్తు ఉండదు. రెండు సందర్భాలు వస్తాయి. పై మూడింటిలో

ఏవేని రెండు ధనాత్మకలు,ఒకటి ఋణాత్మకం,లేదా రెండు ఋణాత్మకాలు,ఒకటి ధనాత్మకం.

సందర్భం. i): $(a-b),(b-c)$లు ధనాత్మకలు

$(c-a)$ఋణాత్మకం అనుకోండి.

$\cot^{-1}\left(\frac{ab+1}{a-b}\right)=\tan^{-1}\left(\frac{a-b}{1+ab}\right)$

$=\tan^{-1}a-\tan^{-1}b$

$\because(ab>0)$

$\cot^{-1}\left(\frac{bc+1}{b-c}\right)=\tan^{-1}\left(\frac{b-c}{1+bc}\right)$

$=\tan^{-1}b-\tan^{-1}(\because bc>0)$

మరియు $\cot^{-1}\left(\frac{ca+1}{c-a}\right)=\pi+\tan^{-1}\left(\frac{c-a}{1+ca}\right)$

$\pi + \left(\tan^{-1} c - \tan^{-1} a\right)$

$\left(\because c - a < 0\right)$

$\therefore \sum \cot^{-1}\left(\frac{ab+1}{a-b}\right) = \left(\tan^{1} a - \tan^{-1} b\right)$

$+\left(\tan^{-1} b - \tan^{-1} c - \tan^{-1} b\right)$

$+\left(\tan^{-1} c - \tan^{-1} a\right) = \pi$

సందర్భం.ii): $(a-b),(b-c)$లు

ఋణాత్మకాలు, $(c-a)$ధనాత్మకం

అప్పుడు $\cot^{-1}\left(\frac{ab+1}{a-b}\right) = \pi - \cot^{-1}\left(\frac{ab+1}{b-a}\right)$

$= \pi - \tan^{-1}\left(\frac{b-a}{1+ba}\right)$

$= \pi - \left(\tan^{-1} - \tan^{-1} a\right)$

ఇదే విధంగా

$\cot^{-1}\left(\frac{bc+1}{b-c}\right) = \pi - \cot^{-1}\left(\frac{bc+1}{c-b}\right)$

$= \pi - \tan^{-1}\left(\frac{c-b}{1+cb}\right)$

$= \pi - \left(\tan^{-1} c - \tan^{-1} b\right)$

$\cot^{-1}\left(\frac{ca+1}{c-a}\right) = \tan^{-1}\left(\frac{c-a}{1+ca}\right)$

$= \tan^{-1} c - \tan^{-1}(a)$

$\therefore \sum \cot^{-1}\left(\frac{ab+1}{a-b}\right) = \pi - \left(\tan^{-1} b - \tan^{-1} a\right)$

$+\pi - \left(\tan^{-1} c - \tan^{-1} b\right)$

$+\left(\tan^{-1} c - \tan^{-1} a\right)$

$= 2\pi$

$\therefore \sum \cot^{-1}\left(\frac{ab+1}{a-b}\right) = \pi \text{ (or) } 2\pi$

12. $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$ **ఐతే** '$x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$ **అని చూపుము.**

సాధన:

$\sin^{-1}x=\alpha \quad \sin^{-1}y=\beta \quad \sin^{-1}z=\gamma$ అనుకొనుము

$\sin\alpha=x \quad \sin\beta=y \quad \sin\gamma=z$

$\alpha+\beta+\gamma=\pi \Rightarrow \alpha+\beta=\pi-\gamma$

$\sin(\alpha+\beta)=\sin\gamma \, and \, \cos\gamma=-\cos(\alpha+\beta)$

$x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}$

$\sin\alpha\sqrt{1-\sin^2\alpha}+\sin\beta\sqrt{1-\sin^2\beta}+\sin\gamma\sqrt{1-\sin^2\gamma}$

$\frac{1}{2}\left[\sin^2\alpha+\sin^2\beta+\sin^2\gamma\right] = \frac{1}{2}\left[2\sin(\alpha+\beta)\cos(\alpha-\beta)+\sin^2\gamma\right]$

$\frac{1}{2}\left[2\sin\gamma\cos(\alpha-\beta)+2\sin\gamma\cos\gamma\right]$

$\sin\gamma\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]=2\sin\alpha\sin\beta\sin\gamma=2xyz$

13. **(i)** $Tan^{-1}x+Tan^{-1}y+Tan^{-1}z=\pi$ **ఐతే** $x+y+z=xyz$ **అని చూపుము.**

(ii) $Tan^{-1}x+Tan^{-1}y+Tan^{-1}z=\frac{\pi}{2}$ **ఐతే** $xy+yz+zx=1$ **అని చూపుము.**

సాధన:

$\tan^{-1}x=\alpha \quad \tan^{-1}y=\beta \quad \tan^{-1}z=\gamma$ అనుకొనుము

$x=Tan\alpha \quad y=Tan\beta \quad z=Tan\gamma$

Given $\alpha+\beta+\gamma=\pi \Rightarrow \alpha+\beta=\pi-x$

$Tan(\alpha+\beta)=Tan(\pi-\gamma)\Rightarrow \frac{Tan\alpha+Tan\beta}{1-Tan\alpha Tan\beta}=Tan\gamma$

$Tan\alpha+Tan\beta=-Tan\gamma+Tan\alpha Tan\beta Tan\gamma \Rightarrow Tan\alpha+Tan\beta+Tan\gamma \; Tan\alpha Tan\beta Tan\gamma$

$=x+y+z=xyz$

(ii)సాధన:

$$\alpha+\beta+\gamma=\frac{\pi}{2} \Rightarrow \alpha+\beta=\frac{\pi}{2}-\gamma$$

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\cot\gamma \Rightarrow \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{1}{\tan\gamma}$$

$$\tan\alpha\tan\gamma+\tan\beta\tan\gamma=1-\tan\alpha\tan\beta$$

$$\tan\alpha\tan\beta+\tan\beta\tan\gamma+\tan\gamma\tan\alpha=1$$

$$xy+yz+zx=1$$

14. ఈ క్రింది సమీకరణాలను సాధించండి.

(i) $Tan^{-1}\left(\frac{x-1}{x-2}\right)+Tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

సాధన: $Tan^{-1}\left(\frac{x-1}{x-2}\right)=\alpha \qquad Tan^{-1}\frac{x+1}{x+2}=\beta$

$$Tan\alpha=\frac{x-1}{x-2} \qquad Tan\beta=\frac{x+1}{x+2}$$

$$\alpha+\beta=\frac{\pi}{4} \Rightarrow \frac{Tan\alpha+Tan\beta}{1-Tan\alpha Tan\beta}=1$$

$$\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{(x^2-1)}{(x^2-4)}}=1 \Rightarrow \frac{(x-1)(x+2)+(x+1)(x-2)}{x^2-4-x^2+1}=1$$

$$\frac{x^2+x-2+x^2-x-2}{-3}=1 \Rightarrow 2x^2-4=-3$$

$$2x^2=1 \Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

ii) $Tan^{-1}\left(\frac{1}{2x+1}\right)+Tan^{-1}\left(\frac{1}{4x+1}\right)=Tan^{-1}\left(\frac{2}{x^2}\right)$

సాధన:

$$Tan^{-1}\frac{1}{2x+1}=\alpha\Rightarrow \quad Tan\alpha=\frac{1}{2x+1}$$

$$Tan^{-1}\left(\frac{1}{4x+1}\right)=\beta\Rightarrow Tan\beta=\frac{1}{4x+1}$$

$$\alpha+\beta=Tan^{-1}\frac{2}{x^2}$$

$$\frac{Tan\alpha+Tan\beta}{1-Tan\alpha Tan\beta}=\frac{2}{x^2}$$

$$\frac{\frac{1}{2x+1}+\frac{1}{4x+1}}{1-\frac{1}{(2x+1)(4x+1)}}=\frac{2}{x^2}$$

$$\frac{4x+1+2x+1}{8x^2+6x+x-y}=\frac{2}{x^2}$$

$$6x^3+2x^2=16x^2+12x$$

$$6x^3-14x^2-12x=0$$

$$2x\{3x^2-7x-6\}=0$$

$$2x\{3x^2-9x+2x-6\}=0$$

$$2x(3x+2)(x-3)=0$$

$$1-\frac{5x^2}{4}=\frac{1}{4}-\frac{x^2}{2}\Rightarrow 1-\frac{1}{4}=\frac{5x^2}{4}-\frac{x^2}{2}$$

$$\frac{3}{4}=\frac{5x^2-2x^2}{4}\Rightarrow x=\frac{1}{\sqrt{3}}$$

15 . ఈ క్రింది వానిని నిరూపించండి.

(i) $\tan\left\{arc\cos\frac{1}{x}\right\}=\sin\left\{arc\cot\frac{1}{2}\right\}$

Let $arc\cos\frac{1}{x}$ i.e $\cos\frac{1}{x}=\alpha\Rightarrow\cos\alpha=\frac{1}{x}$

$\cot^{-1}\frac{1}{2}=\beta\Rightarrow\quad\cot\beta=\frac{1}{2}$

$\therefore\tan\alpha=\sin\beta$

$\frac{1-x^2}{x^2}=\frac{4}{5}$

$\sqrt{x^2-1}=\frac{2}{v5}\Rightarrow x^2-1=\frac{4}{5}\Rightarrow x^2=\frac{9}{5}=x=\frac{3}{\sqrt{5}}$

$x=-\frac{3}{\sqrt{5}}$ సమికరణాన్ని తృప్తి పరచదు.

(v) $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}\quad\Rightarrow\sin^{-1}(1-x)=\frac{\pi}{2}+2\sin^{-1}x$

$(1-x)=\sin\left\{\frac{\pi}{2}+2\sin^{-1}x\right\}$

$(1-x)=\cos\left(2\sin^{-1}x\right)$ Let $\sin^{-1}x=\alpha\Rightarrow x-\sin\alpha$

$1-x=\cos^2\alpha\Rightarrow1-x=1-2x^2$

$2x^2-x=0\Rightarrow\ x(x-1)=0\Rightarrow x=0=\frac{1}{2}$

16. $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$ **ఐతే** $x^4+y^4+z^4+4x^2y^2z^2=2\left\{x^2y^2+y^2z^2+z^2x^2\right\}$ **అని చూపుము.**

సాధన:

$\sin^{-1}x=\alpha \quad \sin^{-1}y=\beta \quad \sin^{-1}z=\gamma$ అనుకొనుము

$\sin\alpha=x \quad \sin\beta=y \quad \sin\gamma=z$

$\alpha+\beta+\gamma=\pi \Rightarrow \cos(\alpha+\beta)=\cos(\pi-\gamma)$

$\sqrt{1-x^2}\sqrt{1-y^2}-xy=-\sqrt{1-z^2}$

$\sqrt{1-x^2}\sqrt{1-y^2}=xy-\sqrt{1-z^2}$

$(1-x)^2(1-y)^2=x^2y^2+1-z^2-2xy\sqrt{1-z^2}$

$1-x^2-y^2+x^2y^2=x^2y^2+1-z^2-2xy\sqrt{1-z^2}$

ఇరుపైపులా వర్గం చేయగా $\quad z^2-x^2-y^2=-2xy\sqrt{1-z^2}$

$z^4+x^4+y^4-2x^2z^2+2x^2y^2-2y^2z^2=4x^2y^2(1-z)^2$

$x^4+y^4+z^4-4x^2y^2z^2=2x^2y^2+2y^2z^2+2x^2z^2$

17. $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$ **ఐతే** $\frac{x^2}{az}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$ **అని చూపుము.**

$\cos^{-1}\frac{x}{a}=\theta \quad , \quad \cos^{-1}\frac{y}{b}=\phi$ అనుకొనుము

$\cos\theta=\frac{x}{a} \quad \cos\phi=\frac{y}{b}=\theta$

$\theta+\phi=\alpha$

$\cos(\theta+\phi)=\cos\alpha \Rightarrow \cos\theta\cos\phi-\sin\theta\sin\phi$

$\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}=\cos\alpha$

$$\frac{xy}{ab}-\cos\alpha=\sqrt{1-\frac{x^2}{a^2}}\ \sqrt{1-\frac{y^2}{b^2}}$$

ఇరువైపులా వర్గం చేయగా

$$\frac{x^2y^2}{a^2b^2}+\cos^2\alpha=\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)$$

$$\frac{x^2y^2}{a^2b^2}-\frac{2xy}{ab}\cos\alpha$$

$$\frac{x^2y^2}{a^2b^2}+\cos^2\alpha=1-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{x^2y^2}{a^2y^2}$$

$$\frac{x^2y^2}{a^2b^2}-\frac{2xy}{ab}\cos\alpha$$

$$\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=1-\cos^2\alpha$$

$$\therefore\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=1-\sin^2\alpha$$

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18. కింది సమీకరణాలను సాధించండి.

i) $Tan^{-1}\left(\frac{x-1}{x-2}\right)+Tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}.$

సాధన. $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

$$\tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{x-1}{x-2}+\frac{x+1}{2}}\right]=\frac{\pi}{4}$$

$$\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}=\tan\frac{\pi}{4}$$

$$\frac{2\left(x^2-2\right)}{-3}=1$$

$$x^2-2=-\frac{3}{2}$$

$$x^2=2-\frac{3}{2}=\frac{1}{2}$$

$$\therefore\ x=\pm\frac{1}{\sqrt{2}}$$

ii) $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$**.**

సాధన.

$$\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan\left[\frac{2}{x^2}\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{\left(\frac{1}{2x+1}\right)+\left(\frac{1}{4x+1}\right)}{1-\left(\frac{1}{2x+1}\right)\left(\frac{1}{4x+1}\right)}\right)$$

$$=\tan^{-1}\left(\frac{2}{x^2}\right)$$

$$\Rightarrow \frac{(4x+1)+(2x+1)}{(2x+1)(4x+1)-1}=\frac{2}{x^2}$$

$$\Rightarrow \frac{6x+2}{8^{x^2}+6x+1-1}=\frac{2}{x^2}$$

$$\Rightarrow \frac{2(3x+1)}{2x(4x+3)}=\frac{2}{x^2}$$

$$\Rightarrow x^2(3x+1)=2x(4x+3)$$

$$\Rightarrow x\left[x(3x+1)-2(4x+3)\right]=0$$

$\Rightarrow x=0$ **(లేదా)** $3x^2-7x-6=0$

$\Rightarrow x=0$ **(లేదా)** $3x^2-9x+2x-6=0$

$\Rightarrow x=0$ **(లేదా)** $3x(x-3)+2(x-3)=0$

$\Rightarrow x=0$ **(లేదా)** $(3x+2)(x-3)=0$

$\Rightarrow x=0,$ **(లేదా) 3 (లేదా)** $\frac{-2}{3}$

19. కింది సమీకరణాలను సాధించండి.

i. $\cot^{-1}\left(\frac{1+x}{1-x}\right)=\frac{1}{2}\cot^{-1}\left(\frac{1}{x}\right), x>0, x\neq 1$.

సాధన. $\cot^{-1}\left(\frac{1+x}{1-x}\right)=\alpha=\cot\alpha=\frac{1+x}{1-x}$

$$\cot 2\alpha=\frac{\cot^2\alpha-1}{2\cot\alpha}=\frac{\left(\frac{1+x}{1-x}\right)^2-1}{2\left(\frac{1+x}{1-x}\right)}$$

$$=\frac{(1+x)^2-(1-x^2)^2}{2(1+x)(1-x)}=\frac{4x}{2(1-x^2)}$$

$$\therefore \cot^{-1}\left(\frac{1+x}{1-x}\right)\Rightarrow\frac{1}{2}\cot^{-1}\frac{1}{x}$$

$$2\cot^{-1}\left(\frac{1+x}{1-x}\right)=\cot^{-1}\frac{1}{x}$$

$$\cot 2\alpha=\frac{4x}{2(1-x^2)}=\frac{1}{x}$$

$$2x^2=1-x^2$$

$$3x^2=1\Rightarrow x^2=\frac{1}{3}x=\pm\frac{1}{\sqrt{3}}$$

ii) $\cos^{-1}\left(\sqrt{3}.x\right)+\cos^{-1}x=\frac{\pi}{2}$.

సాధన. $\alpha=\cos^{-1}\left(\sqrt{3x}\right)$

$\Rightarrow\cos\alpha=\sqrt{3x}$ **అనుకుందాం.**

$$\beta=\cos^{-1}x\Rightarrow\cos\beta=x$$

$$\cos(\alpha+\beta)=\cos\frac{\pi}{2}$$

$$\cos\alpha\cos\beta-\sin\alpha\sin\beta=0$$

$$\left(\sqrt{3x}\right)x-\left(\sqrt{1-3x^2}\right)\left(\sqrt{1-x^2}\right)=0$$

$$\sqrt{3x^2}=\sqrt{(1-3x^2)(1-x^2)}$$

ఇరువైపులా వర్గం చేయగా

$$3x^4=1-x^2-3x^2+3x^4$$

$$0=1-4x^2$$

$4x^2 = 1 \qquad x = \frac{1}{2}$

iii) $\sin\left(\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1} x\right) = 1$.

సాధన. $\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}(x) = \sin^{-1}(1) = \frac{\pi}{2}$

let $\alpha = \sin^{-1}\left(\frac{1}{5}\right) \Rightarrow \sin\alpha = \frac{1}{5}$

$\beta = \cos^{-1} x \Rightarrow \cos\beta = x$

$\cos(\alpha+\beta) = \cos\frac{\pi}{2}$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta = 0$

$\left(\sqrt{1-\frac{1}{25}}\right) \times \frac{1}{5}\sqrt{1-x^2} = 0$

$\frac{\sqrt{24}}{5} = \frac{\sqrt{1-x^2}}{5}$

ఇరువైపులా వర్గం చేయగా

$24x^2 = 1 - x^2$

$25x^2 = 1$

$x^2 = \frac{1}{25} \Rightarrow x = \frac{1}{5}$

iv) $\sec^2\left(\cot^{-1} 3\right) + \cos ec^2\left(\tan^{-1} 2\right)$.

సాధన. $\cot^{-1}(3) = \alpha, \tan^{-1}(2) = \beta$ **అనుకుంటే**

$\cot\alpha = 3, \tan\beta = 2$

$\Rightarrow \tan\alpha = \frac{1}{3}, \cot\beta = \frac{1}{2}$

ఇప్పుడు $\sec^2\left(\cot^{-1} 3\right) + \cos ec^2\left(\tan^{-1} 2\right)$

$= \sec^2\alpha + \cos ec^2\beta$

$= \left(1+\tan^2\alpha\right) + \left(1+\cot^2\beta\right)$

$= 1 + \left(\frac{1}{3}\right)^2 + 1 + \left(\frac{1}{2}\right)^2$

$= 2 + \frac{1}{9} + \frac{1}{4}$

$= \frac{72+4+9}{36} = \frac{85}{36}$

v) $\cot^{-1}\frac{1}{2}+\cot^{-1}\frac{1}{3}$ **విలువ కనుక్కోండి.**

సాధన. $\cot^{-1}\left(\frac{1}{2}\right)+\cot^{-1}\left(\frac{1}{3}\right)$

$$=\tan^{-1}(2)+\tan^{-1}(3)$$

$$\because\ x=3, y=2, xy>1$$

$$=\pi+\tan^{-1}\left(\frac{2+3}{1-(2)(3)}\right)$$

$$=\pi+\tan^{-1}\left(\frac{5}{-5}\right)$$

$$=\pi+\tan^{-1}(-1)$$

$$=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$$

19. $\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{7}{25}=\sin^{-1}\frac{117}{125}$ **అని చూపండి.**

సాధన. మొదటి పద్ధతి.

$\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{7}{25}=\beta$ అనుకుంటే

అప్పుడు $\sin\alpha=\frac{4}{5}, \sin\beta=\frac{7}{25},$

$$\alpha, \beta \in\left(0, \frac{\pi}{2}\right)$$

$$\cos(\alpha+\beta)=\cos\alpha, \cos\beta-\sin\alpha\sin\beta$$

$$=\frac{3}{5}.\frac{24}{25}-\frac{4}{5}.\frac{7}{25}$$

$$=\frac{72-28}{125}=\frac{44}{125}>0$$

$$\Rightarrow(\alpha+\beta)\in\left(0, \frac{\pi}{2}\right)$$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$=\frac{4}{5}.\frac{24}{25}+\frac{3}{5}.\frac{7}{25}$

$=\frac{96+21}{125}=\frac{117}{125}$

$\Rightarrow \alpha+\beta=\sin\left(\frac{117}{125}\right)$

$\therefore\ \sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{7}{25}\right)=\sin^{-1}\left(\frac{117}{125}\right)$

20. $\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\left(\frac{16}{25}\right)=\frac{\pi}{2}$ **అని రుజువు చేయండి.**

సాధన. $\sin^{-1}\frac{4}{5}=\alpha\sin^{-1}\frac{5}{13}=\beta$ అనుకుంటే

α,β అల్పకోణాలు

$\sin\alpha=\frac{4}{5},\sin\beta=\frac{5}{13}$

కనుక $\cos=\frac{3}{5},\cos\beta=\frac{12}{3}$

ఇప్పుడు

$\cos(\alpha+\beta)=\cos\alpha+\cos\beta-\sin\alpha\sin\beta$

$=\frac{3}{5}.\frac{12}{13}-\frac{4}{5}.\frac{5}{13}$

$=\frac{16}{65}$

$\therefore\alpha+\beta=\cos^{-1}\left(\frac{16}{25}\right)$

$\Rightarrow\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}=\cos^{-1}\left(\frac{16}{65}\right)$.......(1)

L.H.S

$=\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)$

$=\cos^{-1}\frac{16}{65}+\sin^{-1}\frac{16}{65}=\frac{\pi}{2}$ [(1) నుండి.]

L.H.S = R.H.S

21) $\cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4} = \frac{\pi}{4}$ **అని రుజువు చేయండి.**

సాధన. $\cot^{-1}(9) = \alpha, \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4} = \beta$ అనుకుంటే

$\cot\alpha = 9, \operatorname{cosec}\beta = \frac{\sqrt{41}}{4}$ అవుతాయి.

$\Rightarrow \tan\alpha = \frac{1}{9}, \cot\beta = \sqrt{\operatorname{cosec}^2\beta - 1}$

$= \sqrt{\frac{41}{16} - 1} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

కనుక $0 < \alpha, \beta < \frac{\pi}{2}$

$\therefore \tan\alpha = \frac{1}{9} \tan\beta = \frac{4}{5}$

ఇప్పుడు $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$= \frac{\frac{1}{9} + \frac{4}{5}}{1 - \left(\frac{1}{9}\right)\left(\frac{4}{5}\right)}$

$= \left(\frac{5+36}{45-4}\right) = 1$

$\Rightarrow \tan(\alpha, \beta) = \tan\frac{\pi}{4}$

$\Rightarrow \alpha + \beta = \frac{\pi}{4}$

$\Rightarrow \cot^{-1}(9) + \operatorname{cosec}^{-1}\left(\frac{\sqrt{41}}{4}\right) = \frac{\pi}{4}$

22. $\tan\left(\left(2\tan^{-1}\frac{1}{5}\right)-\frac{\pi}{4}\right)$ విలువను కనుక్కోండి.

సాధన. $\tan^{-1}\left(\frac{1}{5}\right)=\alpha$, అనుకుంటే $\tan\alpha=\frac{1}{5}$

$$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2}$$

$$=\frac{2}{5}\times\frac{25}{24}=\frac{5}{12}$$

ఇప్పుడు $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)=\tan\left(2\alpha-\frac{\pi}{4}\right)$

$$=\frac{\tan 2\alpha-\tan\frac{\pi}{4}}{1+\tan 2\alpha\tan\frac{\pi}{4}}$$

$$=\frac{\frac{5}{12}-1}{1+\left(\frac{5}{12}\right)(1)}=\frac{-7}{17}$$

23. $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$ **అని రుజువు చేయండి.**

సాధన. $\sin^{-1}\left(\frac{4}{5}\right)=\alpha, \tan^{-1}\frac{1}{3}=\beta$ అనుకుంటే

అప్పుడు $\sin\alpha=\frac{4}{5}, \tan\beta=\frac{1}{3}$ అవుతాయి.

ఇప్పుడు $\tan 2\beta=\frac{2\tan\beta}{1-\tan^2\beta}$

$$=\frac{2\left(\frac{1}{3}\right)}{1-\frac{1}{9}}=\frac{2}{3}\times\frac{9}{8}=\frac{3}{4}$$

$$\therefore \tan 2\beta=\frac{3}{4}$$

$$\Rightarrow \cos 2\beta = \frac{4}{5}$$

$$\therefore \sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{5}$$

$$\therefore 2\beta = \cos^{-1}\left(\frac{4}{5}\right) \Rightarrow 2\tan^{-1}\left(\frac{1}{3}\right) = \cos^{-1}\frac{4}{5}$$

$$\therefore \sin^{-1}\frac{4}{5} + \tan^{-1}\frac{1}{5}$$

$$= \sin^{-1}\frac{4}{5} + \cos^{-1}\frac{4}{5} = \frac{\pi}{2}$$

24. $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$ అని రుజువు చేయండి.

సాధన. $\tan^{-1}\frac{1}{7} = \alpha, \tan^{-1}\frac{1}{3} = \beta$ అనుకుంటే

అప్పుడు $\tan\alpha = \frac{1}{7}, \tan\beta = \frac{1}{3}$

$$\text{L.H.S} = \cos\left(2\tan^{-1}\frac{1}{7}\right)$$

$$= \cos\left(2ga = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}\right) = \frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2}$$

$$= \frac{48}{50} = \frac{24}{25} \text{..................}(1)$$

$$\because \tan\beta = \frac{1}{3} \Rightarrow \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta}$$

$$= \frac{2\left(\frac{1}{3}\right)}{1-\frac{1}{9}} = \frac{2}{3}\times\frac{9}{8} = \frac{3}{4}$$

$$\text{R.H.S} = \sin\left(4\tan^{-1}\frac{1}{3}\right)$$

$$= \sin(4\beta)$$

$$= \sin(2\times 2\beta)$$

$$=\frac{2\tan(2\beta)}{1+\tan^2 2\beta}$$

$$=\frac{2\times\frac{3}{4}}{1+\left(\frac{3}{4}\right)^2}=\frac{6}{4}\times\frac{16}{25}=\frac{24}{25}\ ....(2)$$

(1), (2)

$$\sin\left(4\tan^{-1}\frac{1}{3}\right)=\cos\left(2\tan^{-1}\frac{1}{7}\right)$$

25. $\cos^{-1}\frac{P}{a}+\cos^{-1}\frac{b}{a}=\alpha$, **అయితే** $\frac{p^2}{a^2}-2\frac{pq}{ab}\cos\alpha+\frac{q^2}{b^2}=\sin^2\alpha$ **అని రుజువు చేయండి.**

సాధన. $\cos^{-1}\frac{P}{a}=\alpha, \cos^{-1}\frac{q}{b}=\beta$ **అనుకుంటే**

$$\cos\alpha=\frac{p}{a}, \cos\beta=\frac{q}{b}$$

$A+B=\alpha$ **అవుతాయి**

ఇప్పుడు

$$\cos\alpha=\cos(A+B)$$

$$=\cos A\cos B-\sin A\sin B$$

$$=\frac{p}{a}.\frac{q}{b}-\sqrt{1-\frac{p^2}{a^2}}\sqrt{1-\frac{q^2}{b^2}}=\left(\frac{pq}{ab}-\cos x\right)$$

ఇరువైపులా వర్గం చేయగా

$$\left(1-\frac{p^2}{a^2}\right)\left(1-\frac{q^2}{b^2}\right)=\frac{p^2q^2}{a^2b^2}-\frac{2pq}{ab}\cos\alpha+\cos^2\alpha$$

$$\Rightarrow\frac{p^2}{a^2}-\frac{q^2}{b^2}=\frac{p^2q^2}{a^2b^2}=\frac{p^2q^2}{a^2b^2}-\frac{2pq}{ab}$$

$$\cos\alpha+\cos^2\alpha$$

$$\Rightarrow\frac{p^2}{a^2}+\frac{q^2}{b^2}-\frac{2pq}{ab}\cos\alpha=1-\cos^2\alpha=\sin^2\alpha$$

$$\therefore\ \frac{p^2}{a^2}-2\frac{pq}{ab}\cos\alpha+\frac{a^2}{b^2}=\sin^2\alpha$$

26. $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0$, **అయితే x అనుకుంటే.**

సాధన: $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0$

$\Leftrightarrow 2\cos^{-1}\left[\cot\left(2\tan^{-1}x\right)\right]=0$ **లేదా** π **లేదా** 2π

$\cos^{-1}$ x మొక్క వ్యాప్తి $[0,\pi]$ కనుక

$\Leftrightarrow \cos^{-1}\left[\cot\left(2\tan^{-1}x\right)\right]=0$ **లేదా** $\frac{\pi}{2}$ **లేదా** π

$\Rightarrow \cot\left(2\tan^{-1}x\right)=1 \quad 0 \quad -1$

$\Rightarrow 2\tan^{-1}x \pm\frac{\pi}{4} \quad \pm\frac{\pi}{2} \quad \pm\frac{3\pi}{8}$

$\therefore \tan^{-1}(x)=\pm\frac{\pi}{8} \quad \pm\frac{\pi}{4} \quad \pm\frac{3\pi}{8}$

$x=\pm\left(\sqrt{2}-1\right) \quad \pm1 \quad \pm\left(\sqrt{2}+1\right)$

27. $\cos\left[\tan^{-1}\left\{\sin\left(\cot^{-1}x\right)\right\}\right]=\sqrt{\frac{x^2+1}{x^2+2}}$ **అని నిరూపించండి.**

సాధన. $\cot^{-1}(x)=\theta$ **అనుకుందాం**

అప్పుడు $\cot\theta=x, 0<x<\pi$

కనుక $\sin\left(\cot^{-1}x\right)=\sin\theta=\frac{1}{\cos ec\theta}$

$=\frac{1}{\sqrt{1+\cot^2\theta}}=\frac{1}{\sqrt{1+x^2}}$

$(\because 0<\theta<\pi)$

ఇప్పుడు

$\tan^{-1}\left(\sin\cot^{01}x\right)=\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)=\alpha$

అనుకుందాం

అప్పుడు $\tan\alpha=\frac{1}{\sqrt{1+x^2}}, 0<\alpha<\frac{\pi}{2}$ **అవుతుంది**

$\therefore \cos\left[\tan^{-1}\left\{\sin\left(\cot^{-1}x\right)\right\}\right]=\cos\alpha$

$=\frac{1}{\sec\alpha}=\frac{1}{\sqrt{1+\tan^2\alpha}}$

$$= \frac{1}{\sqrt{1+\frac{1}{1+x^2}}} = \sqrt{\frac{1+x^2}{2+x^2}}$$