మాత్రికలు

అతిస్వల్ప సమాధాన ప్రశ్నలు

1. ఈ క్రింది వాటిని ఒకే మాత్రికగా వ్రాయండి.

$$A=\begin{pmatrix}-1 & -2 & 3\\ 1 & 2 & 4\\ 2 & -1 & 3\end{pmatrix}, B=\begin{pmatrix}1 & -2 & 5\\ 0 & -2 & 2\\ 1 & 2 & -3\end{pmatrix},$$

$$C=\begin{pmatrix}-2 & 1 & 2\\ 1 & 1 & 2\\ 2 & 0 & 1\end{pmatrix}$$ అయితే $A+B+C$ ని కనుక్కోండి.

సాధన: $$A+B+C=\begin{pmatrix}-1 & -2 & 3\\ 1 & 2 & 4\\ 2 & -1 & 3\end{pmatrix}+\begin{pmatrix}1 & -2 & 5\\ 0 & -2 & 2\\ 1 & 2 & -3\end{pmatrix}+\begin{pmatrix}-2 & 1 & 2\\ 1 & 1 & 2\\ 2 & 0 & 1\end{pmatrix}$$

$$=\begin{bmatrix}-1+1-2 & -2-2+1 & 3+5+2\\ 1+0+1 & 2-2+1 & 4+2+2\\ 2+1+2 & -1+2+0 & 3-3+1\end{bmatrix}$$

$$=\begin{bmatrix}-2 & -3 & 10\\ 2 & 1 & 8\\ 5 & 1 & 1\end{bmatrix}$$

2. $$\mathbf{A}=\begin{bmatrix}3 & 2 & -1\\ 2 & -2 & 0\\ 1 & 3 & 1\end{bmatrix}, \mathbf{B}=\begin{bmatrix}-3 & -1 & 0\\ 2 & 1 & 3\\ 4 & -1 & 2\end{bmatrix}$$ మరియు $\mathbf{X}=\mathbf{A}+\mathbf{B}$ అయితే X.కనుగొనుము .

సాధన: $$X=A+B=\begin{bmatrix}3 & 2 & -1\\ 2 & -2 & 0\\ 1 & 3 & 1\end{bmatrix}+\begin{bmatrix}-3 & -1 & 0\\ 2 & 1 & 3\\ 4 & -1 & 2\end{bmatrix}$$

$$\therefore X=\begin{bmatrix}0 & 1 & -1\\ 4 & -1 & 3\\ 5 & 2 & 3\end{bmatrix}$$

3. $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$ **అయితే** **x, y, z , a లను కనుగొనుము**

సాధన: $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$

$\therefore\ x - 3 = 5 \Rightarrow x = 3 + 5 = 8$

$2y - 8 = 2 \Rightarrow 2y = 8 + 2 = 10 \Rightarrow y = 5$

$z + 2 = -2 \Rightarrow z = -2 - 2 = -4$

$a - 4 = 6 \Rightarrow a = 4 + 6 = 10$

4. $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & -6 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ **అయితే** **B – A , 4A – 5B లను కనుగొనుము**

సాధన: $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & -6 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$$B - A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -1-0 & 2-1 & 3-2 \\ 0-2 & 1-3 & 0-4 \\ 0-4 & 0-5 & -1+6 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ -2 & -2 & -4 \\ -4 & -5 & 5 \end{bmatrix}$$

$$4A - 5B = 4\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & -6 \end{bmatrix} - 5\begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 & 8 \\ 8 & 12 & 16 \\ 16 & 20 & -24 \end{bmatrix} - \begin{bmatrix} -5 & 10 & 15 \\ 0 & 5 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 0+5 & 4-10 & 8-15 \\ 8-0 & 12-5 & 16-0 \\ 16-0 & 20-0 & -24+5 \end{bmatrix} = \begin{bmatrix} 5 & -6 & 7 \\ 8 & 7 & 16 \\ 16 & 20 & -19 \end{bmatrix}$$

5. క్రింది మాత్రికల లబ్ధం కనుగొనుము

1) $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} -2 & -3 & 4 \\ 2 & 2 & -3 \\ 1 & 2 & -2 \end{bmatrix}$

$$= \begin{bmatrix} -4+4+1 & -6+4+2 & 8-6-2 \\ -2+0+2 & -3+0+4 & 4+0-4 \\ -4+2+2 & -6+2+4 & 8-3-4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ii) $\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$$= \begin{bmatrix} 0+abc-abc & b^2c-b^2c & bc^2-bc^2 \\ -a^2c+a^2c & -abc+abc & -ac^2+ac^2 \\ a^2b-a^2b & ab^2-ab^2 & abc-abc \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

6. $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$, $A^2 = 0$ అయితే k కనుగొనుము

సాధన: $A^2 = 0 \Rightarrow \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 4-4 & 8+4k \\ -2-k & -4+k^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow 8 + 4k = 0 \Rightarrow 4k = -8 \Rightarrow k = -2$

7. $\mathbf{A} = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ **అయితే $\mathbf{A}^3$ కనుగొనుము**

సాధన: $A^2 = A.A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 1+5-6 & 1+2-3 & 3+6-9 \\ 5+10-12 & 5+4-6 & 15+12-18 \\ -2-5+6 & -2-2+3 & -6-6+9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 3+15-18 & 3+6-9 & 9+18-27 \\ -1-5+6 & -1-2+3 & -3-6+9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

8. $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, **అయితే $\mathbf{A}^4$ ని కనుక్కోండి.**

సూచన: A వికర్ణ మాత్రిక.

సాధన: $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ అయిన

$$A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}, n \in N$$

$$A^4 = \begin{bmatrix} 3^4 & 0 & 0 \\ 0 & 3^4 & 0 \\ 0 & 0 & 3^4 \end{bmatrix} = \begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix}$$

9. $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ **అయితే**

$i)\quad A^2 = B^2 = c^2 = -I$

$ii)\quad AB = -BA = -C, (i^2 = -1)$ **అని చూపండి.**

(I అనేది రెండో తరగతి యూనిట్ మాత్రిక)

సాధన: $i)\quad A^2 = A.A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

$$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -1$$

$$B^2 = B.B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -I$$

$$C^2 = C.C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

$$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -I$$

$$\therefore A^2 = B^2 = C^2 = -I$$

$i)\quad AB=\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$=\begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}=-\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}=-C$

$BA=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}=\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}=C$

$\therefore AB=-BA=-C$

10. A, B **లు ఒకే తరగతి చతురస్ర మాత్రకలైతే** $AB=0, BA\neq 0$ **అయ్యేటట్లుగా** A, B **లకు ఉదాహరణలు ఇవ్వండి.**

సాధన: $A=\begin{bmatrix} a & 0 \\ a & 0 \end{bmatrix}, B=\begin{bmatrix} 0 & 0 \\ a & a \end{bmatrix}$

$AB=\begin{bmatrix} a & 0 \\ a & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ a & a \end{bmatrix}$

$=\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \quad =0$

$BA=\begin{bmatrix} 0 & 0 \\ a & a \end{bmatrix}\begin{bmatrix} a & 0 \\ a & 0 \end{bmatrix}$

$=\begin{bmatrix} 0+0 & 0+0 \\ a^2+a^2 & 0+0 \end{bmatrix}$

$=\begin{bmatrix} 0 & 0 \\ 2a^2 & 0 \end{bmatrix}\neq 0$

$\therefore AB=0, BA\neq 0$

11..A = $\begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 5 \end{bmatrix}$ **B =** $\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}$ **ఐతే** $(AB^T)^T$ **కనుగొనండి.**

సాధన: $B^T=\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}^T=\begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}$

$AB^T=\begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 5 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}=\begin{bmatrix} -2+0+0 & 0+0-2 \\ 1+1+0 & 0+1-10 \end{bmatrix}=\begin{bmatrix} -2 & -2 \\ 2 & -9 \end{bmatrix}$

$$(AB^T)^T = \begin{bmatrix} -2 & -2 \\ 2 & -9 \end{bmatrix}^T = \begin{bmatrix} -2 & 2 \\ -2 & -9 \end{bmatrix}$$

12. $A = \begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$, **B** = $\begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$ **ఐతే 2A + B^T , 3B^T – A లను కనుగొనండి.**

సాధన: $A = \begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix} \Rightarrow 2A = 2\begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 10 & 0 \\ -2 & 8 \end{bmatrix}$

$$B = \begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow B^T = \begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$$

$$2A + B^T = \begin{bmatrix} -4 & 2 \\ 10 & 0 \\ -2 & 8 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4-2 & 2+4 \\ 10+3 & 0+0 \\ -2+1 & 8+2 \end{bmatrix} = \begin{bmatrix} -6 & 6 \\ 13 & 0 \\ -1 & 10 \end{bmatrix}$$

$$B^T = \begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$$

$$3B^T - A = 3\begin{bmatrix} -2 & 4 \\ 3 & 0 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 12 \\ 9 & 0 \\ 3 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -6+2 & 12-1 \\ 9-5 & 0-0 \\ 3+1 & 6-4 \end{bmatrix} = \begin{bmatrix} -4 & 11 \\ 4 & 0 \\ 4 & 2 \end{bmatrix}$$

13. $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$ **ఐతే $A + A^T$ a, $A.A^T$ లనుకనుగొనండి**

సాధన: $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$

$$\Rightarrow A^T = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & -5 \\ -4 & 3 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -5 \\ -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & -4-5 \\ -5-4 & 3+3 \end{bmatrix} = \begin{bmatrix} 20 & -22 \\ -22 & 34 \end{bmatrix}$$

14. $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ **సౌష్ఠవ మాత్రిక అయితే x కనుగొనుము**

సాధన: A సౌష్ఠవ మాత్రిక $\Rightarrow A^T = A$

$$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$$

$$\Rightarrow \quad x = 6.$$

15. $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ **అసౌష్ఠవ మాత్రిక అయితే x కనుగొనుము**

సాధన: A అసౌష్ఠవ మాత్రిక $\Rightarrow A^T = -A$

$$\begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & x \\ 1 & -2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & 2 \\ 1 & -x & 0 \end{bmatrix}$$

$$\Rightarrow x = 2.$$

16. $A=\begin{bmatrix}1 & 5 & 3\\ 2 & 4 & 0\\ 3 & -1 & -5\end{bmatrix}$, $B=\begin{bmatrix}2 & -1 & 0\\ 0 & -2 & 5\\ 1 & 2 & 0\end{bmatrix}$, **అయితే $3A-4B^T$ కనుగొనుము**

సాధన: $B=\begin{bmatrix}2 & -1 & 0\\ 0 & -2 & 5\\ 1 & 2 & 0\end{bmatrix}$

$$\Rightarrow B^T=\begin{bmatrix}2 & -1 & 0\\ 0 & -2 & 5\\ 1 & 2 & 0\end{bmatrix}^T=\begin{bmatrix}2 & 0 & 1\\ -1 & -2 & 2\\ 0 & 5 & 0\end{bmatrix}$$

$$3A-4B^T=3\begin{bmatrix}1 & 5 & 3\\ 2 & 4 & 0\\ 3 & -1 & -5\end{bmatrix}-4\begin{bmatrix}2 & 0 & 1\\ -1 & -2 & 2\\ 0 & 5 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 15 & 9\\ 6 & 12 & 0\\ 9 & -3 & -15\end{bmatrix}-\begin{bmatrix}8 & 0 & 4\\ -4 & -8 & 8\\ 0 & 20 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3-8 & 15-0 & 9-4\\ 6+4 & 12+8 & 0-8\\ 9-0 & -3-20 & -15-0\end{bmatrix}$$

$$=\begin{bmatrix}-5 & 15 & 5\\ 10 & 20 & -8\\ 9 & -23 & -15\end{bmatrix}$$

17. $A=\begin{bmatrix}7 & -2\\ -1 & 2\\ 5 & 3\end{bmatrix}$, $B=\begin{bmatrix}-2 & -1\\ 4 & 2\\ -1 & 0\end{bmatrix}$ **అయితే AB^T and BA^T కనుగొనుము**

సాధన: $B=\begin{bmatrix}-2 & -1\\ 4 & 2\\ -1 & 0\end{bmatrix}$

$$\Rightarrow B^T=\begin{bmatrix}-2 & -1\\ 4 & 2\\ -1 & 0\end{bmatrix}^T=\begin{bmatrix}-2 & 4 & -1\\ -1 & 2 & 0\end{bmatrix}$$

$$AB^T = \begin{bmatrix} 7 & -2 \\ -1 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} -2 & 4 & -1 \\ -1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -14+2 & 28-4 & -7+0 \\ 2-2 & -4+4 & 1+0 \\ -10-3 & 20+6 & -5+0 \end{bmatrix} = \begin{bmatrix} -12 & 24 & -7 \\ 0 & 0 & 1 \\ -13 & 26 & -5 \end{bmatrix}$$

$$A = \begin{bmatrix} 7 & -2 \\ -1 & 2 \\ 5 & 3 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 7 & -2 \\ -1 & 2 \\ 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 7 & -1 & 5 \\ -2 & 2 & 3 \end{bmatrix}$$

$$BA^T = \begin{bmatrix} -2 & -1 \\ 4 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 7 & -1 & 5 \\ -2 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -14+2 & 2-2 & -10-3 \\ 28-4 & -4+4 & 20+6 \\ -7+0 & 1+0 & -5+0 \end{bmatrix} = \begin{bmatrix} -12 & 0 & -13 \\ 24 & 0 & 26 \\ -7 & 1 & -5 \end{bmatrix}$$

18. ఏదైనా చతురస్ర మాత్రిక A కు AA′ ఒకసౌష్ఠవ మాత్రిక అని చూపండి

సాధన: A చతురస్ర మాత్రిక

$(AA')' = (A')'A' = A \cdot A'$

$\because (AA')' = AA'$

$\Rightarrow AA'$ ఒకసౌష్ఠవ మాత్రిక

19. క్రింది మాత్రికల నిర్ధారకం కనుగొనుము

i) $\begin{bmatrix} 1 & 4 & 2 \\ 2 & -1 & 4 \\ -3 & 7 & 6 \end{bmatrix}$

సాధన: det A = 2(–3 –2) + 1(4 – 1) + 4(8 + 3)

= –10 + 3 + 44 = 37

ii) $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

సాధన: $\det A = a(bc - f^2) - h(ch - fg) + g(hf - bg)$

$= abc - af^2 - ch^2 + fgh + fgh - bg^2$

$= abc + 2fgh - af^2 - bg^2 - ch^2$

iii) $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

సాధన: $\det A = a(bc - a^2) - b(b^2 - ac) + c(ab - c^2)$

$= abc - a^3 - b^3 + abc + abc - c^3$

$= 3abc - a^3 - b^3 - c^3$

iv) $\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$

సాధన: $\det A = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$

$= 1(225 - 256) - 4(100 - 144) + 9(64 - 81)$

$= -31 + 176 - 153 = -184 + 176 = -8$

20. $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ **మరియు $\det A = 45$, అయితే x కనుగొనుము**

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సాధన: $\det A = 45 \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{vmatrix} = 45$

$3x + 24 = 45 \Rightarrow 3x - 45 + 24 = 0$

$\Rightarrow 3x - 21 = 0 \Rightarrow x = \frac{21}{3} = 7$

21. $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ **అయితే** $AA^1 = A^1A = I$ **అని చూపండి.**

సాధన: $A.A^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\begin{bmatrix} \cos^2\alpha + \sin^2\alpha & -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha + \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I_2 \qquad -(1)$$

$$A^1A = \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \cos\alpha\sin\alpha-\sin\alpha\cos\alpha\\ -\sin\alpha\cos\alpha-\cos\alpha\sin\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I_2 \qquad -(2)$$

(1),(2) ల నుండి, A

$A^1 = A^1.A = I_2$

22. I = $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, E = $\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$, అయితే $(aI + bE)^3 = a^3I + 3a^2bE$ అని చూపండి

సాధన: $aI + bE = a\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + b\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}a & b\\0 & a\end{bmatrix}$

$$(aI + bE)^2 = \begin{bmatrix}a & b\\0 & a\end{bmatrix}\begin{bmatrix}a & b\\0 & a\end{bmatrix} = \begin{bmatrix}a^2 & 2ab\\0 & a^2\end{bmatrix}$$

$$(aI + bE)^3 = \begin{bmatrix}a^2 & 2ab\\0 & a^2\end{bmatrix}\begin{bmatrix}a & b\\0 & a\end{bmatrix} = \begin{bmatrix}a^3 & 3a^2b\\0 & a^3\end{bmatrix}$$

$$= \begin{bmatrix}a^3 & 0\\0 & a^3\end{bmatrix} + \begin{bmatrix}0 & 3a^2b\\0 & 0\end{bmatrix}$$

$$= a^3\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + 3a^2b\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$$

$$= a^3I + 3a^2bE$$

23.. సిద్ధాంతము: A విలోమనీయ మాత్రిక అయిన A^T కూడ విలోమనీయ మాత్రిక అయిన $(A^T)^{-1} = (A^{-1})^T$

సాధన: ఉపపత్తి: A విలోమనీయ మాత్రిక $\Rightarrow A^{-1}$ వ్యవస్థితం మరియు

$$AA^{-1} = A^{-1}A = I$$

$(AA^{-1})^T = (A^{-1}A)^T = I^T$

$\Rightarrow (A^{-1})^T A^T = A^T.(A^{-1})^T = I$

$\Rightarrow$ నిర్వచనం నుండి $(A^T)^{-1} = (A^{-1})^T$

24. 1 యొక్క సంకీర్ణ ఘన మూలం ω అయితే $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$ అని నిరూపించండి

సాధన: $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$= \begin{vmatrix} 1+\omega+\omega^2 & 1+\omega+\omega^2 & 1+\omega+\omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} [\because 1+\omega+\omega^2 = 0]$

$= 0$

24. మూడవ తరగతి కి చెందిన అసౌష్ఠవ మాత్రిక నిర్ధారకం విలువ శూన్యం అని చూపండి.

సాధన: మూడవ తరగతి కి చెందిన అసౌష్ఠవ మాత్రిక $A = \begin{bmatrix} 0 & -c & -b \\ c & 0 & -a \\ b & a & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} 0 & -c & -b \\ c & 0 & -a \\ b & a & 0 \end{vmatrix} = (-1)^3 \begin{vmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix}$

$= \begin{vmatrix} 0 & -c & -b \\ c & 0 & -a \\ b & a & 0 \end{vmatrix} \because |B| = |B^T|$

$= -|A| \Rightarrow 2|A| = 0$

25. క్రింది మాత్రిక లకు అనుబంధ మరియు విలోమ మాత్రికలను కనుగొనుము

.

i) $\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$

సాధన: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ అయితే

$$\text{Adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix}$$

$$|A| = 12 - (-12) = 24$$

$$A^{-1} = \frac{\text{AdjA}}{\text{DetA}} = \frac{1}{24}\begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix}$$

ii) $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

సాధన: $\text{Adj } A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}, \det A = 1$

$$A^{-1} = \frac{\text{AdjA}}{\text{DetA}} = \frac{1}{\cos^2\alpha + \sin^2\alpha}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

iii) $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

సాధన: $A_1 = \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = 0 - 2 = -2$

$$B_1 = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -(1-2) = 1$$

$$C_1 = \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} = 2 - 0 = 2$$

$$A_2 = -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -(1-4) = 3$$

$$B_2 = \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} = 2-4 = -2$$

$$C_2 = -\begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} = -(4-2) = -2$$

$$A_3 = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1-0 = 1$$

$$B_3 = -\begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = -(2-2) = 0$$

$$C_3 = \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 0-1 = -1$$

$$\text{Adj}A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & -2 & 0 \\ 2 & -2 & -1 \end{bmatrix}$$

$$\text{Det}A = a_1A_1 + b_1B_1 + c_1C_1$$
$$= 2(-2) + 1(1) + 2(2) = -4+1+4 = 1$$

$$A^{-1} = \frac{\text{Adj}A}{\text{Det}A} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & -2 & 0 \\ 2 & -2 & -1 \end{bmatrix}$$

26. క్రింది మాత్రికల కోటి కనుగొనండి.

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

సాధన: $\text{Det } A = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1-0 = 1 \neq 0$

$\therefore \rho(A) = 2$

2. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

సాధన: $\text{Det } A = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0-0 = 0$

$|1| = 1 \neq 0$

$\therefore\ \rho(A) = 1$

3. $\begin{bmatrix} 1 & 0 & -4 \\ 2 & -1 & 3 \end{bmatrix}$

సాధన: $\begin{vmatrix} 1 & -4 \\ 2 & 3 \end{vmatrix} = 3 + 8 = 11 \neq 0$

$\therefore\ \rho(A) = 2$

4. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

సాధన: Det A = $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= 1(1-0) - 0(0-0) + 0(0-0)$

$= 1 - 0 + 0 = 1 \neq 0$

$\therefore\ \rho(A) = 3$

5. $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

సాధన: Det A = $\begin{vmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{vmatrix}$

$= 1(6-0) - 2(8+1) + 0(0+3)$

$= 6 - 18 = -12 \neq 0$

$\therefore\ \rho(A) = 3$

6. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

సాధన: Det A = $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix}$

$=1(6-4)-2(4-3)+0(8-9)$

$=2-2+0=0$

$\therefore\ \rho(A) \neq 3,\ \rho(A) < 3$

Take $\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0$

$\therefore\ \rho(A) = 2$

6. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, det A = 0, $\rho(A) \neq 3$.

సాధన: All 2×2 sub matrix det is zero.

$\therefore\ \rho(A) \neq 2$

$|1| = 1 \neq 0,\ \therefore\ \rho(A) = 1$

7. $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$

సాధన: ఉప మాత్రిక $B = \begin{vmatrix} 1 & 2 & 0 \\ 3 & 4 & 1 \\ -2 & 3 & 2 \end{vmatrix}$

$= 1(8-3) - 2(6+2)$

$= 5 - 16 = -11 \neq 0$

మాత్రిక కోటి= 3.

27. AB = I or BA = I, అయితే A విలోమనీయ మాత్రిక అనీ $B = A^{-1}$ అని నిరూపించండి.

సాధన: $AB = I \Rightarrow |AB| = |I|$

$= |A|\,|B| = 1$

$= |A| \neq 0$

$\therefore$ A సాధారణ మాత్రిక.

$BA = I \Rightarrow |BA| = |I|$

$\Rightarrow |B|\,|A| = 1 \Rightarrow |A| \neq 0$

$\therefore$ A సాధారణ మాత్రిక.

AB = I or BA = I, అయితే A విలోమనీయ మాత్రిక

$\therefore A^{-1}$ వ్యవస్థితం

$AB = I \Rightarrow A^{-1}AB = A^{-1}I$

$\Rightarrow IB = A^{-1} \Rightarrow B = A^{-1}$

$\therefore B = A^{-1}$

స్వల్ప సమాధాన ప్రశ్నలు

1. సిద్ధాంతము: మాత్రికా గుణకారం సాహచర్య న్యాయాన్ని పాటిస్తుంది. $(i.e.,) A, B, C$ లు మూడు మాత్రికలయితే $(AB)C = A(BC)$ అవుతుంది.

ª ▯^ ▮: ఉపపత్తి: $A = (a_{ij})_{mxn}$, $B = (b_{jk})_{nxp}$,

$C = (a_{kl})_{pxq}$ అనుకోండి.

$AB = (d_{ik})_{mxp}$ ఇచ్చుట $d_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

$(AB)C = (f_{il})_{mxq}$ ఇచ్చుట $f_{il} = \sum_{k=1}^{p} d_{ik} c_{kl}$

$BC = (g_{il})_{nxq}$ ఇచ్చుట $g_{il} = \sum_{k=1}^{p} b_{jk} c_{kl}$

$A(BC) = (h_{il})_{mxq}$ ఇచ్చుట $h_{il} = \sum_{j=1}^{n} a_{ij} g_{jl}$

$$f_{il} = \sum_{k=1}^{p} d_{ik} c_{kl} = \sum_{k=1}^{p} \left(\sum_{j=1}^{n} a_{ij} b_{jk} \right) c_{kl}$$

$$= \sum_{j=1}^{p} a_{ij} \left(\sum_{k=1}^{p} b_{jk} c_{kl} \right) = \sum_{j=1}^{n} a_{ij} h_{jl} = h_{il}$$

$(AB)C = A(BC)$

2. సిద్ధాంతం: A ఏదేని మాత్రిక అయితే $(A^T)^T = A$ అని చూపండి.

సాధన: $A = (a_{ij)})_{mxn}$ అనుకోండి.

$A^T = (a^1_{ji})_{mxn}$, ఇచ్చుట $a^n_{ji} = a_{ij}$

$(A^T)^T = (a^{11}_{ji})_{mxn,}$ ఇచ్చుట $a^{11}_{ij} = a_{ji}$

$a^{11}_{ij} = a^1_{ji} = a_{ij} \qquad \therefore (A^T)^T = A.$

3. **సిద్ధాంతము:** A, B లు రెండూ ఒక తరగతి మాత్రికలు అయితే $(A+B)^T = A^T + B^T$.

సాధన: ఉపపత్తి $A = (a_{ij})_{mxn}, B = (b_{ij})_{mxn}$ అనుకోండి.

$A + B = (C_{ij})_{mxn}$, ఇచ్చుట $C_{ij} = a_{ij} + b_{ij}$

$(A+B)^T = (c'_{ji})_{nxm}, c^1_{ji} = c_{ij}$

$A^T = (a^l_{ji})_{nxm}$, ఇచ్చుట $a'_{ji} = a_{ji}$

$B^T = (b^l_{ji})_{nxm}$, ఇచ్చుట $b'_{kj} = b_{jk}$

$A^T + B^T = (d_{ji})_{nxm}$, ఇచ్చుట $d_{ji} = a'_{ji} + b'_{ji}$

$c'_{ji} = c_{ij} = a_{ij} + b_{ij} = a'_{ji} + b'_{ji} = d_{ji}$

$\therefore (A+B)^T = A^T + B^T$

4. **సిద్ధాంతము** $(AB)^T = B^T A^T$

సాధన: ఉపపత్తి $: A = (a_{ij})_{mxn}, B = (b_{jk})_{nxp}$

$AB = (c_{ik})_{mxp}$, ఇచ్చుట $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

$(AB)^T = (c'_{ki})_{pxm}$, ఇచ్చుట $c'_{ki} = c_{jk}$

$A^T = (a'_{ji})_{nxm}$, ఇచ్చుట $a'_{ji} = a_{ij}$

$B^T = (b'_{ki})_{pxn}$, ఇచ్చుట $b'_{kj} = b_{jk}$

$B^T . A^T = (d_{ki})_{pxm}$ ఇచ్చుట $d_{ki} = \sum_{j=1}^{n} b'_{kj} a'_{ji}$

$c'_{ki} = c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk} = \sum_{j=1}^{n} b'_{kj} a'_{ji} d_{ki}$

$\therefore (AB)^T = B^T A^T$

5. **A=** $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, **అయితే $A^3 - 3A^2 - A - 3I$ కనుగొనుము**

సాధన: Given $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+3 & -2-2-1 & 1+2+1 \\ 0+0-3 & 0+1+1 & 0-1-1 \\ 3-0+3 & -6-1-1 & 3+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -5 & 4 \\ -3 & 2 & -2 \\ 6 & -8 & 5 \end{bmatrix}$$

$$A^3 = A^2\,A = \begin{bmatrix} 4 & -5 & 4 \\ -3 & 2 & -2 \\ 6 & -8 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+12 & -8-5-4 & 4+5+4 \\ -3+0-6 & 6+2+2 & -3-3-2 \\ 6+0+15 & -12-8-5 & 6+8+5 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -17 & 13 \\ -9 & 10 & -7 \\ 21 & -25 & 19 \end{bmatrix}$$

Now $A^3 - 3A^2 - A - 3I$

$$= \begin{bmatrix} 16 & -17 & 13 \\ -9 & 10 & -7 \\ 21 & -25 & 19 \end{bmatrix} - 3\begin{bmatrix} 4 & -5 & 4 \\ -3 & 2 & -2 \\ 6 & -8 & 5 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16-12-1-3 & -17+15+2+0 & 13-12-1-0 \\ -9+9+0-0 & 10-6-1-3 & -7+6+1+0 \\ 21-18-3+0 & -25+24+1+0 & 19-15-1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O_{3\times 3}$$

$\therefore\ A^3 - 3A^2 - A - 3I = 0$

6. $A = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$, **అయితే** $n \geq 1$ **కు** $A^n = \begin{bmatrix} a_1^n & 0 & 0 \\ 0 & a_2^n & 0 \\ 0 & 0 & a_3^n \end{bmatrix}$ **అని చూపండి**

సాధన: $A = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$

గణితానుగమనాన్ని అనుసరించి దీనిని నిరూపిస్తాము.

$$A^n = \begin{bmatrix} a_1^n & 0 & 0 \\ 0 & a_2^n & 0 \\ 0 & 0 & a_3^n \end{bmatrix}$$

$n = 1$ అయితే

$$A^1 = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$$

$n = 1$ దత్త ప్రవచనం నిజం

$n = k$ కు దత్త ప్రవచనం నిజం అనుకోండి.

i.e. $A^k = \begin{bmatrix} a_1^k & 0 & 0 \\ 0 & a_2^k & 0 \\ 0 & 0 & a_3^k \end{bmatrix}$

$$A^{k+1} = A^k \cdot A$$

$$= \begin{bmatrix} a_1^k & 0 & 0 \\ 0 & a_2^k & 0 \\ 0 & 0 & a_3^k \end{bmatrix} \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1^k \cdot a_1 + 0 + 0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0 + a_2^k \cdot a_2 + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+a_3^k \cdot a_3 \end{bmatrix} = \begin{bmatrix} a_1^{k+1} & 0 & 0 \\ 0 & a_2^{k+1} & 0 \\ 0 & 0 & a_3^{k+1} \end{bmatrix}$$

$\therefore$ $n = k + 1$ కు దత్త ప్రవచనం నిజం

గణితానుగమనాన్ని అనుసరించి n యొక్కప్రతి ధన పూర్ణంక విలువకు దత్త ప్రవచనం నిజం

$$A^n = \begin{bmatrix} a_1^n & 0 & 0 \\ 0 & a_2^n & 0 \\ 0 & 0 & a_3^n \end{bmatrix}, \quad n \geq 1.$$

7. $\theta - \phi = \frac{\pi}{2}$, అయితే $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix} = 0$ **అని చూపండి**

సాధన: $\theta - \phi = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{2} + \phi$

$$\cos\theta = \cos\left(\frac{\pi}{2} + \phi\right) = -\sin\phi$$

$$\sin\theta = \sin\left(\frac{\pi}{2} + \phi\right) = \cos\phi$$

$$\therefore \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\phi & -\sin\phi\cos\phi \\ -\sin\phi\cos\phi & \cos^2\phi \end{bmatrix}$$

$$\therefore \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

$$=\begin{bmatrix} \sin^2\phi & -\sin\phi\cos\phi \\ -\sin\phi\cos\phi & \cos^2\phi \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

$$=\begin{bmatrix} \sin^2\phi\cos^2\phi-\sin^2\phi\cos^2\phi & \sin^3\phi\cos\phi-\sin^3\phi\cos\phi \\ -\sin\phi\cos^3\phi+\sin\phi\cos^3\phi & -\sin^2\phi\cos^2\phi+\sin^2\phi\cos^2\phi \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=0$$

8. $A=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ **మరియు n అయితే ధనపూర్ణాంక** $A^n=\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, **అని చూపండి**

సాధన: గణితానుగమనాన్ని అనుసరించి దీనిని నిరూపిస్తాము.

$$A^n=\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$$

$$=1 \Rightarrow A'=\begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix}=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$n=1$ దత్త ప్రవచనం నిజం

$n=k$ కు దత్త ప్రవచనం నిజం అనుకోండి

$$A^k=\begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$$

$$A^{k+1}=A^k\cdot A=\begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$$

$$=\begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$$

$$=\begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$$

$\therefore \quad n = k + 1$ కు దత్త ప్రవచనం నిజం

గణితానుగమనాన్ని అనుసరించి n యొక్కప్రతి ధన పూర్ణంక విలువకు దత్త ప్రవచనం నిజం

9. $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$ **అని నిరూపించండి**

సాధన: $L.H.S. = \begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix}$

$= \begin{vmatrix} bc & b+c & 1 \\ c(a-b) & a-b & 0 \\ b(a-c) & a-c & 0 \end{vmatrix}$ by $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$= (a-b)(a-c)\begin{vmatrix} bc & b+c & 1 \\ c & 1 & 0 \\ b & 1 & 0 \end{vmatrix}$

$= (a - b)\ (a - c)\ (c - b)$

$= (a - b)\ (b - c)\ (c - a) = R.H.S.$

10. $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$ **ఐతే x కనుగొనండి.**

సాధన: $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

$\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_2 - R_1 \end{matrix} \Rightarrow \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ -2 & -6 & -12 \\ -6 & -24 & -60 \end{vmatrix} = 0$

$$(-2)(-6)\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 3 & 6 \\ 1 & 4 & 10 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(30-24)-(2x-3)(10-6)$$
$$+(3x-4)(4-3)=0$$

$$\Rightarrow 6x-12-8x+12+3x-4=0$$

$$x-4=0 \Rightarrow x=4$$

11. $\Delta_1 = \begin{vmatrix} a_1^2+b_1+c_1 & a_1a_2+b_2+c_2 & a_1a_3+b_3+c_3 \\ b_1b_2+c_1 & b_2^2+c_2 & b_2b_3+c_3 \\ c_3c_1 & c_3c_2 & c_3^2 \end{vmatrix}$ *and* $\Delta_2 = \begin{vmatrix} a_1 & b_2 & c_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, **అయితే**

$\frac{\Delta_1}{\Delta_2}$ **విలువ కనుక్కోండి.**

సాధన:

$$\Delta_1 = c_3\begin{vmatrix} a_1^2+b_1+c_1 & a_1a_2+b_2+c_2 & a_1a_3+b_3+c_3 \\ b_1b_2+c_1 & b_2^2+c_2 & b_2b_3+c_3 \\ c_3c_1 & c_3c_2 & c_3^2 \end{vmatrix}$$

$$= R_2 \to R_2 - R_3$$

$$= c_3\begin{vmatrix} a_1^2+b_1+c_1 & a_1a_2+b_2+c_2 & a_1a_3+b_3+c_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= c_3b_2\begin{vmatrix} a_1^2+b_1+c_1 & a_1a_2+b_2+c_2 & a_1a_3+b_3+c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$R_1 \to R_1 - R_2 - R_3$$

$$=c_3b_2\begin{vmatrix} a_1^2 & a_1a_2 & a_1a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$=a_1b_2c_3\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=a_1b_2c_3\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

ఇచ్చినది $\Delta_2=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$$\frac{\Delta_1}{\Delta_2}=a_1b_2c_3\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=a_1b_2c_3$$

12. $\Delta_1=\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$, $\Delta_2=\begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\alpha & 0 \end{vmatrix}$, $\Delta_1=\Delta_2$ **అయితే** $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ **అని చూపండి.**

సాధన:

$$\Delta_1=\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$$=1(1-\cos^2\gamma)-\cos\alpha(\cos\alpha-\cos\beta\cos\gamma)$$
$$+\cos\beta(\cos\alpha\cos\gamma-\cos\beta)$$
$$=1-\cos^2\gamma-\cos^2\alpha+\cos\alpha\cos\beta\cos\gamma)$$
$$+\cos\alpha\cos\beta\cos\gamma-\cos^2\beta$$
$$=1-\cos^2\gamma-\cos^2\alpha-\cos^2\beta+2\cos\alpha\cos\beta\cos\gamma$$

$$\Delta_2 = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\alpha & 0 \end{vmatrix}$$

$$0(0-\cos^2\gamma)-\cos\alpha(0-\cos\gamma\cos\beta)$$

$$\cos\beta(\cos\alpha\cos\gamma-0)$$

$$=\cos\alpha\cos\beta\cos\gamma+\cos\alpha\cos\beta\cos\gamma$$

$$=2\cos\alpha\cos\beta\cos\gamma$$

ఇచ్చినది $\Delta_1 = \Delta_2$

$$1-\cos^2\alpha-\cos^2\beta-\cos^2\gamma+2\cos\alpha\cos\beta\cos\gamma$$

$$=2\cos\alpha\cos\beta\cos\gamma$$

$$1-\cos^2\alpha-\cos^2\beta-\cos^2\gamma=0$$

$$1=\cos^2\alpha+\cos^2\beta+\cos^2\gamma$$

ధీర్ఘ సమాధాన ప్రశ్నలు

10. $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$ **అని నిరూపించండి**

సాధన: $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix}$

$R_1 \to R_1 + R_3$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ -c & -a & -b \\ a & b & c \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ -c & -a & -b \\ a & b & c \end{vmatrix}$$

$$= (a+b+c)[(-ac+b^2) - (-c^2+ab) + (-bc+a^2)]$$

$$= (a+b+c)(-ac+b^2+c^2-ab-bc+a^2)$$

$$= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= a^3+b^3+c^3-3abc$$

11. $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$ **అని నిరూపించండి**

సాధన: L.H.S. = (y + z)[(z + x) (x + y) – yz] – x[y(x + y) – yz] + x[yz – z(z + x)]

$$= (y+z)(zx+yz+x^2+xy-yz)$$
$$-x(xy+y^2-yz)+x(yz-z^2-zx)$$
$$= (y+z)(zx+x^2+xy)-x(xy+y^2-yz)$$
$$+x(yz-z^2-zx)$$
$$= xyz+x^2y+xy^2+xz^2+x^2z+xyz$$
$$-x^2y-xy^2+xyz+xyz-xz^2-x^2z$$
$$= 4xyz$$

12. $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, **ఐతే abc = –1 అని నిరూపించండి**

సాధన: L.H.S $= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$

$$= \begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$\therefore\ 1 + abc = 0 \Rightarrow abc = -1$

13. $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$ అని నిరూపించండి

సాధన: L.H.S. $C_1 \to C_1 + (C_2 + C_3)$

$$= \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$(R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1)$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix}$$

$$= 2(a+b+c)(a+b+c)^2$$

$$= 2(a+b+c)^3 = \text{R.H.S}$$

14. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$ అని నిరూపించండి.

సాధన: $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$$= a(bc-a^2) - b(b^2-ac) + c(ab-c^2)$$

$$= abc - a^3 - b^3 + abc + abc - c^3$$

$$= -(a^3+b^3+c^3-3abc) \qquad ...(1)$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times (-) \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} -a & -b & -c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$= \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} \quad \ldots(2)$$

(1), (2) ల నుండి

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$$

15. $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$ అని నిరూపించండి

సాధన: L.H.S. $= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$ $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$= (a-1)^2[0(6-3)-0[3(a+1)-3)+1(a+1-2)$

$= (a-1)^2(a-1) = (a-1)^3 = $ R.H.S.

16. $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ **= abc (a – b)(b – c)(c – a) అని నిరూపించండి.**

సాధన: L.H.S. = $abc\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$$= abc\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix} \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

$$= abc(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a+b & b+c & c^2 \end{vmatrix}$$

$$= abc(a-b)(b-c)[0(c^2 - c(b+c)$$
$$-0(c^2 - c(a+b) + 1(b+c-a-b)]$$
$$= abc(a-b)(b-c)(c-a)$$

17. $\begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$ **= 4(a + b)(b + c)(c + a) అని నిరూపించండి**

సాధన: Let $\Delta = \begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$

Let a + b = 0, then $\Delta = \begin{vmatrix} -2a & 0 & c+a \\ 0 & 2a & -a+c \\ c+a & c-a & -2c \end{vmatrix}$

Apply $R_1 \to R_1 + R_3$, $R_3 \to R_3 + R_2$

$$\Delta = \begin{vmatrix} c-a & c-a & -c+a \\ 0 & 2a & -a+c \\ c+a & c+a & -c-a \end{vmatrix}$$

$$= (c-a)(c+a)\begin{vmatrix} 1 & 1 & -1 \\ 0 & 2a & c-a \\ 1 & 1 & -1 \end{vmatrix} = 0 \ (\because R_1 \equiv R_3)$$

$\therefore$ (c + a) is a factor for Δ

ఇదే విధంగా

a + b, b + c కారణాంకాలని అని చూపరచ్చును

$\because$ Δ అనేది a, b, c ల లోమూడవ తరగతి సమఘాత ప్రమేయము

$\Delta \equiv k(a + b)(b + c)(c + a)$, k ఒక స్థిరాంకం

a = 1, b = 1, c = 1, అయితే అప్పుడు

$$\begin{vmatrix} -2 & 2 & 2 \\ 2 & -2 & 2 \\ 2 & 2 & -2 \end{vmatrix} = k(1 + 1)(1 + 1)(1 + 1)$$

$\Rightarrow -2(4 - 4) - 2(-4 - 4) + 2(4 + 4) = 8k$

$\Rightarrow 16 + 16 = 8k \Rightarrow k = 4$

$\therefore \Delta = 4(a + b)(b + c)(c + a)$

Hence

$$\begin{vmatrix} -2a & a+b & a+c \\ a+b & -2b & b+c \\ a+c & b+c & -2c \end{vmatrix} = 4(a + b)(b + c)(c + a).$$

18. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$ అని నిరూపించండి.

సాధన: L.H.S. = $\begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

By $R_1 \rightarrow R_1 + (R_2 + R_3)$

11. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$ అని నిరూపించండి.

3.

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_2$

సాధన: L.H.S. $\begin{vmatrix} 1 & a & a^2-bc \\ 0 & b-a & b^2-a^2+bc-ca \\ 0 & c-b & c^2-b^2+ac-ab \end{vmatrix}$

$= (b-a)(c-b)\begin{vmatrix} 1 & a & a^2-bc \\ 0 & 1 & a+b+c \\ 0 & 1 & a+b+c \end{vmatrix}$

$= (b-a)(c-b)\cdot 0\,(R_2, R_3 \text{ are identical})$

$= 0 = \text{R.H.S.}$

19. $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x+2a)(x-a)^2$ అని నిరూపించండి.

సాధన: $\text{L.H.S.} = \begin{vmatrix} x+2a & a & a \\ x+2a & x & a \\ x+2a & a & x \end{vmatrix}$

By $C_1 \to C_1 + (C_2 + C_3)$

$= (x+2a)\begin{vmatrix} 1 & a & a \\ 1 & x & a \\ 1 & a & x \end{vmatrix}$

$= (x+2a)\begin{vmatrix} 1 & a & a \\ 0 & x-a & 0 \\ 0 & 0 & x-a \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$= (x+2a)[1(x-a)^2 - a(0(x-a)-0)]$

$+a[0-0(x-a)]$

$= (x+2a)(x-a)^2 = \text{R.H.S}$

20. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$ **అని నిరూపించండి.**

సాధన: $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$$\begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

$$= 1\begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}$$

$$= (b-a)(c-a)(c+a-b-a)$$

$$= (b-a)(c-a)(c-b)$$

$$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

21. $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ **అని నిరూపించండి.**

సాధన: L.H.S. $= \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$

$$R_1 \Rightarrow R_1 + R_2 + R_3$$

$$= \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 2\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ -b & -c & -a \\ -c & -a & -b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= 2\begin{vmatrix} a & b & c \\ -b & -c & -a \\ -c & -a & -b \end{vmatrix}$$

$$= (2)(-1)(-1)\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \text{R.H.S.}$$

22. $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$ **అని నిరూపించండి.**

సాధన: L.H.S $= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

$R_2 \to R_2 - R_1$,$R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b^2-a^2 & b^2-a^3 \\ 0 & c^2-a^2 & c^3-a^3 \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b+a & b^2+ba+a^2 \\ 0 & c+a & c^2+ca+a^2 \end{vmatrix}$$

$R_2 \to R_2 - R_3$

$$= -(a-b)(c-a)\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b-c & b^2-c^2+a(b-c) \\ 0 & c+a & c^2+ca+a^2 \end{vmatrix}$$

$$= -(a-b)(c-a)(b-c)\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & 1 & b+c+a \\ 0 & c+a & c^2+ca+a^2 \end{vmatrix}$$

$$= -(a-b)(b-c)(c-a)[(c^2+ca+a^2)-(b+c+a)(c+a)]$$

$$= -(a-b)(b-c)(c-a)[c^2+ca+a^2-b(c+a)-(c+a)^2 =$$

$$= -(a-b)(b-c)(c-a)[c^2+ca+a^2-bc-ab-c^2-2ca-a^2]$$

$$= -(a-b)(b-c)(c-a)[-ab-bc-ca]$$

$$= (a-b)(b-c)(c-a)(ab+bc+ca)$$

$$\therefore \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

23. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ అని నిరూపించండి.

సాధన: $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} a-b-c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_2 \to C_2 - C_1$

$C_3 \to C_3 - C_1$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$$= (a+b+c)^3$$

21. $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, అయితే $(A^T)^{-1}$ కనుగొనుము.

సాధన: $A^T = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$$A_1 = \begin{vmatrix} -1 & 2 \\ 4 & 1 \end{vmatrix} = -1-8 = -9$$

$$B_1 = -\begin{vmatrix} -2 & 2 \\ 3 & 1 \end{vmatrix} = -(-2-6) = 8$$

$$C_1 = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix} = (-8+3) = -5$$

$$A_2 = -\begin{vmatrix} 0 & -2 \\ 4 & 1 \end{vmatrix} = -(0+8) = -8$$

$$B_2 = \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = 1+6 = 7$$

$$C_2 = -\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} = -(4-0) = -4$$

$$A_3 = \begin{vmatrix} 0 & -2 \\ -1 & 2 \end{vmatrix} = 0-2 = -2$$

$$B_3 = -\begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = -(2-4) = 2$$

$$C_3 = \begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix} = -1-0 = -1$$

$$\text{Adj}A^T = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$\text{Det}A^T = 1(-9)+0(8)-2(-5) = -9+10 = 1$$

$$(A^T)^{-1} = \frac{\text{Adj}(A^T)}{\det A^T} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

22. $A = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$, **అయితే Adj A = $3A^T$ అని నిరూపించండి మరియు A^{-1} కనుగొనండి.**

సాధన: $A_1 = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = 1-4 = -3$

$$B_1 = -\begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -(2+4) = -6$$

$$C_1 = \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = -4-2 = -6$$

$$A_2 = -\begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -(-2-4) = 6$$

$$B_2 = \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = -1+4 = 3$$

$$C_2 = -\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2+4) = -6$$

$$A_3 = \begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = 4+2 = 6$$

$$B_3 = -\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2+4) = -6$$

$$C_3 = \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -1+4 = 3$$

$$\text{Adj}A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} \ldots(1)$$

$$A^T = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$3A^T = 3\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} \ldots(2)$$

(1) , (2) ల నుండి $\text{Adj } A = 3A^T$

$$\begin{aligned} \text{Det}A &= a_1A_1 + b_1B_1 + c_1C_1 \\ &= (-1)(-3) + (-2)(-6) + (-2)(-6) \\ &= 3+12+12 = 27 \end{aligned}$$

$$A^{-1} = \frac{\text{Adj}A}{\text{Det}A} = \frac{1}{27}\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

23. $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, **అయితే $A^{-1} = A^T$ అని నిరూపించండి.**

సాధన: $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix} \Rightarrow A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$

$$A \cdot A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A.A^T = I$

$\therefore A^{-1} = A^T$

24. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ **అయితే $A^{-1} = A^3$ అని నిరూపించండి.**

సాధన: $A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$

$$A^4 = A^2A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 1$$

$\therefore A^4 = I$

$\det A = 3(1) - 3(-2) + 4(-2) = 1$

$\because A \neq 0 \Rightarrow A^{-1}$ వ్యవస్థితం

A^{-1} చే గుణించగా

$$A^4(A^{-1}) = I(A^{-1})$$

$$\Rightarrow A^3(AA^{-1}) = A^{-1} \Rightarrow A^3(I) = A^{-1}$$

$$\therefore A^{-1} = A^3$$

25. క్రింది సమీకరణ వ్యవస్థలు సంగతమో కాదో పరీక్షించండి. సంగతమైతే పూర్తిగా సాధించండి.

1. $\mathbf{x + y + z = 4}$
$\mathbf{2x + 5y - 2z = 3}$
$\mathbf{X + 7y - 7z = 5}$

సాధన: సర్వమాత్రిక $= A = \begin{bmatrix} 1 & 1 & 1 & 4 \\ 2 & 5 & -2 & 3 \\ 1 & 7 & -7 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1,\ R_2 \to R_3 - R_1$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 6 & -8 & 1 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$ we have $A \sim \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 0 & 0 & 11 \end{bmatrix}$

$\rho(A) = 2, \rho(AB) = 3$

$\rho(A) \neq \rho(AB)$

$\therefore$ దత్తసమీకరణ వ్యవస్థ అసంగతం

2.

$\mathbf{x + y + z = 6}$
$\mathbf{x - y + z = 2}$
$\mathbf{2x - y + 3z = 9}$

సాధన: సర్వమాత్రిక $A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \\ 2 & -1 & 3 & 9 \end{bmatrix}$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - 2R_1$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -2 & 0 & -4 \\ 0 & -3 & 1 & -3 \end{bmatrix}$$

$\rho(A) = 3 = \rho(AB)$

$\therefore$ దత్తసమీకరణ వ్యవస్థ సంగతం

$$\therefore A \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -2 & 0 & -4 \\ 0 & -3 & 1 & -3 \end{bmatrix}$$

$R_2 \to R_2 \left(-\frac{1}{2}\right)$,

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & -3 & 1 & -3 \end{bmatrix}$$

By $R_1 \to R_1 - R_2$, $R_3 \to R_3 + 3R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

By $R_1 \to R_1 - R_3$, we have

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$\therefore$ సాధన $x = 1, y = 2, z = 3$.

3. $\mathbf{x + y + z = 1}$
$\mathbf{2x + y + z = 2}$
$\mathbf{x + 2y + 2z = 1}$

సాధన: సర్వమాత్రిక $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 \end{bmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$,

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 - R_3,\quad A \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

$R_2 \rightarrow R_2 - R_1,\quad A \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

$\rho(A) = 2 = \rho(AB) < 3$

దత్తసమీకరణ వ్యవస్థ సంగతంమరియు అనేక సాధనలుంటాయి

సాధన సమితి $[(x, y, z)1x = y + z = 0]$

4. $\mathbf{x + y + z = 9}$
$\mathbf{2x + 5y + 7z = 52}$
$\mathbf{2x + y - z = 0}$

సాధన: సర్వమాత్రిక $A = \begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{bmatrix}$

$R_2 \rightarrow R_2 - 2R_1,\ R_3 \rightarrow R_3 - 2R_1,$

$A \sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & -18 \end{bmatrix}$

$R_3 \rightarrow R_3\,(-1),$

$A \sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 1 & 3 & 18 \end{bmatrix}$

$R_1 \rightarrow R_1 - R_3,\ R_2 \rightarrow R_2 - 3R_3,$

$A \sim \begin{bmatrix} 1 & 0 & -2 & -9 \\ 0 & 0 & -4 & -20 \\ 0 & 1 & 3 & 18 \end{bmatrix}$

$R_2 \rightarrow R_2\left(-\frac{1}{4}\right),$

$A \sim \begin{bmatrix} 1 & 0 & -2 & -9 \\ 0 & 0 & 1 & 5 \\ 0 & 1 & 3 & 18 \end{bmatrix}$

$R_1 \rightarrow R_1 + 2R_2,\ R_3 \rightarrow R_3 - 3R_2,$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 3 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$, $A \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

$\therefore\ \rho(A) = \rho(AB) = 3$

దత్తసమీకరణ వ్యవస్థ సంగతం ఒకేఒక సాధన ఉంటుంది.

$\therefore$ సాధన $x = 1, y = 3, z = 5.$

5. $x - 3y - 8z = -10$
$3x + y - 4z = 0$
$2x + 5y + 6z = 13$

సాధన: సర్వమాత్రిక

$$A = \begin{bmatrix} 1 & -3 & -8 & -10 \\ 3 & 1 & -4 & 0 \\ 2 & 5 & 6 & 13 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - 2R_1$

$$A \sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 10 & 20 & 30 \\ 0 & 11 & 22 & 33 \end{bmatrix}$$

$R_2 \to R_2\left(\dfrac{1}{10}\right),\ R_3 \to R_3\left(\dfrac{1}{11}\right)$

$$A \sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$R_3 \to R_3 - R_2,$

$$A \sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 + 4R_2,$

$$A \sim \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = \rho(AB) = 2 < 3$

∴ దత్తసమికరణ వ్యవస్థ సంగతంమరియు అనేక సాధనలుంటాయి పై మాత్రిక నుండి

$x + y = 2$, $y + 2z = 3$

$z = k$ గా తీసుకుంటే $y = 3 – 2z = 3 – 2k$

$x = 2 – y = 2 – (3 – 2k) = 2 – 3 + 2k = 2k – 1$

∴ సాధన సమితి $x = –1 + 2k$,

$y = 3 – 2k$, $z = k$, k వాస్తవ సంఖ్య.

26. క్రింది సమికరణ వ్యవస్థలను క్రామర్ నియమం తోనూ, మాత్రికా విలోమ పద్ధతి లోనూ, గాస్–జోర్డాన్ పద్ధతి లోనూ సాధించండి.

1. $5x – 6y + 4z = 15$
$7x + 4y – 3z = 19$
$2x + y + 6z = 46$

Hint : $x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}$

సాధన: **i) Cramer's rule :**

$$\Delta = \begin{vmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 1 & 6 \end{vmatrix}$$

$= 5(24+3)+6(42+6)+4(7-8)$

$= 135+288-4 = 419$

$$\Delta_1 = \begin{vmatrix} 15 & -6 & 4 \\ 19 & 4 & -3 \\ 46 & 1 & 6 \end{vmatrix}$$

$= 15(24+3)+6(114+138)+4(19-184)$

$= 405+1512-660 = 1917-660 = 1257$

$$\Delta_2 = \begin{vmatrix} 5 & 15 & 4 \\ 7 & 19 & -3 \\ 2 & 46 & 6 \end{vmatrix}$$

$= 5(114+138)-15(42+6)+4(322-38)$

$= 1260-720+1136 = 1676$

$$\Delta_3 = \begin{vmatrix} 5 & -6 & 15 \\ 7 & 4 & 19 \\ 2 & 1 & 46 \end{vmatrix}$$

$= 5(184-19)+6(322-38)+15(7-8)$

$= 825+1074-15 = 2529-15 = 2514$

$x = \frac{\Delta_1}{\Delta} = \frac{1527}{419} = 3$

$y = \frac{\Delta_2}{\Delta} = \frac{1676}{419} = 4$

$z = \frac{\Delta_3}{\Delta} = \frac{2514}{419} = 6$

సాధనసమితి x = 3, y = 4, z = 6.

ii) మాత్రికా విలోమ పద్ధతి

Hint : $A^{-1} = \frac{\text{Adj} A}{\det A}$

$$A = \begin{bmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 1 & 6 \end{bmatrix}$$

$A_1 = \begin{vmatrix} 4 & -3 \\ 1 & 6 \end{vmatrix} = 24 + 3 = 27$

$B_1 = -\begin{vmatrix} 7 & -3 \\ 1 & 6 \end{vmatrix} = -(42 + 6) = -48$

$C_1 = \begin{vmatrix} 7 & 4 \\ 2 & 1 \end{vmatrix} = 7 - 8 = -1$

$A_2 = -\begin{vmatrix} -6 & 4 \\ 1 & 6 \end{vmatrix} = -(-36 - 4) = 40$

$B_2 = \begin{vmatrix} 5 & 4 \\ 2 & 6 \end{vmatrix} = 30 - 8 = 22$

$C_2 = -\begin{vmatrix} 5 & -6 \\ 2 & 1 \end{vmatrix} = -(5 + 12) = -17$

$A_3 = \begin{vmatrix} -6 & 4 \\ 4 & -3 \end{vmatrix} = 18 - 16 = 2$

$B_3 = -\begin{vmatrix} 5 & 4 \\ 7 & -3 \end{vmatrix} = -(-15 - 28) = 43$

$C_3 = \begin{vmatrix} 5 & -6 \\ 7 & 4 \end{vmatrix} = 20 + 42 = 62$

$$\text{Adj } A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 27 & 40 & 2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}$$

$\text{Det } A = \Delta = 419$

$$A^{-1} = \frac{\text{Adj}A}{\text{Det}A} = \frac{1}{419}\begin{bmatrix} 27 & 40 & 2 \\ 48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}$$

$$x = A^{-1}D = \frac{1}{419}\begin{bmatrix} 27 & 40 & 2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}\begin{bmatrix} 15 \\ 19 \\ 46 \end{bmatrix}$$

$$= \frac{1}{419}\begin{bmatrix} +405+760+92 \\ -720+418+1978 \\ -15-323+2852 \end{bmatrix}$$

$$= \frac{1}{419}\begin{bmatrix} 1257 \\ 1676 \\ 2514 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$\therefore$ సాధనసమితి $x = 3, y = 4, z = 6.$

iii) Gauss-Jordan method :

$$\text{సర్వమాత్రిక } A = \begin{bmatrix} 5 & -6 & 4 & 15 \\ 7 & 4 & -3 & 19 \\ 2 & 1 & 6 & 46 \end{bmatrix}$$

$R_2 \to 5R_2 - 7R_1, R_3 \to 5R_3 - 2R_1$

$$A \sim \begin{bmatrix} 5 & -6 & 4 & 15 \\ 0 & 62 & -43 & -10 \\ 0 & 17 & 22 & 200 \end{bmatrix}$$

$R_1 \to 31R_1 + 3R_2, R_3 \to 62R_3 - 17R_2$

$$A \sim \begin{bmatrix} 155 & 0 & -5 & 435 \\ 0 & 62 & -43 & -10 \\ 0 & 0 & 2095 & 12570 \end{bmatrix}$$

$$R_3 \to R_3\left(\frac{1}{2095}\right)$$

$$A \sim \begin{bmatrix} 155 & 0 & -5 & 435 \\ 0 & 62 & -43 & -10 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$R_1 \to R_1 + 5R_3,\ R_2 \to R_2 + 43R_3$

$$A \sim \begin{bmatrix} 155 & 0 & 0 & 465 \\ 0 & 62 & 0 & 248 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$$R_1 \to R_1\left(\frac{1}{155}\right) R_2 \to R_2\left(\frac{1}{62}\right)$$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$\therefore$ ఏకైకసాధన ఉంటుంది

సాధనసమితి $x = 3,\ y = 4,\ z = 6.$

2. $\mathbf{x + y + z = 1}$
$\mathbf{2x + 2y + 3z = 6}$
$\mathbf{x + 4y + 9z = 3}$

I. i) Cramer's rule

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix}$$

$$= 1(18-12) - 1(18-3) + 1(8-2)$$

$$= 6 - 15 + 6 = -3$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 2 & 3 \\ 3 & 4 & 9 \end{vmatrix}$$

$$= 1(18-12) - 1(54-9) + 1(24-6)$$

$$= 6 - 45 + 18 = -21$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 6 & 3 \\ 1 & 3 & 9 \end{vmatrix}$$

$$= 1(54-9) - 1(18-3) + 1(6-6)$$

$$= 45 - 15 = 30$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 6 \\ 1 & 4 & 3 \end{vmatrix}$$

$$= 1(6-24) - 1(6-6) + 1(8-2)$$

$$= -18 - 0 + 6 = -12$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-21}{-3} = 7$$

$$y = \frac{\Delta_2}{\Delta} = \frac{30}{-3} = -10$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-12}{-3} = 4$$

సాధనసమితి $x = 7$, $y = -10$, $z = 4$.

ii) Matrix inversion method :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$$

$$A_1 = \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 18 - 12 = 6$$

$$B_1 = -\begin{vmatrix} 2 & 3 \\ 1 & 9 \end{vmatrix} = -(18-3) = -15$$

$$C_1 = \begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} = 8 - 2 = 6$$

$$A_2 = -\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -(9-4) = -5$$

$$B_2 = \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 9-1 = 8$$

$$C_2 = -\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -(4-1) = -3$$

$$A_3 = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3-2 = 1$$

$$B_3 = -\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -(3-2) = -1$$

$$C_3 = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2-2 = 0$$

$$\text{Adj}A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}$$

$$\text{Det}A = \Delta = -3$$

$$A^{-1} = \frac{\text{Adj}A}{\text{Det}A} = -\frac{1}{3}\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}$$

$$x = A^{-1}D = \frac{1}{3}\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} 6-30+3 \\ -15+48-3 \\ 6-18+0 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -21 \\ 30 \\ -12 \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \\ 4 \end{bmatrix}$$

∴ సాధనసమితి $x = 7, y = -10, z = 4$.

iii) Gauss-Jordan method :

$$\text{సర్వమాత్రిక } A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 6 \\ 1 & 4 & 9 & 3 \end{bmatrix}$$

$$R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_1$$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 3 & 8 & 2 \end{bmatrix}$$

$R_3 \to R_3 - 8R_2, R_1 \to R_1 - R_2$

$$A \sim \begin{bmatrix} 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 3 & 0 & -30 \end{bmatrix}$$

$R_3 \to R_3\left(\frac{1}{3}\right)$

$$A \sim \begin{bmatrix} 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & -10 \end{bmatrix}$$

$R_1 \to R_1 - R_3, R_2 \leftrightarrow R_3$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

ఏకైకసాధన ఉంటుంది.

సాధన $x = 7, y = -10, z = 4$.

6. $2x - y + 8z = 13$
$3x + 4y + 5z = 18$
$5x - 2y + 7z = 20$

సాధన: i) Cramer's rule :

$$\Delta = \begin{vmatrix} 2 & -1 & 8 \\ 3 & 4 & 5 \\ 5 & -2 & 7 \end{vmatrix}$$

$= 2(28+10) + 1(21-25) + 8(-6-20)$
$= 76 - 4 - 208 = -136$

$$\Delta_1 = \begin{vmatrix} 13 & -1 & 8 \\ 18 & 4 & 5 \\ 20 & -2 & 7 \end{vmatrix}$$

$= 13(28+10) + 1(126-100) + 8(-36-80)$
$= 494 + 26 - 928 = -408$

$$\Delta_2 = \begin{vmatrix} 2 & 13 & 8 \\ 3 & 18 & 5 \\ 2 & 20 & 7 \end{vmatrix}$$
$$= 2(126-100) - 13(21-25) + 8(60-90)$$
$$= 52 + 52 - 240 = -136$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 13 \\ 3 & 4 & 18 \\ 5 & -2 & 20 \end{vmatrix}$$
$$= 2(80+36) + 1(60-90) + 13(-6-20)$$
$$= 232 - 30 - 338 = -136$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-408}{-136} = 3$$
$$y = \frac{\Delta_2}{\Delta} = \frac{-136}{-136} = 1$$
$$z = \frac{\Delta_3}{\Delta} = \frac{-136}{-136} = 1$$

$\therefore$ సాధనసమితి $x = 3, y = 1, z = 1$.

ii) Matrix inversion method :

Let $A = \begin{bmatrix} 2 & -1 & 8 \\ 3 & 4 & 5 \\ 5 & -2 & 7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $D = \begin{bmatrix} 13 \\ 18 \\ 20 \end{bmatrix}$

$$A_1 = \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = 28 + 10 = 38$$

$$B_1 = -\begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = -(21-25) = 4$$

$$C_1 = \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -6 - 20 = -26$$

$$A_2 = -\begin{vmatrix} -1 & 8 \\ -2 & 7 \end{vmatrix} = -(-7+16) = -9$$

$$B_2 = \begin{vmatrix} 2 & 8 \\ 5 & 7 \end{vmatrix} = (14-40) = -26$$

$$C_2 = -\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -(-4+5) = -1$$

$$A_3 = \begin{vmatrix} -1 & 8 \\ 4 & 5 \end{vmatrix} = -5 - 32 = -37$$

$$B_3 = -\begin{vmatrix} 2 & 8 \\ 3 & 5 \end{vmatrix} = -(10 - 24) = 14$$

$$C_3 = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 8 + 3 = 11$$

$$\text{Adj}\,A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 38 & -9 & -37 \\ 4 & -26 & 14 \\ -26 & 1 & 11 \end{bmatrix}$$

$$\begin{aligned} \text{Det}\,A &= a_1A_1 + b_1B_1 + c_1C_1 \\ &= 2 \cdot 38 + (-1)4 + 8(-26) \\ &= 76 - 4 - 208 = -136 \end{aligned}$$

$$A^{-1} = \frac{\text{Adj}\,A}{\text{Det}\,A} = -\frac{1}{136}\begin{bmatrix} 38 & -9 & -37 \\ 4 & -26 & 14 \\ -26 & 1 & 11 \end{bmatrix}$$

$$X = A^{-1}D = -\frac{1}{280}\begin{bmatrix} 38 & -9 & -37 \\ 4 & -26 & 14 \\ -26 & 1 & 11 \end{bmatrix}\begin{bmatrix} 13 \\ 18 \\ 20 \end{bmatrix}$$

$$= -\frac{1}{136}\begin{bmatrix} 494 & -162 & -740 \\ 52 & -468 & +280 \\ -338 & -18 & +220 \end{bmatrix}$$

$$= -\frac{1}{136}\begin{bmatrix} -408 \\ -136 \\ -136 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

సాధనసమితి $x = 3, y = 1, z = 1$.

iii) Gauss Jordan method :

$$\text{సర్వమాత్రిక } A = \begin{bmatrix} 2 & -1 & 8 & 13 \\ 3 & 4 & 5 & 18 \\ 5 & -2 & 7 & 20 \end{bmatrix}$$

$R_2 \to 2R_2 - 3R_1, R_3 \to 2R_3 - 5R_2$ we get

$$A \sim \begin{bmatrix} 2 & -1 & 8 & 13 \\ 0 & 11 & -14 & -3 \\ 0 & 1 & -26 & -25 \end{bmatrix}$$

$R_1 \to R_1 + R_3, R_2 \to R_2 - 11R_3,$ we get

$$A \sim \begin{bmatrix} 2 & 0 & -18 & -12 \\ 0 & 0 & 272 & 272 \\ 0 & 1 & -26 & -25 \end{bmatrix}$$

$R_2 \to R_2\left(\frac{1}{272}\right)$ we get

$$A \sim \begin{bmatrix} 2 & 0 & -18 & -12 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -26 & -25 \end{bmatrix}$$

$R_1 \to R_1 + 18R_2, R_3 \to R_3 + 26R_2,$ we get

$$A \sim \begin{bmatrix} 2 & 0 & 0 & 6 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3, R_1\left(\frac{1}{2}\right), A \sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

∴ ఏకైకసాధన ఉంటుంది

సాధనసమితి x = 3, y = 1, z = 1.

7. 2x – y + 3z = 8
–x + 2y + z = 4
3x + y – 4z = 0

సాధన: i) Cramer's rule :

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{vmatrix}$$

$$= 2(-8-1) + 1(4-3) + 3(-1-6)$$

$$= -18 + 1 - 21 = -38$$

$$\Delta_1 = \begin{vmatrix} 8 & -1 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -4 \end{vmatrix}$$

$$= 8(-8-1) + 1(-16-0) + 3(4-0)$$

$$= -72 - 16 + 12 = -76$$

$$\Delta_2 = \begin{vmatrix} 2 & 8 & 3 \\ -1 & 4 & 1 \\ 3 & 0 & -4 \end{vmatrix}$$

$$= 2(-16-0) - 8(4-3) + 3(-0-12)$$

$$= -32 - 8 - 36 = -76$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 8 \\ -1 & 2 & 4 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= 2(0-4) + 1(0-12) + 8(-1-6)$$

$$= -8 - 12 - 56 = -76$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-76}{-38} = 2$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-76}{-38} = 2$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-76}{-38} = 2$$

$\therefore$ సాధనసమితి $x = 2, y = 2, z = 2$.

ii) Matrix inversion method :

Let $A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, D = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$

$$A_1 = \begin{vmatrix} 2 & 1 \\ 1 & -4 \end{vmatrix} = -8 - 1 = -9$$

$$B_1 = -\begin{vmatrix} -1 & 1 \\ 3 & -4 \end{vmatrix} = -(4-3) = -1$$

$$C_1 = \begin{vmatrix} -1 & 2 \\ 3 & 1 \end{vmatrix} = -1 - 6 = -7$$

$$A_2 = -\begin{vmatrix} -1 & 3 \\ 1 & -4 \end{vmatrix} = -(4-3) = -1$$

$$B_2 = \begin{vmatrix} 2 & 3 \\ 3 & -4 \end{vmatrix} = -8-9 = -17$$

$$C_2 = -\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = -(2+3) = -5$$

$$A_3 = \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix} = -1-6 = -7$$

$$B_3 = -\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} = -(4+3) = -7$$

$$C_3 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 4-1 = 3$$

$$\text{Adj}A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} -9 & -1 & -7 \\ -1 & -17 & -7 \\ -7 & -5 & 3 \end{bmatrix}$$

$$\begin{aligned} \text{Det } A &= a_1A_1 + b_1B_1 + c_1C_1 \\ &= 2(-9) - 1(-1) + 3(-7) \\ &= -18 + 1 - 21 = -38 \end{aligned}$$

$$A^{-1} = \frac{\text{Adj}A}{\text{Det}A} = -\frac{1}{38}\begin{bmatrix} -9 & -1 & -7 \\ -1 & -17 & -7 \\ -7 & -5 & 3 \end{bmatrix}$$

$$X = A^{-1}D = -\frac{1}{38}\begin{bmatrix} -9 & -1 & -7 \\ -1 & -17 & -7 \\ -7 & -5 & 3 \end{bmatrix}\begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

$$= -\frac{1}{38}\begin{bmatrix} -72-4 \\ -8-68 \\ -56-20 \end{bmatrix} = -\frac{1}{38}\begin{bmatrix} -76 \\ -76 \\ -76 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

సాధనసమితి $x = 2, y = 2, z = 2.$

iii) Gauss Jordan method :

$$\text{సర్వమాత్రిక } A = \begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix}$$

$R_1 \to R_1 + 2R_2$, $R_3 \to R_3 + 3R_2$, we get

$$A \sim \begin{bmatrix} 0 & 3 & 5 & 16 \\ -1 & 2 & 1 & 4 \\ 0 & 7 & -1 & 12 \end{bmatrix}$$

$R_2 \to 3R_2 - 2R_1, R_3 \to 3R_3 - 7R_1$, we have

$$A \sim \begin{bmatrix} 0 & 3 & 5 & 16 \\ -3 & 0 & -7 & -20 \\ 0 & 0 & -38 & -76 \end{bmatrix}$$

$R_3 \to R_3\left(-\frac{1}{38}\right)$, we get

$$A \sim \begin{bmatrix} 0 & 3 & 5 & 16 \\ -3 & 0 & -7 & -20 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 \to R_1 - 5R_2, R_2 \to R_2 + 7R_3$, we get

$$A \sim \begin{bmatrix} 0 & 3 & 0 & 6 \\ -3 & 0 & 0 & -6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 \to R_1\left(\frac{1}{3}\right), R_2 \to R_2\left(-\frac{1}{3}\right), R_1 \to R_2$ we get

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

∴ ఏకైకసాధన ఉంటుంది

సాధనసమితి $x = 2, y = 2, z = 2$.

27.క్రింది సమఘాత సమీకరణ వ్యవస్థను సాధించండి.

1. $2x + 3y - z = 0$
   $x - y - 2z = 0$
   $3x + y + 3z = 0$.

సాధన: గుణక మాత్రిక = $\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$

$$\det \text{ of } \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix} = \begin{vmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{vmatrix}$$

$= 2(-3+2) - 3(3+6) - 1(1+3)$

$= -2 - 27 - 4 = -33 \neq 0, \rho(A) = 3$

కాబట్టి దత్త వ్యవస్థ కు $x = y = z = 0$ అనే తృణప్రాయ సాధన మాత్రమే ఉంటుంది.

2. $x + y - 2z = 0$
$2x + y - 3z = 0$
$5x + 4y - 9z = 0$

సాధన: గుణక మాత్రిక $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & 9 \end{bmatrix}$

$$\begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & 9 \end{vmatrix} = 1(-9+12) - 1(-18+15) - 2(8-5)$$

$= 3 + 3 - 6 = 0$

ఉప మాత్రిక $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ సాధారణ మాత్రిక.

$\therefore$ A కోటి $= 2$,

$\rho(A) < 3$.

కాబట్టి దత్త వ్యవస్థ కు తృణప్రాయం కాని సాధన ఉంటుంది.

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & 9 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1,$

$$A \simeq \begin{pmatrix} 1 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & -1 & -1 \end{pmatrix}$$

తుల్య సమీకరణ వ్యవస్థ

$x + y - 2z = 0$

$-y + z = 0$

Let $z = k \Rightarrow y = k, x = k$

$\therefore x = y = z = k$,k వాస్తవ సంఖ్య.

28. $\mathbf{A} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ **అయితే n యొక్క ప్రతి ధనపూర్ణాంక విలువకు**

$A^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$ **అని చూపుము.**

సాధన: దత్త ప్రవచనాన్ని S(n) అనుకోండి.

$$A^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$$

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\Rightarrow A^1 = \begin{bmatrix} \cos 1\theta & -\sin 1\theta \\ \sin 1\theta & \cos 1\theta \end{bmatrix} \Rightarrow$ $n = 1$. దత్త ప్రవచనం నిజం

S(1) నిజం

$n = k$ కు దత్త ప్రవచనం నిజం అనుకోండి

S(k) నిజం అనుకోండి

$$\therefore A^k = \begin{bmatrix} \cos k\theta & -\sin k\theta \\ \sin k\theta & \cos k\theta \end{bmatrix}$$

Now $A^{k+1} = A^k A$

$$= \begin{bmatrix} \cos k\theta & -\sin k\theta \\ \sin k\theta & \cos k\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\theta\cos\theta - \sin k\theta\sin\theta & -\cos k\theta\sin\theta - \sin k\theta\cos\theta \\ \sin k\theta\cos\theta + \cos k\theta\sin\theta & -\sin k\theta\sin\theta + \cos k\theta\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k\theta+\theta) & -\sin(k\theta+\theta) \\ \sin(k\theta+\theta) & \cos(k\theta+\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & -\sin(k+1)\theta \\ \sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

$\therefore$ S(k + 1) నిజం

n యొక్కప్రతి ధన పూర్ణంక విలువకు $A^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$.

29. Fi A = $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ విలోమాన్ని కనుగొనుము.

సాధన: $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

Det A = 1(4 – 3) – 2(6 – 3) + 1(3 – 2)
= 1 – 6 + 1 = –4

$A_{11} = +(4-3) = 1$

$A_{12} = -(6-3) = -3$

$A_{13} = +(3-2) = 1$

$A_{21} = -(4-1) - 3$

$A_{22} = +(2-1) = 1$

$A_{23} = -(1-2) = 1$

$A_{31} = +(6-2) = 4$

$A_{32} = -(3-3) = 0$

$A_{33} = +(2-6) = -4$

$$\therefore \text{Adj}\,A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 1 & -3 & 4 \\ -3 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{Adj}A}{\det A} = -\frac{1}{4}\begin{bmatrix} 1 & -3 & 4 \\ -3 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$$

30. Theorem : A ఒక సాధారణ మాత్రిక అయితే A విలొమనీయమని మరియు $A^{-1} = \frac{\text{Adj}A}{\det A}$ అని చూపండి.

సాధన: **Proof :** $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ ఒక సాధారణ మాత్రిక అనుకోండి.

$\therefore \det A \neq 0.$

$$\operatorname{Adj} A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix}$$

$$A \cdot \operatorname{Adj} A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1A_1 + b_1B_1 + c_1C_1 & a_1A_2 + b_1B_2 + c_1C_2 & a_1A_3 + b_1B_3 + c_1C_3 \\ a_2A_1 + b_2B_1 + c_2C_1 & a_2A_2 + b_2B_2 + c_2C_2 & a_2A_3 + b_2B_3 + c_2C_3 \\ a_3A_1 + b_3B_1 + c_3C_1 & a_3A_2 + b_3B_2 + c_3C_2 & a_3A_3 + b_3B_3 + c_3C_3 \end{bmatrix}$$

$$= \begin{bmatrix} \det A & 0 & 0 \\ 0 & \det A & 0 \\ 0 & 0 & \det A \end{bmatrix} = \det A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \det A \, I$$

$$\therefore \; A \cdot \frac{\operatorname{Adj} A}{\det A} = I$$

ఇదే విధంగా $A \cdot \dfrac{\operatorname{Adj} A}{\det A} = I$

$$\therefore \; A^{-1} = \frac{\operatorname{Adj} A}{\det A}$$

31. $\begin{bmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{bmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$ అని చూపండి.

సాధన: $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \dfrac{1}{abc} \begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix}$

$$= \frac{1}{abc} abc \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$\underset{R_2 \to R_2 - R_3}{\overset{R_1 \to R_1 - R_3}{=}} \begin{vmatrix} 0 & a^2 - c^2 & a^3 - c^3 \\ 0 & b^2 - c^2 & b^3 - c^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$\underset{R_2 \to R_2 - R_1}{=} (a-c)(b-c)\begin{vmatrix} 0 & a+c & a^2+ac+c^2 \\ 0 & b-c & b^2+bc+c^2 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$= (a-c)(b-c)\begin{vmatrix} 0 & a+c & a^2+ac+c^2 \\ 0 & b-c & b^2-a^2+bc-ac \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$= (a-c)(b-c)(b-a)\begin{vmatrix} 0 & a+c & a^2+ac+c^2 \\ 0 & 1 & c+a+b \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$= (a-c)(b-c)(b-a)\begin{vmatrix} a+c & a^2+ac+c^2 \\ 1 & a+b+c \end{vmatrix}$$

$$= (a-c)(b-c)(b-a)(ab+bc+ca)$$

32. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ **అని చూపండి.**

సాధన: $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$= abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 0 \\ 1/b & 1 & 0 \\ 1/c & 0 & 1 \end{vmatrix} \begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$$

33. $A=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, B=\frac{1}{2}\begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$ **అయితే** ABA^{-1} **ఒక వికర్ణ మాత్రిక అని చూపండి.**

సాధన. A^{-1} ను కనుక్కొందాం.

$$A_1 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0-1 = -1$$

$$B_1 = -\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -(0-1) = 1$$

$$C_1 = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

$$B_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$C_2 = -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -(0-1) = 1$$

$$A_3 = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

$$B_3 = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -(-1) = 1$$

$$C_3 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$\begin{aligned} DetA &= a_1A_1 + b_1B_1 + c_1C_1 \\ &= 0(-1) + 1(1) + 1(1) \\ &= 1 + 1 = 2 \end{aligned}$$

$$A^{-1} = \frac{AdjA}{DetA} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} . \frac{1}{2}\begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} c-b+b-c & c+a+a-c & a-b+a+b \\ b+c+b-c & c-a+a-c & b-a+a+b \\ b+c+c-b & c-a+c+a & b-a+a-b \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 2a & 2a \\ 2b & 0 & 2b \\ 2c & 2c & 0 \end{bmatrix}$$

$$ABA^{-1} = \frac{1}{2}.\begin{bmatrix} 0 & 2a & 2a \\ 2b & 0 & 2b \\ 2c & 2c & 0 \end{bmatrix} . \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 2a+2a & -2a+2a & 2a-2a \\ -2b+2b & 2b+2b & 2b-2b \\ -2c+2c & 2c-2c & 2c+2c \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4a & 0 & 0 \\ 0 & 4b & 0 \\ 0 & 0 & 4c \end{bmatrix}$$

$$= \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$ వికర్ణ మాత్రిక

అభ్యాసం-3(జి)

1. $$\begin{aligned} x+y+4z&=6\\ 3x+2y-2z&=9\\ 5x+y+2z&=13 \end{aligned}$$

సాధన. సర్వ మాత్రిక $[AD]=\begin{bmatrix}1&1&4&6\\3&2&-2&9\\5&1&2&13\end{bmatrix}$

$R_2 \to R_2-3R_1 \to R_2-5R_1$ చేస్తే

$$\sim\begin{bmatrix}1&1&4&6\\0&-1&-14&-9\\0&-4&-18&-17\end{bmatrix}$$

$R_3 \to R_3-4R_2$ చేస్తే

$$\sim\begin{bmatrix}1&1&4&6\\0&-1&-14&-9\\0&0&38&19\end{bmatrix}$$

$R_2 \to R_2(-1), R_3 \to R_3\left[\frac{1}{2}\right]$ చేస్తే

$$\sim\begin{bmatrix}1&1&4&6\\0&1&14&9\\0&0&1&\frac{1}{2}\end{bmatrix}$$

$R_1 \to R_1-4R_3, R_2 \to R_2-14R_3$

$$\sim\begin{bmatrix}1&1&0&4\\0&1&0&2\\0&0&1&\frac{1}{2}\end{bmatrix}$$

$$\sim\begin{bmatrix}1&0&0&2\\0&1&0&2\\0&0&1&\frac{1}{2}\end{bmatrix}$$

$\rho(A)=\rho(AB)=3$

$\therefore$ దత్త వ్యవస్థ సంగతము ఏకైక సాధనం ఉంటుంది.

$\therefore$ సాధన $x = 2, y = 2, z = \frac{1}{2}$

అభ్యాసం-3(హెచ్)

1. $$\begin{aligned} 2x - y + 3z &= 9 \\ x + y + z &= 6 \\ x - y + z &= 2 \end{aligned}$$

సాధన. $i)$ క్రేమర్ నియమం.

$$\Delta = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= 2(1+1) + 1(1-1) + 3(-1-1)$$

$$= 4 + 0 - 6 = -2$$

$$\Delta_1 = \begin{bmatrix} 9 & -1 & 3 \\ 6 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$= 9(1+1) + 1(6-2) + 3(-6-2)$$

$$= 18 + 4 - 24$$

$$= -2$$

$$\Delta_2 = \begin{bmatrix} 2 & 9 & 3 \\ 1 & 6 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= 2(6-2) - 9(1-1) + 3(2-6)$$

$$= 8 - 012 = -4$$

$$\Delta_3 = \begin{bmatrix} 2 & -1 & 9 \\ 1 & 1 & 6 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= 2(2+6) + 1(2-6) + 9(-1-1)$$

$$= 16 - 4 - 18 = -6$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-2}{-2} = 1, y = \frac{\Delta_2}{\Delta} = \frac{-4}{-2} = 2,$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-6}{-2} = 3$$

సాధన $x = 1, y = 2, z = 3$

ii) **మాత్రిక విలోమ పద్ధతి:**

$$A=\begin{bmatrix}2 & -1 & 3\\1 & 1 & 1\\1 & -1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, D=\begin{bmatrix}9\\6\\2\end{bmatrix}$$

$$A_1=\begin{vmatrix}1 & 1\\-1 & 1\end{vmatrix}=1+1=2$$

$$B_1=\begin{vmatrix}1 & 1\\1 & 1\end{vmatrix}=0$$

$$C_1=\begin{vmatrix}1 & 1\\1 & -1\end{vmatrix}=-1-1=-2$$

$$A_2=\begin{vmatrix}1 & 3\\-1 & 1\end{vmatrix}=-(-1+3)=-2$$

$$B_2=\begin{vmatrix}2 & 3\\1 & 1\end{vmatrix}=2-3=-2$$

$$C_2=\begin{vmatrix}2 & -1\\1 & -1\end{vmatrix}=-(-2+1)=1$$

$$A_3=\begin{vmatrix}1 & 3\\-1 & 1\end{vmatrix}=-1-3=-4$$

$$B_3=\begin{vmatrix}2 & 3\\1 & 1\end{vmatrix}=-(2-3)=1$$

$$C_3=\begin{vmatrix}2 & -1\\1 & -1\end{vmatrix}=2+1=3$$

$$AdjA=\begin{bmatrix}A_1 & A_2 & A_3\\B_1 & B_2 & B_3\\C_1 & C_2 & C_3\end{bmatrix}=\begin{bmatrix}2 & -2 & -4\\0 & -1 & -\\02 & 1 & 3\end{bmatrix}$$

$$DetA=a_1A_1+b_1B_1+c_1C_1$$
$$=2(2)-1.0+3(-2)=4-6=-2$$

$$A^{-1} = \frac{AdjA}{DetA} = -\frac{1}{2}\begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$X = A^{-1}D = -\frac{1}{2}\begin{bmatrix} 0 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 18-12-8 \\ -6+2 \\ -18+6+6 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore x = 1, y = 2, z = 3$

$\therefore$ సాధన $x = 1, y = 2, z = 3$

iii) గౌస్-జోర్డాన్ పద్ధతి:

సర్వ మాత్రిక $A = \begin{bmatrix} 2 & -1 & 3 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{bmatrix}$

$R_1 \to R_1 - 2R_2, R_3 \to R_3 - R_1$

$$A \sim \begin{bmatrix} 0 & -3 & 1 & -3 \\ 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \end{bmatrix}$$

$R_3 \to R_1 + 3R_3, R_2 \to R_2 - R_3$

$$A \sim \begin{bmatrix} 0 & 0 & 1 & 3 \\ 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$R_2 \to R_2 - R_1, A \sim \begin{bmatrix} 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{bmatrix}$

$By\, R_1 \leftrightarrow R_2, R_2 \leftrightarrow R_3, A \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\therefore$ దత్త వ్యవస్థ సంగతం.

ఏకైక సాధన $x = 1, y = 2, z = 3$.

సాధించిన సమస్యలు:

1. $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}3 & 7\\ 7 & 2\end{bmatrix}, 2x+A=B$ **అయితే మాత్రిక Xను కనుక్కోండి.**

సాధన. $2x+A=B \Rightarrow 2X=B-A$

$$=\begin{bmatrix}3 & 8\\ 7 & 2\end{bmatrix}-\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}2 & -6\\ 4 & -2\end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix}2 & 6\\ 4 & -2\end{bmatrix} \therefore X=\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$$

2. **ఒక 3×2 మాత్రిక మూలకాలు $a_{ij}=\frac{1}{2}|i-3j|$ గా నిర్వచిస్తే, ఆ మాత్రికను నిర్మించండి.**

సాధన. సాధారణంగా 3×2 మాత్రికను

$$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\\ a_{31} & a_{32}\end{bmatrix}$$ గా సూచిస్తాం.

$$a^{ij}=\frac{1}{2}|i-3j| \qquad i=1,2,3; j=1,2.$$

$$a_{11}=\frac{1}{2}|1-(3\times 1)|=1 a_{12}=\frac{1}{2}|1-(3\times 2)|=\frac{5}{2}$$

$$a_{21}=\frac{1}{2}|2-(3\times 1)|=\frac{1}{2} a_{22}=\frac{1}{2}|2-(3\times 2)|=2$$

$$a_{31}=\frac{1}{2}|3-(3\times 1)|=0$$

$$a_{32}=\frac{1}{2}|3-(3\times 2)|=\frac{3}{2}$$

$$\therefore A=\begin{bmatrix}1 & \frac{5}{2}\\ \frac{1}{2} & 2\\ 0 & \frac{3}{2}\end{bmatrix}$$

3. నిర్ధారకాన్ని విస్తరించకుండా

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{matrix} a & b & c \\ b & c & a \\ c & a & b \end{matrix}$$

అని చూపండి.

సాధన.

$$L.H.S = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$R_1 \Rightarrow R_1 + R_2 + R_3$

$$= 2\begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= 2\begin{matrix} a+b+c & a+b+c & a+b+c \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{matrix}$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$= 2\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ -b & -c & -a \\ -c & -a & -b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= 2\begin{matrix} a & b & c \\ -b & -c & -a \\ -c & -a & -b \end{matrix} = 2(-1)(-1)\begin{matrix} a & b & c \\ b & c & a \\ c & a & b \end{matrix}$$

$$= 2\begin{matrix} a & b & c \\ b & c & a \\ c & a & b \end{matrix} = R.H.S$$