

## అతి పరావలయ సమీకరణాలు

$$1. \sin hx = \frac{e^x - e^{-x}}{2}$$

$$2. \cos hx = \frac{e^x + e^{-x}}{2}$$

$$3. \tan hx = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$4. \cos ec hx = \frac{2}{e^x - e^{-x}}$$

$$5. \sec hx = \frac{2}{e^x + e^{-x}}$$

$$6. \cot hx = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$7. \cos h^2 x - \sin h^2 x = 1$$

$$8. 1 - \tan h^2 x = \sec h^2 x$$

$$9. \cot h^2 x - 1 = \cos ec h^2 x$$

$$1. \sin h^{-1} x = \log \left\{ x + \sqrt{x^2 + 1} \right\} \text{అనిచూపుము}$$

Let  $\sin h^{-1} x = y \Rightarrow x = \sin h y$

$$x = \frac{e^y - e^{-y}}{2} = 2x = e^y - \frac{1}{e^y} \Rightarrow 2xe^y = (e^y)^2 - 1$$

$$(e^y)^2 - 2xe^y - 1 = 0 \Rightarrow e^y = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = e^y = x + \sqrt{x^2 + 1} \Rightarrow y = \log_e \left( x + \sqrt{x^2 + 1} \right)$$

$$\boxed{\therefore \sin h^{-1} x = \log_e \left( x + \sqrt{x^2 + 1} \right)}$$

$$2. \cos h^{-1} x = \log_e \left( x - \sqrt{x^2 - 1} \right) \text{అనిచూపుము}$$

**Ans.**  $\cos h^{-1} x = y \Rightarrow x = \cos hy$

$$x = \frac{e^y + e^{-y}}{2} \Rightarrow 2x = e^y + \frac{1}{e^y}$$

$$2xe^y = (e^y)^2 + 1 \Rightarrow (e^y)^2 - 2xe^y + 1$$

$$e^y = \frac{2x \pm \sqrt{4x^2 - 4}}{2} \Rightarrow e^y = x \pm \sqrt{x^2 - 1}$$

$$e^y = x + \sqrt{x^2 - 1} \Rightarrow y = \log_e \left( x + \sqrt{x^2 - 1} \right)$$

$$\boxed{\cos h^{-1} x = \log_e \left( x + \sqrt{x^2 - 1} \right)}$$

3.  $\operatorname{Tanh}^{-1}x = \frac{1}{2} \log_e \left( \frac{4x}{1-x} \right)$  అనిచూపుము

Let  $\operatorname{Tanh}^{-1}x = y \Rightarrow x = \operatorname{tanh} h y$

$$x = \frac{e^y - e^{-y}}{e^y + e^{-y}} \Rightarrow \frac{1}{x} = \frac{e^y + e^{-y}}{e^y - e^{-y}}$$

Using component and dividend

$$\frac{1+x}{1-x} = \frac{e^y + e^{-y} + e^y - e^{-y}}{e^y + e^{-y} - e^y + e^{-y}}$$

$$\frac{1+x}{1-x} = \frac{2e^y}{2e^{-y}} \Rightarrow \frac{1+x}{1-x} = e^{2y}$$

$$2y = \left\{ \frac{1+x}{1-x} \right\} \Rightarrow y = \frac{1}{2} \log \left( \frac{1+x}{1-x} \right)$$

$$\boxed{\cot h^{-1}x = \frac{1}{2} \log \left( \frac{x+1}{x-1} \right)}$$

$$\rightarrow \sec h^{-1}x = \log \left\{ \frac{1+\sqrt{1-x^2}}{x} \right\}$$

$$\rightarrow \cos ec h^{-1}x = \log \left( \frac{1-\sqrt{1+x^2}}{x} \right) x < 0$$

$$= \log \left\{ \frac{1-\sqrt{1+x^2}}{x} \right\} x > 0$$

$$\rightarrow \sin h(x+y) = \sin hx \cos hy + \cosh x \sin hy$$

$$\rightarrow \sin h(x-y) = \sin hx \cos hy + \cos hx \sin hy$$

$$\rightarrow \cos h(x+y) = \cos hx \cos hy + \sin hx \sin hy$$

$$\rightarrow \cos h(x-y) = \cos hx \cos hy - \sin hx \sin hy$$

$$\rightarrow \sin h2x = 2 \sin hx \cos hx = \frac{2 \operatorname{Tanh} hx}{1 - \operatorname{Tanh}^2 x}$$

$$\rightarrow \cosh 2x = \cosh^2 x + \sinh^2 x = 2 \cosh^2 x - 1 = 1 + 2 \sinh^2 x = \frac{1 + \operatorname{Tanh}^2 x}{1 - \operatorname{Tanh}^2 x}$$

$$\rightarrow \operatorname{Tanh}(x+y) \frac{\operatorname{Tanh} x + \operatorname{Tanh} y}{1 - \operatorname{Tanh} x \operatorname{Tanh} y}$$

$$\rightarrow \operatorname{Tanh}(x-y) \frac{\operatorname{Tanh} x - \operatorname{Tanh} y}{1 + \operatorname{Tanh} x \operatorname{Tanh} y}$$

$$\rightarrow \cot h(x+y) = \frac{\cot hx \cot hy + 1}{\cot hy + \cot hx}$$

$$\rightarrow \cot h(x-y) = \frac{\cot hx \cot hy - 1}{\cot hy - \cot hx}$$

$$\tan h2x = \frac{2 \tan hx}{1 + \tanh^2 x}$$

$$\rightarrow \cot h2x = \frac{\coth^2 x + 1}{2 \coth x}$$

### Some problems

1.  $\sin hx = \frac{3}{4}$  என்க  $\cos h2x$ ,  $\sin h2x$  எனும் கந்திமமு.

**Solution:**

$$\sin hx = \frac{3}{4}$$

$$\cos h^2 x = 1 + \sin h^2 x$$

$$\cos h^2 x = 1 + \frac{9}{16} \Rightarrow \cosh^2 x = \frac{25}{16}$$

$$\cos hx = \frac{5}{4}$$

$$\sin^2 x = 2 \sin hx \cos hx = 2 \times \frac{3}{4} \times \frac{5}{4} = \frac{15}{8}$$

$$\cos^2 x = \cosh^2 x + \sin h^2 x = \frac{25}{16} + \frac{9}{16} = \frac{34}{16} = \frac{17}{8}$$

2.  $\sin hx = 3$  என்க  $x = \log(3 - 1\sqrt{10})$  அனிசுபுமு

**Solution:**

$$\sin hx = 3 \Rightarrow x = \sin h^{-1} 3$$

$$\sin h^{-1} y = \cos(y + \sqrt{y^2 + 1})$$

$$\sin h^{-1} 3 = \log \{3 + \sqrt{9+1}\}$$

$$= \log \{3 + \sqrt{10}\}$$

3.  $\tan h(x-y) = \frac{\tan hx - \tan hy}{1 - \tan hx \tan hy}$  అనిచూపుము

$$\text{R.H.S} = \frac{\tan hx - \tan hy}{1 - \tan hx \tan hy} = \frac{\frac{\sin hx}{\cos hx} - \frac{\sin hy}{\cos hy}}{1 - \frac{\sin hx}{\cos hx} \cdot \frac{\sin hy}{\cos hy}}$$

$$\frac{\sin hx \cos hy - \cos hx \sin hy}{\cos hx \cos hy - \sin hx \sin hy} = \frac{\sin h(x-y)}{\cos h(x-y)} = \tan h(x-y)$$

4.  $\cot h(x-y) = \frac{\cot hx \cot hy - 1}{\cot hy - \cot hx}$  అనిచూపుము

$$\begin{aligned} \text{RHS} \quad & \frac{\cot hx \cot hy - 1}{\cot hy - \cot hx} = \frac{\frac{\cos hx}{\sin hx} \cdot \frac{\cos hy}{\sin hy} - 1}{\frac{\cos hy}{\sin hy} - \frac{\cos hx}{\sin hx}} \\ & \frac{\cos hx \cos hy - \sin hx \sin hy}{\sin hx \sin hy} \\ & \frac{\sin hx \sin hy}{\sin hx \cos hy - \cos hx \sin hy} = \frac{\cos h(x-y)}{\sin h(x-y)} = \cot h(x-y) \end{aligned}$$

5.  $(\cos hx - \sin hx)^n = \cos hn x - \sin hn x$  అనిచూపుము

**Solution:**

$$(\cos hx - \sin hx)^n = \left\{ \frac{e^x + e^{-x}}{2} - \frac{e^x - e^{-x}}{2} \right\}^n$$

$$= \left\{ \frac{e^x + e^{-x} - e^x + e^{-x}}{2} \right\} = (e^{-x}) = e^{-nx}$$

$$\text{RHS} = \cosh nx - \sinh nx = \frac{e^{nx} + e^{-nx}}{2} - \left( \frac{e^{nx} - e^{-nx}}{2} \right)$$

$$= \frac{e^{nx} + e^{-nx} - e^{nx} + e^{-nx}}{2} = e^{-nx}$$

6.  $(\cos hx + \sin hx)^n = \cos h nx + \sin h nx$  அனிசுப்பும்

$$\text{LHS} = (\cos hx + \sin hx)^n = \left( \frac{e^x + e^{-x} + e^x - e^{-x}}{2} \right)^n = (e^x)^n = e^{nx}$$

$$\text{RHS } \cos h nx + \sin h nx \frac{e^{nx} + e^{-nx} + e^{nx} - e^{-nx}}{2} = e^{nx}$$

$$\text{LHS} = \text{RHS}$$

7.  $\frac{\tan hx}{\sec hx - 1} + \frac{\tan hx}{\sec hx + 1} = -2 \operatorname{cosech} hx$  அனிசுப்பும்

$$\frac{\tan hx}{\sec hx - 1} + \frac{\tan hx}{\sec hx + 1} = \frac{\tan hx(\sec hx + 1) + \tan hx(\sec hx - 1)}{(\sec hx - 1)(\sec hx + 1)}$$

$$\frac{\tan hx \{ \sec hx + y + \sec hx - y \}}{\sec h^2 x - 1}$$

$$\frac{2 \tan hx \cdot \sec hx}{-\tanh^2 x} = \frac{-2}{\frac{\cos hx}{\frac{\sin hx}{\cos hx}}} = -\operatorname{cosech} hx$$

8.  $\frac{\cos hx}{1 - \tan hx} + \frac{\sin hx}{1 - \cot hx} = \sin hx + \cos hx$  அனிசுப்பும்

**Solution:**

$$\frac{\cos hx}{1 - \tan hx} + \frac{\sin hx}{1 - \cot hx}$$

$$\frac{\cos hx}{1 - \frac{\sin hx}{\cos hx}} + \frac{\sin hx}{1 - \frac{\cos hx}{\sin hx}} = \frac{\cosh^2 x}{\cos hx - \sin hx} + \frac{\sin h^2 x}{\sin hx - \cos hx}$$

$$\begin{aligned}
 &= \frac{\cosh^2 x}{\cos hx - \sin hx} - \frac{\sin h^2 x}{\cos hx - \sin hx} \\
 \frac{\cosh^2 x - \sin h^2 x}{\cos hx - \sin hx} &= \frac{(\cos hx + \sin x)(\cos hx - \sin hx)}{(\cos hx - \sin hx)} = \cos hx + \sin hx
 \end{aligned}$$

9.  $\cos h^4 x - \sin h^4 x = \cos h^2 x$  అనిచూపుము

**Solution:**

$$\begin{aligned}
 \cos h^4 x - \sin h^4 x &= (\cos h^2 x)^2 - (\sin h^2 x)^2 \\
 \cos h^4 x - \sin h^4 x &= (\cos h^2 x)^2 - (\sin h^2 x)^2 \\
 &= (\cos h^2 x + \sin h^2 x)(\cos h^2 x - \sin h^2 x) \\
 &= (\cosh^2 x)(1) = \cos h2x
 \end{aligned}$$

10.  $\mu = \log_e \left\{ \tan \left( \frac{\pi}{4} + \frac{\pi}{2} \right) \right\}$  ఇతే  $\cos h\mu = \sec \theta$  అనిచూపుము

**Solution:**

$$\mu = \log_e \left\{ \tan \left( \frac{\pi}{4} + \frac{\pi}{2} \right) \right\} \Rightarrow e^\mu = \tan \left( \frac{\pi}{4} + \frac{\theta}{2} \right)$$

$$e^{-\mu} = \frac{1}{e^\mu} = \cot \left( \frac{\pi}{4} + \frac{\pi}{2} \right)$$

$$\cosh \mu = \frac{e^\mu + e^{-\mu}}{2} = \frac{1}{2} \left( \tan \left( \frac{\pi}{4} + \frac{\theta}{2} \right) + \cot \left( \frac{\pi}{4} + \frac{\theta}{2} \right) \right)$$

$$= \frac{1}{2} \left( \frac{\sin \left( \frac{\pi}{4} + \frac{\theta}{2} \right)}{\cos \left( \frac{\pi}{4} + \frac{\theta}{2} \right)} + \frac{\cos \left( \frac{\pi}{4} + \frac{\theta}{2} \right)}{\sin \left( \frac{\pi}{4} + \frac{\theta}{2} \right)} \right)$$

$$\begin{aligned}
 &= \frac{1}{2} \left( \frac{\sin^2\left(\frac{\pi}{4} + \frac{\theta}{2}\right) \cos^2\left(\frac{\pi}{4} + \frac{\theta}{2}\right)}{\cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right) \sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right)} \right) \\
 &= \frac{1}{2 \sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right) \cos\left(\frac{\pi}{4} + \frac{\theta}{2}\right)} \\
 &= \frac{1}{\sin\left\{2\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right\}} = \frac{1}{\sin\left(\frac{\pi}{4} + \theta\right)} = \frac{1}{\cos\theta} = \sec\theta \\
 \therefore \cos h4 &= \sec\theta
 \end{aligned}$$

11.  $\operatorname{Tanh}^{-1} \frac{1}{2} = \frac{1}{2} \log_e (3)$  అనిచూపుము

$$\operatorname{Tanh}^{-1} x = \frac{1}{2} \log_e \left( \frac{1+x}{1-x} \right)$$

$$\operatorname{Tanh}^{-1} \frac{1}{2} = \frac{1}{2} \log_e \left( \frac{1+\frac{1}{2}}{1-\frac{1}{2}} \right)$$

$$= \frac{1}{2} \log_e \left( \frac{\frac{3}{2}}{\frac{1}{2}} \right) = \frac{1}{2} \log_e (3)$$

4.  $\cos hx = \frac{5}{2}$  ఇతీ  $\cos h2x, \sin h2x$  లను కనుగొనుము.

5.  $\sin h3x = 3 \sin hx + 4 \sinh^3 x$  అనిచూపుము

6.  $\tan h3x = \frac{3 \tan hx + \tanh^3 x}{1 + 3 \tan h^2 x}$  అనిచూపుము

1.  $\sin hx = 5$  అట్లి  $x = \log(5 + \sqrt{20})$  అనిచూపుము

2.  $\tan h^{-1}\left(\frac{1}{2}\right) = \frac{1}{2} \log_e^3$  అనిచూపుము

$x = \log \left\{ \cot \left( \frac{\pi}{4} + \theta \right) \right\}$  అట్లి  $\cos hx = \sec 2\theta$  and  $\sin hx = -\tan 2\theta$  అనిచూపుము

3.

4.  $\sin h3x = 3\sin hx + 4\sinh^3 x$  అనిచూపుము

5.  $\tan h3x = \frac{3\tan hx + \tanh^3 x}{1 + 3\tan h^2 x}$  అనిచూపుము