Kinetic Molecular Model of Gas

I. Maxwell and Boltzmann proposed a Mathematical theory to explain the behavior of gases. The following are the postulates of this model.

1) A gas consists of a large number of identical molecules of mass m; the dimensions of these molecules are very small compared to the space between them. The actual volume occupied by gas molecule is negligible then compared to the total volume.

2) There are practically no attractive forces between the molecules.

3) The gas molecules are in constant random motion with high velocities. They move in straight lines with uniform velocity and change direction on collision with other molecule or the walls of container

4) All collisions are perfectly elastic hence there is loss of kinetic energy of molecules during a collision.

5) The pressure of a gas is the result of collisions of molecules with the walls of container.

6) The average kinetic energy of colliding molecules is directly proportional to its absolute temperature.

II. A gas that confirms to the assumption of the kinetic theory of gases is called an ideal gas.

The real gases as $H_2, O_2, N_2, ...$ are opposed to the assumption 1, 2.

Kinetic gas Equation

Based on the postulates of kinetic molecular theory of gases and root mean square velocity 'C', the kinetic gas equations , PV = 1/3mnc^2 is derived. Where P is pressure, V is volume, m is mass of gas molecule, n is number of molecules and C is RMS velocity of gas.

Significance of RMS velocity in Kinetic Gas equation

Velocity is a vector quantity. It is represented by positive sign in any particular direction, and negative sign in the opposite direction. Due to randomness molecules may collide with each other. As a result, the velocity of molecules changes frequently. Thus some times the average velocity of the molecules may become negative or zero. To avoid this kind of anomaly, the velocities of molecules are squared which will be always positive. The square root of the average of squares of velocities i.e RMS velocity gives exact velocity molecules.. Hence, it is used in kinetic gas equation.

Kinetic Energy

Kinetic energy of a gas is in general taken as translational kinetic energy.

1. Kinetic energy is related to the number of moles (n) and RMS velocity (C) as (K.E.) = (3/2) nRT.

2. Kinetic energy per one mole of a gas is generally called average kinetic energy or kinetic energy per mol.

   (K.E.) = (3/2) RT

   *Average kinetic energy does not depend on the nature of the gas but depends only on temperature.

3. Kinetic energy per one molecule of a gas is given as

   (K.E.) = (3/2) RT/N = (3/2) kT

   Where k is the Boltzmann constant. It is the value of universal gas constant for one molecule of a gas.It is nuemerically 1.38 x 10^{-23}J K^{-1} molecule $^{-1}$ (or) 1.38 x 10^{-16} erg K^{-1} molecule $^{-1}$

Distribution of molecular velocities

All molecules is a given sample of a gas don't have the same speed due to collisions between them and walls on the container. Even though the speed of individual molecule is constantly changing at a given temperature the fraction of the molecules with a particular speed remains constant. This is known as Maxwell Boltzmann distribution law. Maxwell derived the following equation for the distribution of Molecular velocities

$$\frac{dN_c}{N} = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} e^{\frac{-MC^2}{2RT}}.C^2dc$$

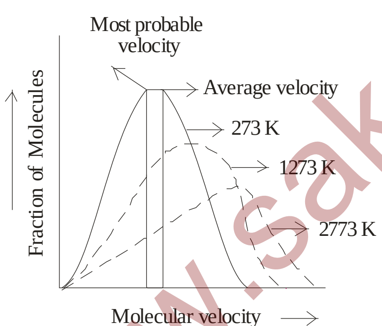
Where dN_c = number of molecules having velocities between c and c+dc

N= total number of molecules

M = molar mass

T = absolute temperature

The distribution of velocities of gas at different temperatures may be shown below.



1) Very small fraction of molecules has either very low (close to zero) or very high velocities.

2) Most intermediate fractions of molecules have velocities close to an average velocity represented by the peak of the curve. This velocity is called most probable velocity.

3) At higher temperatures, the whole curve shifts to the right this shows that at higher temperature more molecules have higher velocities and fewer molecules have lower velocities

Different kinds of velocities: In our study of kinetic theory we come across three different kinds of molecular velocities.

1) Average velocity 2) Root mean square velocity 3) Most probable velocity

1. Most probable velocity

It is the velocity possessed by maximum number of molecules present in a given sample of gasat a given temparature. **It is denoted by C_p.**

Most probable velocity calculated as follows.

$$C_p = \sqrt{\frac{2RT}{M}} \quad \textbf{(Or)} \quad C_p = \sqrt{\frac{2PV}{M}}$$

2. Average velocity

It is the ratio of sum of the velocities of all gas molecules to the total number of moleules. **It is also called mean velocity. It is denoted by** $\bar{C}$**.**

$$\bar{C} = \frac{c_1 + c_2 + c_3 + \dots\dots + c_n}{n}$$

If n_1 molecules of a gas possess c_1 velocity, n_2 molecules c_2 velocity, n_3 molecules c_3 velocity, etc., the average velocity is represented as

$$\bar{C} = \frac{n_1c_1 + n_2c_2 + n_3c_3 \dots\dots\dots\dots}{n_1 + n_2 + n_3 + \dots\dots\dots\dots}$$

Average velocity is calculated by using

$$\bar{C} = \sqrt{\frac{8RT}{\pi M}} (or) \sqrt{\frac{8PV}{\pi M}} (Or) \sqrt{\frac{8P}{\pi d}}$$

3. RMS velocity

It is the square root of mean of squares of velocities of molecules present in the gas. **It is denoted by C. It is given by**

$$C = \sqrt{\frac{c_1^{\,2} + c_2^{\,2} + c_3^{\,2} + \ldots\ldots + c_n^2}{n}}$$

***RMS velocity is the most accurate velocity among the three types of velocities of a gas, because it represents the velocity of all gas molecules.**

$$C = \sqrt{\frac{3RT}{M}}(or)\sqrt{\frac{3pv}{M}}(or)\sqrt{\frac{3p}{d}} \text{ (or)C=} 1.58\times10^4\sqrt{\frac{T}{M}}\text{cm / sec}$$

a. If the units of pressure are expressed in dynes cm^{-2} and volume in cm^3 the units of 'C'will be cms^{-1}. If the units of pressure are Newton $meter^{-2}$ and volume in dm^3, the units of 'C' will be meter sec^{-1}.

b. If the value of R used is 8.314×10^7 erg mol^{-1} K^{-1} and 'M' in grams mol^{-1} the units of 'C' willbe in cm s^{-1}. If the value of R used is 8.314 joule mol^{-1} K^{-1}, the units of 'C' will be in ms^{-1} and M is in kg mol^{-1}.

c. If the units of pressure are dyne cm^{-2}, and the units of'd' are in gm cm^{-3}, the units of 'C' will be in cm sec^{-1}. If the units of pressure are in newton $meter^{-2}$ and the units of'd' are kg $meter^{-3}$, the units of 'C' will be meter sec^{-1}.

Ratio of molecular velocities

The ratio of three types of molecular velocities of a gas is given as,

i. $C_p : \bar{C} : C = 1 : 1.128 : 1.224.$

$C = 1.128 \times \bar{C}$ and $C = 1.224 \times C_p$.

Or

ii. C:$\bar{C}$: C_p = 1 : 0.9213 : 0.8166.

$\bar{C}$ = 0.9213 × C and C_p = 0.8166 × C.

iii. For two gases at different temperatures.the ratio of their velocities is

$$\frac{C_1}{C_2} = \sqrt{\frac{T_1 M_2}{T_2 M_1}}$$

iv. If two gases have same velocity then $\frac{T_1}{M_1} = \frac{T_2}{M_2}$

v. As temperature increases all three types of velocities increase.

vi. At given temperature the value of R.M.S> Average velocity> Most probable velocity.

vii. At given temperature the fraction of molecules having R.M.S< Average velocity <Most probable velocity.

viii. At given temperature the value of R.M.S, Average velocity& Most probable velocity is independent of pressure.