MATHEMATICS PAPER - 1B

COORDINATE GEOMETRY (2D &3D) AND CALCULUS

TIME: 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A, B and C.

SECTION-A

Very Short Answer Type Questions. **10X2 =20**

1. Find the value of k, if the straight lines $6x-10y+3=0$ and $kx-5y+8=0$ are parallel.

2. Find the condition for the points $(a,0),(h,k)$ and $(0,b)$ where $ab\neq 0$ to be collinear

3. Find the fourth vertex of the parallelogram whose consecutive vertices are $(2,4,-1),(3,6,-1)$ and $(4,5,1)$

4. Find the angle between the planes $2x-y+z=6$ and $x+y+2z=7$

5. Compute $\lim_{x\to 0}\left(\frac{3^x-1}{\sqrt{1+x}-1}\right)$

6. Computute $\lim_{x\to\infty}\frac{8|x|+3x}{3|x|-2x}$.

7. If $y=\log(\sin(\log x))$, find $\frac{dy}{dx}$

8. If the increase in the side of a square is 4% find the percentage of change in the area of the square.

9. Find the value of 'C' in the Rolle's theorem for the function $f(x) = x^2 + 4$ on $[-3, 3]$

10. $Find\ \frac{dy}{dx}, if\ y = \left(Cot^{-1}x^3\right)^2$

SECTION-B

Short Answer Type Questions **5X4 =20**

Note: Answer any FIVE questions. Each question carries 4 marks.

11. The ends of the hypotenuse of a right angled triangle are $(0,6)$ and $(6,0)$. Find the equation of locus of its third vertex.

12. When the axes are rotated through an angle 45^0, the transformed equation of curve is $17x^2 - 16xy + 17y^2 = 225$. Find the original equation of the curve.

13. If the straight line $ax + by + c = 0, bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent, then prove that $a^3 + b^3 + c^3 = 3abc$

14. Find derivative of the function $\sin 2x$ form the first principles w.r.to x.

15. Show that the length of the subnormal at any point on the curve $xy = a^2$ varies as the cube of the ordinate of the point. $xy = a^2$

16. A point P is moving on the curve $y = 2x^2$. The x co-ordinate of P is increasing at the rate of 4 units per second. Find the rate at which the y co-ordinate is increasing when the point
is at $(2,8)$

17. Compute $\lim_{x \to 0}\left(\frac{\cos ax - \cos bx}{x^2}\right)$

SECTION C

Long answer type questions. **5X7 =35M**

Note: Answer any Five of the following. Each question carries 7 marks.

18. Find the circumcentre of the triangle with the vertices $(-2,3),(2,-1)$ and $(4,0)$

19. Show that the lines joining the origin to the points of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$ are mutually perpendicular.

20. If the equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of distinct (ie., intersecting) lines, then the combined equation of the pair of bisectors of the angles between these lines is $h\left(x^2 - y^2\right) = (a-b)xy$. $ax^2 + 2hxy + by^2 =$

21. Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$.

22. Find the derivative of $(\sin x)^{\log x} + x^{\sin x}$ with respective x.

23. Show that the curves $6x^2 - 5x + 2y = 0$ and $4x^2 + 8y^2 = 3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

24. Show that when the curved surface of right circular cylinder inscribed in a sphere of radius 'r' is maximum, then the height of the cylinder is $\sqrt{2}r$.

SOLUTIONS

Section A - VSAQ'S

1. Find the value of k, if the straight lines
   $6x - 10y + 3 = 0$ and $k x - 5y + 8 = 0$ are parallel.

Sol. Given lines are $6x - 10y + 3 = 0$ and

$k x - 5y + 8 = 0$

lines are parallel $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow -30 = -10k \quad \Rightarrow \quad k = 3$

2. Find the condition for the points (a, 0), (h, k) and (0,b)where $ab \neq 0$ to be collinear.

Sol. A(a, 0), B(h, k), C(0, b) are collinear.

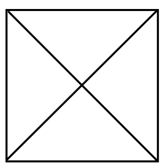
$\Rightarrow$ Slope of AB = Slope of AC

$$\frac{k-0}{h-a} = \frac{-b}{a} \Rightarrow ak = -bh + ab$$

$$bh + ak = ab \Rightarrow \frac{h}{a} + \frac{k}{b} = 1$$

3. Find the fourth vertex of the parallelogram whose consecutive vertices are (2, 4, –1), (3, 6, –1) are (4, 5, 1).

Sol.



ABCD is a parallelogram

where

A = (2, 4, –1), B = (3, 6, –1), C = (4, 5, 1)

Suppose D(x, y, z) is the fourth vertex

A B C D is a parallelogram

Midpoint of AC = Midpoint of BD

$$\left(\frac{2+4}{2},\frac{4+5}{7},\frac{-1+1}{2}\right)=\left(\frac{3+x}{2},\frac{6+y}{2},\frac{-1+z}{2}\right)$$

$$\frac{3+x}{2}=\frac{6}{2}\Rightarrow x=3$$

$$\frac{6+y}{2}=\frac{9}{2}\Rightarrow y=3$$

$$\frac{z-1}{2}=\frac{0}{2}\Rightarrow z=1$$

$\therefore$ Coordinates of the fourth vertex are :

D (3, 3, 1)

4. Find the angle between the planes 2x -y + z =6 and x + 2y + 2z = 7.

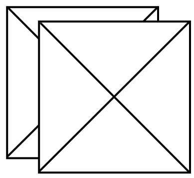
Sol. Equation of the plane are 2x -y + z =6 and x + 2y + 2z = 7.

Let θ be the angle between the planes, then

$$\cos\theta=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$=\frac{|2.1-1.2+1.2|}{\sqrt{4+1+1}\sqrt{1+1+4}}=\frac{2}{6}=\frac{1}{3}$$

$$\theta=\cos^{-1}\frac{1}{3}$$



5. $\underset{x\to 0}{Lt}\left[\frac{3^x-1}{\sqrt{1+x}-1}\right]$

$$\underset{x\to 0}{Lt}\frac{3^x-1}{\sqrt{1+x}-1}$$

Sol: $= \underset{x\to 0}{Lt}\frac{3^x-1}{\sqrt{1+x}-1}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$ (*rationalise* Dr.) $= \underset{x\to 0}{Lt}\frac{(3^x-1)(\sqrt{1+x}+1)}{1+x-1}$

$= \underset{x\to 0}{Lt}\frac{3^x-1}{x}.\underset{x\to 0}{Lt}\left(\sqrt{1+x}+1\right)$

$= (\log 3)\left(\sqrt{1+0}+1\right) = 2.\log 3$

6. $\underset{x\to 0}{Lt}\frac{8|x|+3x}{3|x|-2x}$

Sol : *as* $x\to\infty\Rightarrow|x|=x$ ($\because$ *here* x is positive)

$$\underset{x\to\infty}{Lt}\frac{8|x|+3x}{3|x|-2x}=\underset{x\to\infty}{Lt}\frac{8x+3x}{3x-2x}=\underset{x\to\infty}{Lt}\frac{11x}{x}=11$$

7. If $y = \log(\sin(\log x))$, find $\frac{dy}{dx}$.

$y = \log(\sin(\log x))$

$\frac{dy}{dx} = \frac{d}{dx}\log(\sin(\log x)) == \frac{1}{(\sin(\log x))}\frac{d}{dx}(\sin(\log x))$

$= \frac{1}{(\sin(\log x))}(\cos(\log x))\frac{d}{dx}\log x$

$= \frac{1}{(\sin(\log x))}(\cos(\log x))\frac{1}{x}$

8. Let x be the side and A be the area of the Square .

percentage error in x is $\frac{\delta x}{x}\times 100 = 4$

Area A= x^2

Applying logs on both sides Log A = 2 logx

Taking differentials on both sides

$\frac{1}{A}\delta A = 2.\frac{1}{x}\delta x \Rightarrow \frac{\delta A}{A}\times 100 = 2.\frac{\delta x}{x}\times 100$

= 2 x 4 =8.

Therefore percentage error in A is 8%

9. Let f(x) = $x^2 + 4$.

f is continuous on [–3, 3]

since f(–3) = f(3) and

f is differentiable on [–3, 3]

∴ By Rolle's theorem $\exists c \in (-1,1)$ Such that f′(c) = 0

f′(x) = 2x = 0

∴ = f′(c) = 0

2c = 0 ⇒ c = 0

The point c = 0 ∈ (–3, 3)

10. If $y = (\cot^{-1} x^3)^2$, find $\frac{dy}{dx}$.

Sol. $u = \cot^{-1} x^3, u = x^3, y = u^2$

$$\frac{du}{dv} = -\frac{1}{1+u^2}, \frac{du}{dx} = 3x^2$$

$$\frac{dy}{dx} = 2u = 2\cot^{-1}(x^3) = -\frac{1}{1+x^6}$$

$$\frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

$$= 2\cot^{-1}(x^3)\left(-\frac{1}{1+x^6}\right)3x^2$$

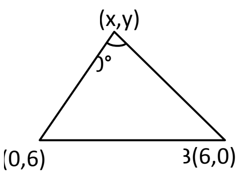
$$= -\frac{6x^2}{1+x^6}\cot^{-1}(x^3)$$

SECTION B- SAQ'S

11. The ends of the hypotenuse of a right angled triangle are (0, 6) and (6, 0). Find the equation of locus of its third vertex.

Ans. Given points A(2, 3), B(–1, 5).

Let P(x, y) be any point in the locus.



Given condition is :∠APB = 90°

$\Rightarrow$ (slope of $\overline{AP}$) (slope of $\overline{BP}$) = –1

$$\Rightarrow \frac{y-6}{x-0} \cdot \frac{y-0}{x-6} = -1$$

$$(y)(y-6) + (x)(x-6) = 0$$

$$x^2 + y^2 - 6x - 6y = 0$$

$\therefore$ Locus of P is $x^2 + y^2 - 6x - 6y = 0$

12. When the axes are rotated through an angle $45°$, the transformed equation of a curve is $17x^2 - 16xy + 17y^2 = 225$. Find the original equation of the curve.

Sol. Angle of rotation is $\theta = 45^0$. Let (X,Y) be the new coordinates of (x, y)

$X = x\cos\theta + y\sin\theta = x\cos 45 + y\sin 45 \quad = \dfrac{x+y}{\sqrt{2}}$

$Y = -x\sin\theta + y\cos\theta = -x\sin 45 + y\cos 45 \quad = \dfrac{-x+y}{\sqrt{2}}$

The original equation of $17X^2 - 16XY + 17Y^2 = 225$ is

$\Rightarrow 17\left(\dfrac{x+y}{\sqrt{2}}\right)^2 - 16\left(\dfrac{x+y}{\sqrt{2}}\right)\left(\dfrac{-x+y}{\sqrt{2}}\right) + 17\left(\dfrac{-x+y}{\sqrt{2}}\right)^2 = 225$

$\Rightarrow 17\dfrac{\left(x^2+y^2+2xy\right)}{2} - 16\dfrac{\left(y^2-x^2\right)}{2} + 17\dfrac{\left(x^2+y^2-2xy\right)}{2} = 225$

$\Rightarrow 17x^2 + 17y^2 + 34xy - 16y^2 + 16x^2 + 17x^2 + 17y^2 - 34xy = 450$

$\Rightarrow 50x^2 + 18y^2 = 450 \Rightarrow \; 25x^2 + 9y^2 = 225$ is the original equation

13. If the straight lines $ax+by+c=0$, $bx+cy+a=0$ and $cx+ay+b=0$ are concurrent, then prove that $a^3+b^3+c^3=3abc$.

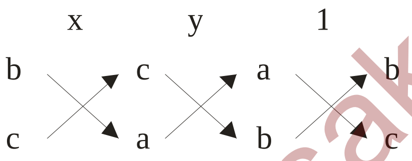
Sol: The equations of the given lines are

$ax+by+c=0$ ---(1)

$bx+cy+a=0$ ---(2)

$cx+ay+b=0$ ---(3)

Solving (1) and (2) points of intersection is got by



$\dfrac{x}{ab-c^2} = \dfrac{y}{bc-a^2} = \dfrac{1}{ca-b^2}$

Point of intersection is $\left(\dfrac{ab-c^2}{ca-b^2}, \dfrac{bc-a^2}{ca-b^2}\right)$

$c\left(\dfrac{ab-c^2}{ca-b^2}\right) + a\left(\dfrac{bc-a^2}{ca-b^2}\right) + b = 0$

$c\left(ab-c^2\right) + a\left(bc-a^2\right) + b\left(ca-b^2\right) = 0$

$abc - c^3 + abc - a^3 + abc - b^3 = 0$

$\therefore a^3+b^3+c^3 = 3abc$.

14. $f^1(x) = \underset{h\to 0}{Lt}\frac{f(x+h)-f(x)}{h} = \underset{h\to 0}{Lt}\frac{SIN\,2(x+h)-\cos 2x}{h}$

$= \underset{h\to 0}{Lt}\frac{2\cos\frac{2x+2h+2x}{2}.\sin\frac{2x+2h-2x}{2}}{h}$

$= \underset{h\to 0}{Lt}\frac{2\cos\left(2x+\frac{2h}{2}\right).\sin\left(\frac{2h}{2}\right)}{h}$

$\underset{h\to 0}{Lt}\,2\cos(2x+h)\underset{h\to 0}{Lt}\frac{\sin(h)}{h}$

$= 2\cos 2x.1 = 2\cos 2x$

15. Show that the length of sub-normal at any point on the curve xy = a^2 varies as the cube of the ordinate of the point.

Sol: Equation of the curve is xy = a^2.

$\Rightarrow\ y = \frac{a^2}{x} \Rightarrow \quad \frac{dy}{dx} = \frac{-a^2}{x^2} = m$

Let P(x,y) be a point on the curve.

Length of the sub-normal = $|\,\boldsymbol{y_1.m}\,|$ $= \left|y\left(\frac{(-a)^2}{x^2}\right)\right| = \left|-a^2 y\frac{y^2}{a^4}\right| = \left|\frac{y^3}{a^2}\right|\left(\because x = \frac{a^2}{y}\right)$

$\therefore l.s.t\ \alpha\ y^3$ i.e. cube of the ordinate.

16. equation of the curve $y = 2x^2$

Diff .w.r.t.t, $\frac{dy}{dt} = 4x.\frac{dx}{dt}$

Given $x = 2$ and $\frac{dx}{dt} = 4.$

$\frac{dy}{dt} = 4(2).4 = 32$

y co-ordinate is increasing at the rate of 32 units/sec.

17. $\underset{x\to 0}{Lt}\, f(x) = \underset{x\to 0}{Lt}\frac{\cos ax - \cos bx}{x^2}$

$= \underset{x\to 0}{Lt}\frac{2\sin\frac{(a+b)x}{x}\sin\frac{(b-a)x}{2}}{x^2}$

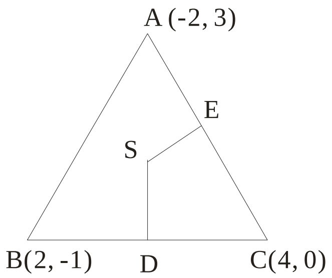
$$= 2 \underset{x\to 0}{Lt} \frac{\sin(a+b)\frac{x}{2}}{x} \underset{x\to 0}{Lt} \frac{\sin(b-a)\frac{x}{2}}{x}$$

$$= \frac{2(b+a)}{2}\frac{(b-a)}{2} = \frac{b^2-a^2}{2}$$

18. $A(-2,3)$, $B(2,-1)$, $C(4,0)$ are the vertices of ΔABC.

Let S be the circumcentre of the ΔABC.

Let D be the midpoint of BC



$$\Rightarrow D = \left(\frac{2+4}{2}, \frac{-1+0}{2}\right) = \left(3, \frac{-1}{2}\right)$$

$$\Rightarrow \text{Slope of BC} = \frac{-1-0}{2-4} = \frac{-1}{-2} = \frac{1}{2}$$

$\Rightarrow$ SD is perpendicular to BC

Slope of SD $= -\frac{1}{m} = -2$

Equation of SD is $y + \frac{1}{2} = -2(x-3)$

$\Rightarrow 2y+1 = -4(x-3) = -4x+12$

$\Rightarrow 4x - 2y - 11 = 0$ ---(1)

Let E be the midpoint of AC

Co-ordinates of E are $\left(\frac{-2+4}{2}, \frac{3+0}{2}\right) = \left(1, \frac{3}{2}\right)$

Slope of $AC = \frac{3-0}{-2-4} = -\frac{3}{6} = -\frac{1}{2}$

$\Rightarrow$ SE is perpendicular to AC

$\Rightarrow$ Slope of SE $= -\frac{1}{m} = 2$

Equation of SE is $y - \frac{3}{2} = 2(x-1)$

$\Rightarrow 2y-3=4(x-1)=4x-4$

$\Rightarrow 4x-2y-1=0$ ---(2)

$\Rightarrow 4x+2y-11=0$ ---(1)

Adding (1), (2) $\Rightarrow 8x-12=0$

$8x=12$

$\Rightarrow x=\frac{12}{8}=\frac{3}{2}$

Substitute this x in (1),

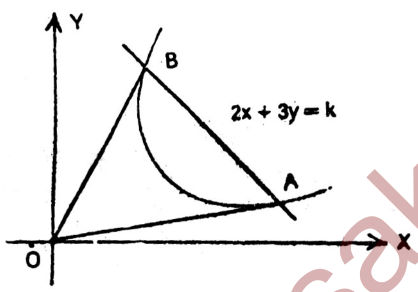
$2y=11-4x=11-4.\frac{3}{2}=11-6=5$ $\Rightarrow y=\frac{5}{2}$

$\therefore$ Co-ordinates of S are $\left(\frac{3}{2},\frac{5}{2}\right)$

SECTION C - LAQ'S

19. Show that the lines joining the origin to the points of intersection of the curve $x^2-xy+y^2+3x+3y-2=0$ and the straight line $x-y-\sqrt{2}=0$ are mutually perpendicular.

Sol.



Le t A,B the the points of intersection of the line and the curve.

Equation of the curve is $x^2-xy+y^2+3x+3y-2=0$ …….(1)

Equation of the line AB is $x-y-\sqrt{2}=0$

$\Rightarrow$ $x-y=\sqrt{2} \Rightarrow \frac{x-y}{\sqrt{2}}=1$ ….(2)

Homogenising, (1) with the help of (2) combined equation of OA, OB is

$x^2-xy+y^2+3x.1+3y.1-2.1^2=0$

$\Rightarrow x^2-xy+y^2+3(x+y)\frac{x-y}{\sqrt{2}}-2\frac{(x-y)^2}{2}=0$

$\Rightarrow x^2-xy+y^2+\frac{3}{\sqrt{2}}(x^2-y^2)-(x^2-2xy+y^2)=0$

$$\Rightarrow x^2 - xy + y^2 + \frac{3}{\sqrt{2}}x^2 - \frac{3}{\sqrt{2}}y^2 - x^2 + 2xy - y^2 = 0$$

$$\Rightarrow \frac{3}{\sqrt{2}}x^2 + xy - \frac{3}{\sqrt{2}}y^2 = 0$$

$\Rightarrow$ coefficient of x^2 +coefficient of y^2 $= a + b = \frac{3}{\sqrt{2}} - \frac{3}{\sqrt{2}} = 0$

$\therefore$ OA, OB are perpendicular.

20. Let $ax^2 + 2hxy + by^2 = 0$ represent the lines $l_1x + m_1y = 0$ -- (1) and $l_2x + m_2y = 0$ -- (2).

Then $l_1l_2 = a$, $l_1m_2 + l_2m_1 = 2h$, $m_1m_2 = b$.

The equations of bisectors of angles between (1) and (2) are $\frac{l_1x + m_1y}{\sqrt{l_1^2 + m_1^2}} = \pm\frac{l_2x + m_2y}{\sqrt{l_2^2 + m_2^2}}$

$$\Rightarrow \frac{l_1x + m_1y}{\sqrt{l_1^2 + m_1^2}} - \frac{l_2x + m_2y}{\sqrt{l_2^2 + m_2^2}} = 0 \text{ and}$$

$$\frac{l_1x + m_1y}{\sqrt{l_1^2 + m_1^2}} + \frac{l_2x + m_2y}{\sqrt{l_2^2 + m_2^2}} = 0$$

The combined equation of the bisectors is

$$\left(\frac{l_1x + m_1y}{\sqrt{l_1^2 + m_1^2}} - \frac{l_2x + m_2y}{\sqrt{l_2^2 + m_2^2}}\right)\left(\frac{\ell_1x + m_1y}{\sqrt{\ell_1^2 + m_1^2}} + \frac{\ell_2x + m_2y}{\sqrt{\ell_2^2 + m_2^2}}\right) = 0$$

$$\Rightarrow \left(\frac{l_1x + m_1y}{\sqrt{l_1^2 + m_1^2}}\right)^2 - \left(\frac{l_2x + m_2y}{\sqrt{l_2^2 + m_2^2}}\right)^2 = 0$$

$$\Rightarrow \left(l_2^2 + m_2^2\right)(l_1x + m_1y)^2 - \left(l_1^2 + m_1^2\right)(l_2x + m_2y)^2 = 0$$

$$\Rightarrow x^2\left[l_1^2\left(l_2^2 + m_2^2\right) - l_2^2\left(l_1^2 + m_1^2\right)\right] y^2\left[m_2^2\left(l_1^2 + m_1^2\right) - m_1^2\left(l_2^2 + m_2^2\right)\right] \quad -2xy\left[l_2m_2\left(l_1^2 + m_1^2\right) - l_1m_1\left(l_2^2 + m_2^2\right)\right] = 0$$

$$\Rightarrow x^2\left(l_1^2l_2^2 + l_1^2m_2^2 - l_1^2l_2^2 - l_2^2m_1^2\right) - y^2\left(l_1^2m_2^2 + m_1^2m_2^2 - m_1^2l_2^2 - m_1^2m_2^2\right) - 2xy\left(l_2m_2l_1^2 + l_2m_2m_1^2 - l_1m_1l_2^2 - l_1m_1m_2^2\right) = 0$$

$$\Rightarrow x^2\left(l_1^2m_2^2 - l_2^2m_1^2\right) - y^2\left(l_1^2m_2^2 - l_2^2m_2^2\right) = 2xy\left[l_1l_2(l_1m_2 - l_2m_1) - m_1m_2(l_1m_2 - l_2m_1)\right]$$

$$\Rightarrow (x^2 - y^2)\left(l_1^2m_2^2 - l_2^2m_1^2\right) = 2xy\left(l_1l_2 - m_1m_2\right)\left(l_1m_2 - l_2m_1\right) \Rightarrow (x^2 - y^2)(l_1m_2 + l_2m_1) = 2xy(l_1l_2 - m_1m_2)$$

$$\Rightarrow 2h(x^2 - y^2) = 2xy(a - b)$$

$$\therefore h(x^2 - y^2) = (a - b)xy \qquad \text{OR} \quad \frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

21. Given $3l+m+5n=0$

$6mn\ -2nl+5lm=0$

From (1), $m=-(3l+5n)$

Substituting in (2)

$\Rightarrow -6n(3l+5n)-2nl-5l(3l+5n)=0$

$\Rightarrow -18\ln-30n^2-2nl-15l^2-25\ln=0$

$\Rightarrow -15l^2-45\ln-30n^2=0$

$\Rightarrow l^2+3\ln+2n^2=0$

$\Rightarrow (l+2n)(l+n)=0$

$\Rightarrow l+2n=0 \text{ or } l+n=0$

Case (i) :

$l_1+n_1=0 \Rightarrow n_1=-l_1; \Rightarrow n_1=-l_1; \Rightarrow \frac{l_1}{1}=\frac{n_1}{-1}$

But $m_1=-(3l_1+5n_1)=-(-3n_1+5n_1)=-2n_1$

$\therefore \frac{m_1}{+2}=\frac{n_1}{-1}$

$\therefore \frac{l_1}{1}=\frac{m_1}{2}=\frac{n_1}{-1}$

D.rs of the first line l_1 *are* $(1,2,-1)$

Case (ii) : $l_2+2n_2=0$

$\Rightarrow l_2=-2n_2 \Rightarrow \frac{l_2}{-2}=\frac{n_2}{1}$

$\Rightarrow m_2=-(3l_2+5n_2)=-(-6n_2+5n_2)=n_2$

$\frac{m_2}{1}=\frac{n_2}{1}$

$\therefore \frac{l_2}{-2}=\frac{m_2}{1}=\frac{n_2}{1}$

D.rs of the second line l_2 *are* $(-2,1,1)$

Suppose 'θ' is the angle between the lines l_1 *and* l_2

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\frac{|1(-2)+2.1+(-1).1|}{\sqrt{1+4+1}\sqrt{4+1+1}}$$

$= \frac{1}{6} \Rightarrow \theta = \cos^{-1}(1/6)$

22. Let $u = (\sin x)^{\log x}, v = x^{\sin x}$ so that $y = u + v$,

$u = (\sin x)^{\log x}$ applying logs,

$\Rightarrow \log u = \log\left\{(\sin x)^{\log x}\right\} = \log x.\log(\sin x)$

Differentiating w. r. to x

$\Rightarrow \frac{d}{dx}\log u = \frac{d}{dx}\log x.\log(\sin x)$

$\frac{1}{u}.\frac{dy_1}{dx} = \log x\frac{1}{\sin x}.\cos x + \log(\sin x)\frac{1}{x}$

$\frac{dy_1}{dx} = u\left[\cot x.\log x + \frac{\log(\sin x)}{x}\right]$

$= (\sin x)^{\log x}\left[\cot x.\log x. + \frac{\log \sin x}{x}\right]$

$v = x^{\sin x}$

$\log v = (\log x)^{\sin x} = \sin x.\log x$

$\frac{1}{v}.\frac{dv}{dx} = \sin x.\frac{1}{x} + (\log x.\cos x)$

$\frac{dv}{dx} = v\left[\frac{\sin x}{x}.\cos x.\log x\right] = x^{\sin x}\left[\frac{\sin x}{x} + \cos x.\log x\right]$

$\sin ce\ \ y = u+v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= (\sin x)^{\log x}\left(\cot x.\log x. + \frac{\log \sin x}{x}\right) + x^{\sin x}\left(\frac{\sin x}{x} + \cos x.\log x\right)$

23. Show that the curves $\mathbf{6x^2 - 5x + 2y = 0}$ and $\mathbf{4x^2 + 8y^2 = 3}$ touch each other at $\left(\mathbf{\frac{1}{2}, \frac{1}{2}}\right)$.

Sol: Equation of the first curve is $6x^2 - 5x + 2y = 0$

$\Rightarrow 2y = 5x - 6x^2 \Rightarrow \quad 2.\frac{dy}{dx} = 5 - 12x \quad \Rightarrow \frac{dy}{dx} = \frac{5-12x}{2}$

$m_1 = \left(\frac{dy}{dx}\right)_{atP\left(\frac{1}{2},\frac{1}{2}\right)} = \frac{5-12.\frac{1}{2}}{2} = \frac{5-6}{2} = -\frac{1}{2}$

Equation of the second curve is $4x^2 + 8y^2 = 3$

$$\Rightarrow 8x + 16y.\frac{dy}{dx} = 0 \Rightarrow 16y.\frac{dy}{dx} = -8x$$

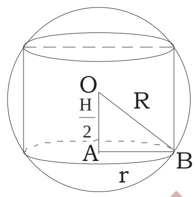
$$\Rightarrow \frac{dy}{dx} = \frac{-8x}{16y} = -\frac{x}{2y}$$

$$m_2 = \left(\frac{dy}{dx}\right)_{atP\left(\frac{1}{2},\frac{1}{2}\right)} = \frac{-\frac{1}{2}}{2\left(\frac{1}{2}\right)} = -\frac{1}{2}$$

$$\therefore m_1 = m_2$$

The given curves touch each other at $P\left(\frac{1}{2},\frac{1}{2}\right)$

24. Let r be the radius and h be the height of the cylinder.



From ΔOAB, $OA^2 + AB^2 = OB^2$

$$\Rightarrow r^2 + \frac{h^2}{4} = R^2;\ r^2 = R^2 - \frac{h^2}{4}$$

Curved surface area $= 2\pi rh$

$$= 2\pi\sqrt{R^2 - \frac{h^2}{4}}.h$$

$$= \pi h\sqrt{4R^2 - h^2}$$

Let $f(h) = \pi h\sqrt{4R^2 - h^2}$

$$f'(h) = \pi\left[h.\frac{1}{2\sqrt{4R^2 - h^2}}(-2h) + \sqrt{4R^2 - h2.1}\right]$$

$$= \pi.\frac{-h^2 + 4R^2 - h^2}{\sqrt{4R^2 - h^2}} = \frac{2\pi\left(2R^2 - h^2\right)}{\sqrt{4R^2 - h^2}}$$

For max or min $f'(h) = 0$

$$\Rightarrow \frac{2\pi\left(2R^2 - h^2\right)}{\sqrt{4R^2 - h^2}} = 0$$

$$\therefore 2R^2 - h^2 = 0$$

$\Rightarrow h^2 = 2R^2 \Rightarrow h = \sqrt{2}R$

$\Rightarrow \sqrt{4R^2 - h^2}\,(-2h) + \left(2R^2 - h^2\right)$

$$f''\left(\text{when } h = \sqrt{2}R\right) = 2\pi \frac{\frac{d}{dh}\sqrt{4R^2 - h^2}}{4R^2 - h^2}$$

$$= -\frac{4\pi h + 0}{\sqrt{4r^2 - h^2}} < 0$$

$f(h)$ is greatest when $h = \sqrt{2}R$

i.e., Height of the cylinder $= \sqrt{2}R$