ENGINEERING MECHANICS

VECTORS

VECTOR NOTATIONS

So far we have determined the resultant of concurrent force system by using analytical method i.e., we have learned how to resolve a vector into its rectangular components.

For a general development of the theoretical aspects of mechanics, however, a more rigorous treatment is possible by using vector analysis.

A vector may be denoted by drawing a short arrow above the letter used to represents it.

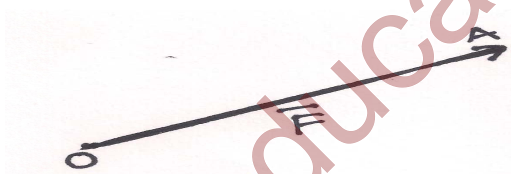


Fig. 1 Representation of vector

Vector OA=F, and will be represented as $\bar{F}$

UNIT VECTOR

A unit vector is defined as a vector of unit magnitude in a specified direction. We shall denote them by placing a ***circumflex*** (^) over them.

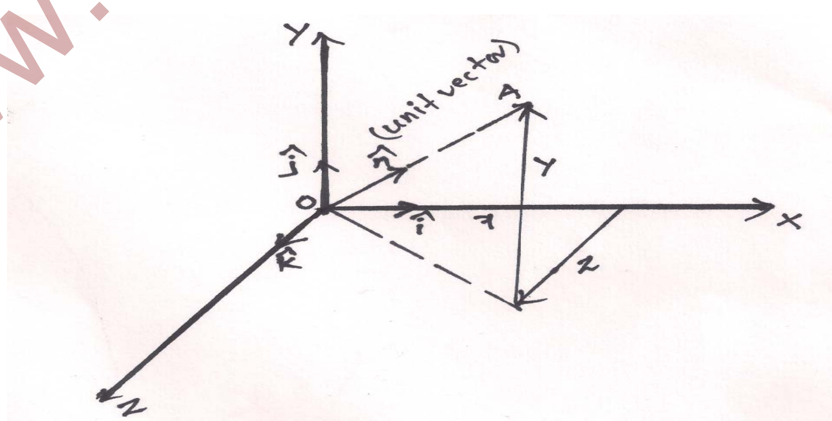


Fig.2. Representation of unit vector

From above figure the unit vector $\hat{n}$ in the direction from O to A is the vector $\bar{d}$ divided by the magnitude 'd' of the distance from O to A.

Multiplying a unit vector by a scalar denotes a vector having the direction of the unit vector and magnitude equal to that of scalar.

$\hat{\imath}$, $\hat{\jmath}$, *and* $\hat{k}$ represents vector of unit length directed along the positive senses of X, Y, Z coordinate axes respectively as sown in above figure.

By using this, any vector can be written as

$$\bar{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$

This is known as standard Cartesian form of representing a vector

$$\hat{n} = \frac{\bar{d}}{d} = \frac{x\hat{\imath}+y\hat{\jmath}+z\hat{k}}{d} = \frac{1}{d}(x\hat{\imath} + y\hat{\jmath} + z\hat{k}), \text{ where } \boldsymbol{d = \sqrt{(x^2 + y^2 + z^2}}$$

Therefore, any vector $\bar{F}$ can be written as

$$\bar{F} = \text{F}.\,\hat{n}$$

$$\bar{F} = \text{F}.\,\frac{x\hat{\imath}+y\hat{\jmath}+z\hat{k}}{d} = \frac{F}{d}(x\hat{\imath} + y\hat{\jmath} + z\hat{k})$$

$$\bar{F} = \boldsymbol{F_m}(\,x\hat{\imath} + y\hat{\jmath} + z\hat{k})$$

Where, $\boldsymbol{F_m}$ = Force Multiplier **(Detailed note given in topic:2)**

VECTOR ALGEBRA

There are two types of vector multiplication

1. Dot product or scalar product

   It is to find the component of vector along any special direction

2. Cross product or Vector product

   It is to find the moment of vector about any centre.

Note*:*

1. In our past studies like intermediate we have studied about vectors, hence I am not going to discuss much about that, we will concentrate on subject of Engineering Mechanics.

2. Now we will try to solve the Exercise problem in simplified manner from F L Singer (problem No: 2-8.9, Page No: 47). While solving this problem we are going to take pounds (lb) as Newton (N) and inches (in) as meters (m).

Let us try to solve some problems on component of forces in space by using the vector algebra

Q.1 In figure a boom AC is supported by a ball and socket joint at C and by the cables BE and AD. If the force multiplier of a force F acting from B to E is $F_m = 10N/m$, find the component of F that is perpendicular to the plane DAC

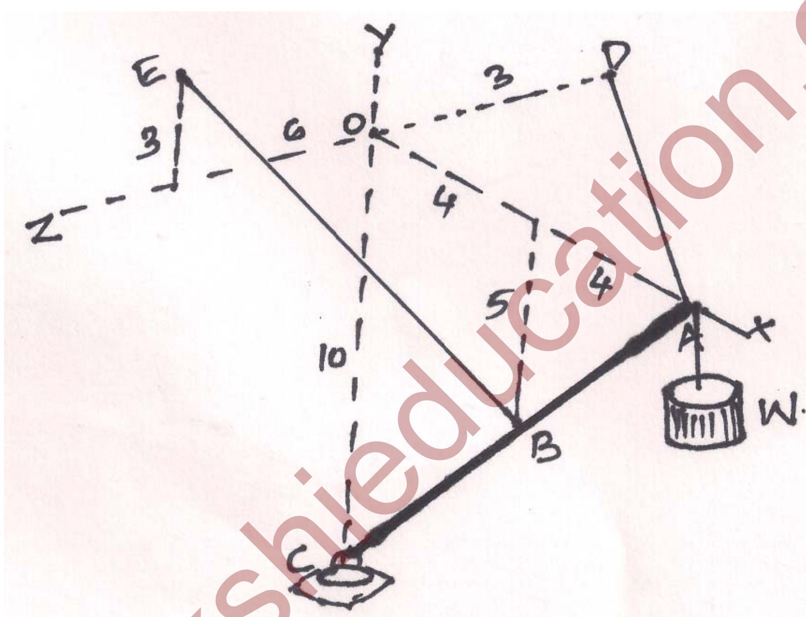


Fig.3 Schematic diagram of problem

Let us write the coordinate points in the tabular format:

Supporting Point	X-Direction ($\hat{\imath}$)	Y- Direction ($\hat{\jmath}$)	Z- Direction ($\hat{k}$)
A	8	0	0
B	4	-5	0
C	0	-10	0
D	0	0	-3
E	0	3	6

above coordinate points can also be written as A(8,0,0), B(4,-5,0), C(0,-10,0), D(0,0,-3) and E(0,3,6)

Vector $\overline{AD}$=-8$\hat{\imath} + 0\hat{\jmath} - 3\hat{k}$

Vector $\overline{AC}$=-8$\hat{\imath} - 10\hat{\jmath} + 0\hat{k}$

Above vectors are made from coordinates of A&D and A&C

As given in the problem, the force $\overline{F}$ is acting from B to E

Vector $\overline{BE}$=-4$\hat{\imath} + 8\hat{\jmath} + 6\hat{k}$

$$\overline{F} = \boldsymbol{F_m}(x\hat{\imath} + y\hat{\jmath} + z\hat{k})$$

$$\overline{F} = 10(-4\hat{\imath} + 8\hat{\jmath} + 6\hat{k})$$

Consider Vector $\overline{N}$ perpendicular to plane DAC

therefore $\overline{N} = \overline{AD} \times \overline{AC}$ (we have to solve by using cross product)

$$\overline{N} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -8 & 0 & -3 \\ -8 & -10 & 0 \end{vmatrix}$$

$$\overline{N} = -30\hat{\imath} + 24\hat{\jmath} + 80\hat{k}$$

The unit vector along the $\overline{N}$; $\hat{n}_N = \frac{\overline{N}}{N} = \frac{\boldsymbol{-30\hat{\imath}+24\hat{\jmath}+80\hat{k}}}{\sqrt{(\boldsymbol{30^2+24^2+80^2}}}$

$$\hat{n}_N = \frac{\boldsymbol{-30\hat{\imath}+24\hat{\jmath}+80\hat{k}}}{\sqrt{\boldsymbol{7876}}}$$

Component of force $\overline{F}_N = \overline{F} \cdot \hat{n}_N$

$$\overline{F}_N = 10(-4\hat{\imath} + 8\hat{\jmath} + 6\hat{k}) \cdot \frac{\boldsymbol{-30\hat{\imath}+24\hat{\jmath}+80\hat{k}}}{\sqrt{\boldsymbol{7876}}} = \frac{10}{\sqrt{7876}} \cdot (-30 \times -4 + (24 \times 8) + (80 \times 6))$$

$$\overline{F}_N = \frac{10}{\sqrt{7876}} \cdot (120 + 192 + 480) = 89.242 \text{ N}$$

Component of force F that is perpendicular to DAC; $\overline{F}_N$= 89.242 N Ans.

***Note**: Now we will try to solve the Exercise problem in simplified manner from F L Singer (problem No: 2-7.4, Page No: 40). This problem will give you the detailed understanding of solving the problems with vector notations; this solution is little bigger because it covers many parameters to be found. While solving this problem we are going to take pounds (lb) as Newton (N) and inches (in) as meters (m).*

Q. 2. In the system shown in figure it is found that the force multiplier of force F acting from B to D is $F_m = 150N/m$, and that of force P is acting from A to E is $P_m = 100N/m$. Find the component of each force along AC. What angle does each force make with AC?

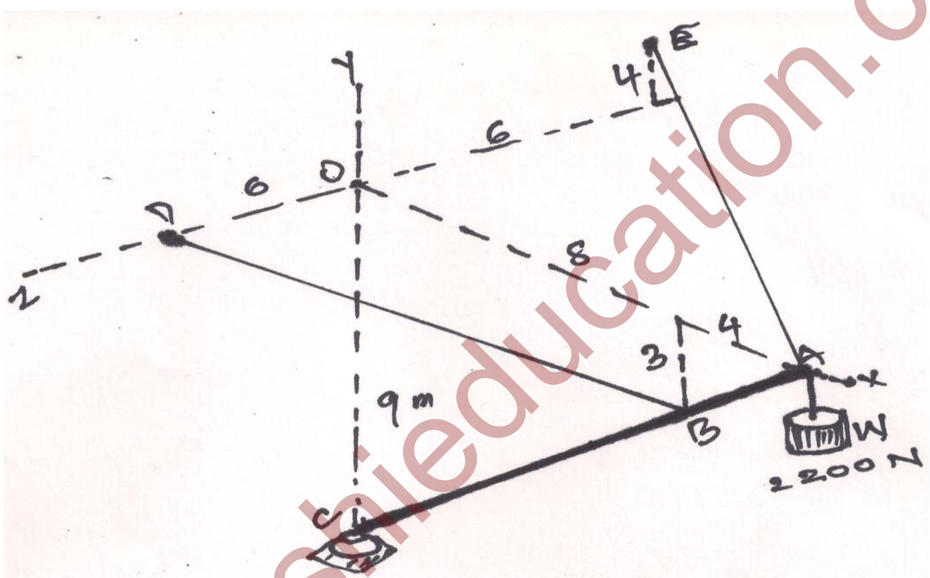


Fig.4 Schematic diagram of problem

Let us write the coordinate points in the tabular format:

Supporting Point	X-Direction ($\hat{\imath}$)	Y- Direction ($\hat{\jmath}$)	Z- Direction ($\hat{k}$)
A	12	0	0
B	8	-3	0
C	0	-9	0
D	0	0	6
E	0	4	-6

above coordinate points can also be written as A(12,0,0), B(8,-3,0), C(0,-9,0), D(0,0,6) and E(0,4,-6)

(i) As given in the problem, the force $\overline{F}$ is acting from B to D

Vector $\overline{BD}$=-$8\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

This vector is made from coordinates of B (8,-3, 0), & D (0, 0, 6)

Now the force: $\bar{F}$ = $\boldsymbol{F_m}(x\hat{\imath} + y\hat{\jmath} + z\hat{k})$

$\bar{F}$ =150 $(-8\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

Magnitude if force F; $\bar{F}$ =150 $\sqrt{(\mathbf{8^2 + 3^2 + 6^2}}$

$\bar{F}$=1566.046 N

(ii) Similarly we can solve for the force P is acting from A to E

Vector $\overline{AE}$=-$12\hat{\imath} + 4\hat{\jmath} - 6\hat{k}$

This vector is made from coordinates of A (12, 0, 0), & E (0, 4,-6)

Now the force: $\bar{P}$ = $\boldsymbol{P_m}(x\hat{\imath} + y\hat{\jmath} + z\hat{k})$

$\bar{P}$ =100 $(-12\hat{\imath} + 4\hat{\jmath} - 6\hat{k})$

Magnitude if force P; $\bar{P}$ =150 $\sqrt{(\mathbf{12^2 + 4^2 + 6^2}}$

$\bar{P}$=1400 N

(iii) Calculation of component of force F along AC

Vector $\overline{AC}$=-$12\hat{\imath} - 9\hat{\jmath} + 0\hat{k}$

This vector is made from coordinates of A(12,0,0), & C(0,-9,0)

Unit Vector along $\overline{AC}$: $\hat{n}_{AC}$=$\frac{\overline{AC}}{AC}$=$\frac{\mathbf{-12\hat{\imath}-9\hat{\jmath}+0\hat{k}}}{\sqrt{(\mathbf{12^2+9^2+0^2}}}$

$\hat{n}_{AC}$=$\frac{\mathbf{-12\hat{\imath}-9\hat{\jmath}+0\hat{k}}}{\mathbf{15}}$

Component of force $\bar{F}$ along $\overline{AC}$=F_{AC}=$\bar{F}.\hat{n}_{AC}$

$$F_{AC} = 150\,(-8\hat{\imath} + 3\hat{\jmath} + 6\hat{k})\ .\ \frac{-12\hat{\imath}-9\hat{\jmath}+0\hat{k}}{\mathbf{15}}$$

$$F_{AC} = \frac{150}{\mathbf{15}}.(12 \times 8 - (3 \times 9) + (0 \times 6))$$

Component of force $\bar{F}$ along $\overline{AC}$; F_{AC} = 690 N Ans.

(iv) Calculation of component of force P along AC

Component of force $\bar{P}$ along $\overline{AC}$=P_{AC}=$\bar{P}.\hat{n}_{AC}$

$$P_{AC} = 100\,(-12\hat{\imath} + 4\hat{\jmath} - 6\hat{k}).\ \frac{-12\hat{\imath}-9\hat{\jmath}+0\hat{k}}{\mathbf{15}}$$

$$P_{AC} = \frac{150}{\mathbf{15}}.(12 \times 12 - (4 \times 9) - (0 \times 6))$$

Component of force $\bar{P}$ along $\overline{AC}$, P_{AC} =720 N

Component of force $\bar{P}$ along $\overline{AC}$, P_{AC} =720 N Ans.

(v) Calculation of angle of force F making with AC

From definition of dot product

$$\bar{F}\ .\ \overline{AC} = \text{F. AC.} \cos\theta_F$$

Where, θ_F= Angle of forces F making with AC

$$\cos\theta_F = \frac{\bar{F}\ .\ \overline{AC}}{\text{F .AC}} = \frac{\bar{F}\ .\ \hat{n}_{AC}}{\text{F}} = \frac{690}{1566.046}$$

$$\theta_F = \cos^{-1}\frac{690}{1566.046} = 63.86°$$

Angle of forces F making with AC: θ_F=63.86° Ans.

(vi) Calculation of angle of force P making with AC

$$\bar{P}\ .\ \overline{AC}=\text{P. AC.}\cos\theta_P$$

Where θ_P= Angle of forces P making with AC

$$\cos\theta_P=\frac{\bar{P}\ .\ \overline{AC}}{\text{P .AC}}=\frac{\bar{P}\ .\ \hat{n}_{AC}}{\text{P}}=\frac{720}{1400}$$

$$\theta_P=\cos^{-1}\frac{720}{1400}$$

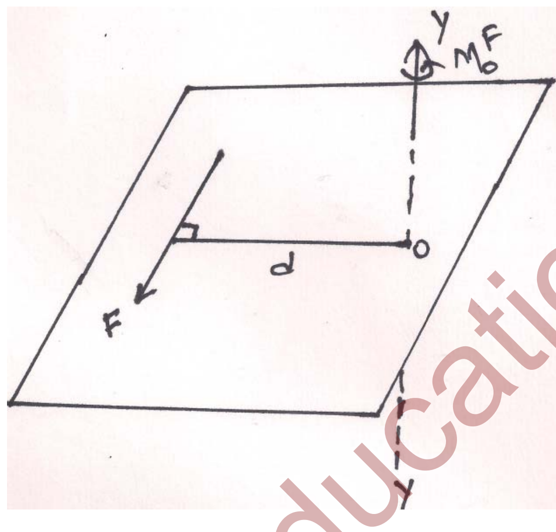
$$\theta_F=59.05°$$

Angle of forces P making with AC: $\boldsymbol{\theta_P}$=59.05° Ans.

MOMENT OF FORCE

Definition the moment of force (of any vector) about a point is defined as the product of the magnitude of the force by the perpendicular distance from the point to the action line of the force

For better understanding refer the below figure



Where d=moment arm

F=Magnitude of the moment of force

The moment of force about a centre 'O' can be expressed as:

$$\boldsymbol{M_O^F}\textbf{=F.d}$$

The moment of force about a point represents the tendency to rotate the moment arm about an axis which is perpendicular to the plane defined by the force and its moment arm.

PRINCIPLE OF MOMENTS: (Varignon's theorem)

It states that the moment of a force is equal to the moment sum of its components. This almost self-evident statement is known as Varignon's theorem

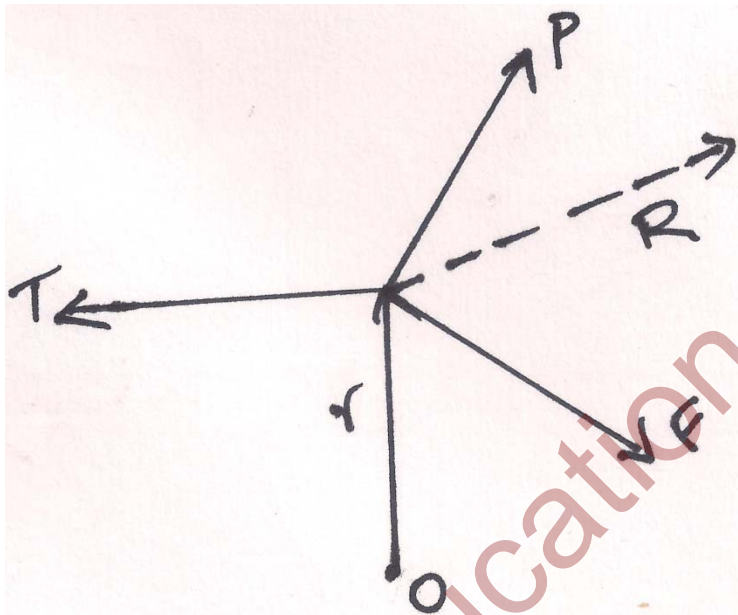


Fig.4 Presentation of principle of moments

- Let R be the resultant of the concurrent forces P, F and T
- The force system may be either coplanar or spatial, but it must be concurrent
- From above figure R=P+F+T about
- About any point 'O' as a moment centre, the moment of these force is

r x R=r x (P+F+T) = r x P + r x F + r x T

Resultant and components must be concurrent

A general symbolic statement of the theorem is: $\boldsymbol{M_O^R}=\sum \boldsymbol{M_0}=\sum(\boldsymbol{r} \times \boldsymbol{F})$

COUPLES

Definition A couple is a pair of equal, parallel, oppositely directed forces

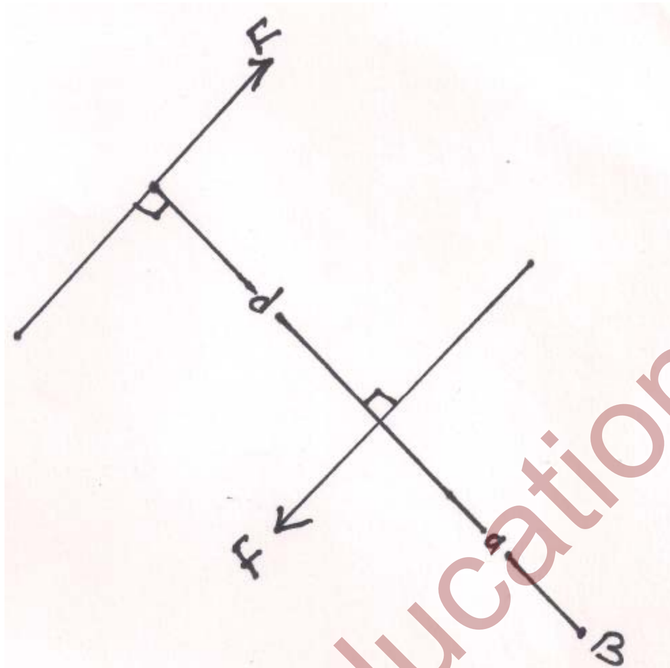


Fig.5 Presentation of couple

Where, d=moment arm couple of the forces, i.e. distance between the action lines of the force

We must know some of the facts about the couple:

- The vector sum of the two forces in the couple is ***zero***
- But moment sum of the two forces is in the couple ***not zero***
- The effect of the couple on a body is a tendency to rotate the body about an axis perpendicular to the plane of the couple
- A unique property of the couple is that the moment sum of its forces is constant and independent of any moment centre

Moment of a couple C is equal to the product of one of the forces composing the couple multiplied by the perpendicular distance between their action lines

$$|C| = \mathbf{F.\ d}$$

SPATIAL COUPLE

Definition A couple is a pair of equal, parallel, oppositely directed spatial forces (F & -F) moment sum also is constant and independent of any moment centre

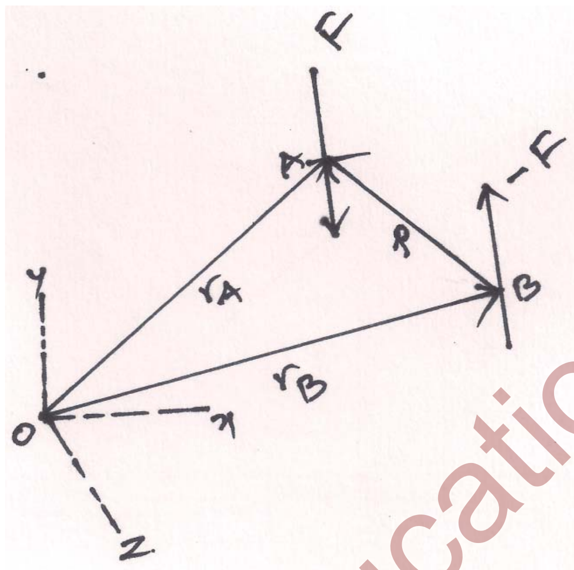


Fig.6 Presentation of spatial couple

We can understand from figure, with respect to any origin 'O' draw position vector r_A to any point **A** on F and draw r_B to any point **B** on -F. Adding the moment of each force about O, the expression will be as below

$$\mathbf{M} = \boldsymbol{r_A} \times \mathbf{F} + \boldsymbol{r_B} \times (-\mathbf{F}) = (\boldsymbol{r_A} - \boldsymbol{r_B}) \times \mathbf{F} = \boldsymbol{\rho} \times \mathbf{F}$$

Where ρ=Position vector $(r_A - r_B)$

Since the only effect of a couple is to produce a moment that is independent of the moment centre, the effect of couple is unchanged if

1. The couple is rotated through any angle in its plane

2. The couple is shifted to any other position in its plane

3. The couple is shifted to any other plane

4. The couple is replaced by another pair of force in its plane whose product (F. d) and sense of rotation is unchanged