Oscillations

Equations

2011

1. Out of the following functions representing motion of a particle which represents SHM

I) $y = \sin \omega t - \cos \omega t$

II) $y = \sin^3 \omega t$

III) $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$

IV) $y = 1 + \omega t + \omega^2 t^2$

a) Only IV does not represent SHM
b) I and III
c) I and II
d) Only I

2. The motion which is not simple harmonic is

a) Vertical oscillations of a spring
b) Motion of simple pendulum
c) Motion of a planet around the sun
d) Oscillation of liquid column in a U-tube

2009

9. Which one of the following equations of motion represents simple harmonic motion?

a) Acceleration $= -k_0 x + k_1 x^2$
b) Acceleration = -k (x + a)
c) Acceleration = k (x + a)
d) Acceleration = kx

2008

4. The function $\sin^2(\omega t)$ represents

a) A periodic, but not simple harmonic motion with a period $2\pi / \omega$

b) A periodic, but not simple harmonic motion with a period π / ω

c) A simple harmonic motion with a period $2\pi / \omega$

d) A simple harmonic motion with a period π / ω

2007

5. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

a) $\frac{T}{4}$ b) $\frac{T}{8}$ c) $\frac{T}{12}$ d) $\frac{T}{2}$

2006

6. The motion of a particle varies with time according to the relation

$y = a(\sin \omega t + \cos \omega t)$

a) The motion is oscillatory but not SHM b) The motion is SHM with amplitude a

c) The motion is SHM with amplitude $a\sqrt{2}$ d) The motion is SHM with amplitude 2a

2005

7. Which of the following functions represents a simple harmonic oscillation?

a) $\sin \omega t - \cos \omega t$ b) $\sin^2 \omega t$ c) $\sin \omega t + \sin 2\omega t$ d) $\sin \omega t - \sin 2\omega t$

8. The minimum phases difference between two simple harmonic oscillations

$y_1 = \frac{1}{2}\sin \omega t + \frac{\sqrt{3}}{2}\cos \omega t$; $y_2 = \sin \omega t + \cos \omega t$ **is**

a) $\frac{7\pi}{12}$ b) $\frac{\pi}{12}$ c) $-\frac{\pi}{6}$ d) $\frac{\pi}{6}$

2003

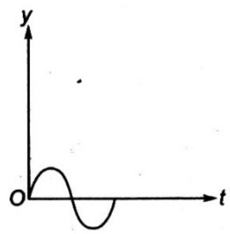
9. The displacement of a particle from its mean position (in metre) is given by

$y = 0.2\sin(10\pi t + 1.5\pi)\cos(10\pi t + 1.5\pi)$ **. The motion of the particle is**

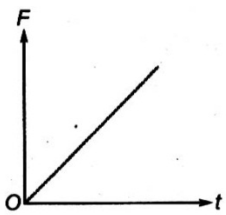
a) Periodic but not SHM b) Non periodic

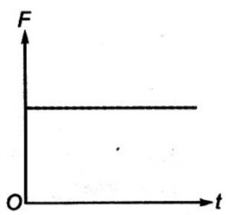
c) Simple harmonic motion with period 0.1s d) Simple harmonic motion with periodic 0.2s

10. The displacement time graph of a particle executing SHM is as shown in the figure

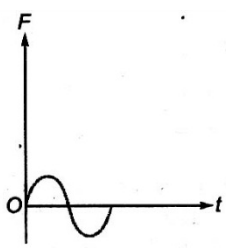


The corresponding force-time graph of the particle is

a) 

b) 

c) 

d) 

Velocity, Acceleration and Energy

2008

11. Two simple harmonic motions of angular frequency 100 and $1000\, rad\, s^{-1}$ have the same displacement amplitude. The ratio of their maximum accelerations is

a) 1 : 10 b) $1:10^2$ c) $1:10^3$ d) $1:10^4$

12. A point performs simple harmonic oscillation of period T and the equation of motion is given by $x = a\sin(\omega t + \pi/6)$. After the elapses of what fraction of the time period the velocity of the point will be equal to half of its maximum velocity

a) $\frac{T}{8}$ b) $\frac{T}{6}$ c) $\frac{T}{3}$ d) $\frac{T}{12}$

13. **Two points are located at a displacement of 10m and 15m from the source of oscillation. The period of oscillation is 0.05s and the velocity of the wave is $300ms^{-1}$. What is the phase difference between the oscillations of two points?**

a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) π d) $\frac{\pi}{6}$

14. **A particle is executing SHM. Then, the graph of velocity as a function of displacement is a/an**

a) Straight line b) Circle c) Ellipse d) Hyperbola

2007

15. **The particle executing simple harmonic motion has a kinetic energy $K_0 \cos^2 \omega t$. The maximum values of the potential energy and the total energy are respectively**

a) 0 and $2K_0$ b) $\frac{K_0}{2}$ and K_0 c) K_0 and $2K_0$ d) K_0 and K_0

16. **Which one of the following statements is true for the speed v and the acceleration a, of a particle executing simple harmonic motion?**

a) When v is maximum, a is maximum

b) Value of a is zero, whatever may be the value of v

c) When v is zero, a is zero

d) When v is maximum, a is zero

2006

17. A particle executes SHM; its time period is 16s. If it passes through the centre of oscillation then its velocity is $2ms^{-1}$ at times 2s. Then amplitude will be

a) 7.2 m b) 4 cm c) 6 cm d) 0.72 m

18. A particle executing SHM has amplitude 0.01 m and frequency 60 Hz. The maximum acceleration of particle is

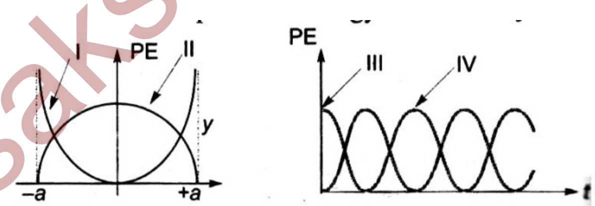
a) $60\pi^2 ms^{-2}$ b) $80\pi^2 ms^{-2}$ c) $120\pi^2 ms^{-2}$ d) $144\pi^2 ms^{-2}$

2004

19. The magnitude of acceleration of particle executing SHM at the position of maximum displacement is

a) Zero b) Minimum c) Maximum d) None of these

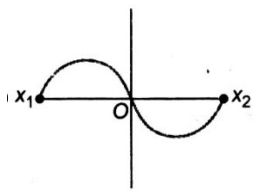
20. If for a particle executing SHM, the equation of SHM is given as $y = a\cos\omega t$. Then which of the following graph represents the variation in its potential energy?

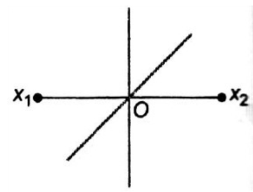


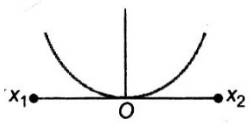
a) II, IV b) I, III c) III, IV d) I, II

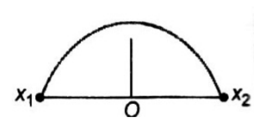
2003

21. A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equilibrium position being O. Its potential energy is plotted. It will be as given below in the graph

a) 

b) 

c) 

d) 

Time Period and Frequency

2011

22. A particle of mass m is located in a one dimensional potential field where potential energy is given by V(x) = A (1 – cos px) where A and p are constants. The period of small oscillations of the particle is

a) $2\pi\sqrt{\frac{m}{Ap}}$ b) $2\pi\sqrt{\frac{m}{Ap^2}}$ c) $2\pi\sqrt{\frac{m}{A}}$ d) $\frac{1}{2\pi}\sqrt{\frac{Ap}{m}}$

2010

23. A body is executing SHM when its displacement large the mean position are 4cm and 5cm it has velocity $10ms^{-1}$ and $8ms^{-1}$ respectively. Its periodic time t is

a) $\frac{2\pi}{3}\sec$ b) $\pi\sec$ c) $\frac{3\pi}{2}\sec$ d) $2\pi\sec$

24. One-fourth length of a spring of force constant k is cut away. The force constant of the remaining spring will be

a) $\frac{3}{4}k$ b) $\frac{4}{3}k$ c) k d) 4k

25. The equation of a damped simple harmonic motion is $m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$. Then the angular frequency of oscillation is

a) $\omega = \left(\frac{k}{m} - \frac{b^2}{4m^2}\right)^{1/2}$ b) $\omega = \left(\frac{k}{m} - \frac{b}{4m}\right)^{1/2}$ c) $\omega = \left(\frac{k}{m} - \frac{b^2}{4m}\right)^{1/2}$ d) $\omega = \left(\frac{k}{m} - \frac{b^2}{4m^2}\right)$

2009

26. Assertion (A): The periodic time of a hard spring is less as compared to that of a soft spring.

Reason (R): The periodic time depends upon the spring constant, and spring constant is large for hard spring.

a) Both assertion and reason are true and reason is the correct explanation of assertion.

b) Both assertion and reason are true but reason is not the correct explanation of assertion.

c) Assertion is true but reason is false.

d) Both assertion and reason are false.

27. A body executes simple harmonic motion under the action of force F_1 with a time period $\frac{4}{5}s$. If the force is changed to F_2 it executes simple harmonic motion with time period $\frac{3}{5}s$. If both forces F_1 and F_2 act simultaneously in the same direction on the body, its time period will be

a) $\frac{12}{25}s$ b) $\frac{24}{25}s$ c) $\frac{35}{24}s$ d) $\frac{15}{12}s$

28. A simple pendulum performs simple harmonic motion about x = 0 with an amplitude a, and time period T. The speed of the pendulum at $x = \frac{a}{2}$ will be

a) $\frac{\pi a\sqrt{3}}{2T}$ b) $\frac{\pi a}{T}$ c) $\frac{3\pi^2 a}{T}$ d) $\frac{\pi a\sqrt{3}}{T}$

29. A simple pendulum of length l has a maximum angular displacement θ. The maximum kinetic energy of the bob is

a) $mgl(1-\cos\theta)$ b) 0.5 mgl c) mgl d) 2 mgl

2008

30. A particle of mass m is executing oscillation about the origin on the x-axis. Its potential energy is $U(x)=k[x]^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is

a) Proportional to $\frac{1}{\sqrt{a}}$

b) Independent of a

c) Proportional to $\sqrt{a}$

d) Proportional to $a^{3/2}$

31. Two pendulums have time periods T and 5T/4. They start SHM at the same time from the mean position. What will be the phase difference between them after the bigger pendulum completed one oscillation

a) 45^0 b) 90^0 c) 60^0 d) 30^0

2007

32. The maximum displacement of the particle executing SHM is 1cm and the maximum acceleration is $(1.57)^2 cm\,s^{-2}$. Its time period is

a) 0.25 s b) 4.0 s c) 1.57 s d) 3.14 s

2006

33. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium it undergoes oscillation with a time period T. Then

a) $T \propto \sqrt{\rho}$ b) $T \propto \frac{1}{\sqrt{A}}$ c) $T \propto \frac{1}{\rho}$ d) $T \propto \frac{1}{\sqrt{m}}$

34. A particle executes simple harmonic motion with a frequency f. Then frequency with which the potential energy oscillates is

a) f b) f/2 c) 2f d) Zero

Simple Pendulum

2010

35. Two simple pendulum first of bob mass M_1 and length L_1, second of bob mass M_2 and length L_2. $M_2 = M_2$ and $L_1 = 2L_2$ If the vibrational energies of both are same. Then which is correct?

a) Amplitude of B greater than A
b) Amplitude of B smaller than A
c) Amplitude will be same
d) None of the above

36. The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (if it a second's pendulum on earth)

a) $1/\sqrt{2}s$
b) $2\sqrt{2}s$
c) 2s
d) 1/2s

37. Assertion (A): The percentage change in time period is 1.5%, if the length of simple pendulum increases by 3%.

Reason (R): Time period is directly proportional to length of pendulum.

a) Both assertion and reason are true and reason is the correct explanation of assertion.
b) Both assertion and reason are true but reason is not the correct explanation of assertion.
c) Assertion is true but reason is false.
d) Both assertion and reason are false.

38. A clock S is based on oscillation of a spring and a clock p is based on pendulum motion. Both clock run at the same rate on earth and on a planet having the same density as earth but twice the radius.

a) S will run faster than P.
b) P will run faster than S.
c) Both will run at the same rate as on the earth.
d) Both will run at the same rate which will be different from that on the earth.

39. The time period of a simple pendulum of length L as measured in an elevator descending with acceleration $\frac{g}{3}$ is

a) $2\pi\sqrt{\frac{3L}{2g}}$ b) $\pi\sqrt{\frac{3L}{g}}$ c) $2\pi\sqrt{\frac{3L}{g}}$ d) $2\pi\sqrt{\frac{2L}{3g}}$

40. A pendulum has time period T in air when it is made to oscillation in water, it acquired a time period $T' = \sqrt{2}T$. Then density of the pendulum bob is equal to (density of water = 1)

a) $\sqrt{2}$ b) 2 c) $2\sqrt{2}$ d) None of these

2008

41. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time

a) At the mean position of the platform b) For an amplitude of $\frac{g}{\omega^2}$

c) For an amplitude of $\frac{g^2}{\omega^2}$ d) At the highest position of the platform

42. A heavy small-sized sphere is suspended by a string of length l. The sphere rotates uniformly in a horizontal circle with the string making an angle θ with the vertical. Then, the time-period of this conical pendulum is

a) $t = 2\pi\sqrt{\frac{l}{g}}$ b) $t = 2\pi\sqrt{\frac{l\sin\theta}{g}}$ c) $t = 2\pi\sqrt{\frac{l\cos\theta}{g}}$ d) $t = 2\pi\sqrt{\frac{l}{g\cos\theta}}$

43. The length of the second's pendulum is decreased by 0.3cm when it is shifted to Chennai from London. If the acceleration due to gravity at London is $981 cm\, s^{-2}$, the acceleration due to gravity at Chennai is (assume $\pi^2 = 10$)

a) $981 cm\, s^{-2}$ b) $978 cm\, s^{-2}$ c) $984 cm\, s^{-2}$ d) $975 cm\, s^{-2}$

2007

44. The time period of a simple pendulum in a stationary train is T. The time period of a mass attached to a spring is also T. The train accelerates at the rate $5ms^{-2}$. If the new time periods of the pendulum and spring be T_p and T_s respectively, then

a) $T_p = T_s$ b) $T_p > T_s$ c) $T_p < T_s$ d) Cannot be predicted

45. Assertion (A): Water in a U-tube executes SHM, the time period for mercury filled up to same height in the U-tube be greater than that in case of water.

Reason (R): The amplitude of an oscillating pendulum goes on increasing.

a) Both assertion and reason are true and reason is the correct explanation of assertion.

b) Both assertion and reason are true but reason is not the correct explanation of assertion.

c) Assertion is true but reason is false.

d) Both assertion and reason are false.

46. Time period of a simple pendulum is T. If its length increases by 2%, the new time period becomes

a) 0.98T b) 1.02 T c) 0.99T d) 1.01T

2005

47. The amplitude of an oscillating simple pendulum is 10cm and its period is 4s. Its speed after 1s when its passes through its equilibrium position is

a) Zero b) $2.0ms^{-1}$ c) $0.3ms^{-1}$ d) $0.4ms^{-1}$

48. A simple second pendulum is mounted in a rocket. Its time period will decrease when the rocket is

a) Moving up with uniform velocity

b) Moving up with uniform acceleration

c) Moving down with uniform acceleration

d) Moving around the earth in geostationary orbit

49. If the length of a pendulum is made 9 times and mass of the bob is made 4 times, then the value of times period becomes

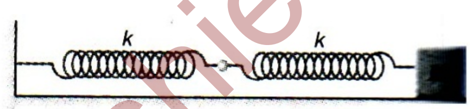
a) 3T b) 3/2T c) 4T d) 2T

2004

50. The period of oscillation of a simple pendulum is T in a stationary lift. If the lift moves upwards with acceleration of 8g, the period will

a) Remain the same b) Decreases by T/2 c) Increase by T/3 d) None of these

51. Two spring are connected to a block of mass M placed on a frictionless surface as shown below. If both the springs have a spring constant k, the frequency of oscillation of block is



a) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ b) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ c) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ d) $\frac{1}{2\pi}\sqrt{\frac{M}{k}}$

2003

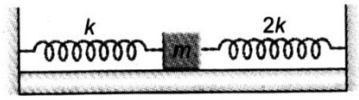
52. The time period of a mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from the of the parts, then the new time period will be

a) $\frac{T}{2}$ b) 2T c) $\frac{T}{4}$ d) T

53. Pendulum after some time becomes slow in motion and finally stops due to

a) Air Friction b) Earth's Gravity c) Mass of Pendulum d) None of these

54. Two springs of force constants k and 2k are connected to a mass as shown below. The frequency of oscillation of the mass is



a) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ c) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ d) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$

55. Assertion (A): The amplitude of an oscillating pendulum decreases gradually with time.

Reason (R): The frequency of the pendulum decreases with time.

a) Both assertion and reason are true and reason is the correct explanation of assertion.

b) Both assertion and reason are true but reason is not the correct explanation of assertion.

c) Assertion is true but reason is false.

d) Both assertion and reason are false.

Key

1) **c**	2) **c**	3) **b**	4) **b**	5) **c**	6) **c**	7) **a**	8) **b**	9) **c**	10) **c**
11) **b**	12) **d**	13) **b**	14) **c**	15) **d**	16) **d**	17) **a**	18) **d**	19) **c**	
20) **b**	21) **c**	22) **b**	23) **b**	24) **b**	25) **a**	26) **a**	27) **a**	28) **d**	29) **a**
30) **a**	31) **b**	32) **b**	33) **b**	34) **c**	35) **b**	36) **b**	37) **c**	38) **b**	39) **a**
40) **b**	41) **b**	42) **c**	43) **b**	44) **c**	45) **d**	46) **d**	47) **a**	48) **a**	49) **a**
50) **c**	51) **b**	52) **a**	53) **a**	54) **c**	55) **c**				

Hints

Equations

1. $\frac{d^2y}{dt^2} \propto -y$

$y = \sin \omega t - \cos \omega t$

And $y = 5\cos\left(\frac{3\pi}{4} - 3\omega t\right)$ are satisfying this

Condition and equation $y = 1 + \omega t + \omega^2 t^2$ is not periodic and $y = \sin^3 \omega t$ is periodic but not SHM

2. Concept.
3. Concept
4. Here $y = \sin^2 \omega t$

$\frac{dy}{dt} = 2\omega \sin \omega t \cos \omega t = \omega \sin 2\omega t$

$\frac{d^2y}{dt^2} = 2\omega^2 \cos 2\omega t$

For SHM, $\frac{d^2y}{dt^2} \propto -y$

$t = \frac{\pi}{\omega}$

5. $y = a \sin \omega t$

$y = \frac{a}{2}$

$\frac{a}{2} = a \sin \omega t$

Or $\sin \omega t = \frac{1}{2} = \sin \frac{\pi}{6}$

Or $\omega t = \frac{\pi}{6} \text{ or } t = \frac{\pi}{6\omega}$

Or $t = \frac{T}{12}$ $\left(\because \omega = \frac{2\pi}{T}\right)$

6. $y = a(\sin\omega t + \cos\omega t)$

Or $y = a\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t + \frac{1}{\sqrt{2}}\cos\omega t\right)$

Or $y = a\sqrt{2}\left(\cos\frac{\pi}{4}\sin\omega t + \frac{\sin\pi}{4}\cos\omega t\right)$

Or $y = a\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$

This is the equation of simple harmonic motion with amplitude $a\sqrt{2}$

7. Let $y = \sin\omega t - \cos\omega t$

$\frac{dy}{dt} = \omega\cos\omega t + \omega\sin\omega t$

$\frac{d^2y}{dt^2} = -\omega^2\sin\omega t - \omega^2\cos\omega t$

Or $a = -\omega^2(\sin\omega t - \cos\omega t)$

Or $a = -\omega^2 y$

Or $a \propto -y$

9. Given $y = 0.2\sin(10\pi t + 1.5\pi)\cos(10\pi t + 1.5\pi)$

$y = 0.1\sin 2(10\pi t + 1.5\pi)$

$y = 0.1\sin 2(10\pi t + 3\pi)$

This equation represents simple harmonic motion of angular frequency 20π .

$\therefore$ Time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10} = 0.1s$

10. Concept

Velocity, Acceleration and Energy

11. $a_{max} = -\omega^2 A$

Or $\frac{(a_{max})_1}{(a_{max})_2} = \frac{\omega_1^2}{\omega_2^2}$

Or $\frac{(a_{max})_1}{(a_{max})_2} = \frac{(100)^2}{(1000)^2} = \left(\frac{1}{10}\right)^2 = 1:10^2$

12. $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$

$v = \frac{dx}{dt} = a\omega\cos\left(\omega t + \frac{\pi}{6}\right)$

Or $\frac{a\omega}{2} = a\omega\cos\left(\omega t + \frac{\pi}{6}\right)$

Or $t = \frac{\pi}{6\omega} = \frac{\pi \times T}{6 \times 2\pi} = \frac{T}{12}$

Thus, at $\frac{T}{12}$ velocity of the point will be equal to half of its maximum velocity

13. Path different $\Delta x = 15 - 10 = 5m$

Time period, T = 0.05s

$\Rightarrow$ Frequency $\nu = \frac{1}{T} = \frac{1}{0.05} = 20Hz$

Velocity, $v = 300ms^{-1}$

$\therefore$ Wavelength $\lambda = \frac{v}{\nu} = \frac{300}{20} = 15m$

Hence, phase difference

$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{15} \times 5 = \frac{2\pi}{3}$

14. $v^2 = \omega^2(a^2 - x^2)$

$\Rightarrow v^2 + \omega^2 x^2 = \omega^2 a^2 \Rightarrow \frac{v^2}{a^2\omega^2} + \frac{x^2}{a^2} = 1$

Which is the equation of an ellipse

15. Concept

16. Concept

17. $v = a\omega \cos \omega t$

$\therefore 2 = a.\frac{2\pi}{16}.\cos\frac{2\pi}{16}.2$

Or $a = \frac{16\sqrt{2}}{\pi} = 7.2cm$

18. Maximum acceleration of particle $= a\omega^2 = a(2\pi f)^2$

$$= 4a\pi^2 f^2 = 4\times 0.01\times \pi^2 \times (60)^2 = 144\pi^2 ms^{-2}$$

19. Concept

20. The potential energy is maximum at extreme position (where $y = \pm a$) and zero at mean position so, graph I is correct. Also, from

$y = a\cos\omega t$

$y = \pm a$ at time t = 0

Hence, graph III is also correct

21. $U = \frac{1}{2}kx^2$

At equilibrium position (x = 0), potential energy is minimum. At extreme position x_1 and x_2, its potential energies are

$U_1 = \frac{1}{2}kx_1^2$ and $U_2 = \frac{1}{2}kx_2^2$

Time Period and Frequency

22. V (x) = A (1-cos px)

$F = -\frac{dV}{dx} = -Ap\sin px$

For small oscillations, we have

$F \approx -Ap^2 x$

Hence, the acceleration would be given by

$$a = \frac{F}{m} = -\frac{Ap^2}{m}x$$

Also, $a = \frac{F}{m} = -\omega^2 x$

But, $\omega = \sqrt{\frac{Ap^2}{m}}$

Or $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{Ap^2}}$

23. $v = \sqrt{\omega^2(a^2 - y^2)}$

$10^2 = \omega^2(a^2 - 4^2)$

And $8^2 = \omega^2(a^2 - 5^2)$

So, $10^2 - 8^2 = \omega^2(5^2 - 4^2) = (3\omega)^2$

Or $\omega = 2$

$\therefore$ Time, $t = \frac{2\pi}{\omega}$

$\therefore t = \frac{2\pi}{2} = \pi \sec$

24. $k \propto \frac{1}{l}$

$k' = \frac{4}{3}k$

25. Displacement of damped oscillator is given by $x = x_m e^{-bt/2m}\sin(\omega' t + \phi)$ where $\omega' =$ angular frequency of damped oscillator

$$= \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

26. Concept

29. Height of bob at maximum angular displacement

$h = l - l\cos\theta = l(l - \cos\theta)$

Also, PE = KE

$mgh = mgl(1-\cos\theta)$

30. $U = k|x|^3 \Rightarrow F = -\frac{dU}{dx}$

$= -3k|x|^2$ ………….. (i)

Also, \

$x = a\sin\omega t$

And $\frac{d^2x}{dt^2} + \omega^2 x = 0$

$\Rightarrow$ Acceleration, $a = \frac{d^2x}{dt^2} = -\omega^2 x$

$\Rightarrow F = ma = m\frac{d^2x}{dt^2}$

$= -m\omega^2 x$ ……………… (ii)

From eqns (i) and (ii) $\omega = \sqrt{\frac{3kx}{m}}$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{3kx}} \quad = 2\pi\sqrt{\frac{m}{3k(a\sin\omega t)}} \Rightarrow T \propto \frac{1}{\sqrt{a}}$

31. Concept

32. Maximum acceleration $= A\omega^2$

$\Rightarrow \left(\frac{\pi}{2}\right)^2 = 1\left(\frac{2\pi}{T}\right)^2 \quad \left(\because 1.57 = \frac{\pi}{2}\right)$

$\Rightarrow T^2 = \frac{4\times 4\pi^2}{\pi^2} \qquad \Rightarrow T = 4s$

33. Up thrust (upwards) $= -Ax\rho g$

$\therefore ma = -Ax\rho g$

Or $a = -\frac{A\rho g}{m}x = -\omega^2 x$

This is the equation of simple harmonic motion. Time period of oscillation

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{A\rho g}} \Rightarrow T \propto \frac{1}{\sqrt{A}}$$

Simple Pendulum

35. Frequency, $n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

Or $n \propto \frac{1}{\sqrt{l}}$

$\therefore \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{L_2}{2L_2}}$

$n_2 = \sqrt{2}n_1$

$\Rightarrow n_2 > n_1$

Energy, $E = \frac{1}{2}m\omega^2 a^2$

$= 2\pi^2 mn^2 a^2$

And $a^2 \propto \frac{1}{mn^2}$

$\therefore \frac{a_1^2}{a_2^2} = \frac{m_2 n_2^2}{m_1 n_1^2}$

Given, $n_2 > n_1$ and $m_1 = m_2 \Rightarrow a_1 > a_2$.

So, amplitude of B smaller than A

36. Gravity $g = \frac{GM}{R^2}$

$\therefore \frac{g_{earth}}{g_{planet}} = \frac{M_e}{M_p} \times \frac{R_p^2}{R_e^2} \Rightarrow \frac{g_e}{g_p} = \frac{2}{1}$

Also, $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}}$

$\Rightarrow \frac{2}{T_p} = \sqrt{\frac{1}{2}}$

$T_p = 2\sqrt{2}s$

37. $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$

$\therefore \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$

$\frac{\Delta T}{T} = \frac{1}{2}\times 3 = 1.5\%$

38. $g = \frac{4}{3}\pi G\rho R$ Or $g \propto R$

For pendulum block, g will increase on the planet so time period will decreases. But for spring clock, it will not change. Hence, P will run faster than S

39. The effective acceleration in a lift descending with acceleration $\frac{g}{3}$ is

$g_{eff} = g - \frac{g}{3} = \frac{2g}{3}$

Time period of simple pendulum

$\therefore T = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{L}{2g/3}} = 2\pi\sqrt{\frac{3L}{2g}}$

40. The effective acceleration of a bob in air and water are given as

$T = 2\pi\sqrt{\frac{l}{g}}$ and $T' = 2\pi\sqrt{\frac{l}{g'}}$

$\therefore \frac{T}{T'} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{g\left(1-\frac{\sigma}{\rho}\right)}{g}}$

$= \sqrt{1-\frac{\sigma}{\rho}} = \sqrt{1-\frac{1}{\rho}}$ $[\because \sigma = 1]$

Putting $\frac{T}{T'} = \frac{1}{\sqrt{2}}$

$\frac{1}{2} = 1 - \frac{1}{\rho} \Rightarrow \rho = 2$

42. Concept

43. $L_1 = \frac{g_1 T^2}{4\pi^2} = \frac{g_1}{\pi^2}$

$L_2 = \frac{g_2 T^2}{4\pi^2} = \frac{g_2}{\pi^2}$

Since, length is decreased g_2 is less than g_1

$\therefore L_1 - L_2 = \frac{g_1 - g_2}{\pi^2}$

Or $(L_1 - L_2)\pi^2 = g_1 - g_2$

Or $0.3 \times 10 = g_1 - g_2$

$\therefore g_2 = 981 - 3 = 978 cms^{-2}$

44. Time period of simple pendulum placed in a train accelerating at the rate of ams^{-2} is given by

$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$

It is independent of g as well as a. hence, when the train acceleration, the time period of the simple pendulum decreases and that of spring remains unchanged.

Hence, $T_p < T$ and $T_s < T$

i.e, $T_p < T_s$

45. Concept

46. $T \propto l^{1/2}$

$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta l}{l}\right)$

$= \frac{\Delta T}{T} = \frac{1}{2}(2\%) = 1\%$

$\Rightarrow \frac{T' - T}{T} = \frac{1}{100}$

$\Rightarrow T' = T + 0.01T$

$\Rightarrow T' = 1.01T$

47. A = 10cm, = 0.1m, T = 4s, t = 1s

$$y = A\sin\omega t$$

$$\frac{dy}{dt} = v = A\omega\cos(\omega t) = A\times\frac{2\pi}{T}\cos\left(\frac{2\pi}{T}t\right) = \frac{2\pi\times 0.1}{4}\cos\left(\frac{2\pi\times 1}{4}\right) = \frac{2\pi\times 0.1}{4}\cos\frac{\pi}{2} = 0$$

48. $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \frac{l}{\sqrt{g}}$

Since, time period of second pendulum decreases, so, it implies that effective value of g is increasing. Thus, it means that rocket is acceleration upwards.

49. $T = 2\pi\sqrt{\frac{l}{g}}$

$T_1 = T\ ,\ l_1 = l\ \ l_2 = 9l$

$\therefore \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

$\Rightarrow T_2 = 3T_1 = 3T$

50. $\therefore g' = g + 8g = 9g$

$T \propto \frac{1}{\sqrt{g}}$

$\therefore T_1^2 g = T_2^2 \times 9g \Rightarrow T_2 = \frac{T_1}{3} = \frac{T}{3}$

51. $\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k}$

$k_{eq} = \frac{k}{2}$

Frequency of oscillation $f = \frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ $\left(\because f = \frac{1}{2\pi}\sqrt{\frac{k}{M}}\right)$

52. $T = 2\pi\sqrt{\frac{m}{k}}$ …………… (i)

Now we know that,

Spring constant $\propto \frac{1}{length}$

Or $k \propto \frac{1}{x}$ ………………….. (ii)

$k' = 4k$

So, new time period of same mass suspended from one of the parts,

$$T' = 2\pi\sqrt{\frac{m}{4k}} = \frac{1}{2}.2\pi\sqrt{\frac{m}{k}} = \frac{T}{2}$$

53. Concept

54. Let F_1 and F_2 be the restoring forces produced then

$F_1 - kx$ and $F_2 - 2kx$

Total restoring force is

$$F = F_1 + F_2 = -kx - 2kx = -(3k)x$$

Hence, frequency

$$n = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

55. Concept