Moment of Inertia

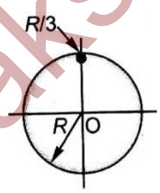
2011

1. **The moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its mid-point and perpendicular to its length is I_0. It moment of inertia about an axis passing through one of its ends and perpendicular to its length is**

   a) $I_0 + ML^2/4$ b) $I_0 + 2ML^2$ c) $I_0 + ML^2$ d) $I_0 + ML^2/2$

2010

2. **From a circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed from the disc, the moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is**



   a) $4MR^2$ b) $\frac{40}{9}MR^2$ c) $10MR^2$ d) $\frac{37}{9}MR^2$

3. **The ratio of the radii of gyration of a circular disc and a circular ring of the same radii about a tangential axis perpendicular to plane of disc or ring is**

   a) 1 : 2 b) $\sqrt{5}:\sqrt{6}$ c) 2 : 3 d) $\frac{\sqrt{3}}{2}$

2008

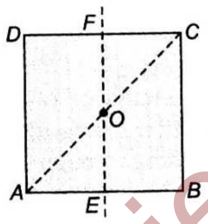
4. The ratio of the radii of gyration of a circular disc to that of a circular ring, each of same mass and radius, around their respective axes is

a) $\sqrt{3}:\sqrt{2}$ b) $1:\sqrt{2}$ c) $\sqrt{2}:1$ d) $\sqrt{2}:\sqrt{3}$

5. A thin rod of length L and mass M is bent at its mid-point into two halves so that the angle between them is 90^0. The moment of inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by the two halves of the rod is

a) $\frac{ML^2}{24}$ b) $\frac{ML^2}{12}$ c) $\frac{ML^2}{6}$ d) $\frac{\sqrt{2}ML^2}{24}$

6. For the given uniform square lamina ABCD, whose centre is O



a) $\sqrt{2}I_{AC} = I_{EF}$ b) $I_{AC} = 3I_{EF}$ c) $I_{AC} = 4I_{EF}$ d) $I_{AC} = \sqrt{2}I_{EF}$

7. Moment of inertia of circular loop of radius R about the axis of rotation parallel to horizontal diameter at a distance R/2 from it is

a) MR^2 b) $\frac{1}{2}MR^2$ c) $2MR^2$ d) $\frac{3}{4}MR^2$

8. The radius of gyration of a rod of length L and mass M about an axis perpendicular to its length and passing through a point at a distance L/3 from one of ends is

a) $\frac{\sqrt{7}}{6}L$ b) $\frac{L^2}{9}$ c) $\frac{L}{3}$ d) $\frac{\sqrt{5}}{2}L$

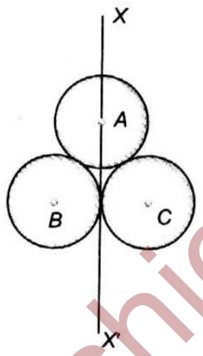
2007

9. The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is

a) 2 : 3 b) 2 : 1 c) $\sqrt{5}:\sqrt{6}$ d) $1:\sqrt{2}$

2006

10. Three rings each of mass M and radius R are arranged as shown in figure. The moment of inertia of the system about the axis XX' will be



a) $\frac{7}{2}MR^2$ b) $3R^2$ c) $\frac{3}{2}MR^2$ d) $5MR^2$

11. The moment of inertia of a rod about an axis through its centre and perpendicular to it is $\frac{1}{12}ML^2$ (where M is the mass and L the length of the rod). The rod is bent in the middle so that two halves make an angle of 60^0. The moment of inertia of the bent rod about the same axis would be

a) $\frac{1}{48}ML^2$ b) $\frac{1}{12}ML^2$ c) $\frac{1}{24}ML^2$ d) $\frac{ML^2}{8\sqrt{3}}$

12. Five particles of mass 2kg are attached to the rim of a circular disc of radius 0.1m and negligible mass. Moment of inertia of the system about the axis passing through the centre of the disc and perpendicular to is place is

a) $1kg-m^2$ b) $0.1kg-m^2$ c) $2kg-m^2$ d) $0.2kg-m^2$

2004

13. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is

a) $\frac{I_2\omega}{I_1+I_2}$ b) ω c) $\frac{I_1\omega}{I_1+I_2}$ d) $\frac{(I_1+I_2)\omega}{I_1}$

14. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

a) $I\sin^2\theta$ b) $I\cos^2\theta$ c) I d) $I\cos^2\left(\frac{\theta}{2}\right)$

Torque, Couple and Angular Momentum

15. **The instantaneous angular position of a point on a rotating wheel is given by the equation** $Q(t) = 2t^3 - 6t^2$ **. The torque on the wheel becomes zero at**

a) t = 0.5s b) t = 0.25s c) t = 2s d) t = 1s

2010

16. **A small object of mass m is attached to a light string which passes through a hollow tube. The tube is hold by one hand and the string by the other. The object is set into rotation in a circle of radius R and velocity v. The string is then pulled down, shortening the radius of path of r. What is conserved?**

a) Angular Momentum b) Linear Momentum

c) Kinetic Energy d) None of These

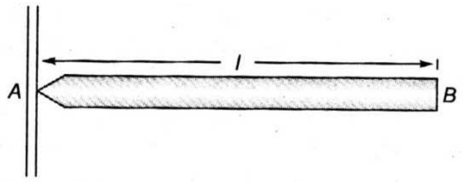
17. **A particle of mass m is projected with a velocity v making an angle of** 45^0 **with the projectile about an axis of projection when the particle is of maximum height h is**

a) Zero b) $\frac{mv^3}{4\sqrt{2}g}$ c) $\frac{mv^2}{\sqrt{2}g}$ d) $m(2gh^3)$

2007

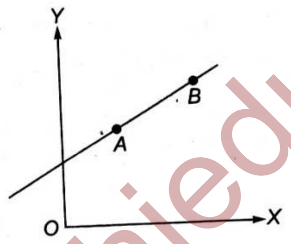
18. **A uniform rod AB of length l and mass m is free to rotate about point A. The rod is released from rest in the horizontal position. Given that the moment of**

inertia of the rod about A is $\frac{ml^2}{3}$, the initial angular acceleration of the rod will be



a) $\frac{2g}{3l}$ b) $mg\frac{l}{2}$ c) $\frac{3}{2}gl$ d) $\frac{3g}{2l}$

19. **A particle of mass m moves in the XY plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is L_A when it is at A and L_B when it is at B, then**



a) $L_A > L_B$

b) $L_A = L_B$

c) The relationship between L_A and L_B depends upon the slope of the line AB

d) $L_A < L_B$

20. **A thin rod of mass m and length 2l is made to rotate about an axis passing through its centre and perpendicular to it. If it angular velocity changes from 0 to ω in time t, the torque acting on it is**

a) $\frac{ml^2\omega}{12t}$ b) $\frac{ml^2\omega}{3t}$ c) $\frac{ml^2\omega}{t}$ d) $\frac{4ml^2\omega}{3t}$

21. **A ring and a disc of different masses are rotating with the same kinetic energy. If we apply a retarding torque τ on the ring, it stops after completing n**

revolutions in all. If same torque is applied to the disc, how many revolutions would it complete in all before stopping?

a) 4n b) 2n c) n d) n/2

22. A wheel having moment of inertia $2kg-m^2$ about its vertical axis rotates at the rate of 60rpm about this axis. The torque which can stop the wheels rotation in one minute would be

a) $\frac{2\pi}{15}Nm$ b) $\frac{\pi}{12}Nm$ c) $\frac{\pi}{15}Nm$ d) $\frac{\pi}{18}Nm$

2006

23. A particle performs uniform circular motion with an angular momentum L. If the frequency of particle motion is doubled and its KE is halved the angular momentum becomes

a) 2L b) 4L c) $\frac{L}{2}$ d) $\frac{L}{4}$

24. A particle of mass m = 5 units is moving with a uniform speed $v = 3\sqrt{2}$ units in the XOY plane along the line y = x +4. The magnitude of the angular momentum of the particle about the origin is

a) 60 unit b) $40\sqrt{2}\,unit$ c) Zero d) 7.5 unit

2005

25. A horizontal platform is rotating with uniform angular velocity around the vertical axis passing through its centre. At some instant of time a viscous fluid of mass m is dropped at the centre and is allowed to spread out and finally fall, the angular velocity during this period

a) Decreases continuously

b) Decreases initially and increases again

c) Remains unaltered

d) Increases continuously

26. Assertion (A): For a system of particle under central force field, the total angular moment is conserved

Reason (R): The torque acting on such a system is zero.

a) Both assertion and reason are true and reason is the correct explanation of assertion.

b) Both assertion and reason are true but reason is not the correct explanation of assertion.

c) Assertion is true but reason is false.

d) Both assertion and reason are false.

27. A uniform disc of mass M and radius R is mounted on an axle supported in frictionless bearings. A light cord is wrapped around the rim of the disc and a steady downward pull T is exerted on the cord. The angular acceleration of the disc is

a) $\frac{MR}{2T}$ b) $\frac{2T}{MR}$ c) $\frac{T}{MR}$ d) $\frac{MR}{T}$

28. The angular momentum of a rotating body changes from A_0 to $4A_0$ in 4min. The torque acting on the body is

a) $\frac{3}{4}A_0$ b) $4A_0$ c) $3A_0$ 4) $\frac{3}{2}A_0$

2004

29. If a particle of mass m is moving in horizontal uniform circular motion, then the angular momentum of the particle is constant about

a) Radius of the circle b) Centre of the circle

c) Tangent of the circle d) None of the above

Rotational Energy and Power

2010

30. A body of mass 10kg moves with a velocity v of $2ms^{-1}$ along a circular path of radius 8m. The power produced by the body will be

a) $10Js^{-1}$ b) $98Js^{-1}$ c) $49Js^{-1}$ d) Zero

31. A coin is of mass 4.8kg and radius 1m rolling on a horizontal surface without sliding with angular velocity 600rot/min. What is total kinetic energy of the coin?

a) 360J b) $1440\pi^2 J$ c) $4000\pi^2 J$ d) $600\pi^2 J$

32. A person, with outstretched arms, is spinning on a rotating stool. He suddenly brings his arms down to his sides. Which the following is true about his kinetic sides which the following is true about his kinetic energy K and angular momentum L

a) Both K and L increase b) Both K and L remain unchanged

c) K remains constant, L increases d) K increases but L remains constant

2007

33. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be

a) $\frac{K^2}{K^2+R^2}$ b) $\frac{R^2}{K^2+R^2}$ c) $\frac{K^2+R^2}{R^2}$ d) $\frac{K^2}{R^2}$

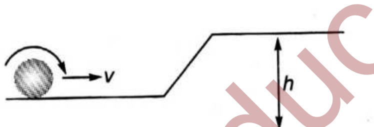
2006

34. A sphere of diameter 0.2m and mass 2kg is rolling on an inclined plane with velocity $y = 0.5ms^{-1}$. The kinetic energy of the sphere is

a) 0.1J b) 0.3J c) 0.5J d) 0.42J

2005

35. Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio



a) 1 : b) $\sqrt{2}:1$ c) 2 : 1 d) $1:\sqrt{2}$

36. A solid sphere is rolling on a frictionless surface, shown in figure with a translational velocity vms^{-1}. If it is to climb the inclined surface then v should be

a) $\geq\sqrt{\frac{10}{7}gh}$ b) $\geq\sqrt{2gh}$ c) 2gh d) $\frac{10}{7}gh$

2004

37. If the moments of inertia of two freely rotating bodies A and B are I_A and I_B respectively such that $I_A > I_B$ and their angular momenta are equal. If K_A and K_B are their kinetic energies, then

a) $K_A < K_B$ b) $K_A > K_B$ c) $K_A = K_B$ d) $K_A = 2K_B$

Rolling Motion

2008

38. Assertion (A): The velocity of a body at the bottom of an inclined plane of given height is more when it slides down the plane, compared to, when it rolling down the same plane.

Reason (R): In rolling down, a body acquires both, kinetic energy of translation and rotation.

a) Both assertion and reason are true and reason is the correct explanation of assertion.

b) Both assertion and reason are true but reason is not the correct explanation of assertion.

c) Assertion is true but reason is false.

d) Both assertion and reason are false.

2005

39. A drum of radius R and mass M, rolls down without slipping along an inclined plane of angle θ. The frictional force

a) Converts translational energy to rotational energy

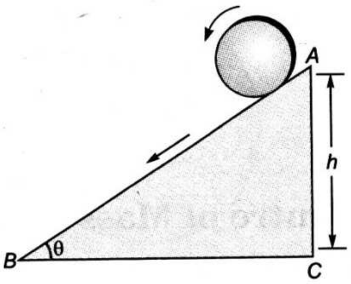
b) Dissipates energy as heat

c) Decreases the rotation motion

d) Decreases the rotational and translational motion

2004

40. If a sphere rolling on an inclined plane with velocity v without slipping, the vertical height of the incline in terms of velocity will be



a) $\frac{7v}{10g}$ b) $\frac{7v^2}{10g}$ c) $\frac{2v^2}{5g}$ d) $\frac{3v}{5g}$

Key

1) **a**	2) **a**	3) **d**	4) **b**	5) **b**	6) **c**	7) **d**	8) **c**	9) **c**	10) **a**
11) **b**	12) **b**	13) **c**	14) **c**	15) **d**	16) **a**	17) **b**	18) **d**	19) **b**	20) **b**
21) **c**	22) **c**	23) **d**	24) **a**	25) **b**	26) **b**	27) **b**	28) **a**		
29) **b**	30) **a**	31) **b**	32) **d**	33) **a**	34) **d**	35) **d**	36) **a**	37) **a**	38) **b**
39) **a**	40) **b**								

Hints

2. $I_{remaining} = I_{whole} - I_{removed}$

Or $I = \frac{1}{2}(9M)(R)^2 - \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + \frac{1}{2}m\left(\frac{2R}{3}\right)^2\right]$

Here $m = \frac{9M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$

$\therefore I = 4MR^2$

3. Radius of gyration $K = \sqrt{\frac{k}{m}}$

$K_{disc} = \sqrt{\frac{\frac{1}{2}mR^2 + mR^2}{m}} = \sqrt{\frac{3}{2}}R$

$K_{ring} = \sqrt{\frac{mR^2 + mR^2}{m}} = \sqrt{\frac{3}{2}}R$

$\frac{K_{disc}}{K_{ring}} = \frac{\sqrt{\frac{3}{2}}}{\sqrt{2}} = \frac{\sqrt{3}}{2}$

4. Radius of gyration is given by $K = \sqrt{\frac{I}{M}}$

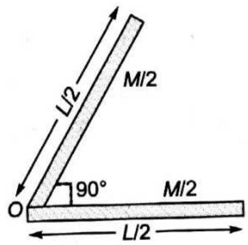
$\frac{K_{disc}}{K_{ring}} = \sqrt{\frac{I_{disc}}{I_{ring}}}$

But I_{disc} (about its axis) $= \frac{1}{2}MR^2$

And I_{ring} (about its axis) $= MR^2$

$\therefore \frac{K_{disc}}{K_{ring}} = \sqrt{\frac{\frac{1}{2}MR^2}{MR^2}} = 1:\sqrt{2}$

5.



Moment of inertia of each part through its one end $=\frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2$

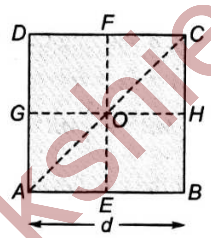
Net moment of inertia through its middle point O is

$$I=\frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2=\frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2=\frac{1}{3}\left[\frac{ML^2}{8}+\frac{ML^2}{8}\right]=\frac{ML^2}{12}$$

6. Let the each side of square lamina is d

So, $I_{EF}=I_{GH}$ (due to symmetry)

And $I_{AC}=I_{BD}$ (due to symmetry)



Now, according to theorem of perpendicular axis

$I_{AC}+I_{BD}=I_0$

Or $2I_{AC}=I_0$ ………….(i)

And $I_{EF}+I_{GH}=I_0$

Or $2I_{EF}=I_0$ ………….(ii)

Form eqs (i) and (ii), $I_{AC}=I_{EF}$

$$\therefore I_{AD}=I_{EF}+\frac{md^2}{4}=\frac{md^2}{12}+\frac{md^2}{4}\left(as\ I_{EF}=\frac{md^2}{12}\right)$$

$\therefore I_{AD} = \frac{md^2}{3} = 4I_{EF}$

7. From parallel axis theorem,

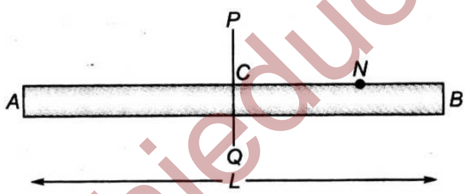
$$I = I_{CM} + M\left(\frac{R}{2}\right)^2$$

$$I = \frac{1}{2}MR^2 + \frac{MR^2}{4}$$

$$I = \frac{3}{4}MR^2$$

8. Moment of inertia of the rod about a perpendicular axis PQ passing through centre of mass C

$$I_{CM} = \frac{ML^2}{12}$$



Let N be the point which divides the length of rod AB in ratio 1:3. This point will be at a distance $\frac{L}{6}$ from C. Thus, the moment of inertia I' about an axis parallel to PQ and passing through the point N

$$I' = I_{CM} + M\left(\frac{L}{6}\right)^2$$

$$= \frac{ML^2}{12} + \frac{ML^2}{36} = \frac{ML^2}{9}$$

If K be the radius of gyration, then

$$K = \sqrt{\frac{I'}{M}} = \sqrt{\frac{L^2}{9}} = \frac{L}{3}$$

9. Moment of inertia of a disc and circular ring about a tangential axis in their planes are respectively

$I_d = \frac{5}{4} M_d R^2$

$I_r = \frac{3}{2} M_r R^2$

But $I = MK^2$

Or, $K = \sqrt{\frac{I}{M}}$

$\therefore \frac{K_d}{K_r} = \sqrt{\frac{I_d}{I_r} \times \frac{M_r}{M_d}}$

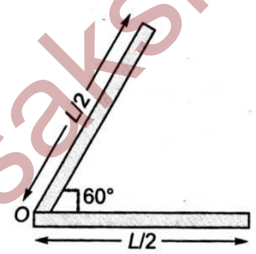
$\Rightarrow \frac{I_d}{I_r} = \sqrt{\frac{(5/4)M_d R^2}{(3/2)M_r R^2} \times \frac{M_r}{M_d}} = \sqrt{\frac{5}{6}}$

10. The axis XX' is a diametric for ring A and tangent in the plane of ring for ring B and ring C

$\therefore I = I_A + I_B + I_C = \frac{1}{2} MR^2 + \frac{3}{2} MR^2 + \frac{3}{2} MR^2 = \frac{7}{2} MR^2$

11.

Moment of inertia of each part through its one end



$= \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2$

Hence, not moment of inertia through its middle poit O is

$I = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 + \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{3}\left(\frac{ML^2}{8} + \frac{ML^2}{8}\right) = \frac{ML^2}{12}$

12. Moment of inertia of the system = moment of inertia of particles

$= 5 \times 2 \times (0.1)^2 = 0.1 kg - m^2$

13. From the law of conservation of angular momentum,

$L_1 = L_2$

$I_1\omega = (I_1 + I_2)\omega'$

$\Rightarrow \omega' = \dfrac{I_1\omega}{I_1 + I_2}$

14. Concept

Torque, Couple and Angular Momentum

15. $\theta(t) = 2t^3 - 6t^2$

$\dfrac{d\theta}{dt} = 6t^2 - 12t$

$\dfrac{d^2\theta}{dt} = 12t - 12 \qquad \left(\because \alpha = \dfrac{d^2\theta}{dt^2} = 0\right)$

12t – 12 = 0

t = 1s

16. Concept

17. Maximum height attained $= \dfrac{v^2 \sin^2\theta}{2g} = \dfrac{v^2 \sin^2 45^0}{2g} = \dfrac{v^2}{4g}$

At height point, momentum = $mv\cos 45^0 = \dfrac{mv}{\sqrt{2}}$

Angular momentum $= \dfrac{mv}{\sqrt{2}} \times \dfrac{v^2}{4g} = \dfrac{mv^3}{4\sqrt{2}g}$

18. $I = \dfrac{ml^2}{3}$

Where m is mass of rod and l is length

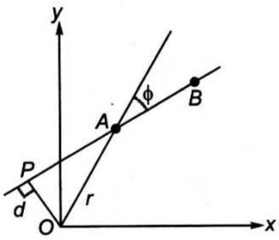
Torque $(\tau = I\alpha)$ acting on centre of gravity of rod is given by

$\tau = mg\dfrac{l}{2}$

Or $I\alpha = mg\frac{l}{2}$

Or $\frac{ml^2}{3}\alpha = mg\frac{1}{2}$ or $\alpha = \frac{3g}{2l}$

19. $L = r \times p = rmv\sin\phi(-k)$



Therefore, the magnitude of L is

$L = mvr\sin\phi = mvd$

Where $d = r\sin\phi$ is the distance of closest approach of the particle to the origin. As d is same for both particle, hence $L_A = L_B$

20. $\tau = I\alpha$

So, $\tau = \left[\frac{m(2l)^2}{12}\right]\left(\frac{\omega}{t}\right)$ or $\tau = \frac{m \times 4l^2 \times \omega}{12 \times t}$

Or $\tau = \frac{4ml^2\omega}{12t} = \left(\frac{ml^2\omega}{3t}\right)$

21. Loss in kinetic energy = work done by torque $= \tau.\theta = \tau.2\pi n$

Hence both ring and disc stop after completing equal number of revolutions n.

22. $I = 2kg - m^2,\ \omega_0 = \frac{60}{60} \times 2\pi\, rad\, s^{-1}$,

$\omega = 0$, t = 60s

The torque required to stop the wheel's rotation is

$\tau = I\alpha = I\left(\frac{\omega_0 - \omega}{t}\right)$

$\therefore \tau = \frac{2 \times 2\pi \times 60}{60 \times 60} = \frac{\pi}{15} N - m^{-1}$

23. $L = mvr = mr^2\omega$

Also kinetic energy $K = \frac{1}{2}mv^2$

Or $K = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}mr^2\omega^2$

$K = \frac{1}{2}\frac{L}{\omega}\omega^2 = \frac{L\omega}{2}$

$\Rightarrow L = \frac{2K}{\omega}$

$\because \omega' = 2\omega$

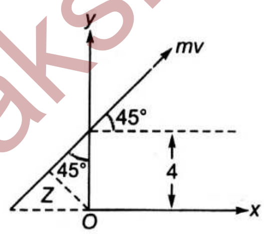
Or $K' = \frac{1}{2}K$

$\therefore L' = \frac{2K'}{\omega'} = \frac{2\left(\frac{1}{2}K\right)}{2\omega} = \frac{L}{4}$

24. Momentum of the particle = $mv = (5)\times(3\sqrt{2}) = 15\sqrt{2}$

The direction of momentum in the XOY plane is given by

y = x + 4



Slope of the line $=1=\tan\theta$

ie, $\theta = 45^0$

Intercept of its straight line = 4

Length of the perpendicular z from the origin of the straight line

$= 4\sin 45^0 = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

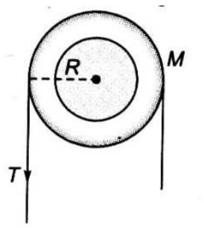
Angular momentum = momentum x perpendicular length

$=15\sqrt{2}\times2\sqrt{2}=60unit$

25. Concept

26. Concept

27. The torque exerted on the disc is given, by



$\tau=TR$

Also $\tau=I\alpha$

From eqs (i) and (ii) ,

$I\alpha=TR$

$\alpha=\frac{TR}{I}=\frac{2TR}{MR^2}$

Or $\alpha=\frac{2T}{MR}$

28. Torque = rate of change of angular momentum

Or $\tau=\frac{dJ}{dt}$

$dJ=4A_0-A_0=3A_0$ dt = 4 min

$\therefore\tau=\frac{3}{4}A_0$

29. Concept

5. Rotational Energy and Power

30. Power $p=\frac{dE}{dt}$

$$p = \frac{d}{dt}(F \times d) = \frac{d}{dt}(0) = 0$$

31. $\omega = 600 rot/\min = \frac{600 \times 2\pi}{60} rads^{-1} = 20\pi\, rads^{-1}$

$$K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}\times\frac{1}{2}mr^2\omega^2 + \frac{1}{2}m(r\omega)^2$$

$$= \frac{1}{4}\times 4.8 \times (1)^2 (20\pi)^2 + \frac{1}{2}\times 4.8 \times (20\pi \times 1)^2 = 1440\pi^2 J$$

32. Concept

33. Kinetic energy if rotation $K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}MK^2\frac{v^2}{R^2}$

Kinetic energy of translation $K_{trans} = \frac{1}{2}Mv^2$

Total energy

$$E = K_{rot} + K_{trans} = \frac{1}{2}MK^2\frac{v^2}{R^2} + \frac{1}{2}Mv^2 = \frac{1}{2}Mv^2\left(\frac{K^2}{R^2} + 1\right)$$

$$= \frac{1}{2}Mv^2(K^2 + R^2)$$

Hence, $\frac{K_{rot}}{K_{trans}} = \frac{\frac{1}{2}MK^2\frac{v^2}{R^2}}{\frac{1}{2}\frac{Mv^2}{R^2}(K^2+R^2)} = \frac{K^2}{K^2+R^2}$

34. E_K = Translation kinetic energy + rotational kinetic energy

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$E_K = \frac{1}{2}\times 2 \times (0.5)^2 + \frac{1}{2}\times\frac{2}{3}\times 2 \times (0.1)^2 \times \left(\frac{0.5}{0.1}\right)^2$ = 0.25 + 0.17 = 0.42 J

35. $\frac{1}{2}I_1\omega_1^2 = \frac{1}{2}I_2\omega_2^2$

Or $\frac{1}{2I_1}(I_1\omega_1)^2 = \frac{1}{2I_2}(I_2\omega_2)^2$

Or $\frac{L_1}{L_2} = \sqrt{\frac{I_1}{I_2}}$

But, $I_1 = I$, $I_2 = 2I$

$\therefore \frac{L_1}{L_2} = \sqrt{\frac{I}{2I}} = \frac{1}{\sqrt{2}}$

Or $L_1 : L_2 = 1 : \sqrt{2}$

36. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Or $mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right).\frac{v^2}{R^2}$

Or $mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$

Or $v = \sqrt{\frac{10}{7}gh}$

$\therefore v \geq \sqrt{\frac{10}{7}gh}$

37. Since, $K = \frac{L^2}{2I}$ and $L_A = L_B$

Hence, $\frac{K_A}{K_B} = \frac{I_B}{I_A} < 1 \Rightarrow K_A < K_B$

6. Rolling Motion

38. Concept

39. Concept

40. $v = \sqrt{\left[\frac{2gh}{(1 + I / MR^2)}\right]}$

Where, I = moment of inertia of sphere,

M = mass of sphere

And R = radius of sphere

The moment of inertia of solid sphere about its diameter

$$I = \frac{2}{5}MR^2$$

$$\therefore v = \sqrt{\left[\frac{2gh}{\left(1+\frac{2}{5}\right)}\right]} = \sqrt{\left(\frac{10}{7}gh\right)}$$

$$\therefore h = \frac{7v^2}{10g}$$